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SPECIFYING OPTIMUM EXAMINEES FOR ITEM PARAMETER ESTIMATION IN ITEM RESPONSE THEORY

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Abstract

Information functions are used to find the optimum ability levels and maximum contributions to information for estimating item parameters in three commonly used logistic item response models. For the three and two parameter logistic models, examinees who contribute maximally to the estimation of item difficulty contribute little to the estimation of item discrimination. This suggests that in applications that depend heavily upon the veracity of individual item parameter estimates (e.g. adaptive testing or test construction), better item calibration results may be obtained (for fixed sample sizes) from examinee calibration samples in which ability is widely dispersed.

Keywords: Item response theory
IRT item parameters
Optimum examinees
Information functions

Introduction

The success of applications in Item Response Theory (IRT) depends upon the accuracy with which individual item parameters can be estimated. This dependence is especially important for those applications that depend heavily upon the veracity of individual item parameter estimates in contexts perhaps unrelated to the calibration setting in which estimates were obtained. Two recent examples of such applications are adaptive testing (see, for example, Lord (1980, chap. 10); Stocking (1988)) and IRT-based test development (see, for example, van der Linden & Boekkooi-Timminga (1987)).

Typically, calibration samples are selected with an eye to convenience as random subsamples from a larger set of data (see, for example, Cook, Petersen, & Stocking (1983)). It may be possible to obtain better, i.e., more accurate, item parameter estimates if the planning for calibration sample selection explicitly includes considerations of the accuracy of the resulting item parameter estimates. This idea was suggested, based on the analogy between item parameter estimation and estimation problems in regression, 20 years ago by Lord (1968). More recently, Wingersky and Lord (1984) provide theoretical support.

One approach to issues of the accuracy of item parameter estimation considers the asymptotic expected covariance matrix. Lord (1980, p. 191) presents formulas from which this matrix can be derived for the three-parameter logistic (3PL) model in the finite sample case with known abilities. Thissen and Wainer (1982) consider the properties of the asymptotic covariance matrix for item parameters when a particular distribution of ability is assumed in the calibration example. Lord and Wingersky (1985) consider the asymptotic properties of the covariance matrix for the 3PL model when both item parameters and abilities are estimated jointly by maximum likelihood.

In this paper we explore the relationship between examinee ability and the accuracy of maximum likelihood item parameter estimation in terms of the expected (Fisher) information (Kendall & Stuart, 1979, p. 10). The advantage to this approach is that it is possible to study directly the relationship between examinee ability and contribution to information in some detail and with some surprising results. This theoretical examination depends upon the assumption that examinee abilities, as well as some item parameters, are known. However, the results can be translated into general guidelines for situations in which samples can be selected on the basis of some observed score for the purpose of obtaining accurate parameter estimates for a collection of items whose properties are imperfectly known.

The Theory

Suppose the probablility of a correct response to an item is specified by a logistic function, either one parameter logistic (1PL), two parameter logistic (2PL) or three parameter logistic (3PL). Suppose further that estimates of the item parameters are obtained by maximum likelihood. If the model fits the data, and true abilities are known, then formulas exist for the amount of information in the sample for estimating item parameters (Lord, 1980, eqs. 12-8 through 12-13)).

Of course, the model never fits any set of real data, and true abilities are never known. Therefore, the information computed from the Lord formulas is an estimate of the maximum possible information, and will be larger than what can be realistically obtained (Mislevy & Sheehan, 1987). Nevertheless, an examination of what the theory predicts may be useful in planning a calibration sample to estimate item parameters accurately.

If the probability of a correct response by examinee a to an item is P_a, the log of the likelihood of observed responses to the item for N examinees is

$$\ell = \sum_{a=1}^{N} [u_a \ell n P_a + (1 - u_a) \ell n Q_a] \quad ,$$

where $u_a = 0$ if the response in incorrect, $u_a = 1$ if the response is correct, and $Q_a = 1 - P_a$. The maximum likelihood estimates of each item parameter χ are located at a point where the partial derivatives of the log likelihood are zero. The expected second partial derivatives of the log likelihood can be computed at this point. (The inverse of the negative of this matrix is the asymptotic variance/covariance matrix used by Thissen and Wainer (1982).) The negative of a diagonal element of the matrix of expected second partial derivatives is referred to as the information in the sample for estimating an item parameter. For any item parameter χ, this information function has the form (Lord, 1980, equations 12-8 to 12-13)

$$I_{\chi\chi} = \sum_{a=1}^{N} \frac{1}{P_a Q_a} \left(\frac{\partial P_a}{\partial \chi} \right)^2 = \sum_{a=1}^{N} i_{\chi\chi_a} \quad .$$

The information is composed of individual additive contributions from each examinee,

$$i_{\chi\chi} = \frac{1}{PQ} \left(\frac{\partial P}{\partial \chi} \right)^2 \tag{1}$$

where the subscript a has been dropped for convenience. By examining the properties of an individual contribution as a function of ability, we can determine what values of ability provide the most (and least) information for estimating an item parameter, assuming other item parameters are fixed.

Results

The 3PL Model

The 3PL item response function is

$$P = c + \frac{1 - c}{1 + e^{-Da(\theta - b)}} \quad .$$

where

a is a function of the slope of the item response function in the neighborhood of the item difficulty;

b characterizes the difficulty or location of the item on the ability continuum;

c is the lower bound of probabilities of correct response, even from low ability examinees;

θ is examinee ability;

D is a scaling constant commonly used with the value of 1.7.

The partial derivatives of P with respect to the item parameters are given by Lord (1980, Equation 12-2). Substituting the derivative with respect to c into Equation 1 gives an examinee's contribution to the information for estimating c as

$$i_{cc} = \frac{1}{(1 - c)^2} \frac{Q}{P} \quad . \tag{2}$$

Values of ability that give local minima and maxima of Equation 2 are found where

$$\frac{\partial i_{cc}}{\partial \theta} = \frac{-Da}{(1 - c)^3} \frac{1}{P^2} Q(P - c) = 0 \quad .$$

This derivative is zero when $Q = 0$, in which case $\theta = +\infty$, and i_{cc} is zero. It is also zero when $P = c$, in which case $\theta = -\infty$ and $i_{cc} = \frac{1}{(1 - c)c}$. This latter value forms an upper asymptote to the amount of information from all ability levels. The higher the value of c, the more examinees, even optimal examinees, are required for estimation of c.

The information for estimating b is obtained by substituting the derivative of P with respect to b into Equation 1, giving

$$i_{bb} = \frac{D^2 a^2}{(1 - c)^2} (P - c)^2 \frac{Q}{P} \quad , \tag{3}$$

with partial derivative

$$\frac{\partial i_{bb}}{\partial \theta} = \frac{D^3 a^3}{(1 - c)^3} \frac{Q(P - c)^2}{P^2} [-2P^2 + P + c] \quad . \tag{4}$$

A root of this derivative is found where $Q = 0$, in which case $\theta = +\infty$ and $i_{bb} = 0$. A second root is found when $P = c$, where $\theta = -\infty$ and $i_{bb} = 0$. Examinees with abilities far away from the item's location are useless for estimating that location. A final root of this derivative is found where $-2P^2 + P + c = 0$. The root of this latter quantity, P^*, where $c \le P^* \le 1$, is $P^* = \frac{1 + \sqrt{1 + 8c}}{4}$. The optimal ability θ^* is then

$$\theta^* = b + \frac{1}{Da} \ln\left(\frac{P^* - c}{1 - P^*} \right) \tag{5}$$

and

$$i_{bb} = \frac{D^2 a^2}{(1 - c)^2} (P^* - c)^2 \frac{Q^*}{P^*} \quad , \qquad (6)$$

where $Q^* = 1 - P^*$.

The optimal ability level is at $\theta^* = b$ when $c = 0$. When $0 < c < 1$, the optimal ability level is greater than b by an amount that depends upon both a and c. For a fixed value of c, higher values of a mean that the optimum location is closer to but still greater than b. Since P^* is a function of c alone, the maximum amount of information from an examinee for estimating b, Equation 6, depends only on a and c, and for fixed c, is proportional to the square of a.

By analagous computations, the information from each examinee (Equation 1) for estimating a for the 3PL is

$$i_{aa} = \frac{D^2}{(1 - c)^2} (\theta - b)^2 (P - c)^2 \frac{Q}{P} \quad , \qquad (7)$$

with partial derivative

$$\frac{\partial i_{aa}}{\partial \theta} = \frac{D^2}{(1 - c)^2} \frac{(\theta - b)(P - c)^2 Q}{P^2} [(\theta - b) \frac{Da}{(1 - c)} (-2P^2 + P + c) + 2P] . \qquad (8)$$

Three roots of Equation 8 are at $\theta = b$, $\theta = -\infty$, and $\theta = +\infty$. For all three of these roots, $i_{aa} = 0$. Low and high ability examinees, as well as examinees whose abilities may be close to optimum for estimating b are of no use in the estimation of a. The location of optimal abilities is found from the last factor in Equation 8. This expression is zero when the optimal ability

$$\theta^{**} = b + \frac{2(1 - c)P^{**}}{Da(2P^{**2} - P^{**} - c)} . \qquad (9)$$

The optimal ability is not given explicitly by this expression, but values of θ^{**} can be found by numerical methods. The top panel of Figure 1 shows P plotted as a function of θ - b, from Equation 9. When $P = c$, $\theta - b = - \frac{1}{Da}$, and when $P = 1$, $\theta - b = \frac{2}{Da}$. The plot suggests that these values form upper and lower bounds respectively for two regions in which the optimal ability might be found: $\theta < b - \frac{1}{Da}$ and $\theta > b + \frac{2}{Da}$.

Insert Figure 1 about here

Numerical Examples

Equations 2, 3, and 7 are plotted as functions of ability in the top panel of Figure 2 for an item with $a = 1$, $b = 0$, and $c = .2$. Additive contributions to information for estimating a or b from examinees at various ability levels must be read from the right-hand scale; those for estimating c must be read from the left-hand scale. Figure 2 demonstrates graphically what we have learned analytically: 1) the contribution of examinees to the information available for estimating item difficulty is asymetric around b, with higher ability examinees contributing more information; 2) examinees who are most informative in the estimation of item difficulty are of little use in estimating item discrimination; 3) examinees who do contribute to the estimation of discrimination are asymetrically distributed around the item difficulty; and 4) only low ability examinees contribute much information for the estimation of c, and there is a limit to this contribution.

Insert Figure 2 about here

The effects of different values of a, for fixed b and c, on the additive contribution of examinees to information for estimating a are shown in the top left panel of Figure 3. Each examinee contributes substantially more information to the estimation of a if the value of a is low rather than high. More lower ability examinees than higher ability examinees are required to obtain a given amount of information for the estimation of a, regardless of the value of a, since lower ability examinees contribute less information than higher ability examinees.

Insert Figure 3 about here

The middle left panel of Figure 3 shows the effects of different values of b, for fixed a and c, on the additive contribution of examinees to information for estimating b. Changing b shifts the location of optimal examinees, but not the maximum contribution. The effects of different c's on the additive contribution for estimating c, for fixed a and b, are shown in the bottom left panel of Figure 3. For lower ability levels, the higher the guessing parameter, the less the additive contribution for each examinee; more examinees are required to obtain a given amount of information about c. Regardless of the value of c, higher ability examinees do not contribute much. However, they contribute more if c is higher than if c is lower.

Numerical values of optimal abilities and maximal contributions to information (in parentheses) are presented for typical values of a and b for $c = .2$ in Table 1. Optimal abilities and maximal contributions for b come directly from Equations 5 and 6. Those for a come from the application of numerical methods to Equations 9 and 7.

Insert Table 1 about here

The 2PL and 1PL Models

The 2PL item response function can be considered as a special case of the 3PL with $c = 0$. In this case,

$$P = \frac{1}{1 + e^{-Da(\theta - b)}} \quad .$$

The 1PL item response function is also a special case of the 3PL with $c = 0$ and constant a. The expression for P is identical to that for the 2PL immediately above, but the item discrimination is constant for all items.

The information for estimating b for the 2PL is algebraically the same as for the 1PL:

$$i_{bb} = D^2a^2PQ \quad , \tag{10}$$

with derivative with respect to ability

$$\frac{\partial i_{bb}}{\partial \theta} = D^3a^3PQ(1 - 2P) \quad .$$

These are obtained form Equations 3 and 4 for the 3PL by setting $c = 0$. Examinees with abilities of $\pm\infty$ contribute nothing to the estimation of b in these models. Examinees with $\theta^* = b$ $(P^* = .5)$ contribute the maximum amount of information, $.25D^2a^2$. The additive contribution from each examinee depends upon the square of the discrimination of the item.

For the 2PL model, the information for estimating a is

$$i_{aa} = D^2(\theta - b)^2PQ \tag{11}$$

with derivative

$$\frac{\partial i_{aa}}{\partial \theta} = D^2(\theta - b)PQ[Da(\theta - b)(1 - 2P) + 2] \quad ,$$

again obtained from the 3PL results (Equations 7 and 8) by setting $c = 0$. As with the 3PL, examinees with low and high abilities, as well as examinees whose abilities are optimal for estimating b are useless for estimating a. The location of optimal abilities is found by setting $Da(\theta - b)(1 - 2P) + 2 = 0$ in which case the optimal ability θ^{**} is

$$\theta^{**} = b + \frac{2}{Da(2P - 1)} \quad . \tag{12}$$

The bottom panel of Figure 1 shows P plotted as a function of $\theta - b$ for the 2PL in Equation 12. When $P = 0$, $\theta - b = -\frac{2}{Da}$, and when $P = 1$, $\theta - b = \frac{2}{Da}$. As with the 3PL, there are two regions in which θ^{**} might be found: $\theta < b - \frac{2}{Da}$ and $\theta > b + \frac{2}{Da}$. In contrast to the 3PL, these regions are symmetric around $\theta - b = 0$.

The additive contribution to estimating a for optimal examinees is

$$i_{aa} = .25D^2(\theta^{**} - b)^2 - \frac{1}{a^2} \quad . \tag{13}$$

Both terms in Equation 13 decrease for higher values of a since the optimal θ is closer to the difficulty for increased a, given fixed b. In contrast with the 3PL, the contributions of examinees whose true abilities are symmetric around the item difficulty are identical.

Numerical Examples

Equation 10 for the 2PL and 1PL and Equation 11 for the 2PL are plotted as fuctions of ability in the bottom panel of Figure 2 for an item with $a = 1$ and $b = 0$. Additive contributions to information for estimating a (for fixed b) or b (for fixed a) are higher in models that do not contain a guessing parameter. This is seen by comparing the two panels in Figure 2.

The effects of different a values (with fixed b) on the additive contributions for estimating a in the 2PL (Equation 11) are shown in the top right panel of Figure 3. Examinees symmetrically located around the item difficulty contribute equally to information. As in the 3PL (top left panel), each examinee contributes more to information for estimating a when a is low. In contrast to the 3PL, examinee's additive contributions are higher when the model does not contain a guessing parameter (compare the two top panels of Figure 3). Equation 10 for the 2PL with fixed a or the 1PL is plotted for different values of b in the middle right panel of Figure 3. As with the 3PL (middle left panel), changing values of b shifts the location of optimal examinees but not the maximum contribution to information for estimating item difficulty.

Numerical values of optimal abilities and maximum contributions to information (in parentheses) are presented for typical values of a and b for the 2PL and 1PL in Table 2. Optimal abilities and maximum contributions for estimating b come directly from Equation 10 and its consequences. Comparable data for information for estimating a for the 2PL comes from the application of numerical methods to Equation 12 and Equation 13.

Insert Table 2 about here

Discussion

The focus of this paper has been to explore the contribution of examinees with different true abilities to the expected information available for estimating item parameters. This is not the same as an exploration of the *accuracy* with which item parameters may be estimated, but has the advantage of directly suggesting strategies of calibration sample selection. In terms of making statements about the accuracy with which item parameters are estimated, the analyses presented here are immediately relevant to the problem of estimating a single item parameter, conditional on other item parameters being fixed at their true values. However, statements about the accuracy of item parameter estimates are usually made in a broader context in which all three item parameters must be estimated and a particular distribution of true ability may be assumed. Such considerations then take into account the covariances among the item parameters, and depend upon the assumed distribution of ability. The results need not hold for different ability distributions.

As an example, the results presented here show that in both the 2PL and 3PL, optimal examinees contribute more information for estimating item discrimination when that discrimination is low. This cannot be interpreted to mean that low a's are necessarily more accurately estimated than high a's, for a fixed sample size or a fixed distribution of ability. Thissen and Wainer (1982), using the expected asymptotic variance/covariance matrix and assuming a normal distribution of ability, show that, for fixed b, the asymptotic standard error of a decreases with a for the 2PL. However, for fixed b in the 3PL model, the asymptotic error of a is larger for both small and large values of a than it is for more moderate values of a. These results depend on the assumed distribution of ability as well as the covariances among the item parameters.

As an illustration of this phenomenon, Figure 4 shows the Thissen and Wainer asymptotic standard error of b plotted against b for the 3PL (solid curve). This is a partial reproduction of Thissen and Wainer's Figure 1, with a = 1.5, c = 0., and a standard normal calibration sample of N = 2500. The results presented here suggest that a calibration sample with a larger spread of abilities will improve the accuracy of the estimation of high and low difficulties. Figure 4 shows the same information for the same item when a calibration sample of the same size is drawn from a normal distribution of ability with a variance of 4 (dashed line). Although the sample si~e is the same, the accuracy of estimation is much improved for the more extreme difficulties.

Insert Figure 4 about here

Given the knowledge of the location of optimal examinees for estimating the parameters for a single item in three different item response models presented here, what suggestions can we make for calibration sample selection that would aid practitioners who must jointly estimate the item parameters for many items simultaneously? If the practitioner chooses the 1PL as the appropriate model, then examineees whose true ability is equal to item difficulty are most informative. If a collection of items to be calibrated is thought to have a broad spread of difficulties, based perhaps on the conventional proportions correct, then the distribution of true abilities in the calibration sample should also be broad. Such a sample could possibly be selected based on some available observed auxiliary information. If the range of abilities is too small, the sample will provide information only for middle difficulty items; little information will be provided to estimate the difficulty for easy and hard items.

If the 2PL is the appropriate model, estimation of both a and b requires a wider range of true abilities than for the 1PL. This is so because only examinees with ability not equal to b are informative in the estimation of a. If the range of abilities is too small, information for the estimation of difficulties for middle level items may be provided, but information for estimating their discrimination may not be. Information for the estimation of discriminations for easy and hard items may be provided, but information for estimating their difficulties may not be.

If the 3PL is the most appropriate model, only abilities well below the item difficulty are informative about c. Abilities below and above the difficulty are most informative about a and abilities slightly above the difficulty are most informative about b. Even if all items are of equal difficulty, a normal ditribution of abilities centered slightly above the item difficulty may not provide much information for the estimation of all parameters simultaneously. If the items have a spread of difficulties, better results may be obtained by sampling all ability levels equally. Wingersky and Lord (1984) show that when item and ability parameters are estimated simultaneously, a sample of abilities drawn from a uniform distribution produces standard errors nearly as small as a sample of abilities four times as large drawn from a bell-shaped distribution.

Calibration samples, particularly for the more complex models, typically consist of several thousand examinees. Depending upon the nature of the collection of items to be calibrated, which can be roughly assessed through the use of conventional item statistics, such samples, although large, may not prove useful for estimating the paramters of all items. If the success of a particular application of IRT depends heavily on the veracity of item level data, it seems worthwhile to consider selecting more informative samples.

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Table 1

Optimum Abilities and Maximum Contributions (in Parentheses)

to Information for Estimating a or b Assuming Fixed Values

for the Other Parameters for the 3PL Model with c = .2

a	b	lower optimal θ for a	upper optimal θ for a	optimal θ for b
.5	-2	-3.96	.86	-1.69
(i_{aa} = .58, 1.38)	-1	-2.96	1.86	-.69
	0	-1.96	2.86	.31
(i_{bb} = .12)	1	-.96	3.86	1.31
	2	.04	4.86	2.31
1.0	-2	-2.98	-.57	-1.84
(i_{aa} = .14, .35)	-1	-1.98	.43	-.84
	0	-.98	1.43	.16
(i_{bb} = .49)	1	.02	2.43	1.16
	2	1.02	3.43	2.16
1.5	-2	-2.65	-1.05	-1.90
(i_{aa} = .06, .15)	-1	-1.65	-.05	-.90
	0	-.65	.95	.10
(i_{bb} = 1.11)	1	.35	1.95	1.10
	2	1.35	2.95	2.10
2.0	-2	-2.49	-1.28	-1.92
(i_{aa} = .04, .09)	-1	-1.49	-.28	-.92
	0	-.49	.72	.08
(i_{bb} = 1.97)	1	.51	1.72	1.08
	2	1.51	2.72	2.08
2.5	-2	-2.39	-1.43	-1.94
(i_{aa} = .02, .06)	-1	-1.39	-.43	-.94
	0	-.39	.57	06
(i_{bb} = 3.08)	1	.61	1.57	1.06
	2	1.61	2.57	2.06

Table 2

Optimum Abilities and Maximum Contributions (in Parentheses) to Information for Estimating a or b Assuming Fixed Values for the Other Parameters for the 2PL and 1PL Models with c = 0

a	b	lower optimal θ for a	upper optimal θ for a	optimal θ for b
.5	-2	-4.82	.82	-2.00
(i_{aa} = 1.76, 1.76)	-1	-3.82	1.82	-1.00
	0	-2.82	2.82	.00
(i_{bb} = .18)	1	-1.82	3.82	1.00
	2	-.82	4.82	2.00
1.0	-2	-3.41	-.59	-2.00
(i_{aa} = .44, .44)	-1	-2.41	.41	-1.00
	0	-1.41	1.41	.00
(i_{bb} = .72)	1	-.41	2.41	1.00
	2	.59	3.41	2.00
1.5	-2	-2.94	-1.06	-2.00
(i_{aa} = .20, .20)	-1	-1.94	-.06	-1.00
	0	-.94	.94	.00
(i_{bb} = 1.63)	1	.06	1.94	1.00
	2	1.06	2.94	2.00
2.0	-2	-2.70	-1.30	-2.00
(i_{aa} = .11, .11)	-1	-1.70	-.30	-1.00
	0	-.70	.70	.00
(i_{bb} = 2.90)	1	.30	1.70	1.00
	2	1.30	2.70	2.00
2.5	-2	-2.56	-1.44	-2.00
(i_{aa} = .07, .07)	-1	-1.56	-.44	-1.00
	0	-.56	.56	.00
(i_{bb} = 4.53)	1	.44	1.56	1.00
	2	1.44	2.56	2.00

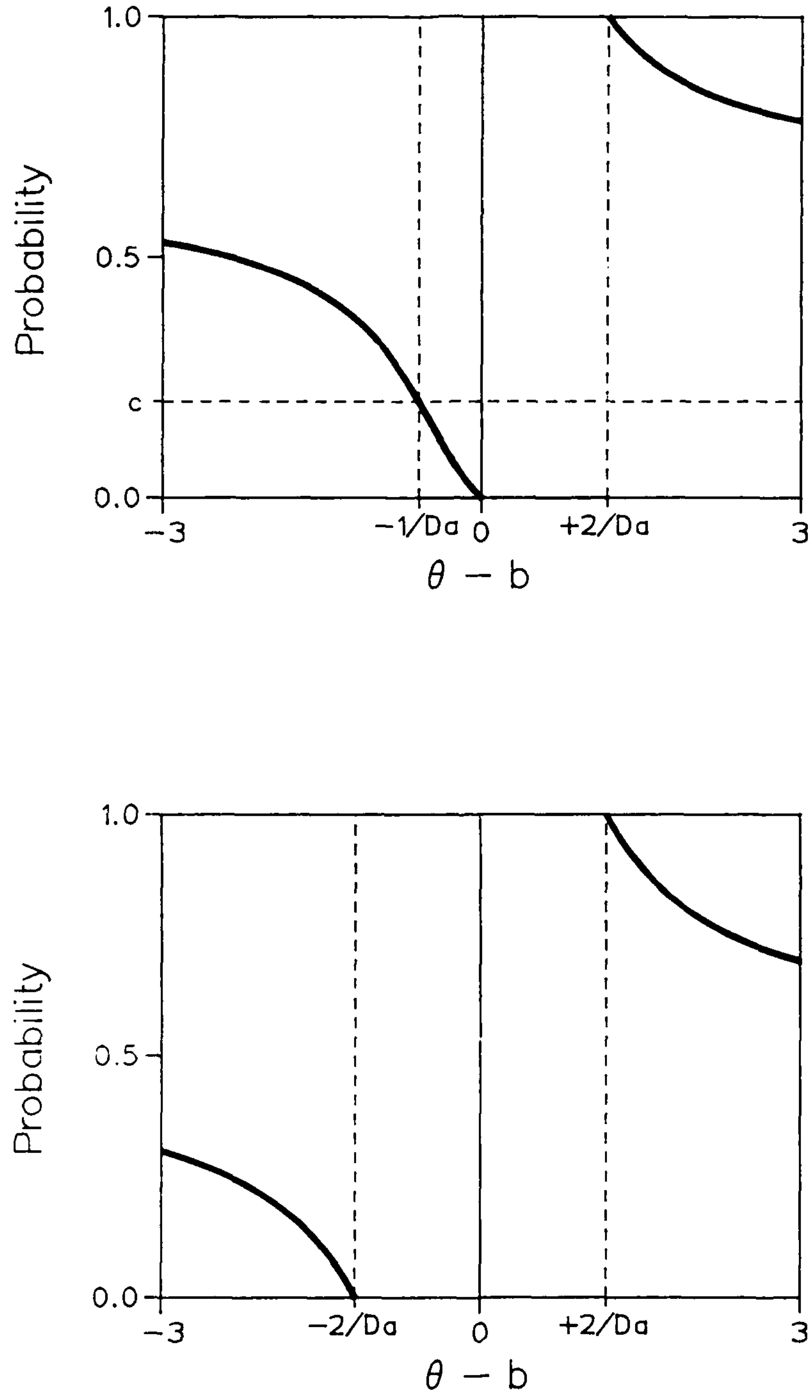

Figure 1. The top panel shows P as a function of $\theta - b$ from Equation 9 for the 3PL (solid curves). The bottom panel shows P as a function of $\theta - b$ from Equation 12 for the 2PL (solid curves).

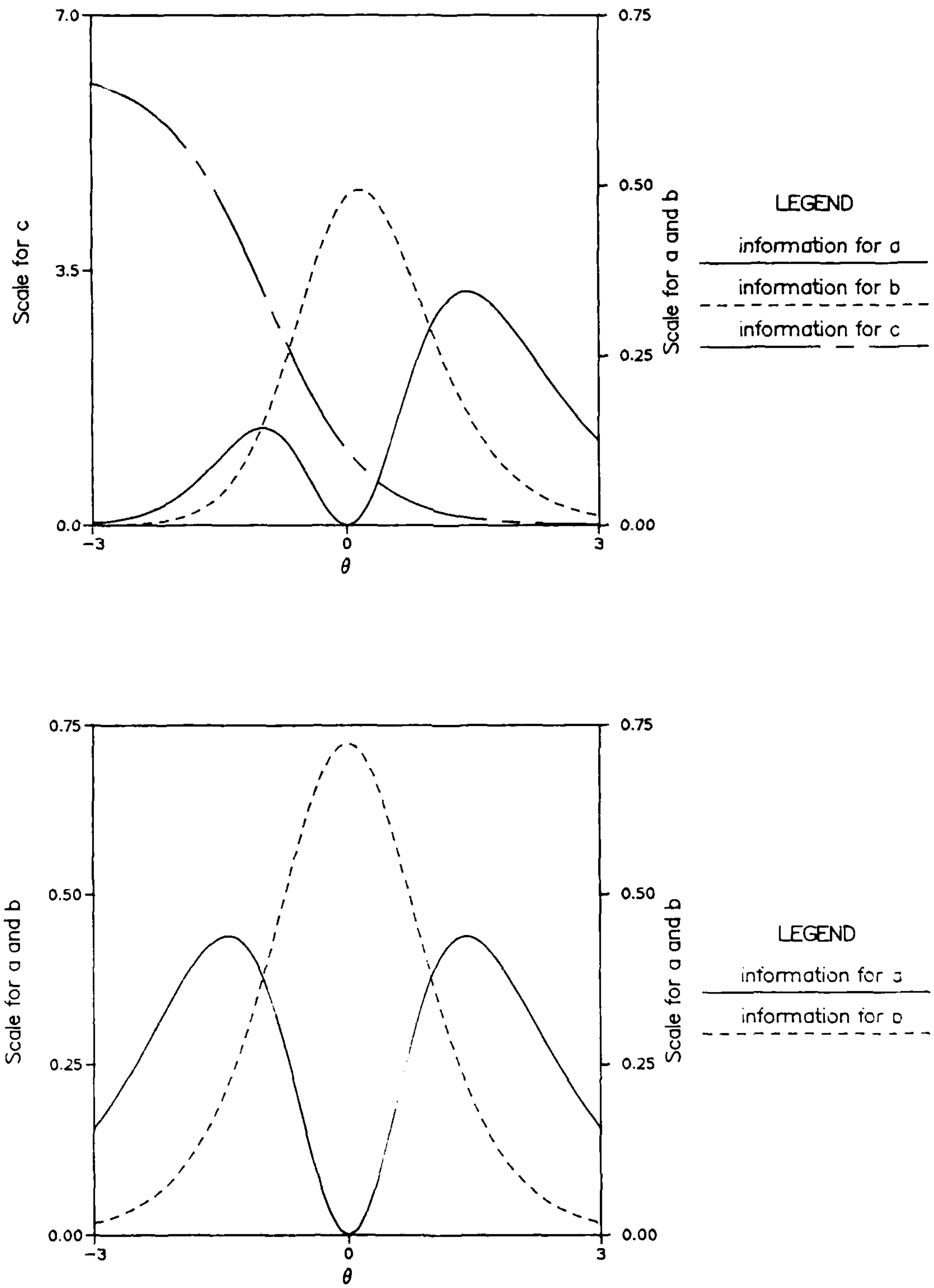

Figure 2. Contributions to information for estimating item parameters as functions of ability for the 3PL (top; a=1, b=0, c=.2), the 2PL and 1PL (bottom; a=1, b=0).

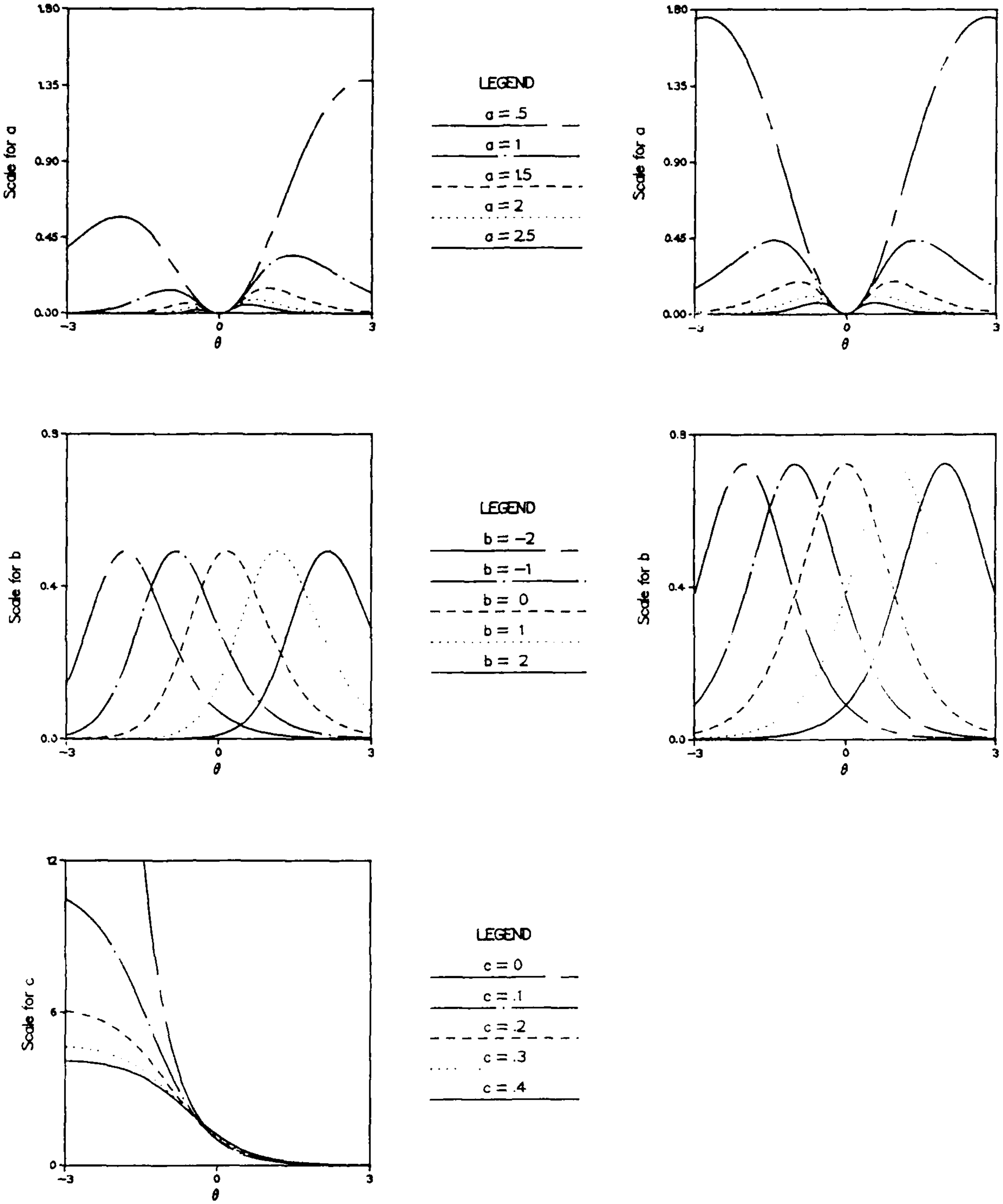

Figure 3. Contributions to information for estimating item parameters for the 3PL (left) and 2PL and 1PL (right). The effects of varying a conditional on fixed b=0 (and c=.2 for the 3PL) are shown in the top row. The effects of varying b for fixed a=1 (and c=.2 for the 3PL) are shown in the middle row. The effects of varying c for the 3PL conditional on a=1 and b=0 are shown in the bottom row.

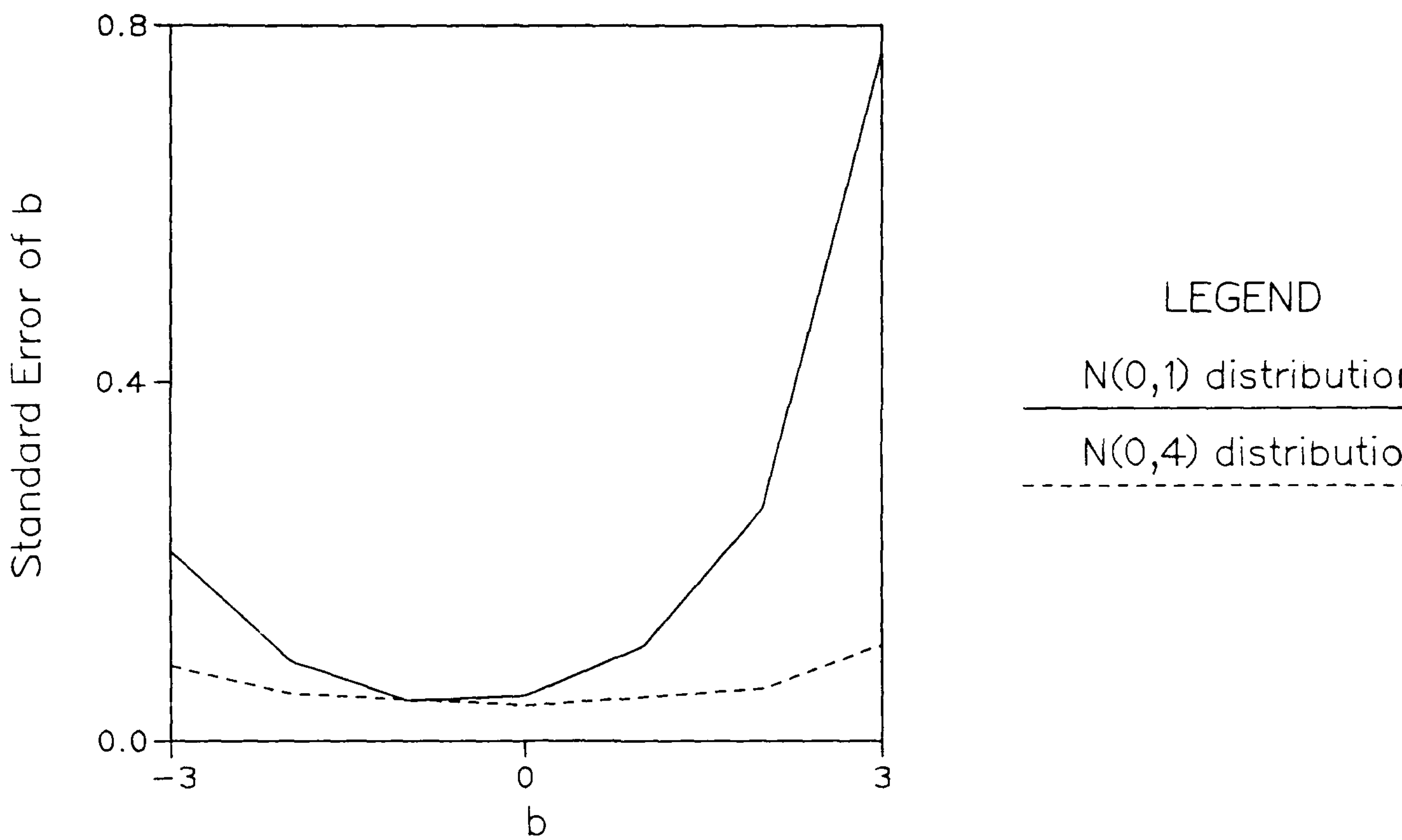

Figure 4. Standard errors of estimated item difficulties for the 3PL model, a=1.5, c=0. A calibration sample of N=2,500 is drawn from an N(0,1) distribution (solid) and from an N(0,4) distribution (dotted).

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