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This annual report summarizes the principal investigator's achievements on the research project during the period September 1, 1986 - March 31, 1988. These include new results for wave equations and plate equations on the following problems: exact controllability, strong and uniform stabilization, structural damping, Algebraic Riccati equation, and numerical aspects thereof.

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FINAL TECHNICAL REPORT

Grant No.: AFOSR-84-0365

Principal Investigators: I. Lasiecka and R. Triggiani

Title: Increasing the margin of stability of arbitrarily finite modes of flexible large space structures with damping

1. General overview of research project and its components. During its four year life span, the original research thrust of the present research project, as expressed by its title above, has expanded considerably as to encompass various other aspects of the dynamics, control and stability of large space flexible structures. A main common theme which has weaved together in a coherent unity the different facets of research under the present project, is the issue of (uniform) exponential decay rates of the underlying dynamics, which are sought by means of suitable feedback operators exercised on the boundary (or part thereof) of the structures. This ultimate achievement may acquire two distinct forms or modes: it may either result through a boundary feedback explicitly given, or found, a priori; or else through a boundary feedback which is based on the Riccati operator solution of the Algebraic Riccati equation associated with the corresponding optimal quadratic cost problem over an infinite horizon. The Principal Investigators, after solving the original problem of the title -- which had been raised of the NASA Workshop held at the Langley Research Center in April 1982 -- have proceeded to investigate other aspects of the dynamics, stability and control of canonical components of large space flexible structures (membranes, beams, plates etc.). Our research efforts encompass a broad range of theoretical and numerical studies. Numerical investigations were primarily targeted for the computation of the Riccati operator via finite dimensional approximations.

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We list below a sample of problems that were specifically investigated under the AFOSR grant, along with a summary of the major findings that were obtained. Reference to the original research articles, where a full treatment may be found, is also given. Major themes of research performed under the grant include, but are not restricted to, the following problems:

(i) increasing the margin of stability of arbitrarily finite modes of damped wave equations. Allocation of spectrum and of Riesz basis properties of eigenvectors;

(ii) Uniform stabilization (linear case) and strong stabilization (non-linear case) by a-priori, explicit boundary feedbacks for waves and plates;

(iii) exact boundary controllability for waves and plates;

(iv) study of the optimal quadratic cost problem for waves and plates, in particular of the associated Algebraic Riccati Equation which produces a boundary feedback based on the Riccati operator which uniformly stabilizes the system [compare with (ii)];

(v) structural damping for elastic systems under a natural, broad class of damping operators.

(vi) numerical aspects related to some of topics listed above, in particular related to the computation of the Riccati operator in case of boundary control problems for waves and plates

We finally point out that the AFOSR grant has supported a graduate student, Jerry Bartolomeo, who is now completing his Ph.D. thesis (Ph.D. degree in Mathematics, University of Florida, Gainesville, expected December 1988) in the area of uniform stabilization for a class of plates problems by means of explicit boundary feedbacks (topic (ii) above).

2. Specific results

2.1 Increasing the margin of stability of arbitrarily finite modes of damped wave equations with viscous damping. Use of a non-local, explicit boundary feedback consisting of one boundary actuator and two interior observation functions[L-T.3], [L-T.4], [T.6], [B.1]

This is the original research project for which a rather complete theoretical solutions is given in [L-T.3], [L-T.4], [T.6] with some numerical testing reported in [B.1] and performed by our graduate student J. Bartolomeo. It was shown that the problem raised at the NASA Workshop in 1982 admits an affirmative answer, which is moreover given in a constructive manner with implementable numerical algorithms. A related, in fact more general problem which was solved in [L-T.4], consists in pre-assigning desirable spectral properties of the closed loop feedback system: i.e. both the location of the spectrum (subject to certain necessary constraints) as well as forcing the corresponding eigenvectors to form a Riesz basis. This way, a unified framework is produced whose solution encompasses various other cases previously considered by the Principal Investigators and others.

2.2 Boundary feedback stabilization

Linear case: uniform stabilization. The principal investigators results on uniform stabilization for the wave equation with feedback in the Dirichlet boundary conditions was already noted above [L-T.1]. In addition we cite a new proof which provides further insight into J. Lagnese's result on uniform stabilization with feedback in the Neumann boundary conditions given by the principal investigators in [T.1]. Work on uniform stabilization for certain plates is part of a Ph.D. thesis by J. Bartolomeo [B-T.1]. Uniform stabilization for other plates was considered in [L-T.12]. A sharp result on the lack of uniform stabilization was also achieved in [T.3] and [T.4].

Nonlinear case: strong stabilization. Strong stabilization results by the principal investigators for waves and plates with nonlinear boundary damping defined by monotone functions are given in [L.1 - L.3]. These results extend, by means of a general technique applicable to spatial domains of any dimension, recent results of the literature obtained only in the one-dimensional case.

Nonlinear case: uniform exponential stabilization. The following question is addressed and solved in

[L.2]: does a linear stabilizing feedback operator provide a robust mechanism (at least locally) in the presence of nonlinear undesirable perturbations? An affirmative answer is given and applied to waves and plates in feedback form.

2.3 Exact boundary controllability for waves and plates

Waves It was already noted in Section I of the research proposal (continuation) submitted to the AFOSR in January 1987, that the first time that exact controllability of the wave equation with Dirichlet boundary control was obtained in its natural state space was in the principal investigators' paper [L-T.1], as a corollary of a related uniform stabilization result. Further contributions followed, by J. L. Lions and his associates, and by the principal investigators, see [T.2]. More recent contributions by the principal investigators include a rather thorough study of exact boundary controllability of wave equation with Neumann boundary control [L-T.3], and a study of these questions when the equation contains damping [T.5].

Plate equations In a recent paper [L-T.7 , first announced in [L-T.6], the principal investigators have solved an open problem, raised by J. L. Lions, on exact controllability of a plate equation with control action only in the Dirichlet boundary conditions, while the corresponding Neumann boundary conditions are homogeneous. Further work by the principal investigators on plates with different boundary conditions is included in [L-T.9] (maximal, optimal regularity) and in [L-T.8] (exact controllability). The case with damping is also under study with preliminary results announced at the International Conference on Differential Equations held at Columbus, Ohio, March 1988, see [T.5].

2.4 Optimal quadratic cost problem over an infinite horizon

The principal investigators have recently completed, in collaboration with F. Flandoli, a rather thorough and comprehensive study on the optimal quadratic cost problem over an infinite horizon for an abstract unifying model which includes waves and plates [L-T.5]. This work extends prior work by the principal investigators on the same problem for wave equations with Dirichlet boundary control (SIAM J. Control, 1985).

2.5 Structural damping for elastic systems

As already noted in the proposal (continuation) submitted to the AFOSR in January 1987, work by the primcipal investigators in collaboration with S. Chen, has succeeded in proving two conjectures, raised by G. Chen and D. Russell in 1982, that if the damping operator behaves like $A^{1/2}$ (in a technical sense), where A is the elastic operator, then the elastic system displays structural damping, see [C-T.1]. Further work by the same authors shows that the same conclusion holds if the damping operator behaves like A^{α}, $1/2 \leq \alpha$, see [C-T.2]. Indeed, even a more general result holds true. The result is false if, instead, $0 \leq \alpha < 1/2$.

2.6 Numerical aspects

Work in the area of numberical analysis can be divided into two parts, reviewed separately below.

(i) Numerical approximations of Riccati equations. The ultimate goal of proving exponential stability, which is uniform with respect to the parameter of approximation, of stabilizing feedbacks based on the algebraic Riccati equations was achieved by the principal investigators in the case of arbitrary analytic semigroups with (strongly) unbounded feedbacks, see [L-T.10-11]. Examples include parabolic equations with Dirichlet boundary controls and strongly damped wave equations with boundary control.

(ii) Numerical schemes for the computation of the solutions of wave equations with rough (nonsmooth) data. Convergence and stability of the algorithm, along with numerical illustrations and examples are obtained in [L-S.1-2].

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