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REPORT SRL-P-01-1989

COMPACT, LIGHTWEIGHT CO_2 LASERS FOR SDIO APPLICATIONS

Prepared by
Dr. Jonah Jacob

SCIENCE RESEARCH LABORATORY, INC.
15 Ward Street
Somerville, MA 02143

31 January 1989

PROGRESS TECHNICAL REPORT
Period for September 30, 1988 to January 31, 1989
Contract Number N00014-88-C-0727

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Research Supported by
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RF STABILIZATION OF MOLECULAR DISCHARGES

Two rate equations are important in determining the stability of the discharge. The first is the electron production and loss

$$\frac{dn_e}{dt} = \nu n_m n_e - \alpha n_e^2 - \beta n_e \tag{1}$$

and the second is the metastable production and loss

$$\frac{dn_m}{dt} = \langle \sigma v \rangle n_e n_a - n_m/\tau_m \tag{2}$$

where n_e is the electron density, n_m is the metastable density, α is the recombination rate, β is the attachment rate, $\langle \sigma v \rangle$ is the electron impact metastable production rates constant and τ_m is the metastable lifetime. The ionization rate constant ν in Eq. (1) is assumed to be the result of metastable ionization. Since the metastable levels have a much smaller ionization energy than the ground state the ionization rate of the discharge is dominated by electron impact ionization of the metastables. The stability of Eqs. (1) and (2) will be analyzed assuming that the applied discharge is sinusoidally varying. Hence the various rates are determined by the local RF electric field of frequency w which is generated externally by the system shown in Fig. 1. From the analysis performed in Phase I Science Research Laboratory (SRL) has shown that a current source is volumetrically stable and so the initial analysis will assume an RF current source.

We shall assume now that the rates describing the variation of n_e and n_m in Eqs. (1) and (2) (i.e. νn_m, αn_e, β, $\gamma n_e/n_m$) are much smaller than $\omega/2\pi$, so that one can use **time averaged** rate constants in these equations. On the other hand, we shall assume that $\omega/2\pi$ is much smaller than the electron energy distribution equilibration rate ($\nu_{inelastic}$), and therefore in the above mentioned time averaging process one can use the rates corresponding to the stationary discharge with the electric field equal everywhere to the field in the RF discharge at a given time. Thus we assume

$$\nu n_m << \omega/2\pi << \nu_{inelastic} \tag{3}$$

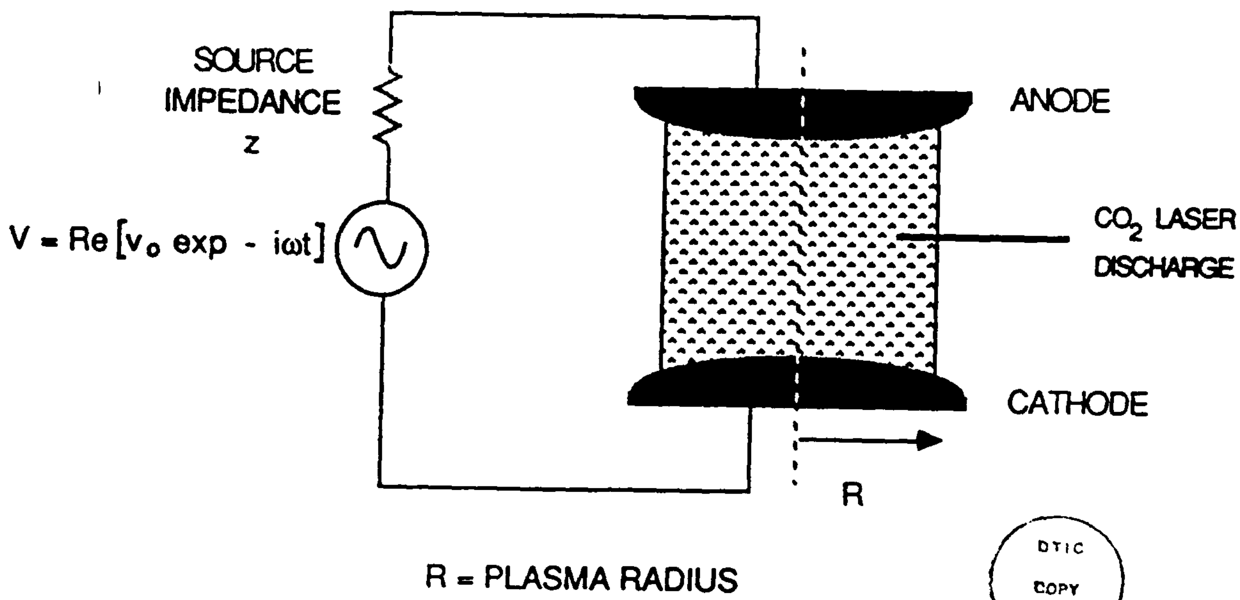
SOURCE
IMPEDANCE
z
V = Re [v_o exp - iωt]
ANODE
CO2 LASER
DISCHARGE
CATHODE
R
R = PLASMA RADIUS

Figure 1

Assuming a 100 Torr discharge, $\nu n_m \sim 10^6$ sec^{-1} and $\nu_{coll}^{in} = 10^{11}$ sec^{-1}, so that 1 MHz $<< f = \omega/2\pi << 10^2$ GHz.

At this stage it will also be assumed that the vacuum wavelength of our oscillator ($\lambda = 2\pi c/\omega$) is much larger than L, so that in the vicinity of the discharge one can neglect the relativistic retardation of time due to the extended volume, and treat the electromagnetic problem as being quasistationary. For $L = 1$ m, the condition $L << \lambda$ yields $f << c/L = 3 \cdot 10^8$ sec^{-1}. So that we are limiting our discussion to frequencies in the range 10 MHz $< f <$ 100 MHz.

The quasistationarity condition means that in the Maxwell equation for $\nabla \times \vec{B}$ one can neglect the displacement current and use the "magnetstatic" law

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{j}\,(t) \tag{4}$$

while $\vec{E}$ is given by the Faraday law and the quasi-neutrality

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{5}$$

$$\nabla \cdot \vec{E} = 0 \tag{6}$$

Equations (4), (5) and (6) yield

$$\nabla^2 \vec{E} = \frac{\pi}{c^2}\frac{\partial \vec{j}}{\partial t} \tag{7}$$

where the current density in the plasma is given by $\vec{j} = \sigma \vec{E}$, with the conductivity $\sigma = -en_e\mu$. The simplest model for the mobility μ will be used to give

$$\sigma = \frac{e^2 n_e}{m\nu_{coll}} \tag{8}$$

Now we shall consider the steady state of Eqs. (1-2) assuming cylindrical symmetry and that

$$\vec{E} = Re(E_o e^{-\omega t})\hat{e}_z$$

Therefore, all the parameters of the steady state (n_{eo}, n_{mo}, σ_o,E_o) are functions of r alone. In steady state one obtains

$$n_{mo} = \gamma\tau n_{eo} \tag{9}$$

$$n_{eo} = \frac{\beta}{\nu\gamma\tau - \alpha} \tag{10}$$

All the rate constants are temporally averaged over the voltage oscillation.

Returning back to Eq. (7), the steady state equation gives

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dE_o}{dr}\right) + i\frac{4\pi e^2 n_{eo}}{mc^2}\frac{\omega}{\nu_{coll}}E_o = 0 \tag{11}$$

The solution of this equation is (assuming $n_{eo} \simeq \mathrm{const}(r)$)

$$E_o = AJ_o(\chi r) \tag{12}$$

where

$$\chi^2 = i\frac{4\pi e^2 n_{eo}}{mc^2}\frac{\omega}{\nu_{coll}} = \frac{2i}{\delta^2} \tag{13}$$

and δ is the collisional skin-depth

$$\delta = \frac{c}{\sqrt{2\pi\sigma\omega}} = \left(\frac{cm\nu_{coll}}{2\pi e^2 n_{eo}\omega}\right)^{1/2} \tag{14}$$

The constant A in Eq. (12) can be found from the condition that the magnetic field on the surface of the plasma equals to $2I_o/cR$, where $I_o = V_o/Z$ is the current given by the external current source. From the Faraday law

$$\frac{i\omega}{c}B_\phi = (\nabla\times \vec{E})_\phi = -\frac{\partial E_o}{\partial r} \tag{15}$$

so that

$$B_\phi = -\frac{ic}{\omega}\chi AJ_1(\chi r) \tag{16}$$

and therfore

$$A = i\frac{2I_o\omega}{c^2R\chi j_1(\chi R)} \tag{17}$$

In the limit of low frequencies, when

$$\frac{R}{\delta} << 1 \longrightarrow \omega << \frac{c^2 m\nu_{coll}}{2\pi e^2 n_{eo}R^2} \tag{18}$$

we can use the small argument expansion of the Bessel function to obtain the expression for E_o:

$$E_o(r) \simeq \frac{2I_o\omega}{c^2}\left(\frac{\delta}{R}\right)^2\left[1 - \frac{i}{2}\left(\frac{r}{\delta}\right)^2\right] \tag{19}$$

Condition (18) is important, since, as follows from (19), it allows one to have a radially uniform discharge in the steady state. For example, in the case of a 100 Torr laser discharge ($\nu_{coll} \sim 3\cdot 10^{11}$ sec^{-1}), $R = 3$ cm and $n_{eo} = 10^{12}$ cm^{-3}, (18) yields $\omega << 2{\cdot}10^{10}$ ($f << 3$ GHz) which is within the quasistationary limit ($f < 100$ MHz) described earlier.

When (18) is satisfied, Eq. (19) yields

$$E_o(r) \simeq \frac{2I_o\omega}{c^2}\left(\frac{\delta}{R}\right)^2 \sim \frac{1}{n_{eo}} = \mathrm{const}(r)$$

It is precisely this inverse dependence of E_o on n_e which assures the **global** stability of the discharge in the current source scheme. Any increase (or decrease) of the total density leads to the decrease (or increase) of E_o and therfore to the decrease (or increase) of the electron and metastable production rates. This suppresses the initial perturbation. The details of such a global stabilization by the current source are similar to those already considered previously for the DC discharge. In contrast to the DC discharge, however, the RF current source also allows to control the local, streamer type instabilities via the skin-effect. This analysis will be presented next.

1.1 Stability Analysis

Assume that the large skin-depth condition (18) is satisfied for a given steady state so that n_{eo}, n_{mo}, σ_o, E_o are "almost" (to order $(R/\delta)^2$) uniform throughout the plasma volume. One can introduce a small local streamer type density perturbation of a characteristic transverse size Λ. The diffusion term will be added to the rate equations since non-uniformities are of interest. Hence one can write

$$\frac{\partial n_e}{\partial t} = \gamma n_m n_e - \alpha n_e^2 - \beta n_e + D_a \nabla^2 n_e \qquad (20)$$

$$\frac{\partial n_m}{\partial t} = \gamma n_e - \frac{n_m}{\tau} + D_m \nabla^2 n_m \qquad (21)$$

where D_a and D_m are the ambipolar and metastable atomic diffusion coefficients respectively. D_m is of the order $1/3 v_{th}\bar{\lambda} \sim 10^2$ cm^2/sec/Torr while D_a may probably be 10 times larger ($D_a \sim 10^3$ cm^2/sec Torr).

The perturabation in n_e leads to a local conductivity perturbation which in turn results in the decrease or increase in the (rms) electric field value, depending on the sign of the density perturbation. The effect is similar to the skin-effect and can be used to suppress a possible absolute instability in the discharge (as that associated with the attachment). This stabilization phenomenon will be considered in more details. In what follows, the diffusion of metastables will be neglected for simplicity. It will also be assumed that τ^{-1} is the **fastest** rate in Eqs. (20 and (21), i.e.,

$$\frac{1}{\tau} >> \nu n_{mo}, \quad \alpha n_{eo}, \quad \beta, \quad D_a/\Lambda^2 \tag{22}$$

Typically the rate τ^{-1} is in the few MHz region, while the other rates belong to the sub-MHz scale. For times $t > 1\mu$s, condition (22) Eq. (21) to be replaced by

$$n_m(t) \simeq \tau\gamma n_e(t) \tag{23}$$

(this is the fast scale analysis of the previous study) Then Eq. (20) becomes

$$\frac{\partial n_e}{\partial t} = (\nu\gamma\tau - \alpha)n_e^2 - \beta n_e + D_a\nabla^2 n_e \tag{24}$$

If $n_e =$ const, Eq. (24) again yields the steady state Eq. (10).

At this point the perturbation analysis, will be employed i.e. solution of Eqs. (7) and (24) in the form

$$\left.\begin{aligned} n_e &= n_{eo} + n_{e1} \\ \vec{E} &= (E_o + E_1)e^{i\omega t}\hat{e}_z \end{aligned}\right\} \tag{25}$$

where all the perturbations (n_{e1}, E_1) vary **slowly** in time

$$\left(\frac{1}{\omega}\left|\frac{d\ln\xi}{dt}\right| << 1, \xi = n_{e1}, E_1\right) \quad .$$

The perturbed equations become (after the linearization):

$$\nabla^2 E_1 = -i\frac{4\pi e^2\omega}{c^2 m\nu_{coll}}(n_e 1 E_o + n_{eo}E_1) + \left[\frac{4\pi e^2}{c^2 m\nu_{coll}}\left(\frac{\partial n_{e1}}{\partial t}E_o + n_{eo}\frac{\partial E_1}{\partial t}\right)\right] \tag{26}$$

$$\frac{\partial n_{e1}}{\partial t} = \left[2(\nu\gamma\tau - \alpha)n_{eo} - \beta\right] n_{e1} + D_a\nabla^2 n_{e1} + \left[(\nu\gamma\tau - \alpha)' n_{eo}^2 - \beta' n_{eo}\right] E_1 \tag{27}$$

Here $(...)' = \partial/\partial E_o\ (...)$. Finally, neglecting the selected term in Eq. (26) and using the steady state relation (10) in rewriting Eq. (27) one obtains

$$\nabla^2 E_1 = -i\frac{2}{\delta^2}\left(\frac{n_{e1}}{n_{eo}}E_o + E_1\right) \tag{28}$$

$$\frac{\partial n_{e1}}{\partial t} = \beta n_{e1} + D_a \nabla^2 n_{e1} + \beta n_{eo}\left[\frac{(\nu\gamma\tau - \alpha)'}{(\nu\gamma\tau - \alpha)} - \frac{\beta'}{\beta}\right] E_1 \tag{29}$$

The skin-depth δ has again been introduced in Eq. (28).

At this stage a solution of Eqs. (28)-(29) in the form $n_{e1}, E_1 \sim e^{i\vec{k}\cdot\vec{r}}$ are searched for, where $|\vec{k}|^2$ is of $O(2\pi/\Lambda)$ Then one has

$$\left[-k^2 + \frac{2i}{\delta^2}\right] E_1 = -\frac{2i}{\delta^2}\frac{E_o}{n_{eo}} n_{e1} \tag{30}$$

$$\frac{dn_{e1}}{dt} = (\beta - D_a k^2) n_{e1} + \beta n_{eo} B' E_1 \tag{31}$$

where

$$B' = \frac{(\nu\gamma\tau - \alpha)'}{(\nu\gamma\tau - \alpha)} - \frac{\beta'}{\beta} \tag{32}$$

By substituting E_1 from (30) into (31) one obtains

$$\frac{dn_{e1}}{dt} = [(\beta - D_a k^2 - \frac{4\beta n_{eo} E_0 B'}{k^4\delta^4 + 4}) + i\frac{2\beta n_{eo} E_0 B' k^2 \delta^2}{k^4\delta^4 + 4}] n_{e1} \tag{33}$$

Since $k > 2\pi/R$ *and* $S >> R$ one finds $(k\alpha)^4 >> 4$ and therefore (33) reduces to

$$\frac{d(\ln n_{e1})}{dt} = \beta - D_a k^2 - \frac{4\beta n_{eo} E_0 B'}{k^4\delta^4} + L \quad \frac{2\beta n_{eo} E_o B'}{k^2\delta^2} \tag{34}$$

The last equation shows that a stable operation corresponds to the case

$$\Phi \equiv \beta - D_a k^2 - \frac{4\beta n_{eo} E_o B'}{K^4\delta^4} < 0 \tag{35}$$

One observes that there exist two stabilizing phenomena in the system. The first is due to the diffusion and is effective in stabilizing **short scale** density perturbations The diffusion simply smoothes the short scale non-uniformities. The second phenomenum is basically the skin effect. It is more important in suppressing larger scale perturbations, espesially those of size comparable to the radious of the plasma column.

The interplay between the two stabilizing processes can be demonstrated by drawing the curve showing the dependence of $\Phi = \Phi(k^2)$ (see Eq. (35)). A typical situation is shown in Fig. 2.

$\Phi(k^2)$ reaches its maximum value

$$\Phi_m = \beta - 2.05\left(\frac{4\beta n_{eo} E_o B' D_a^2}{\delta^4}\right)^{1/3} \tag{36}$$

at

$$k_m^2 = \left(\frac{8\beta n_{eo} E_o B'}{D_a \delta^4}\right)^{1/3} \tag{37}$$

Therefore, in case $k_m > 2\pi/R$, the stability condition for the entire k - space becomes

$$\beta < 2.9\left(\frac{4 n_{eo} E_o B' D_a^2}{\delta^4}\right)^{1/2} \equiv G(\sim w n_{eo}^{3/2}) \tag{38}$$

Note that the stable operation is easier at higher frequencies and plasma densities. Note also that the final value of Da is important, since (38) can not be satisified for $D_a \longrightarrow 0$.

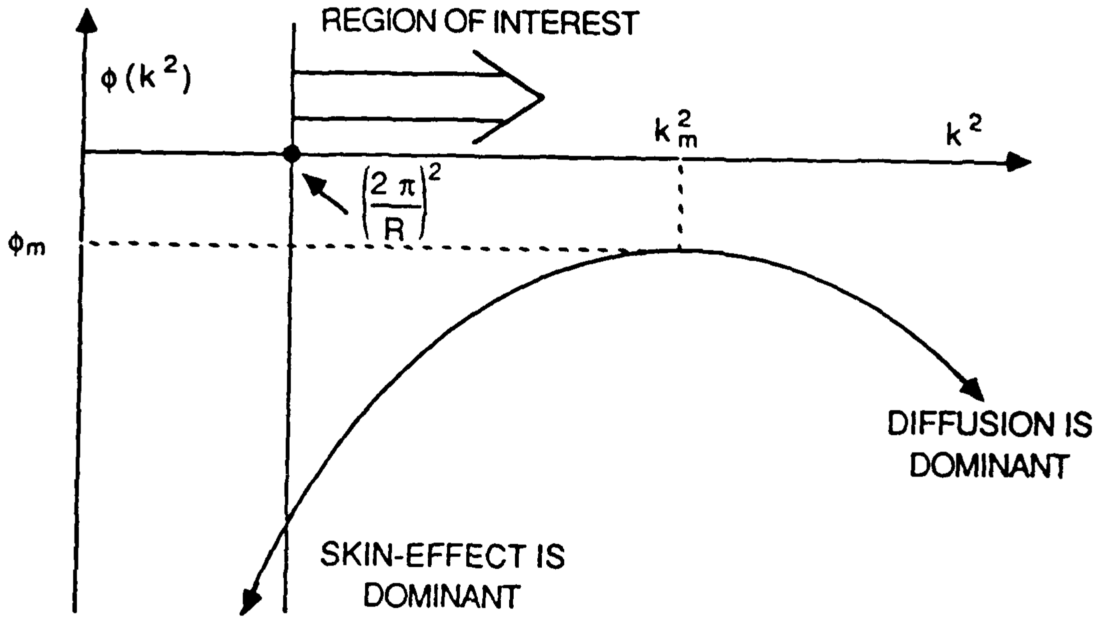
REGION OF INTEREST
$\phi(k^2)$
k_m^2
k^2
$\left(\frac{2\pi}{R}\right)^2$
ϕ_m
DIFFUSION IS DOMINANT
SKIN-EFFECT IS DOMINANT

Figure 2