

SOFT CONSTRAINED LMS ALGORITHMS: A ROBUST PROCEDURE FOR SIGNAL MISALIGNMENT

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ABSTRACT

This paper presents a new algorithm for both broadband and narrowband adaptive beamforming which is robust to both desired signal misalignment and array imperfections. By utilizing available *a priori* information, the new algorithm minimizes desired signal cancellation by introducing new soft constraints which are incorporated into the LMS adaptation algorithm.

1. INTRODUCTION

Adaptive beamformers can be used, with minimal *a priori* information, to greatly improve the signal-to-noise ratio of a signal in the presence of hostile interference (jammers). However, when the actual signal angle of arrival does not coincide with the estimated angle of arrival, or when the actual geometry of the array differs from the geometry assumed by the array designer, then the performance of the array may be degraded due to adaptation. This phenomena is due to the fact that if there is misalignment or geometry imperfections, in the absence of a pilot signal, the usual adaptive array attempts to minimize total output power regardless of the type of signal present. While this is desirable if the received signal is mostly jammer and receiver noise, when the desired signal component becomes large, the receiver attempts to reject it, thus significantly degrading performance [2,3].

The most common way of avoiding the problem of rejecting the signal along with the noise is to impose some sort of linear constraints on the ways in which the array is allowed to adapt. Many methods have been proposed to implement these types of algorithms, but the most common are the Frost algorithm and the Generalized Sidelobe Canceller (GSC) structure [4,5]. Both of these types of arrays impose constraints on the array patterns which they may generate, usually either fixed direction gain constraints, or constraints on the derivative of gain vs. angle at a fixed look direction. In the broadband case, these constraints may become quite complex in some instances.

The array processor proposed in this study is designed to allow the cancellation of interferers away from the desired signal, while being very robust with respect to both array geometry and signal direction. This is achieved by use

of the available *a priori* knowledge of the signal direction and power spectral density to generate soft constraints on the processor weights.

2. SYSTEM MODEL

For the remainder of the paper, the structure of the adaptive array under consideration will be that of a zeroth order GSC. This structure has been shown to converge to the same steady state solution as the Frost constrained adaptation algorithm with one directional constraint. We will assume we have sources emitting only plane wave signals (i.e. far field approximation). We will also assume one jammer with unknown direction, and a signal whose direction is random, but known *a priori* not to be uniformly distributed over the entire $(-\pi, \pi)$ angle. In addition, we assume that we have some knowledge of the time autocorrelation of the desired signal. It will later be shown that exact knowledge of these quantities is not required for good performance. We will also assume that the antenna array is linear, with uniform $\lambda/2$ spacing between antennas. The gain of each antenna is assumed to be a complex Gaussian random variable with inphase and quadrature parts i.i.d. with a distribution $N(1, \sigma^2)$. This model is also appropriate for an array where the antenna placements are random, but close to linear.

It should be noted that the assumptions of a uniform linear array and one jammer are made purely for mathematical expediency, and are not necessary for the results of this paper to hold.

The sidelobe cancellation matrix B is designed to be an $N \times N-1$ matrix of rank $N-1$, such that

$$B\vec{s} = \vec{0} \quad (1)$$

where $\vec{s}$ is the N dimensional vector of delays seen for a plane wave signal coming from the constraint direction. Normally, the array is assumed to be *pre-steered* so that the constraint direction is broadside to the array, and thus:

$$\vec{s} = \vec{1} \quad (2)$$

We must define a new received vector, taking into account the delayed versions of the data. With this in mind, we define new NK dimensional vectors $\vec{x}_i$ and $\vec{s}_i$ as:

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$$\vec{x}_i' = [\vec{x}'(0), \vec{x}'(1), \dots, \vec{x}'(K-1)]$$

$$\vec{s}_i' = [\vec{s}'(0), \vec{s}'(1), \dots, \vec{s}'(K-1)]$$

Where the vectors $\vec{x}(i)$ and $\vec{s}(i)$ are the vectors associated with the data and signal delayed by i sampling periods, respectively. We must now define a new beamforming from the matrix B . The new matrix, $\bar{B}$, will now be an $(N-1)K \times NK$ block diagonal matrix, of the form:

$$\bar{B} = \text{diag}(B, \dots, B)$$

The output of the GSC, using our above stated assumptions on the gain of the antenna elements, is given by the expression:

$$\vec{1}_{K/2}^T \bar{G} \vec{x}_i - \vec{w}^T \bar{B} \bar{G} \vec{x}_i \quad (3)$$

where $\vec{1}_{K/2}$ is a vector with N ones in the $K/2$ block and zeros everywhere else, and $\vec{w}$ is the $(N-1)K$ dimensional vector of weights used in weighting the outputs of the $N-1$ tapped delay lines following the B matrix.

The antenna gains will be incorporated into a matrix G :

$$G = \text{diag}(1 + \Delta g_1, 1 + \Delta g_2, \dots, 1 + \Delta g_N) = I + \Delta G$$

$\bar{G}$ is the $(NK \times NK)$ block diagonal matrix of the antenna gains, and can be written as:

$$\bar{G} = \text{diag}(G, \dots, G)$$

and Δg_i is a $N(0, \sigma_g^2)$ complex random variable. It should be mentioned that a given realization of the matrix $\bar{G}$ may be dependent on the directional properties of the data vector $\vec{x}_i$, but in all cases the statistical properties of the matrix remain identical.

We now define the output of the beamforming matrix as $\vec{u}$:

$$\vec{u} = \bar{B} \bar{G} \vec{x}_i$$

$\vec{u}$ will be an $(N-1)K$ dimensional vector. The optimization problem of minimizing the total output power of the GSC translates to the problem:

$$\text{MIN}_{\vec{w}} E[|\vec{1}_{K/2}^T \bar{G} \vec{x}_i - \vec{w}^T \vec{u}|^2] \quad (4)$$

The solution to this problem is the well known Wiener Filter solution. The optimum weight vector $\vec{w}$ is given by the equation:

$$\vec{w} = R_{uu}^{-1} \vec{r}_{ud} \quad (5)$$

The matrix R_{uu} is defined as:

$$R_{uu} = E[\vec{u} \vec{u}^H]$$

And the vector $\vec{r}_{ud}$ is of the form:

$$\vec{r}_{ud} = \bar{B} \bar{G} R_{xx} \vec{1}_{K/2}$$

$$\text{Where } R_{xx} = E[\vec{x}_i \vec{x}_i^H]$$

And $\vec{x}_i^H$ and $\vec{x}_i^*$ denote complex conjugate transpose and complex conjugate of $\vec{x}_i$ respectively.

3. ROBUST ADAPTATION ALGORITHM

If the signal direction vector $\vec{s}$ is not exactly equal to $\vec{1}$, or if the variance of the antenna gains is not very small, the array pattern generated by (5) will attempt to null out a strong desired signal. To deal with the problem of random antenna gains, Jablon and Widrow [1] have proposed using a soft constraint type of optimization. A suitable penalty function is chosen, and the optimization problem is then changed to minimizing the original functional, (in this case $E[|\vec{1}_{K/2}^T \bar{G} \vec{x}_i - \vec{w}^T \vec{u}|^2]$) plus some penalty constant times the penalty function. If the penalty constant is chosen to be large enough, then the optimization problem will have a solution very close to that of a hard constraint problem, where the constraint is defined as the penalty function being forced to take on the value zero.

We wish to generate a penalty function which preserves the desired signal while allowing the adaptive processor to eliminate interfering sources. In a previous paper [8] we have dealt with this problem in the narrowband situation. We wish now to extend this technique to the case of broadband arrays. We will be interested specifically in the case of preserving the signal in the presence of array imperfections and desired signal misalignment.

We will define our penalty function $p(\vec{w})$ as one which is linear in the amount of power that is transmitted through the cancellation branch of the GSC due to the signal component. The leakage of the signal vector into the cancellation branch is given by the expression:

$$L(\vec{s}_i) = \vec{w}^T \bar{B} \bar{G} \vec{s}_i \quad (6)$$

We will take the expected value of the power of the leakage signal as our penalty function. Thus, $p(\vec{w})$ becomes:

$$p(\vec{w}) = E[\vec{w}^T \bar{B} (\bar{G} \vec{s}_i \vec{s}_i^H \bar{G}^H) \bar{B}^H \vec{w}] \quad (7)$$

Bringing the ensemble expectation inside and utilizing the assumptions that the errors in the antenna gains are independent of each other, and of the possible signal directions, we get the simplified result:

$$p(\vec{w}) = \vec{w}^T \bar{B} (\sigma_s^2 R'_{ss} + R_{ss}) \bar{B}^H \vec{w} \quad (8)$$

Here we have defined two new matrices, R'_{ss} and R_{ss} . The matrix R'_{ss} is a modified time autocorrelation matrix of the signal. It is a block $NK \times NK$ matrix, with blocks of size $N \times N$. Each block is an identity matrix weighted by a time autocorrelation coefficient of the signal. The i th matrices off the diagonal are weighted by the i th time autocorrelation coefficient. In the degenerate case of a narrowband processor, i. e. no tapped delay lines, the matrix R'_{ss} becomes simply an identity matrix multiplied by the desired signal power σ_s^2 .

The matrix R_{ss} can be thought of as the space-time autocorrelation matrix of the signal vector $\vec{s}_i$. The matrix will be a Hermitian Toeplitz block Hermitian Toeplitz matrix, where the n th off diagonal element of the m th off diagonal block is given by the expression:



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$$\int_{-\pi/2}^{\pi/2} R(mT + n d \sin(\theta)) p(\theta) d\theta$$

Where $R(\cdot)$ is the time autocorrelation function of the desired signal, $p(\cdot)$ is the probability distribution of the direction of arrival of the signal, T is the unit time delay in the tapped delay lines, and d is the time for a wave to propagate between adjacent antennas.

We see that in the degenerate case of no tapped delay lines, the integral in the above expression becomes:

$$\int_{-\pi/2}^{\pi/2} R(n d \sin(\theta)) p(\theta) d\theta$$

And in the narrowband case, the autocorrelation function $R(\cdot)$ becomes a complex exponential, causing the integral to take a form similar to that of a Fourier transform.

Utilizing the new penalty function, the new soft constrained optimization problem becomes:

$$\begin{aligned} \text{MIN}_{\vec{w}} \quad & \vec{1}_{K/2}^T \bar{G} \mathbf{R}_{xx} \bar{G}^T \vec{1}_{K/2} - 2\text{Re} \{ \vec{w}^T \bar{B} \bar{G} \mathbf{R}_{xx} \bar{G}^T \vec{1}_{K/2} \} \\ & + \vec{w}^T \bar{B} [\mathbf{R}_{xx} + c (\sigma_g^2 \mathbf{R}'_{ss} + \mathbf{R}_{ss})] \bar{B}^T \vec{w} \end{aligned} \quad (9)$$

At this point, we note that the term is quadratic in the vector $\vec{w}$, and thus any minima of this expression will be global. The matrices $\mathbf{R}_{ss}$ and $\mathbf{R}'_{ss}$ are assumed to be known *a priori*, but as we will show later, any reasonable estimate will suffice for the proposed algorithm.

We now take the instantaneous approximation to the gradient vector of our error surface using the usual LMS methods and subtract some multiple of it from our present solution for $\vec{w}$. This method gives a new adaptive algorithm for the tap weights:

$$\begin{aligned} \vec{w}_{k+1} = & (\mathbf{I} - 2\mu (\vec{u}^* \vec{u}^T + \frac{\eta}{2\mu} \bar{B} [\sigma_g^2 \mathbf{R}'_{ss} + \mathbf{R}_{ss}] \bar{B}^T)) \vec{w}_k \\ & + 2\mu \vec{x}_k^T \vec{1}_{K/2} \bar{B} \vec{x}_k^* \end{aligned} \quad (10)$$

Or, after re-arranging terms:

$$\begin{aligned} \vec{w}_{k+1} = & [\mathbf{I} - \eta \sigma_g^2 \bar{B} \mathbf{R}'_{ss} \bar{B}^T - \eta \bar{B} \mathbf{R}_{ss} \bar{B}^T] \vec{w}_k \\ & + 2\mu y_k \vec{u}^* \end{aligned} \quad (11)$$

Where the term y_k is the output of the array at the k th iteration. This algorithm can be shown to converge for positive values of μ such that:

$$\mu < \frac{1}{\text{tr} [\mathbf{R}_{ss} + \frac{\eta}{2\mu} \bar{B} (\sigma_g^2 \mathbf{R}'_{ss} + \mathbf{R}_{ss}) \bar{B}^T]} \quad (12)$$

The new algorithm requires that a matrix vector multiplication take place during each tap weight vector update. In general, this would be an order $((N-1)^2 K^2)$ operation. However, since the matrix in the multiplication is Hermitian Toeplitz block Hermitian Toeplitz, the multiplication can be seen as a convolution, and thus it can be computed by FFT in order $((N-1)K \log((N-1)K))$, or simplifying, order $((N-1)K \log(N-1) + (N-1)K \log(K))$. By utilizing this fact, the large size of the matrix needed for the robust algorithm does not preclude its implementation.

4. RESULTS

The new algorithm requires the knowledge of the signal space-time autocorrelation matrix $\mathbf{R}_{ss}$, and the desired signal time autocorrelation function $R(\cdot)$ *a priori*. In practice, the time autocorrelation function may be well known. If it is not, then a robust estimate of the autocorrelation function may be used. The desired signal space-time autocorrelation matrix may then be approximated by assuming that the signal will definitely fall within the main beam of the array (or else the array would be missteered), and then taking the maximum variance distribution within this arc. Using this method, the spatial distribution of the desired signal is assumed constant within some arc symmetric about broadside, and zero elsewhere.

The quiescent pattern (ignoring misadjustment) generated by the new adaptation algorithm was calculated analytically for a broadband array with 11 antennas and tapped delay lines of length 5 for signals having a PSD given by figure 2. The output SINR (signal to interference plus noise ratio) was calculated as a function of jammer angle of arrival for a fixed signal location, and again as a function of signal angle of arrival for fixed jammer location. The results show that even with inexact knowledge of signal time autocorrelation and direction of arrival, the new robust algorithm performs quite well, almost eliminating the effects of signal misalignment.

We then simulated both the robust algorithm and a 0th order GSC on an 11-by-5 processor using gaussian data having identical PSD to that in the SINR calculation. The results of the simulation show a marked improvement of the robust algorithm over the 0th order GSC.

5. CONCLUSION

We have developed a new algorithm for robust beamforming for both narrowband and broadband arrays in the presence of uncertain signal directions and array geometries. The new algorithm incorporates available *a priori* information into a penalty function which imposes soft constraints on the adaptation of the array. The new soft constrained algorithm is shown to perform well even with reasonably poor information about the desired signal.

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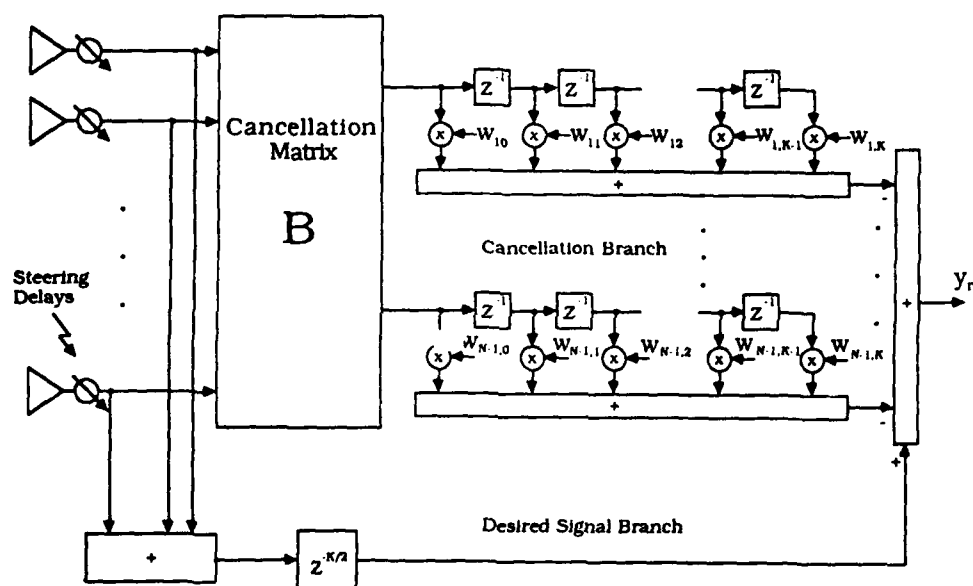


Fig. 1: Broadband GSC

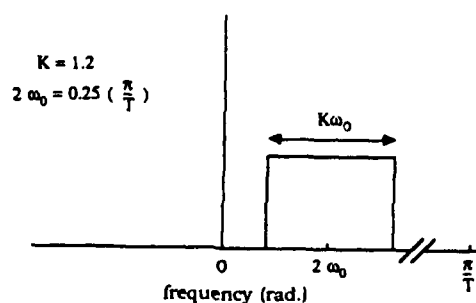


Fig. 2: Power spectral density of simulated signal jammer.

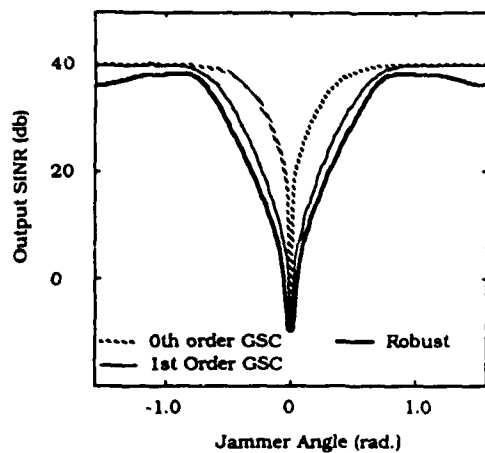


Fig. 3: Output SINR vs. Jammer angle, no misalignment

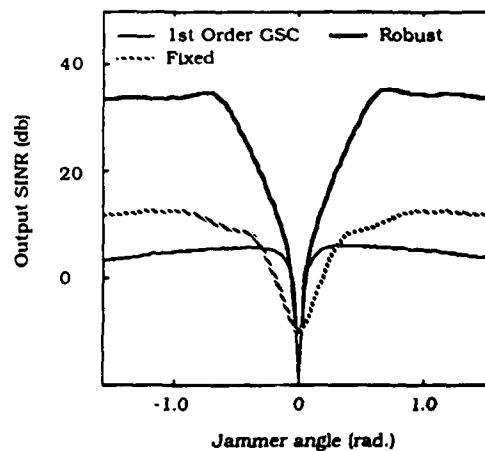


Fig. 4: Output SINR vs. Jammer angle, 0.02 rad. misalignment.

In all graphs, signal power is 30 db
Jammer power is 40 db

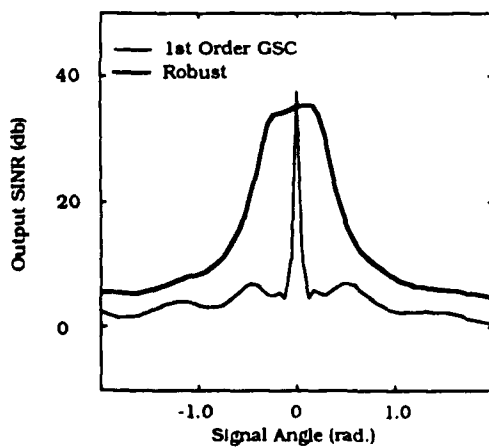


Fig. 5: Output SINR vs. Signal angle, Jammer at 0.65 rad.

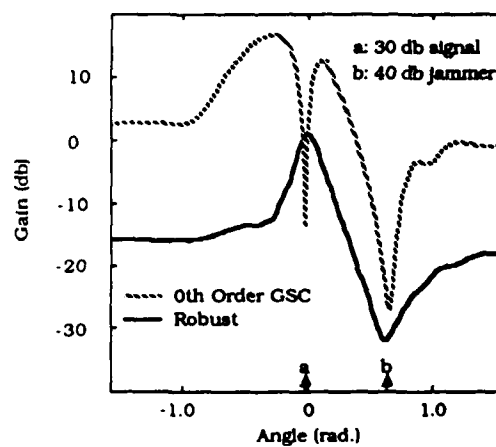


Fig. 6: Array patterns for 0th order GSC, Robust processor.

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