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Logical and mathematical issues of developing, pooling and reconciling multiple models of a given uncertain variable have been addressed. A number of technical papers have been published in refereed journals.

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January 20, 1989

Dr. Julia Abrams
Statistics Program
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Subject: Final Report on ONR Contract No. N00014-85-C-0268 "Theoretical Development for Pooling Multiple Inconsistent Estimates"

Dear Dr. Abrams:

The main product of the subject contract is publications and drafts appended to this report, and whose references and abstracts follow. In addition, the following work is in progress:

Impersonal probabilities as a decision aiding concept. Complementary drafts of material on the connection between beliefs and physical randomness (attached) by each co-PI have been prepared, with a view to merging into a joint paper for publication.

Correlations between experts or pieces of evidence an expert has. While it is not unreasonable to extract judgments about means and variances from people, it is almost impossible to think about correlations. (Variances can be obtained indirectly through the use of quantiles or other fractiles.) However correlations enter into such simple quantities as sums, the variance of a sum involving a covariance, and hence a correlation. Work has therefore continued on studying how variances change when additional expertise or evidence is introduced. This had led to good methods in perhaps the majority of cases. But subtleties remain to be investigated.

The value of decision analytic procedures. In order to better understand the opinions held about and by experts, one has to appreciate the value of the tools used. It has been shown that under very general conditions, an extension to include additional quantities, increases the value of the assessment of the original quantity. Surprisingly the same thing does not happen with Bayes' formula: the posterior probabilities can often be assessed more accurately without the formula. This puzzling result is being investigated further, together with other techniques used in decision analysis. In this way, it is hoped to increase our understanding of the value of a decision analysis.

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A proposal to ONR for a small amount of support to complete the above tasks is under consideration.

The undersigned would be glad to address any questions.

Yours sincerely,

for Rex V. Brown and Dennis V. Lindley
Co-Principal Investigators

Enclosures: Abstracts
Referenced Papers

cc (w/1 encl): Naval Research Laboratory (1 copy)
DTIC (12 copies)

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REFERENCES AND ABSTRACTS OF COMPLETED WORK CREDITING CONTRACT N00014-85-C-0268

Lindley, D.V. The use of probability statements. *Proceedings of the Italian Physical Society*, February 1988.

The problem discussed, with variants, is that of someone who is interested in a quantity whose value is unknown. In order to lessen the uncertainty, several people are consulted who provide opinions about the quantity. How should these opinions be combined to provide an enhanced judgment about the quantity?

Lindley, D.V. Using expert advice on a skew judgmental distribution. *Operations Research, 35*(5), September-October, 1987.

A decision maker is interested in a quantity θ. He consults an expert who provides three fractiles of her probability distribution for θ. The problem discussed in this paper is how the decision maker should use the expert's fractiles to produce his distribution for θ. A key ingredient has to be the decision maker's opinion of the expert. Our analysis shows how the Bayesian approach clearly incorporates this feature. With three fractiles provided, information about the skewness of the distribution of θ is available, in addition to location and scale. Our development uses the skew logistic to accommodate this additional feature, and much of the paper is concerned with skewness, extending the methods of Lindley for the symmetric case. The ideas easily extend to several experts if the unrealistic assumption is made that the decision maker regards them as independent.

Brown, R.V. and Lindley, D.V. Plural analysis: Multiple approaches to quantitative research. *Theory and Decision, 20*, 1986, 133-154.

The dominant approach to decision analysis in particular, and quantitative research in general, is *singular*, i.e., all conclusions are based on a single preferred model or approach. In *plural analysis*, several singular analyses are pursued in parallel, the typically conflicting results are merged. Although it is a central feature of intelligent everyday thinking, plural analysis has largely been ignored by the research community. This paper reports on an ongoing, multidisciplinary, research program, supported by the National Science Foundation and the Office of Naval Research, to establish guidelines, grounded in defensible theory, for designing a plural analysis strategy (where appropriate) and integrating the results of analysis.

Lindley, D.V. *An invariance axiom for probability* (Draft Paper). Falls Church, VA: Decision Science Consortium, Inc., February 1986. [To be submitted for publication.]

An invariance axiom for probability is introduced, saying that a probability on the supposition that an event B is true is the same as that when B is known to be true. The reasonableness of the axiom is discussed and, in particular, it is shown that apparent violations are remedied if due consideration is given to the way in which the knowledge that turned supposition into fact is considered.

Lindley, D.V. *The present position in Bayesian statistics*. The Wald Lectures to the Institute of Mathematical Statistics given in Fort Collins, Colorado, August, 1988.

The aim of these lectures is to explain Bayesian statistics as a new paradigm. In section 1, Wald's work is contrasted with Bayesian, particularly emphasis being placed on the role of the sample space. In section 2 an attempt is made to describe the basic problem of statistics as part of scientific methods, and in section 3 to argue, and to explore the unappreciated consequence, that its solution must be Bayesian. Section 4 extends the discussion to decision-making and section 5 is a foundational glance at statistical practice. A final section 6 contains an attempt to face up to the famous, legitimate objection to Bayesian statistics that the prior may be unknown. It is argued that the measurement of probabilities is a problem that has not been seriously considered, even by probabilists, and a beginning is made on its resolution.

Brown, R.V. *Impersonal probability as a decision aiding concept*. (Draft of a joint paper planned with D. Lindley for submission to the Journal of Risk and Uncertainty.) Reston, VA: Decision Science Consortium, Inc., January, 1989.

The concept of the *impersonal* probability, I(E), of a unique event, E, is already implicit in typical analyses of risk management and similar decisions (for example as "*the*" risk of an accident). We hypothesize that it can be given an explicit interpretation, which is applicable--and useful--in most such cases. It is non-frequentist (i.e. not limited to repetitive events). It does not depend on any particular assessor or his actual knowledge, but its value is normally unknown, subject to personal (second-order) uncertainty.

It is defined as

$$I(E) = P^*(E|K^*)$$

where: E is any unambiguously specified event; K* is unlimited, but feasible (i.e. not "perfect") knowledge, available immediately; and P* is ideal analysis. The conditions under which I(E) is uniquely defined for practical purposes are discussed, in the context both of repetitive (coin tossing) and realistically unique (nuclear accident) events.

Lindley, D.V. *Impersonal Probability as an Aid to Decision Making*. (Draft of joint paper planned with Rex Brown.)

A person, or subject S, contemplating an uncertain event A when in possession of information H often needs to measure the amount of that uncertainty. There are several ways in which this can be done: for example, in terms of belief functions (Shafer) or fuzzy logic (Zadeh). A statistician might use notions of confidence. In the discussion here we shall suppose the uncertainty is measured in terms of probability and refers to the subject's personal probability $p_s(A|H)$ for A, given H. The reasons for this choice have often been rehearsed. The basic reason is that it is only with probability that the assessments of different uncertainties, of A, and of B etc. fit together, or cohere, sensibly. The rules of coherence must be those of the probability calculus and not of fuzzy logic.