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This research has advanced both theoretical and computational aspects of evaluating the reliability of a complex system in terms of its struture and the reliability of its components. This type of problem arises in particular in the design and evaluation of telecommunication and distribution systems, which are commonly modelled as networks. The present research emplous an algebraic approach for studying the reliability of such network systems. This approach has not only unified certain theretical aspects of network reliability problems but has always suggested a number of new algorithms for calculating various reliability measures. Based on this approach, both exact and approximate computational schemes have been developed, together with supporting data structures for implementing the necessary computations in efficient manner. Approximation schemes, also based on anunderlying algebraic structure, have also been developed for evaluating more general measures of system performance such as average delay or throughput.

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"Algebraic and Computational Aspects of Network Reliability Problems"

SUMMARY

This research has advanced both theoretical and computational aspects of evaluating the reliability of a complex system in terms of its structure and the reliability of its components. This type of problem arises in particular in the design and evaluation of telecommunication and distribution systems, which are commonly modelled as networks. The present research employs an algebraic approach for studying the reliability of such network systems. This approach has not only unified certain theoretical aspects of network reliability problems but has also suggested a number of new algorithms for calculating various reliability measures. Based on this approach, both exact and approximate computational schemes have been developed, together with supporting data structures for implementing the necessary computations in an efficient manner. Approximation schemes, also based on an underlying algebraic structure, have also been developed for evaluating more general measures of system performance, such as average delay or throughput.

RESEARCH OBJECTIVES

The objective of this research project is to study network reliability problems by identifying and exploiting certain underlying algebraic structures. These structures enable the exact calculation of reliability using reasonably efficient ("pseudopolynomial") algorithms applicable to certain special classes of networks. The underlying structure governing this situation is that of a semilattice, defined on certain combinatorial objects (paths and cutsets) and for which a general recursive algorithm can be used for calculating system reliability. In addition, approximation of the reliability, or more general measures of system performance, derives from studying a different type of algebraic structure (a distributive lattice) in more general network systems.

ACCOMPLISHMENTS

The calculation of network reliability in general is known to be mathematically difficult (NP-hard). Thus, a more promising line of attack is to investigate the calculation of reliability when the underlying network has certain special structure. One commonly occurring structure is that of a planar network: a network that can be represented in the plane without crossing edges. An algebraic structure (a lattice) governing planar networks has been identified and studied. This algebraic structure enables the "pseudopolynomial" calculation of network reliability; that is, the computational effort is polynomially bounded in terms of the number of paths (or cutsets) joining two specified nodes in the network. To streamline these calculations, efficient techniques for generating paths and cutsets in planar networks have also been developed. The resulting algorithm has been developed and applied to a number of difficult benchmark problems in the literature, providing in many cases the first complete reliability analysis of such problems. In one particular case, the computation time was reduced from the order of hours to the order of minutes. Another feature of the algebraic approach has been that it enables the calculation of the entire reliability polynomial in functional form, rather than simply the numerical value at a particular setting for the input edge reliabilities. This research, conducted jointly by D. Shier, J. P. Jarvis, and D. Whited (Ph.D. student), has resulted in a number of papers [3, 6, 11, 12] as well as the Ph.D. dissertation [10]. Recent collaboration with M. Ball and J. S. Provan has extended these ideas to the pseudopolynomial calculation of reliability for other types of coherent systems: generalizations of consecutive k-out-of-n systems and reliability covering problems.

An algebraic approach has also led to several new methods for approximating the s-t reliability of a network (the probability that an operative path exists between two given nodes s and t in the network). The method developed is an iterative one, which produces both a sequence of lower bounds and a sequence of upper bounds on the exact s-t reliability value. It has been proved that both the lower and the upper bounds will converge to the correct answer after a finite number of iterations. Moreover, the lower bounds produce a nondecreasing sequence of polynomials and the upper bounds produce a nonincreasing sequence of polynomials, converging to the true reliability polynomial. Thus at each step one has an interval guaranteed to contain the correct value; it is then possible to terminate the iterative process once this interval is sufficiently small. This aspect is particularly important since the exact calculation of reliability can be quite time-consuming, even for moderately small networks. The algebraic approach developed here produces a general polynomial expression for the s-t reliability in terms of the input parameters rather than simply a single numerical answer. This symbolic form is especially useful for assessing the "importance" of individual links in the original network, as well as facilitating a general sensitivity analysis. These results, obtained by D. Shier and D. Whited, are discussed in the papers [7, 8].

Another polynomial, related to the reliability polynomial, is the chromatic polynomial of an undirected network whose coefficients reveal a good deal of information about the combinatorial structure of the network. In particular, one of these coefficients gives the "all terminal domination" of the network, an invariant that has been recently studied in some detail in relation to the "all terminal reliability" of a network (the probability that all nodes of the network can communicate). Our research has provided more efficient methods for calculating the chromatic polynomial, based on decomposition of the original network into trees [1] and into chordal subgraphs [4]. This algorithmic research, conducted jointly by D. R. Shier and graduate students S. Frank and N. Chandrasekharan, is reported in [1, 4].

The algebraic focus of this research has also been applied to the calculation of general performance measures (including reliability, delay, throughput) defined on a network with stochastically failing components. Again, since the exact calculation of such measures is known to be mathematically intractable, we have instead developed approaches for approximating the performance of the system, based on generating a relatively small subset of the "states" of the system that covers in probability a large proportion of the total probability. It has been found that an elegant algebraic structure (a distributive lattice) governs the case in which each component assumes one of two modes: "working" or "failed." This lattice turns out to have a number of significant properties; it is rank unimodal, rank symmetric, and satisfies the Sperner property. Study of the theoretical properties of this lattice has yielded a reasonably effective algorithm for generating in order the most probable states of the system, without the need to examine the entire state space. In one fairly typical example having 25 components, only 0.08% of the states of the system needed to be generated to cover 95% of the total probability. Rather interestingly, the computational complexity of the new algorithm turns out to be related to a certain combinatorial invariant of this lattice. A number of papers, written jointly by D. Shier, R. Jamison, and E. Valvo (graduate student), have been devoted to discussing this approach [2, 5, 9]. More recent work, involving J. P. Jarvis, D. Shier, R. Lakin (graduate student), and E. Bibelnieks (graduate student), has considered the more general case in which each component of the system can assume a finite number of states. Such a model is appropriate when a system degrades slowly over time, rather than simply fails. It appears that an algebraic approach will also be successful in this situation.

PAPERS

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[3] D. Shier, "Algebraic aspects of computing network reliability," in *Applications of Discrete Mathematics*, R. Ringeisen and F. Roberts (eds.), SIAM, Philadelphia, 1988.

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[5] D. Shier, E. Valvo, and R. Jamison, "Generating the states of a probabilistic system," Report URI-031, September 1987. Submitted to *SIAM Journal on Discrete Mathematics.*

[6] D. Shier and D. Whited, "Iterative algorithms for generating minimal cutsets in directed graphs," *Networks* **16** (1986), 133-147.

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[9] E. Valvo, D. Shier, and R. Jamison. "Generating the most probable states of a communication system," Proceedings of the IEEE INFOCOM 87 Conference, San Francisco, March 31-April 2, 1987, pp. 1128-1136.

[10] D. Whited, "Calculation of s-t reliability in planar graphs," Ph.D. dissertation, Clemson University, August 1986.

[11] D. Whited, D. Shier, and J. P. Jarvis, "Reliability computations for planar networks," submitted to *ORSA Journal on Computing.*

[12] D. Whited, D. Shier, and J. P. Jarvis, "Discrete structures and reliability computations," submitted to Proceedings of the 20th Symposium on the Interface: Computing Science and Statistics.

PRESENTATIONS

"Algebraic aspects of computing network reliability," Third SIAM Conference on Discrete Mathematics, Clemson, SC, May 1986 (INVITED).

"Generating states of a probabilistic system," IEOR Department, University of California, Berkeley, September 1986 (INVITED).

"Evaluating network performance by efficient state generation," Operations Research Department, Naval Postgraduate School, November 1986 (INVITED).

"Algebraic aspects of generating the most probable states of a probabilistic system," Mathematical Sciences Department, Johns Hopkins University, March 1987 (INVITED).

"Generating the most probable states of a system," Department of Mathematics, College of William and Mary, March 1987 (INVITED).

"Generating the most probable states of a communication system," IEEE INFOCOM 87 Conference, San Francisco, April 1987 (INVITED).

"Generating the states of a communication system," First International Conference on Industrial and Applied Mathematics, Paris, June 1987 (INVITED).

"Algebraic aspects of network reliability," University of Paris, Orsay, July 1987 (INVITED).

"Iterative techniques for approximating network reliability," 11th Triennial Conference on Operations Research, Buenos Aires, August 1987 (INVITED).

"Chordal graphs and chromatic polynomials," Operations Research Society of America, St. Louis, October 1987 (INVITED).

"Approximating reliability in coherent systems," Operations Research Society of America, St. Louis, October 1987 (INVITED).

"Algorithms for computing the chromatic polynomial," 19th SE International Conference on Combinatorics, Graph Theory and Computing, February 1988.

"Algorithms for computing network reliability," Graduate School of Business, University of Texas, Austin, April 1988 (INVITED).

"Discrete structures and reliability computations for planar graphs," 20th Interface Conference, Reston, April 1988 (INVITED).

"Approximate analysis of systems with multimodal components," Operations Research Society of America, Washington, April 1988 (INVITED).

"Exact and approximate algorithms for computing network reliability," Analysis and Control of Large Scale Stochastic Systems, Chapel Hill, May 1988 (INVITED).

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ADDITIONAL CONSULTATIONS

- Attended ORSA/TIMS National Meeting, Miami, October 1986: discussed research with M. Ball (Univ. of Maryland) and J. S. Provan (Univ. of North Carolina).
- During visit at University of California, Berkeley, discussed reliability problems with R. Barlow (Berkeley) and V. Li (Univ. of Southern California).
- Attended ORSA/TIMS National Meeting, New Orleans: discussed research with M. Ball (Univ. of Maryland).
- Visited University of North Carolina, May 1988: discussed joint research with M. Ball (Univ. of Maryland) and J. S. Provan (Univ. of North Carolina).