NUSC Technical Document 8453
8 December 1988

Detection Performance of Normalizer for a Multi-Pulse Signal Subject to Partially-Correlated Fading with Chi-Squared Statistics

A Paper Presented at the 2nd
Joint Meeting of the Acoustical
Societies of America and Japan,
15 November 1988, Honolulu, Oahu

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Preface

This research was conducted under NUSC Project No. A75205, Subproject No. RR00N01, "Applications of Statistical Communication Theory to Acoustic Signal Processing," Principal Investigator Dr. Albert H. Nuttall (Code 304). This technical report was prepared with funds provided by the NUSC In-House Independent Research and Independent Exploratory Development Program, sponsored by the Chief of Naval Research.

Reviewed and Approved: 8 December 1988

W. A. Von Winkle
Associate Technical Director for Technology

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

AD-A204 698

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution is unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S): TD 8453	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION: Naval Underwater Systems Center; 6b. OFFICE SYMBOL (If applicable): Code 304	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code): New London Laboratory, New London, CT 06320	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: Chief of Naval Research; 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code): Department of the Navy, Washington, DC 20362	10. SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
DETECTION PERFORMANCE OF NORMALIZER FOR A MULTI-PULSE SIGNAL SUBJECT TO PARTIALLY-CORRELATED FADING WITH CHI-SQUARED STATISTICS

12. PERSONAL AUTHOR(S)
Albert H. Nuttall

13a. TYPE OF REPORT	13b. TIME COVERED FROM ____ TO ____	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
		1988 December 8	

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Detection Probability; Normalizer; Multi-Pulse Signal; Correlated Fading; Chi-Squared Fading; Unknown Noise Level; Constant False Alarm Rate

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The false alarm and detection probabilities for a multipulse signal subject to partially correlated fading, in the presence of Gaussian noise of unknown level, are derived in closed form. The number K of signal pulses, as well as the number L of noise-only pulses used to estimate the noise background power level, are arbitrary. The power fading is characterized by a chi-squared distribution with $2m$ degrees of freedom and a normalized set of covariance coefficients $\{\rho_{kl}\}$, all of which can be selected arbitrarily, in order to match an experimental realization or an actual measured situation. The performance capability of this processor depends additionally on the received signal-to-noise ratio. This study covers the case of a nonconstant threshold; comparisons of this normalizer with earlier results (for $L = \infty$) enable a quantitative evaluation of the losses incurred by

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL: Albert H. Nuttall	22b. TELEPHONE (Include Area Code): (203) 440-4618; 22c. OFFICE SYMBOL: Code 304

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
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19. ABSTRACT (Cont'd.)

lack of knowledge of the noise level. The important capability of constant false alarm rate is achieved by this normalizer.

DETECTION PERFORMANCE OF NORMALIZER FOR A MULTI-PULSE SIGNAL SUBJECT TO PARTIALLY-CORRELATED FADING WITH CHI-SQUARED STATISTICS

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The following 5 pages give the text of the oral presentation at the JASA meeting.

The succeeding 8 pages constitute the poster presentation. The particular 5 pages that were employed for the oral presentation are labeled with VG1, VG2, VG3, VG4, VG5, at the top center.

VU-GRAPH 1

The underwater propagation of acoustic signals is very adversely affected by the presence of deep fades in time and/or frequency. In an effort to combat fading, diversity in the transmitted signal is often employed, by emitting energy distributed over time and frequency. However, the fading of nearby pulses can be highly dependent, thereby thwarting some of the gains to be expected from diversity combination. This paper outlines the evaluation of the performance of a diversity-combining system in the presence of partially-correlated fading and additive noise of unknown level, in terms of the false alarm and detection probabilities.

VU-GRAPH 2

The transmitted signal consists of K CW tone bursts, each of duration T_1 seconds, distributed in a known pattern over time and frequency. The k-th received signal pulse undergoes an amplitude-scaling r_k and a phase-shift θ_k, both of which are constant over an individual pulse duration of T_1 seconds and bandwidth $1/T_1$ Hertz. That is, we are considering slow non-selective fading.

The additive noise is stationary over the total observation time and has a flat spectrum over the total bandwidth utilized. However, since the noise level is unknown, a group of L noise-only bins, located in time-frequency where there is no signal, is used to extract an estimate of the noise level, N_o.

VU-GRAPH 3

In order to be quantitative about the fading statistics and dependencies, we define the power-scaling variate $q_k = r_k^2$, the square of the amplitude-scaling variate. The first-order probability density of q is taken to be chi-squared with 2m degrees of freedom. Thus m = 1 corresponds to exponential power-fading or Rayleigh amplitude-fading. The parameter 'a' is a measure of the average strength of the fading. In particular, the average power scaling equals m times a. Particularly deep fading is modeled by small values of parameter m, which need not be integer. In fact, some recent measurements yielded fading which was characterized by m near 1/2.

The statistical dependencies between fading of separate signal pulses is modeled by allowing arbitrary covariance coefficients between any two power scaling variates. The exact statistics and dependencies of phase shifts θ_k are not relevant, due to the particular form of receiver processing adopted.

VU-GRAPH 4

The potential-signal channels are matched-filtered and square-law envelope-detected at the instants of peak signal output, if signal is present. The sum of these K squared envelopes is then compared with a similar sum of L envelope-squared outputs of filters which are known to operate in a region of noise-only. If the ratio ν exceeds a threshold, a signal is declared present.

For signal absent, the value of output ν on any trial is independent of the absolute unknown noise level, N_o. Hence, this processor has constant false alarm rate capability; that is, we can realize a specified false alarm probability without knowledge of noise level N_o.

VU-GRAPH 5

The detailed analysis of performance of this system is too lengthy to be presented here. All we can do is to present a sample which illustrates the type of results that are currently available. This plot gives the receiver operating characteristics on normal probability paper, with input signal-to-noise ratio as a parameter. This particular case is for four signal pulses subject to Rayleigh amplitude fading with a covariance coefficient of 0.5, and utilizing 16 noise-only filters. Many additional examples and a general program are available in NUSC Technical Report 8133, from which this paper was drawn.

VG1

DETECTION PERFORMANCE OF NORMALIZER FOR A MULTI-PULSE SIGNAL SUBJECT TO PARTIALLY CORRELATED FADING WITH CHI-SQUARED STATISTICS

NUSC TR 8133

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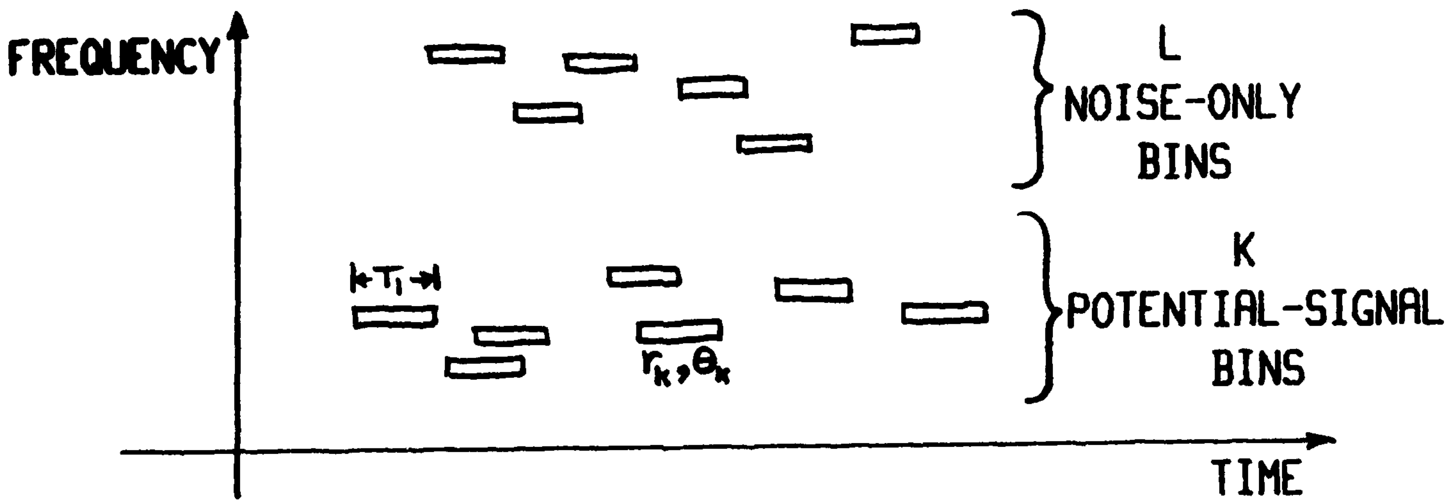
PRESENTED AT
ACOUSTICAL SOCIETY OF AMERICA CONFERENCE
SHERATON-WAIKIKI HOTEL
HONOLULU, HAWAII

15 NOVEMBER 1988

TIME-FREQUENCY OCCUPANCY



VG2

BINS: T_1 SECONDS DURATION
$1/T_1$ HERTZ BANDWIDTH
} CW TONE BURSTS

RECEIVED SIGNAL (IF PRESENT):
AMPLITUDE SCALING r_k
PHASE SHIFT θ_k
} OF k-TH SIGNAL PULSE

ADDITIVE NOISE:
STATIONARY OVER TOTAL OBSERVATION TIME.
FLAT SPECTRUM OVER TOTAL BANDWIDTH UTILIZED.
UNKNOWN LEVEL.

VG3

SIGNAL FADING STATISTICS

r_k, θ_k : CONSTANT OVER EACH INDIVIDUAL PULSE DURATION T_1 (SLOW FADING). ARBITRARY DEPENDENCIES BETWEEN PULSES.

DEFINE $q_k = r_k^2$ = POWER SCALING OF k-TH RECEIVED SIGNAL PULSE.

FIRST-ORDER PDF OF q_k IS CHI-SQUARED WITH 2m DEGREES OF FREEDOM:

$$p_q(u) = \frac{u^{m-1} \exp(-u/a)}{\Gamma(m)\, a^m} \quad \text{for } u > 0$$

AVERAGE POWER SCALING $\bar{q}$ = ma FOR ALL k.

m = 1 CORRESPONDS TO RAYLEIGH AMPLITUDE-FADING OR EXPONENTIAL POWER-FADING.

m NEED NOT BE AN INTEGER.

STATISTICS (CONTINUED):

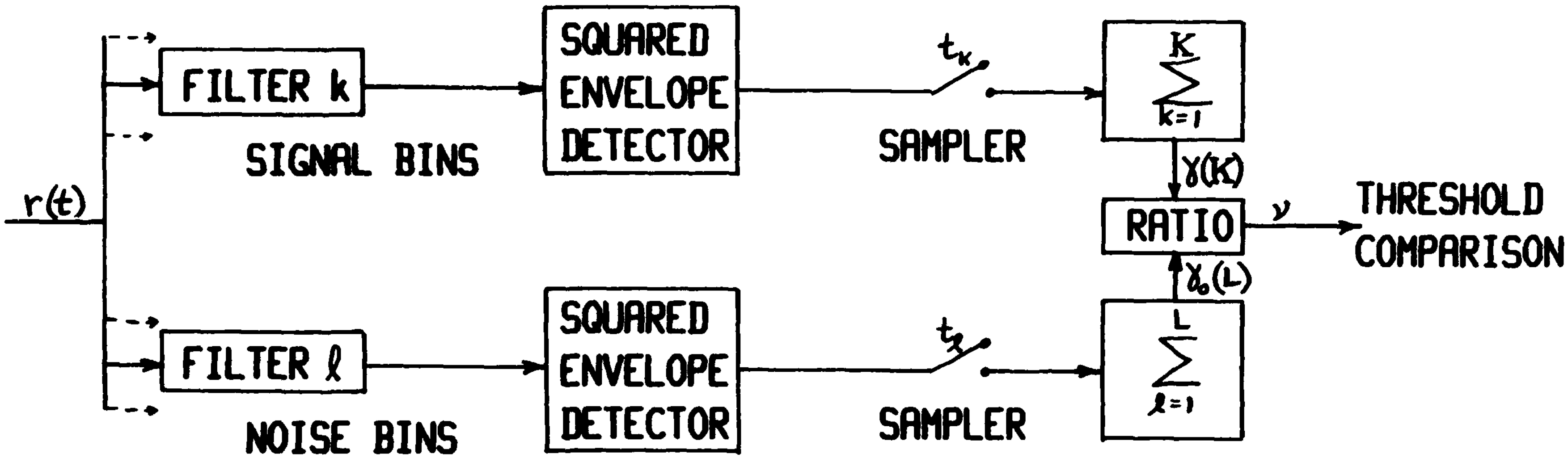
ARBITRARY CORRELATION COEFFICIENTS BETWEEN POWER SCALINGS:

$$\rho_{kj} = \frac{\overline{q_k q_j} - \overline{q_k}\,\overline{q_j}}{\sigma_q^2}$$

PHASE SHIFTS $\{\theta_k\}$ HAVE ARBITRARY STATISTICS AND DEPENDENCIES.

ADDITIVE NOISE SPECTRAL LEVEL, N_o WATTS/HZ, IS UNKNOWN.

RECEIVER PROCESSING



VG-4

TIME AND FREQUENCY SYNCHRONIZATION PRESUMED.
(DELAY AND DOPPLER COMPENSATED)

FILTERING CAN BE ACCOMPLISHED VIA FFTS.

DECISION VARIABLE $\nu = \dfrac{\gamma(K)}{\gamma_0(L)} \gtrless$ THRESHOLD.

PROBLEM DEFINITION

RELEVANT PARAMETERS:

- K = NUMBER OF SIGNAL PULSES
- L = NUMBER OF NOISE PULSES
- m = SIGNAL FADING PARAMETER (NON-INTEGER); $2m$ DEGREES OF FREEDOM
- ρ_{kj} = POWER-FADING COVARIANCE COEFFICIENT
- $\overline{E_1}$ = AVERAGE RECEIVED SIGNAL ENERGY PER PULSE
- N_0 = SINGLE-SIDED RECEIVED NOISE SPECTRAL LEVEL

WANT PROB($\nu >$ THRESHOLD | SIGNAL PRESENT).

CAN THEN DETERMINE P_F AND P_D.

$L = \infty$ CORRESPONDS TO KNOWN NOISE LEVEL.

FOR SIGNAL ABSENT, ν IS INDEPENDENT OF ACTUAL (UNKNOWN) NOISE LEVEL N_0; CFAR PROCESSOR.

CAN ACHIEVE SPECIFIED P_F WITHOUT KNOWLEDGE OF N_0.

ANALYTIC APPROACH

HOLD $\{r_k\}$ AND $\{\theta_k\}$ FIXED.
THEN CONDITIONAL CHARACTERISTIC FUNCTION
(CF) OF SUM $\gamma(K)$ DEPENDS ONLY ON

$$S = \sum_{k=1}^{K} r_k^2 = \sum_{k=1}^{K} q_k .$$

APPROXIMATE S BY A CHI-SQUARED VARIATE
WITH EXACT SAME MEAN AND VARIANCE.

AVERAGE THE CONDITIONAL CF TO GET THE UNCOND. CF OF $\gamma(K)$.

FOURIER TRANSFORM TO GET PDF OF $\gamma(K)$.

COMPUTE PROB($\gamma(K) > T * \gamma_0(L)$) IN TERMS OF A FINITE
SERIES OF TERMINATING GAUSSIAN-HYPERGEOMETRIC FUNCTIONS.

VG5

SAMPLE RESULT

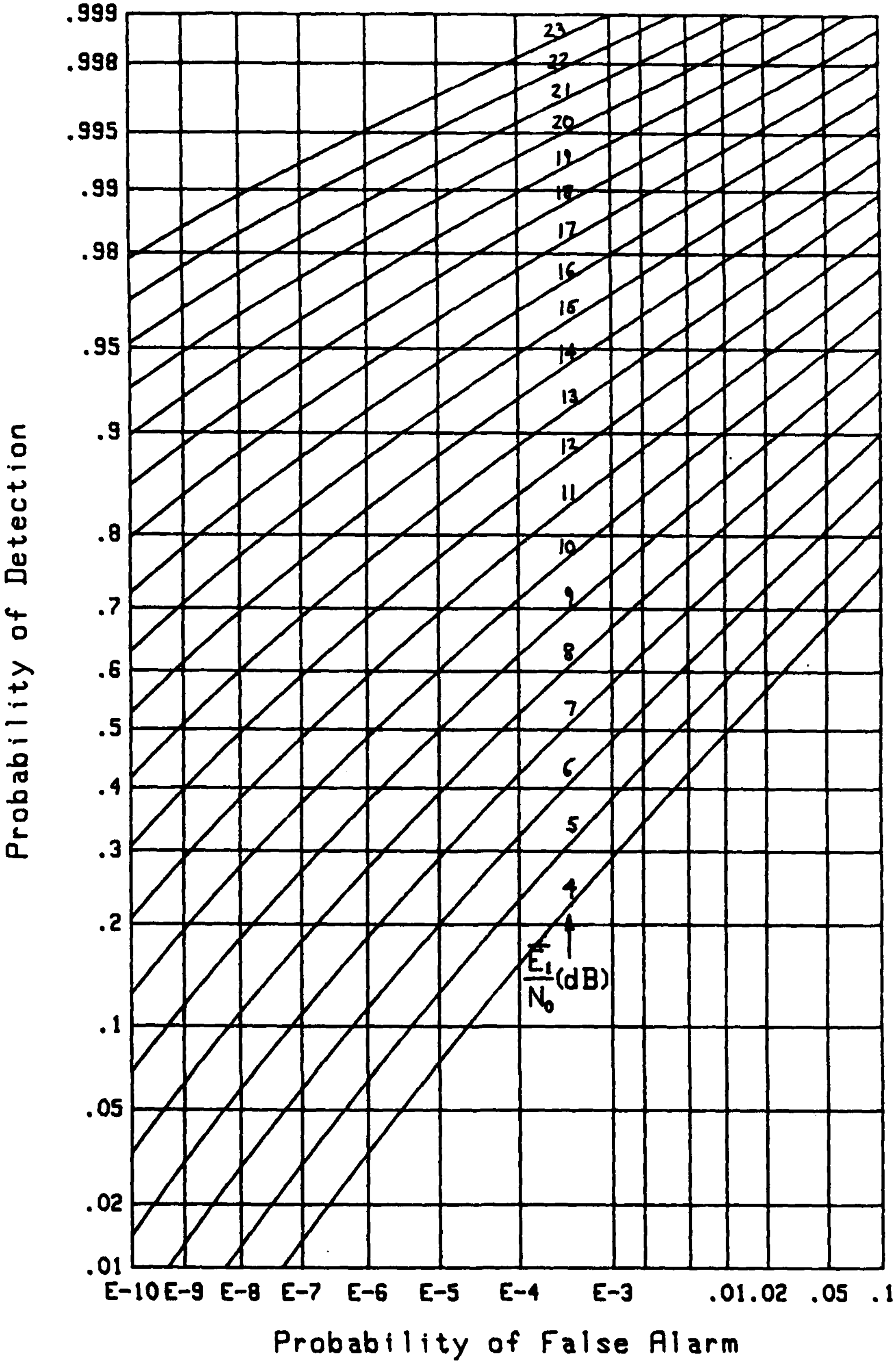


Figure 25. ROC for K=4, m=1, ρ=.5, L=16

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