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**Equations of Cosmic-Ray Transport with
Distributed Acceleration**

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Title: Equations of Cosmic-Ray Transport with Distributed Acceleration

Author: John R. Letaw, Ph.D. and Jacob Roginsky, Ph.D.
Severn Communications Corporation
223 Benfield Park Drive
Millersville, MD 21108

Contract: #N00014-87-C-2251
Naval Research Laboratory
4555 Overlook Avenue, S.W.
Washington, DC 20375-5000

Abstract: The equations of cosmic-ray transport used in our galactic cosmic-ray propagation model are described. The equations include "distributed acceleration" terms associated with weak reacceleration of cosmic-ray ions in supernova remnants. Reacceleration may occur at any time during the diffusion of cosmic rays through the interstellar medium.

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1. Introduction

The purpose of this report is to outline our one-dimensional transport model for cosmic rays propagating in the interstellar medium (ISM). The equations described here have been used to prepare a numerical model of galactic cosmic-ray transport which is used for investigations of cosmic-ray composition, the sensitivity of cosmic-ray abundances to cross section data uncertainties, and weak reacceleration in the interstellar medium.

The basic assumption intrinsic to the equations described here is that cosmic rays, most of which seem to originate in our galaxy, while traveling through the ISM are subject to a number of processes which tend to destroy a sense of direction of propagation with regard to their sources. As a result, instead of dealing with three-dimensional continuity equations which imply knowledge of spatial structure and source distributions, we are invoking a one-dimensional "compressed" view of cosmic-ray diffusion. This view is exemplified by the treatment of "reflections" at the "boundary" of the galaxy which are represented simply as a probability of escape from the galaxy, or escape mean free path. Such an assumption seems to adequately represent much of the available data on cosmic-ray composition, but may be of limited usefulness in representing other features of cosmic radiation such as anisotropy.

Within the one-dimensional transport model there is a further assumption that many of the processes necessary to interpret cosmic-ray composition are insensitive to the detailed behavior of individual particles. Included in this assumption are the use of interaction mean free paths, averaging over electron stripping and attachment processes for the estimate of electron capture decay rates, and representing reacceleration processes with a Green's function.

The processes necessary to interpret cosmic-ray data are approximated using a few parameters which, it is hoped, will reveal some information about the structure of the ISM as well as allowing "background" to be removed in estimating cosmic-ray source abundances. The parameters of principal interest are the mean amount of interstellar material traversed, the mean density of the confinement region, and the average time between acceleration at the source and loss by escape from galactic confinement. These parameters are well-known and often quoted in association with cosmic-ray data analysis.

Additional parameters of cosmic-ray transport are discussed in this report. These parameters include the mean strength of weak shocks in the ISM, the mean time (or pathlength) between encounters with these shocks, and a measure of the mean inhomogeneity in the ISM. These parameters introduce additional physical content into the cosmic-ray transport equations which may explain certain anomalies in the cosmic-ray composition and spectra. The additional parameters may also be helpful in ridding the cosmic-ray transport equations of *ad hoc* parameters and functional dependencies which have been introduced to provide good fits to the cosmic-ray data.

The discussion of this report is arranged into sections. Section 2 summarizes the notation used in this report. Each variable is identified and the usual units are stated. Section 3 contains several defining and kinematic relations among a number of the variables. Sections 4 - 10 each describe a different constituent part of the transport equation, which is given in its entirety in Section 11. Each part represents the infinitesimal change in the cosmic-ray flux density, $J_i(E)$, due to a different process. Various approximate solutions to the cosmic-ray transport equations are possible when a limited number of constituent parts are combined or when a limited number of cosmic-ray species are considered.¹ Section 12 addresses the extension of the transport equations to inhomogeneous media.

¹For example, G. Gloeckler and J.R. Jokipii, "Physical Basis of the Transport and Composition of Cosmic Rays in the Galaxy," Phys. Rev. Lett. 22, 1448-1453 (1969)

2. Notation

A_i	-	isotopic mass number of a cosmic-ray species
A_k	-	isotopic mass number of a component of the transport medium
c	-	speed of light (cm s^{-1})
E	-	kinetic energy per nucleon (MeV)
$J(E)$	-	particle flux spectrum ($\text{particles cm}^{-2} \text{s}^{-1} \text{MeV}^{-1}$)
$J(p)$	-	particle flux spectrum ² ($\text{particles cm}^{-2} \text{s}^{-1} [\text{MeV/c}]^{-1}$)
M	-	mass (MeV/c^2)
m	-	atomic mass unit $\approx 931.5 \text{ MeV}/c^2$
$N(E)$	-	number density spectrum ($\text{particles cm}^{-3} \text{MeV}^{-1}$)
$N(p)$	-	number density spectrum ($\text{particles cm}^{-3} [\text{MeV/c}]^{-1}$)
N_A	-	Avogadro's number
n	-	number density of transport medium (cm^{-3})
n_k	-	number density of a component of the transport medium (cm^{-3})
P	-	momentum (MeV/c)
p	-	momentum per nucleon (MeV/c)
$Q(E)$	-	injection rate spectrum ($\text{particles cm}^{-3} \text{s}^{-1} \text{MeV}^{-1}$)
$Q(p)$	-	injection rate spectrum ($\text{particles cm}^{-3} \text{s}^{-1} [\text{MeV}/c]^{-1}$)
$q(E)$	-	injection spectrum per unit pathlength ($\text{particles cm}^{-1} \text{g}^{-1} \text{MeV}^{-1}$)
R_{process}	-	inverse mean free time or rate of a process (s^{-1})
r_{process}	-	inverse mean free pathlength of a process ($\text{cm}^2 \text{g}^{-1}$)
T	-	total energy (MeV)
t	-	time [independent variable] (s)
v	-	velocity (cm s^{-1})
x	-	pathlength [independent variable] (g cm^{-2})
Z_i	-	isotopic charge number of a cosmic-ray species
γ	-	shock reacceleration index
ρ	-	mass density of transport medium (g cm^{-3})
ρ_k	-	partial mass density of a component of the transport medium (g cm^{-3})
τ	-	half-life for nuclear decay (s)
Ω	-	time rate of energy loss per nucleon (MeV s^{-1})
ω	-	stopping power per nucleon ($\text{MeV cm}^2 \text{g}^{-1}$)

In general, roman subscripts label cosmic-ray species and greek subscripts label components of the transport medium (ISM).

²The two flux spectra mentioned here are different functions and, according to mathematical convention, should be represented as such, i.e., $J_k(E)$ and $J_p(p)$. Since we will have no occasion to mention such functional forms as $J_p(E)$, we have chosen, for the sake of simplicity, to use the simplified notation. Such considerations are also applicable to the number density spectrum, $N(E)$, and the injection rate spectrum, $Q(E)$.

3. Relations

The following relations exist between the quantities defined in Section 2. This is not intended to be a complete or minimal set of equations. Several equations serve as definitions of quantities and several others are standard results of special relativity; the remainder may be derived from these.

$$T^2 = P^2 c^2 + M^2 c^4 \quad [3.1]$$

$$E = (T - M c^2)/A \quad [3.2]$$

$$p = P/A \quad [3.3]$$

$$m = M/A \quad [3.4]$$

$$\frac{dE}{dp} = \frac{pc^2}{(E + mc^2)} = v \quad [3.5]$$

$$x = \rho v t \quad [3.6]$$

$$\frac{\partial}{\partial t} = \rho v \frac{\partial}{\partial x} \quad [3.7]$$

$$J(E) = v N(E) \quad [3.8]$$

$$N(E) dE = N(p) dp \quad [3.9]$$

$$J(E) = N(p) \quad [3.10]$$

$$\frac{\partial N(E)}{\partial t} = \rho \frac{\partial J(E)}{\partial x} \quad [3.11]$$

$$R_{\text{process}} = \rho v \tau_{\text{process}} \quad [3.12]$$

$$vQ(E) = Q(p) \quad [3.13]$$

$$\rho_K = n_K A_K / N_A \quad [3.14]$$

$$\text{Lorentz time dilation factor} = 1 + \frac{E}{mc^2} \quad [3.15]$$

$$q(E) = Q(E)/\rho v \quad [3.16]$$

4. Source Term

The source abundance contribution to the transport of the i^{th} cosmic-ray species is taken to be of the form:

$$\frac{\partial N_i(p)}{\partial t} = Q_i(p) \quad [4.1]$$

where $Q_i(p)$ is the rate of injection of the i^{th} cosmic-ray species into the interstellar medium as a number density spectrum. Often it is assumed that $Q \propto p^\Gamma$, where $\Gamma \geq 2$.³

Using [3.10] and [3.13] the contribution to the i^{th} component of the cosmic-ray flux spectrum is:

$$\frac{\partial J_i(E)}{\partial t} = vQ_i(E) \quad [4.2]$$

Using [3.7], [4.2] may be expressed with pathlength, x , as the independent variable:

$$\frac{\partial J_i(E)}{\partial x} = \frac{Q_i(E)}{\rho} - vq_i(E) \quad [4.3]$$

where $q_i(E)$ is the number density spectrum of cosmic rays injected during the period that the i^{th} species of cosmic rays of energy, E , passes through a unit pathlength in the interstellar medium ([3.16]).

³For example, J.F. Ormes and R.J. Protheroe, "Implications of HEAO 3 Data for the Acceleration and Propagation of Galactic Cosmic Rays," Ap. J. 272, 756-764 (1983)

5. Escape Term

The continuity equation may be structured to approximate diffusion in the galaxy with loss at the boundaries by inclusion of a galactic escape term. A single rate of escape from the system is defined. The rate may be dependent on the energy, charge, and mass of the transported cosmic ray. This transport model is then termed the "leaky-box" model.⁴

The one-dimensional diffusion equation is:

$$\frac{\partial N(E)}{\partial t} = -R_e(E) N(E) \quad [5.1]$$

where $R_e(E)$ is the time-rate of escape from the galaxy, and time and pathlength are related by [3.6]. For convenience we have dropped the species index from both the number density spectrum and the escape rate; it is understood that both quantities may depend on the charge and mass of the transported species. The species index will often be omitted in future sections when no change in composition occurs for a particular process.

Applying relations [3.7], [3.10], and [3.12] as in Section 4 the change in flux with respect to pathlength is:

$$\frac{\partial J(E)}{\partial x} = -r_e(E) J(E) \quad [5.2]$$

where $r_e(E)$ is the inverse mean free pathlength for galactic escape as a function of energy, charge, and mass of the transported species.

More complex diffusion models may be constructed by multiple application of [5.2]. The "nested leaky-box" is an example of one such model. Additional transport parameters are required in multiple leaky box models.

Pathlength distributions are often used for the analysis of cosmic-ray data.⁵ Arbitrary pathlength distributions are not generally consistent with cosmic-ray

⁴R. Cowsik and L.W. Wilson, Proc. 13th Internat. Cosmic Ray Conf. 1, 500 (1973)

⁵H. Garcia-Munoz, J.A. Simpson, T.G. Guzik, J.P. Wefel, and S.H. Margolis, "Cosmic-Ray Propagation in the Galaxy and in the Heliosphere: The Path-Length Distribution at Low Energy," Ap. J. Suppl. 64, 269-304 (1987)

diffusion equations when energy losses and gains are important.* There is no use made of pathlength distributions in this report. To achieve the effects of such distributions on cosmic-ray composition, one must introduce additional physical models which may be represented as terms in the differential equations described in this report. The physical models are likely to be accompanied by free parameters which could have some astrophysical significance.

*J.A. Lezniak, "The Extension of the Concept of the Cosmic-Ray Path-Length Distribution to Nonrelativistic Energies," *Ap. Space Sci.* 63, 279-293 (1979)

6. Acceleration Term

A term in the transport equation for acceleration may be derived from the assumptions that (i) particles are conserved in the acceleration process, (ii) a monochromatic number density spectrum is spread into a power-law spectrum in momentum, p^γ , by a single encounter with a shock, and (iii) particles are not decelerated by this process.⁷ Under these assumptions the following result is derived:

$$\frac{\partial N(p)}{\partial t} = -R_a(p) N(p) + (\gamma-1)p^\gamma \int_0^p R_a(p) p^{\gamma-1} N(p) dp \quad [6.1]$$

where R_a is the inverse mean free time for acceleration events (shock encounter rate), γ is the shock reacceleration index, and p is a dummy integration variable which may be interpreted as the cosmic-ray momentum prior to acceleration.

In terms of the cosmic-ray flux spectrum [6.1] may be rewritten using [3.10]:

$$\frac{\partial J(E)}{\partial t} = -R_a(E) J(E) + (\gamma-1)p^\gamma \int_0^E R_a(E) p^{\gamma-1} v^{-1} J(E) dE \quad [6.2]$$

where p , E , and v are dummy quantities with the same relations as p , E , and v . Converting to pathlength as independent variable using [3.7] and [3.12]:

$$\frac{\partial J(E)}{\partial x} = -r_a(E) J(E) + (\gamma-1)v^{-1}p^\gamma \int_0^E r_a(E) p^{\gamma-1} J(E) dE \quad [6.3]$$

where r_a is the inverse mean free pathlength for acceleration events. The acceleration rates, r_a and R_a , can be species dependent.

By addition of several terms of the form of the right hand side of [6.3], that equation may be generalized to include several different shock strengths each of which is encountered at a different rate.

⁷A. Wandel, D. Eichler, J.R. Letaw, R. Silberberg, and C.H. Tsao, "Distributed Reacceleration of Cosmic Rays," Ap. J. 316, 676-690 (1987)

7. Ionization Loss Term

As cosmic rays move through the ISM some of their energy is lost due to ionization and excitation of interstellar matter. In general, interstellar material may be composed of several atomic species labeled by κ . The rate of energy loss of a cosmic ray is dependent on its charge, mass, and energy, as well as the composition of the interstellar medium. If energy losses are assumed to occur continuously, the ionization loss process is governed by the following equation:

$$\frac{\partial N_i(E)}{\partial t} = - \sum_{\kappa} \frac{\partial}{\partial E} \left[(\rho_{\kappa}/\rho) \Omega_{\kappa}(E) N_i(E) \right] \quad [7.1]$$

where $\Omega_{\kappa}(E)$ is the rate of energy loss per nucleon per unit time in medium κ for one cosmic-ray species. The factor (ρ_{κ}/ρ) weights the stopping powers in each component of the ISM according to their abundance.

In terms of the cosmic-ray flux with pathlength as an independent variable:

$$\frac{\partial J_i(E)}{\partial x} = - \sum_{\kappa} \frac{\partial}{\partial E} \left[(\rho_{\kappa}/\rho) \omega_{\kappa}(E) J_i(E) \right] \quad [7.2]$$

where $\omega_{\kappa}(E)$ is the stopping power of a nuclide in energy per nucleon per unit pathlength in medium κ . Heavy ion stopping powers have been reviewed by Ahlen.⁴

⁴S.P. Ahlen, "Theoretical and experimental aspects of the energy loss of relativistic heavily ionizing particles," Rev. Mod. Phys. 52, 121-173 (1980)

8. Nuclear Fragmentation Term

The change in number density, $N_i(E)$, of species i due to nuclear fragmentation taking place in a medium with several components labeled by κ may be represented using the equation:

$$\frac{\partial N_i(E)}{\partial t} = - \sum_{\kappa} R_i(E)_{\kappa} N_i(E) + \sum_{j, \kappa} \int_0^{\infty} R_i(E, E')_{\kappa} N_j(E') dE' \quad [8.1]$$

where E is the energy of the incident cosmic ray (species j) and E is the energy of the resulting spallation fragment (species i),

$$R_i(E)_{\kappa} = n_{\kappa} v \sigma(E)_{\kappa} \quad [8.2]$$

is the rate of destruction of species i in medium κ at energy E , and $\sigma(E)_{\kappa}$ is the total cross section for species change (nearly equal to the total inelastic cross section at higher energies), and

$$R_i(E, E')_{\kappa} = n_{\kappa} v \sigma(E, E')_{\kappa} \quad [8.3]$$

is the rate of production of species i at energy E from j at energy E' and velocity v in medium κ , and $\sigma(E, E')_{\kappa}$ is the partial cross section for production of species i from species j . Note that both rates for nuclear fragmentation are explicitly dependent on the velocity of the incident nuclide and the number density of the ISM.

In terms of the cosmic-ray flux spectrum with pathlength as the independent variable using [3.8] and [3.11]:

$$\frac{\partial J_i(E)}{\partial x} = - \sum_{\kappa} r_i(E)_{\kappa} J_i(E) + \sum_{j, \kappa} \int_0^{\infty} r_i(E, E')_{\kappa} J_j(E') dE' \quad [8.4]$$

where the r_i are inverse mean free pathlengths corresponding to the rates, R_i , which are obtained by application of [3.12] and [3.14]:

$$r_i(E)_{\kappa} = N_A (\rho_{\kappa}/\rho) \sigma(E)_{\kappa} / A_{\kappa} \quad [8.5]$$

and

$$r_i(E, E')_{\kappa} = N_A (\rho_{\kappa}/\rho) \sigma(E, E')_{\kappa} / A_{\kappa} \quad [8.6]$$

where N_A is Avogadro's number, ρ_κ is the partial density and A_κ is the atomic mass of ISM component κ . We note that pathlength is a convenient choice of the independent variable in cosmic-ray transport computations because the r_i are not explicitly dependent on velocity or on the absolute density of the ISM.

The total and partial cross sections for heavy-ion spallation reactions on components of the interstellar medium, especially on hydrogen and helium, are an important requirement for the analysis of cosmic-ray composition. New measurements of spallation cross sections are reported in the bi-annual Proceedings of the International Cosmic Ray Conference. Semi-empirical formulations have been developed by Silberberg, Tsao, and Letaw.^{9,10}

Two classes of cosmic-ray heavy ions are distinguished according to their history of fragmentation in the ISM. Cosmic-ray primaries are heavy ions which have not fragmented in the ISM. Cosmic-ray secondaries are heavy ions which have arisen from one or more spallation reactions in the ISM. Any nuclide, in principle, can arise from either of these processes; however, H, He, C, O, Ne, Mg, Si, and Fe are mostly primary, while Li, Be, B, F, Na, Al, and nuclei with $21 \leq Z \leq 25$ are mostly secondary.

The ratio of stable secondaries to primaries, such as B/C, is determined by measurable nuclear fragmentation cross sections and the escape rate from the galaxy, and is independent of the ISM density distribution. Such secondary to primary ratios may be used to establish aspects of the interstellar diffusion process, such as the average amount of matter that cosmic rays have passed through.

⁹R. Silberberg and C.H. Tsao, "Partial Cross-Sections in High-Energy Nuclear Reactions, and Astrophysical Applications. I. Targets with $Z \leq 28$. II. Targets Heavier than Nickel.," Ap. J. Suppl. 25, 315 and 335 (1973)

¹⁰J.R. Letaw, R. Silberberg, and C.H. Tsao, "Proton-Nucleus Total Inelastic Cross Sections: An Empirical Formula for $E > 10$ MeV," Ap. J. Suppl. 51, 271-276 (1983)

9. Nuclear Decay Term: Particle Emission

The following equation accounts for the rate of change of a cosmic-ray number density spectrum due to nuclear decay by particle emission:

$$\frac{\partial N_i(E)}{\partial t} = -R_d(E)_i N_i(E) + \sum_j R_d(E)_j N_j(E) \quad [9.1]$$

where $R_d(E)_i$ is the rate of decay of species i at energy E and $R_d(E)_j$ is the rate of decay of species j into species i at energy E .

$$R_d(E)_i = \ln 2 / (1 + E/mc^2) \tau_i \quad [9.2]$$

and

$$R_d(E)_j = \ln 2 / (1 + E/mc^2) \tau_j \quad [9.3]$$

where τ_i is the halflife of species i and τ_j is the halflife of species j for decay into species i . All rates include only particle emission decay modes; electron capture decay modes are specifically excluded and are considered in Section 10.

In terms of cosmic-ray flux with pathlength as the independent variable using [3.11] and [3.12]:

$$\frac{\partial J_i(E)}{\partial x} = -r_d(E)_i J_i(E) + \sum_j r_d(E)_j J_j(E) \quad [9.4]$$

where the r_d are inverse mean free pathlengths corresponding to the R_d .

$$r_d(E)_i = \ln 2 / \rho v (1 + E/mc^2) \tau_i \quad [9.5]$$

and

$$r_d(E)_j = \ln 2 / \rho v (1 + E/mc^2) \tau_j \quad [9.6]$$

The rate of decay of cosmic-ray species depends on the mean density of the ISM. If the mean pathlength in the ISM has been established, then [3.6] may be applied to estimate the mean confinement time. Nuclides such as ^{10}Be , ^{26}Al , and ^{36}Cl with halflives of about one million years have been analyzed for this purpose."

"For example, M.E. Wiedenbeck, "The Isotopic Composition of Cosmic Ray Chlorine," Proc. 19th Internat. Cosmic Ray Conf. 2, 84-87 (1985)

10. Nuclear Decay Term: Electron Capture

Electron capture decay of cosmic rays is suppressed because high-energy heavy ions passing through relatively small amounts of material are likely to be stripped of their electrons. Lower energy and higher charged ions may have a significant probability of having one or more attached electrons. The attached electrons make electron capture decay possible and electron capture decay may ultimately have an observable effect on composition. The importance of electron-capture decay modes is increased in distributed acceleration models because ions which decay freely at low energies are mixed, after acceleration, with high-energy components.

The number density spectra of cosmic rays with no electrons attached, $N^0(E)$, and with one electron attached, $N^1(E)$, are governed by the following coupled equations if the probability of two attached electrons is negligible.¹²

$$\frac{\partial N^0_i(E)}{\partial t} = - \sum_k \left[R_{+}(E)_{ik} N^0_k(E) - R_{-}(E)_{ik} N^1_k(E) \right] + \sum_j R_{ex}(E)_{ji} N^1_j(E) \quad [10.1]$$

$$\frac{\partial N^1_i(E)}{\partial t} = \sum_k \left[R_{+}(E)_{ik} N^0_k(E) - R_{-}(E)_{ik} N^1_k(E) \right] - R_{ex}(E)_i N^1_i(E) \quad [10.2]$$

where

$$R_{-}(E)_i = \ln 2 / (1 + E/mc^2) (r_{ex})_i \quad [10.3]$$

and

$$R_{ex}(E)_j = \ln 2 / (1 + E/mc^2) (r_{ex})_j \quad [10.4]$$

are the rate of electron-capture decay of species i at energy E , and the rate of electron-capture decay of species j into species i at energy E , respectively, and the electron attachment rate, $R_{+}(E)$, and electron stripping rate, $R_{-}(E)$, in each component of the transport medium are related to the attachment and stripping cross sections by:

$$R_{+}(E)_{ik} = n_k v \sigma_{+}(E)_{ik} \quad [10.5]$$

¹²J.R. Letaw, J.H. Adams, R. Silberberg, and C.H. Tsao, "Electron Capture Decay of Cosmic Rays," Ap. Space Sci. 114, 365-379 (1985)

and

$$R(Z)_{\text{ec}} = n_e v \sigma(E)_{\text{ec}} \quad [10.6]$$

The electron-capture decay half lives, (τ_{ec}), and electron stripping (σ_-) and attachment (σ_+) cross sections are discussed by Letaw, Silberberg, and Tsao.¹³ The electron-capture decay half-life is approximately double the laboratory measurement because there is generally only one electron in the K-shell.

[10.1] and [10.2] may be combined to give an expression for the total change in a cosmic-ray species as a result of electron-capture decay:

$$\frac{\partial [N_i^0(E) + N_i^1(E)]}{\partial t} = - R_{\text{ec}}(E)_i N_i^1(E) + \sum_j R_{\text{ec}}(E)_j N_j^1(E) \quad [10.7]$$

The analogous equation for the cosmic-ray flux spectrum with pathlength as an independent variable is:

$$\frac{\partial [J_i^0(E) + J_i^1(E)]}{\partial x} = - r_{\text{ec}}(E)_i J_i^1(E) + \sum_j r_{\text{ec}}(E)_j J_j^1(E) \quad [10.8]$$

where the inverse mean free pathlengths, r_{ec} , have the same relation to the rates, R_{ec} as in [9.5] and [9.6]. It should be noted that the summation in [10.8] will generally have a contribution from only one nuclide, i.e., the species, j , which can decay into species i by electron capture.

In most cases cosmic-rays achieve an equilibrium charge state after passage through an amount of material which is small compared to other propagation parameters such as the interaction mean free path and the escape mean free path. The fraction of particles with a single electron attached is:

$$\left[\frac{J^1}{J^0} \right]_{\text{eq}} = \frac{[(r_- - r_+ + r_{\text{ec}})^2 + 4 r_- r_+]^{1/2} - (r_- - r_+ + r_{\text{ec}})}{2 r_-} \quad [10.9]$$

¹³J.R. Letaw, R. Silberberg, and C.H. Tsao, "Propagation of Heavy Cosmic Ray Nuclei," Ap. J. Suppl. 56, 369-391 (1984)

For each species a function $g_i(E)$ which relates the flux of particles with one electron attached, J^1 , to the total particle flux, $J^0 + J^1$, at equilibrium may be defined based on [10.9].

$$g_i = \left[\frac{J^1}{J^0 + J^1} \right]_{eq} \quad [10.10]$$

This function must be obtained by proper weighting of the stripping and attachment rates over the components of the ISM.

When the attachment rate is much smaller than the stripping rate plus the electron-capture decay rate, that is, when only a small fraction of cosmic rays have an attached electron,

$$g_i(E) \sim (\sum_k r_{+}(E)_{ik}) / (r_{ec}(E)_i + \sum_k r_{-}(E)_{ik}) \quad [10.11]$$

where the inverse mean free pathlength for attachment, $r_{+}(E)$, and for stripping, $r_{-}(E)$, have the same relations to the cross sections as [8.5] and [8.6]. [10.11] is generally applicable for cosmic-ray transport, the exception being low-energy, ultraheavy cosmic rays in the actinide group.

Using the function $g(E)$ to eliminate J^1 , [10.8] may be rewritten:

$$\frac{\partial J_i(E)}{\partial x} = -r_{ec}(E)_i g_i(E) J_i(E) + \sum_j r_{+}(E)_{ji} g_j(E) J_j(E) \quad [10.12]$$

Yiou and Raisbeck¹⁴ have demonstrated that the electron capture decay of ⁷Be is important below 20 MeV per nucleon. Detection within the heliosphere may be impossible at that energy. Similar conclusions apply to secondary nuclei such as ³⁹Ar, ⁴⁰Ca, ⁴⁶V, ⁵⁴Cr, and ⁵⁵Fe in the absence of distributed acceleration processes. Electron capture decay generally occurs for ultraheavy nuclei.

¹⁴F. Yiou and G.M. Raisbeck, Ap. J. 7, L129 (1970)

11. The Equation of Distributed Acceleration

The transport equation described in Sections 4 - 10 is summarized here. For purposes of simplification we assume that (1) nuclear interaction fragments maintain the velocity of their progenitors, the "straight-ahead" approximation, and (2) the interstellar medium consists of one substance, namely hydrogen.

The first assumption is generally applied for studies of cosmic rays heavier than helium and allows

$$\int_0^{\infty} r_i(E, E_0) J_i(E) dE \rightarrow r_i(E)_0 J_i(E) \quad [11.1]$$

where $r_i(E)_0$ is related to the partial cross sections and is different from $r_i(E)_i$ which is related to the total inelastic cross section.

The second assumption is generally applied because of the lack of available cross section data for other target nuclei. Some investigations of the importance of helium as a target in the interstellar medium have been performed.¹⁵

The complete equation of distributed acceleration is:

$$\begin{aligned} \frac{\partial J_i(E)}{\partial x} = & \frac{Q_i(E)}{\rho} - r_i(E)_i J_i(E) + \frac{\partial}{\partial E} \left[\omega(E)_i J_i(E) \right] \\ & - r_i(E)_i J_i(E) + (\gamma-1)v^4 p^\gamma \int_0^E r_i(E)_i p^{\gamma-1} J_i(E) dE \\ & - r_i(E)_i J_i(E) + \sum_j r_i(E)_j J_j(E) \end{aligned}$$

¹⁵For example, P. Ferrando, P. Goré, and A. Soutoul, "On the Importance of Interstellar Helium for the Propagation of Heavy Cosmic Rays," Proc. 19th Internat. Cosmic Ray Conf. 3, 61 (1985)

$$- r_d(E)_i J_i(E) + \sum_j r_d(E)_j J_j(E)$$

$$- r_{sc}(E)_i g_i(E) J_i(E) + \sum_j r_{sc}(E)_j g_j(E) J_j(E) \quad [11.2]$$

12. Extension to an Inhomogeneous Interstellar Medium

Electron capture decay processes are sensitive to density variations in the interstellar medium.¹⁶ Most of these nuclides have a very short mean lifetime compared to the time during which cosmic rays move from one density region to another, hence no effect on composition is observable. A few, such as ⁴⁴Ti, ⁹³Nb, ⁹⁴Hf, and ¹⁶²Tb are quite sensitive to possible density irregularities. Measured abundances of these isotopes, particularly at low energy, may eventually offer the opportunity to study aspects of the density distribution in the ISM by utilizing cosmic-ray spectra.

The procedure for treating one type of density inhomogeneity is discussed here. Cosmic rays are assumed to encounter density fluctuations in the ISM which are randomly distributed in time and space. These fluctuations are described by a distribution function, $F(n)$, of interstellar gas number densities, n . The function is normalized so that:

$$\int_0^{\infty} F(n) \, dn = 1 \quad [12.1]$$

A rough estimate of this distribution function may be made based on models of the interstellar medium,¹⁷ if the entire ISM is accessible to cosmic rays.

The model may be implemented by writing

$$dx(n) \propto f(n) \, dn \quad [12.2]$$

where $f(n)$ is the fraction of total pathlength over which the interstellar gas density is between n and $n+dn$, and $x(n)$ is the portion of the total pathlength with density less than or equal to n . It follows that

$$f(n) = n F(n) / \langle n \rangle \quad [12.3]$$

where $\langle n \rangle$ is the mean density of the ISM relative to the distribution $F(n)$. $f(n)$ will necessarily be normalized to unity if constructed in this manner.

For processes where $\partial J / \partial x$ is independent of the density of the interstellar medium, such as nuclear fragmentation [8.4], we have:

¹⁶J.R. Letaw, R. Silberberg, and C.H. Tsao, "Electron Capture Decay of Cosmic Rays: A Model of the Inhomogeneous Interstellar Medium," Proc. 19th Internat. Cosmic Ray Conf. 3, 33 (1985)

¹⁷For example, C.F. McKee and J.P. Ostriker, Ap. J. 218, 148 (1977)

$$dJ \propto J dx \propto J f(n) dn \quad [12.4]$$

and consequently

$$\ln J \propto 1 \quad [12.5]$$

because $f(n)$ is normalized to unity. The flux of cosmic rays subject to this type of process is independent of the interstellar gas density distribution.

For processes where $\partial J/\partial x$ is inversely proportional to the density of the interstellar medium, such as nuclear decay by particle emission [9.4], we have:

$$dJ \propto J \langle n \rangle^{-1} dx \propto J n F(n) \langle n \rangle^{-1} dn \quad [12.6]$$

and consequently

$$\ln J \propto \langle n \rangle^{-1} . \quad [12.7]$$

The flux of cosmic rays subject to this type of process is dependent on the mean density of the ISM.

In the case of electron capture decay ([10.10] and [10.11])

$$dJ \propto J \frac{k_1}{1 + k_2 \langle n \rangle} dx \quad [12.8]$$

where k_1 and k_2 are energy- and species-dependent functions which are independent of the ISM density distribution. In the two extreme cases, $k_2 \ll 1$ and $k_2 \gg 1$, the solution to this equation is identical in form to [12.5] and [12.7], respectively. For intermediate cases, the solution involves an integral over the density distribution of the interstellar medium which may provide information about inhomogeneity over the cosmic-ray path.