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THEORY AND APPLICATION OF RANDOM FIELDS

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The main results discussed in this report include:

1) The modelling of rough surfaces via Gaussian and non-Gaussian random fields. Development of new classes of non-Gaussian random processes;
2) The distributional properties of the supremum of Gaussian random processes defined on general state spaces. Applications of these results to the theory and application of empirical processes.
3) Investigation and development of the interface between Gaussian and Markovian processes. Results on local time and intersection local time of measure and distribution valued processes; and
4. Preparation of a monograph treating the general theory of continuity and boundedness for Gaussian processes via entropy and majorising measures.

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CONTENTS

1. INTRODUCTION

The following report covers work carried out with support of the AFOSR, under contract 87-0298. This work represents a natural continuation and extension of similar work supported under contracts 83-0068, 84-0104, and 85-0384.

The research program is centered on the study of various properties of random fields (stochastic processes whose "time" parameter is multi-dimensional) and includes the development of the requisite theoretical foundation to enable the application of these properties to specific modelling problems.

The original proposal was divided into four sections, each corresponding to a separate research project. The report is divided into five, one for each of those in the proposal, and an extra one covering additional results not originally anticipated when the proposal was written up.

To make the report self-contained, the following section contains some background material on random fields, and the discussion of each set of results is preceeded by a more detailed discussion of the area and a brief recapitulation of the projects in that area that appeared in the original proposal. A brief discussion of the results we have obtained then follows. Since all the results reported on here have also appeared (or are about to appear) as detailed research papers, we shall not go into too much detail.

A reader who is familiar with the original proposal should probably skip all this background material. A reader whose expertise lies in the areas under discussion should read the research papers. (A list of these appear in Section 5 of the report.) Thus this report is aimed at the reader who has some general knowledge about the subjects under consideration, but whose area of personal expertise may be elsewhere.

2. BACKGROUND MATERIAL

Random fields are stochastic processes, $X(t)$, whose parameter, t, varies over some general space rather than over the real line, in which case t is usually interpreted as time. The simplest of these occur when the parameter space is some multi-dimensional Euclidean space, and it is these fields that will be at the centre of our study. Of these, the most basic arise when the parameter space is the two-dimensional plane, so that we are dealing with some kind of random surface. When the parameter space is three-dimensional then we have a field (such as ore concentration in a geological site) that varies over space, while when the dimension increases to four we are generally dealing with space-time problems.

More complicated examples of random fields arise as the parameter space becomes more esoteric. Typical examples are parameter spaces of classes of sets, such as arise in the statistical theory of multivariate Kolmogorov-Smirnov tests and set indexed empirical processes. Another common example is provided by fields indexed by families of functions. Although these arise, once again, in the theory of empirical processes, they are much more famous for their appearance in Quantum Field Theory in Mathematical Physics. There they appear, among other guises, as continuum limits of such well known discrete parameter random fields as the Ising model of Statistical Mechanics.

There is a common thread running through the study of all random fields, whether they be the simple random surfaces of Materials Science engineers described in the following section, or the elusive and esoteric fields of the mathematical physicist. The common thread is that once the parameter space ceases to be one-dimensional, it also ceases to be totally ordered, and so we can no longer think of a stochastic process as the famous statistician, Sir Ronald Fisher, once did, as "one damn thing after another". This common thread is rarely exploited as well as it could be, for rarely are workers involved in one area of application of random fields aware of the techniques of workers in other areas. Rare, indeed, is the geologist who realises that his locally averaged samples of ore concentration (or, at least, his Statistical model of them) are studied under the name of "generalised processes" by theoretical probabilists. Most of the latter are themselves unaware (as, *a fortiori*, are the geologists) that these same objects have been studied in great detail in Mathematical Physics under the name of Euclidean quantum fields. In the middle of all this sit a variety of engineers working with very often *ad hoc* techniques and without the benefits of the knowledge of the techniques used by others.

Perhaps the simplest most illuminating example of this "common thread" theme lies in the study of the Markov property of planar random fields. For a process defined on the real line, the Markov property is most succinctly described as the independence of the past and future given the present. For a process defined on an unordered space, concepts as simple as "past", "future" and "present" are at best ill-defined, and need to be replaced by something both workable and useful. The replacement is via "outside", "inside" and "boundary", in the sense that if D is a set in the plane with smooth boundary ∂D and complement D^c, then we say that a process X_t is Markov with respect to D if the values of X_t, $t \in D$ are conditionally independent of those of X_t, $t \in D^c$, given the values of X_t, $t \in \partial D$. If X_t is Markov with respect to all smooth D, we call it simply Markov.

All of this seems simple enough, and, up until now, we have not had to get involved with more esoteric processes defined, say, on spaces of functions or spaces of measures on the plane. Now, however, comes the unpleasant surprise. If we ask that our random field be Gaussian and stationary, then there are *none* (that are non-trivial) that are defined continuously on the plane. If, however, we move to generalised processes, then we again find a multitude of candidates.

The importance of this example is the following: There are many applied situations in which one wishes to model Markovian processes on lattices. Our experience from one-dimensional processes tells us that it is often convenient to consider some sort of continuous version of a discrete time process. But the above tells us that, for random fields, we can only do this if we are prepared to consider generalised processes. These are not processes that the average user of stochastic models is familiar with, and hence the need for emphasising and developing the unified approach mentioned above.

Thus the central aim of my research over the past few years has been the continuing development of a broad and general approach to random fields that covers both theory and applications from a number of seemingly different areas. Ultimately, my aim is to build machinery powerful enough to tackle some quite simple sounding problems related to the simplest of all random fields, those defined on two or three dimensional Euclidean space. Thus, to conclude this background section, let us consider these fields in a little more detail.

The basic theory of random fields defined on Euclidean spaces is now reasonably well understood, with four separate monographs on various aspects of the subject having appeared in the past five years. (Adler, (1981a), Rozanov (1982), Vanmarke (1983) and Yadrenko (1983).) Roughly speaking, the theory breaks into two distinct parts.

In the first situation, one assumes that the sample functions (realisations) of the random field satisfy certain basic regularity conditions, such as continuity, differentiability, etc. Until very recently, fields of this kind were always used as models of phenomena such as the structure of rough metallic surfaces, or of a sea surface. It is possible, and interesting, to study both the fine and global structure of such fields.

In particular, interest in this area often centres on how such fields behave in the neighbourhood of local maxima, or on how high global maxima tend to be. These are problems of both theoretical interest and practical importance, as described in the following section.

In the second situation, these basic regularity conditions are no longer assumed, and we enter the class of the so-called fractal surfaces of Mandelbrot (1982) and his coworkers. Here, one usually studies such problems as the Hausdorff dimensions of the fields and sets they generate. While this is a fascinating and hyper-active area, it is not an area that I worked in over the last ten years or so.

In both of the above described situations, until very recently, interest has focussed on one very particular class of random fields - those whose distribution was Gaussian. There were two reasons for this. One was the ever-present appeal to the central limit theorem, made by theoretician and the most applied modeller alike. The second reason (which generally generated the first) was a basic inability to handle anything other than Gaussian fields. These reasons (particularly the second) were considered to be sufficiently strong to justify Gaussian models, even in situations in which experimental data clearly contra-indicated an assumption of Gaussianity. (See, for example, Adler (1981b).) Only in the last few years has attention been directed to non-Gaussian models, and this is now an area of considerable interest and activity, that we shall have more to say about below.

In the five parts of the following section a wide range of problems in the study of random fields are discussed - from very applied, problem oriented studies to quite theoretical investigations. The common thread that runs between them is one that has already shown its strength in the past - that a broad investigation often yields unexpected and surprisingly specific results whose applications lie in areas far from those in which they arose.

3. PROJECTS AND REPORTS

3(a) Rough surfaces.

As was mentioned in the previous section, one of the assumptions that one continually finds in the theory and applications of random fields is that the field under study is Gaussian. Furthermore, while many "good" reasons are given for this assumption, such as a ubiquitious appeal to the central limit theorem, the real reason that generally underlies this assumption is that it is only Gaussian random fields that are easily amenable to analytic study.

While it may not be too serious a problem that theoreticians concentrate on Gaussian fields, it is a major problem when one turns to applications, since Nature is apparently not as familiar with the ubiquity of the central limit theorem as are mathematicians. Along with others, I have made this point on a number of occassions in the past. A particularly nice example is given in Adler and Firman (1981), where we reanalysed some tribological data coming from the study of microscopically rough metallic surfaces. It was shown there that whereas these surfaces were generally assumed to be of a Gaussian nature, this was completely contra-indicated by the data. In Adler (1981b) I showed that by using non-Gaussian models one could actually replicate, from a theoretical viewpoint, laboratory data on the contact area of surfaces under load, something that was impossible from a Gaussian model.

The model used there, and studied in considerable detail since (Aronowich and Adler (1985, 86, 87)), is that of the so called χ^2−random field, defined as

$$Z(t) = \sum_{i=1}^{n} X_i^2(t), \qquad t \in \Re^d,$$

where the X_i are independent, Gaussian random fields. The importance of χ^2 fields lies essentially in two of their properties. The first is that they behave in a distinctly non-Gaussian fashion, and so can often provide useful models in situations in which Gaussian fields are inappropriate. The second lies in the fact that because they are mathematically related to Gaussian fields, their mathematics, while not as elegant or simple of that of their Gaussian relatives, is still tractable.

Over the past few years, and generally together with students, a considerable amount of work has been done on establishing the general properties of these processes. Related, complementary, work has been undertaken by Georg Lindgren in Sweden, again, generally together with students. Overall, we now have excellent knowledge of how these fields behave, and perhaps the best indication of our success is the fact that they are beginning to appear in applied works as useful models.

Two different, but related, research directions were suggested in the proposal being discussed here. The first was the application of χ^2 random fields to the actual study of rough surfaces. We have made little progress in this area, but have, over the past year, begun to feel that the suggestion itself was at worst ill conceived and at best naive. Applications of this kind, if they are to be of lasting value, need to be made by true engineers, who are aware of the problems of current interest and importance, and not by (essentially pure) mathematicians seeking to dress up their work in self designed pseudo-applications. Thus, we have not attacked this project with any seriousness. On the other hand - and this, I believe, speaks far more strongly for the applied value of χ^2 fields and the research we have been involved in over the past few years - a significant number of papers have appeared over the past year or two applying the theoretical analyses we have developed to areas as far apart as surface science, statistical physics, and physical astronomy. The appearence of these papers has been a particularly rewarding experience.

The second direction discussed in the proposal was the development of further non-Gaussian models for stochastic processes and random fields. Work is proceeding apace in this direction, and current study centers on a class of processes described as follows:

Consider a Gaussian process $X(t)$ defined on a finite set $T = \{t_1, \ldots, t_M\}$. Then all the properties of X on T depend only on the joint probability density $p_T^X(x_1, \ldots, x_M)$ of $\langle X(t_1), \ldots, X(t_M)\rangle$. Define the function

$$F(X) = F(X(t_1), \ldots, X(t_M)) = \sum_{i=1}^{M} X^4(t_i),$$

and define a new stochastic process Y on T to be the process whose joint probability density is given by

$$p_T^Y(x_1, \ldots, x_M) = \frac{e^{-\beta F(x)}}{C_{T,M}^{\beta}} p_T^X(x_1, \ldots, x_M),$$

where $C_{T,M}^{\beta}$ is an appropriate normalising constant, and $\beta > 0$ is a parameter which measures "departure from normality". A number of properties of the new process Y are transparent. For a start, Y is definitely *not* Gaussian. If X is stationary on T, so is Y. The tails of the distribution of Y are not as heavy as those of X, i.e. Y tends to take smaller values than X. Many calculations on Y are going to be more difficult than those on X, but, when all is done, all the calculations are still "Gaussian" in some sense, and therefore will be possible.

The family of processes defined in this fashion is both analytically tractable, and physically interesting. Indeed, what we have just described is a peasant's version of the "$P(\phi^4)$ field" of Euclidean quantum field theory, an object that has been studied in substantial detail by mathematical physicists. We are currently working to bring this study into the realm of the more down to earth user of random field models, both in terms of notation, terminology, and results.

So far, none of the work done on this project is at the stage where it can be written up. Nevertheless, another year or so should see the development of publishable results.

3(b) Gaussian maxima and empirical processes.

Our main research activities here have been in the specific subject area of empirical measures rather than Gaussian maxima *per se*.

Recall that if $X_1, \ldots, X_n$ is a sample of n independent observations of a d-dimensional random variable from a common parent measure μ, then their empirical measure is the process

$$W_n(A) := n^{-\frac{1}{2}} \left[\sum_{i=1}^{n} 1_A(X_i) - \mu(A) \right]$$

defined on a class $\mathcal{A}$ of subsets of $\Re^d$.

One theme underlying the original research proposal was the study of various properties of W_n, and the corresponding limit process W_∞, when $n \to \infty$. Of particular interest was the case when $\mathcal{A}$ was a family of translates in $\Re^d$ of some basic set, say B.

We have made considerable progress in this study. In particular, together with Larry Brown (Cornell University) we have shown how to use theory that was published in previous joint work with Brown in 1986 to carry out Kolmogorov-Smirnov tests in two dimensions. This methodology comes complete with a set of statistical tables enabling practical implementation. The results appear in publication (1) of §5.

We have also started tentative work on a project related to the Poisson structure of level crossings and local maxima, (or their equivalent) for processes defined on very general parameter spaces. Whereas there is a rich theory for Gaussian processes defined on Euclidean spaces (see, for example, the exhaustive treatment of Leadbetter, Lindgren and Rootzén (1983)) we believe that this theory should have a natural extension to Gaussian processes defined on more general spaces. The analogy that should be drawn here is the extension from continuity questions of processes on Euclidean spaces to those for processes on general spaces, and the development of the concepts of metric entropy (e.g. Dudley (1973)) and, more recently, majorising measures (e.g. Talagrand (1987)). The more general theories not only have provided nice mathematics, but have simplified the treatment of the older processes, by concentrating on the important, intrinsic questions of Gaussian continuity without peripheral considerations related to the specific geometry of the parameter space. (The latter point is treated in detail in the set of lecture notes (7) listed in §5.)

The next twelve months should see the development of publishable results in this direction.

3(c) The Markov process - Gaussian field interface.

In order to recall what this project was about, we start by defining two functions. If $X(u)$ is a zero mean Gaussian process, taking values in $\Re^1$ but with its parameter u in $\Re^k$, $k \geq 1$, then its covariance function $R(u,v)$ is defined by

$$R(u,v) = EX(u).X(v).$$

If $W(t)$ is a Markov process defined on $\Re^1$, but taking values in $\Re^k$, and with stationary, symmetric, transition density $p_t(u,v)$, then its *Green's function*, $g(u,v)$ is defined by

$$g(u,v) = \int_0^\infty e^{-t} p_t(u,v)\, dt.$$

It is a simple fact that every Green's function can serve as the covariance function of some (perhaps generalised) Gaussian field, and that the covariance functions of many Gaussian fields are also the Green's functions of symmetric Markov processes. This obvious, indeed, almost trite, fact has been known at least since Hammersley (1967), but had not been properly exploited until Dynkin, in a series of papers (1980 - 1984), used it to study Gaussian random fields from the viewpoint of Markov processes. Although Dynkin's approach was purely formal - i.e. it relied only on the fact that Green's functions and covariance functions were essentially equivalent objects - a simple physical bridge between the Markov and Gaussian situations also exists, and this was developed and discussed in detail in Adler and Epstein (1987).

The importance of such a link is almost self obvious. In both the theories of Markov processes and Gaussian random fields there exist tremendously powerful technologies, but in both there also exist substantial open problems. One hopes, therefore, that by translating a problem from one of these areas into a corresponding problem in the other solutions may become available. Thus it is natural that there has been substantial interest in this area over the last 4-5 years. This interest has also extended to the study of measure valued stochastic processes, whose values at a fixed point of time provide fascinating examples of new classes of *non*-Gaussian random fields.

Measure valued processes arise in much the same way as empirical processes, but involve measures that change with time. To give an example of this, let $W_t^1, W_t^2, \ldots$, be a sequence of independent Brownian motions starting at the points of a homogeneous Poisson process in $\Re^d$ of intensity λ. Let $\sigma^1, \sigma^2, \ldots$ be an independent sequence of Rademacher random variables, and set

$$\mu_t^\lambda(A) := \lambda^{-\frac{1}{2}} \sum_i \sigma^i 1_A(W_t^i).$$

The $\lambda \to \infty$ limit of μ_t^λ is called the *Brownian density process*, and can be thought of as a signed, measure valued process. (Although, formally, of course, it must be treated as a distribution.)

We have a number of results of interest on these processes, including detailed information on the self-intersection local time of the Brownian density process, and fluctuation theory results that discuss what happens when the W^i above are not independent. These results appear in papers (2)-(4) of §5, with some more in preparation in (8).

I have also been involved a joint project with Mike Marcus (City University of New York) and Joel Zinn (Texas A& M), on limit theorems for the local times of independent Markov processes, and the so called "isomorphism theorem" of Dynkin linking Gaussian and Markovian processes. This work resulted in item (5) of §5.

3(d) Computer generation of random fields.

The principal aim of this project was the production of a "picture book" of examples of two dimensional random fields, incorporating both Gaussian and non-Gaussian fields, stationary and non-stationary, isotropic and non-isotropic, etc. The idea was that given all the time that is being spent developing various theoretical models, it would be useful to have a catalogue of pictorial examples that would enable both theoreticians and practitioners to develop a heuristic "feel" for what different fields looked like.

Virtually no progress was made on this project, for a pair of independent reasons. Perhaps the more important of the two was a personel problem. At the beginning of the year a 12 month appointment of a research assistant was made. The responsibilities of this assistant centered around the above project. Let it suffice to say that this was the least successfull appointment that I have been involved with in over 15 years in academia, and no useful results were generated by his employment.

The second reason, however, is somewhat more comforting, and relates to the point made above about the applications of (my kind of) random fields made in a variety of different disciplines over the past two years. Along with these applications has been the development of a variety of graphical representations of random fields of various kinds, both planar and spatial. Now that the users of random fields have taken this task upon themselves, it somehow seems best to leave it in their hands, following the principle that they are motivated by models from Nature, and we only by our own imagination. Nature is generally the more inventive of this pair.

3(e) New projects.

There were essentially two new projects that were not mentioned in the original proposal.

The first grew out of an extended visit to the Center for Stochastic Processes in Chapel Hill in 1985, and concerns the Markov structure of stable random processes. While this topic is somewhat peripheral to the main thrust of our work, it is connected insofar as it is a study of how close certain non-Gaussian processes can be to Gaussian ones. This project resulted in paper (6) of §5. The results presented there catalogue the Markovian behaviour of a wide variety of stable processes. For example, while it is known that there is only one stationary, Gaussian, Markovian process on $\Re$ (the Ornstein-Uhlenbeck process) this is not the case for stable processes.

The second grew out of a visit to the Department of Mathematical Statistics of the University of Lund in February 1988. At the invitation and prompting of Georg Lindgren and (especially) Holger Rootzén, I gave a series of twelve lectures on the "modern" theory of continuity and boundedness for Gaussian processes. This theory is based on the concepts of entropy and majorising measures, as has already been mentioned above.

The notes of these lectures were written up, and I am now building them into a short monograph. (To date, 75 pages have been typed.) I believe that these notes may be the first time that the modern theory has been presented in a format that is not directed only at the expert, and feel that it may be one of the most useful projects carried out during the last year. (Twelve more months should see the completion of this project.) The notes appear as item (7) in §5.

4. REFERENCES

1. Adler, R. J. (1981a) *The Geometry of Random Fields*, Wiley, London.
2. Adler, R. J. (1981b) Random field models in surface science, (Invited paper) *Bull. Int. Statist. Inst.* **49** 660-681.
3. Adler, R. J. and Brown, L. D. (1986) Tail behaviour for the suprema of empirical processes, *Ann. Probability* **14** 1-30.
4. Adler, R. J. and Epstein, R. (1987) A central limit theorem for Markov paths and some properties of Gaussian random fields, *Stoch. Proc. Appls.* **25** 157-202.
5. Adler, R. J. and Firman, D. (1981) A non-Gaussian model for random surfaces, *Phil. Trans. Roy. Soc.* **303** 433-462.
6. Aronowich, M. and Adler, R. J. (1985) The behaviour of chi-squared processes at critical points, *Adv. Appl. Prob.* **17** 280-297.
7. Aronowich, M. and Adler, R. J. (1986) Extrema and level crossings of chi-squared processes, *Adv. Appl. Prob.* **18** 901-920.
8. Aronowich, M. and Adler, R. J. (1987) Sample path behaviour of chi-squared surfaces at extrema, *Adv. Appl. Prob.* **19** to appear.
9. Cramér, H and Leadbetter, M. R. (1967) *Stationary and Related Stochastic Processes*, Wiley, NY.
10. Dudley, R. M. (1973) Sample functions of the Gaussian process, *Ann. Probability* **1** 66-103.
11. Dynkin, E. B. (1980) Markov processes and random fields, *Bull. Amer. Math. Soc., (New Series)* **3** 975-999.
12. Dynkin, E. B. (1983) Markov processes as a tool in field theory, *J. Functional Anal.*, **50** 167-187.
13. Dynkin, E. B. (1983) Gaussian and non-Gaussian random fields associated with Markov processes, *J. Functional Anal.*, **55** 344-376.
14. Dynkin, E. B. (1984) Polynomials of the occupation field and related raandom fields, *J. Functional Anal.*, **58** 20-52.
15. Hammersley, J. M. (1967) Harnesses, *Proc. Sixth Berkeley Symp. Math. Statist. Prob.* **III** 89-118.
16. Leadbetter, M. R., Lindgren, G. and Rootzén, H. (1983) *Extremes and Related Properties of Random Sequences and Processes*, Springer, New York.
17. Rozanov, Yu. A. (1982) *Markov Random Fields*, Springer, New York.
18. Talagrand, M. (1987) Regularity of Gaussian processes, *Acta Math.* **159** 99-149.
19. Vanmarcke, E. (1983) *Random Fields - Analysis and Synthesis*, MIT Press, Cambridge.
20. Yadrenko, M. I. (1983) *Spectral Theory for Random Fields*, Springer, New York.

5. PUBLICATIONS UNDER THE GRANT

1. R. J. Adler, L. D. Brown and K-L. Lu, Tests and confidence bounds for bivariate distribution functions. Submitted.
2. R. J. Adler, The net charge process for interacting, signed diffusions. *Annals of Probability*, to appear.
3. R. J. Adler and R. Epstein, An integral representation for the intersection local time of a system of planar Brownian motions. Submitted.
4. R. J. Adler and M. Lewin, Intersection local time for the Brownian density process. Submitted.
5. R. J. Adler, M. B. Marcus and J. Zinn, The central limit theorem for products of Gaussian processes and for local times of Markov processes. Submitted.
6. R. J. Adler, S. Cambanis, and G. Samorodnitsky, The Markov property for stable processes, Under revision.
7. R. J. Adler, *An Introduction to Continuity and Gaussian Processes for General Gaussian Processes.* (75 page version currently prepared)
8. R. J. Adler and M. Lewin, A Tanaka-like formula for the intersection local time of measure valued diffusions. In preparation.

6. CONFERENCES ATTENDED AND PROFESSIONAL VISITS

During the year I attended two conferences under the support of the grant. These were:

1. The 17th Conference on Stochastic Processes and Their Applications; Rome, Italy, June 26 - 31, 1988.
2. IMS Symposium on Probability and Stochastic Processes, Fort Collins, Colorado, August 16 - 19, 1988.

In both cases, AFOSR funds were used to cover both travel and local expenses.

I also visited the following universities, where I gave seminars (and in the first case a series of 12 lectures) and conferred on issues related to the research proposal with the following academics:

1. Department of Mathematical Statistics, University of Lund, Lund, Sweden. February 7 - 25, 1988. (Georg Lindgren, Holger Rootzén).
2. Center for Stochastic Processes, Chapel Hill, North Carolina. August 21 - September 1, 1988. (M. Ross Leadbetter, Stamatis Cambanis, Gopniath Kallianpur).

In both cases, AFOSR funds were used to help support travel expenses, and local expenses were covered by the host institution.