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THE APPLICATION OF BRIAN'S METHOD TO THE SOLUTION OF TRANSIENT HEAT CONDUCTION PROBLEMS IN CYLINDRICAL GEOMETRIES

by

Karl R. Heinz

December 1988

Thesis Advisor: Yogendra Joshi

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The Application of Brian's Method to the Solution of Transient Heat Conduction Problems in Cylindrical Geometries

by

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Submitted in partial fulfillment of the
requirements for the degree of

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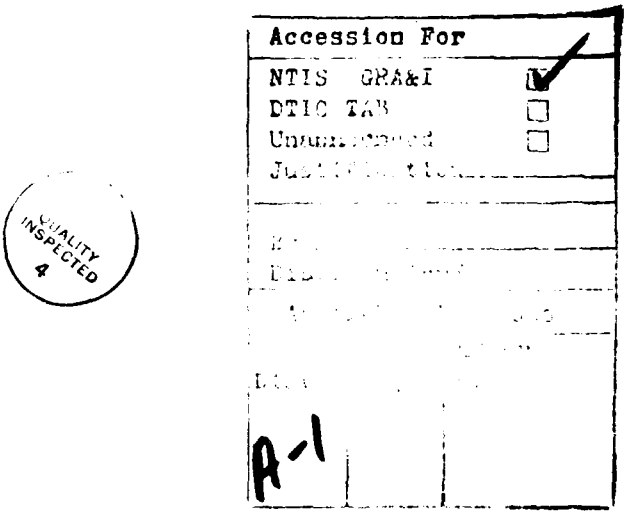
ABSTRACT

A FORTRAN 77 computer code employing an adaptation of the finite differencing algorithm proposed by Brian was developed for the solution of transient heat conduction problems in cylindrical geometries. Validation of code was accomplished by comparison with an analytic solution derived for a model with symmetric, linear boundary conditions. Accuracy of results for asymmetric and non-linear boundary conditions was determined by comparison with a similarly validated code employing the explicit method. Code effectiveness was then demonstrated by conducting a transient temperature analysis for a simulated earth-orbiting satellite. Brian's method demonstrated unconditional stability with associated significant reductions in execution time compared to the explicit method. The effects of discretization error on the accuracy of results require further investigation.



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I. INTRODUCTION

A. BACKGROUND

Analysis of transient heat-conduction problems necessarily involves solution of the heat diffusion equation. In the absence of internal generation of energy, this is represented by

$$\nabla \cdot (k \nabla T) = \rho \, C_p \frac{\partial T}{\partial x} \tag{1.1}$$

Although analytic solutions may be obtained to certain simple transient problems [Ref. 1:p. 212], many of the practical problems encountered by engineers involve one or a combination of nonlinearities, complex geometries, complex boundary conditions, or systems of coupled partial differential equations of which any one may necessitate the use of numerical methods. Although Monte-Carlo (probability sampling) and finite-element methods have been applied to the solution of heat-conduction problems, the approximation of partial derivatives by finite differences presents a straightforward and popular approach [Ref. 2:p. 471]. Two fundamental finite-difference techniques are the implicit and explicit methods and their derivatives.

B. PROBLEM

The analysis of transient heat-conduction problems in three dimensions is of great importance to many fields of science and engineering, but solutions are often quite costly in terms of computational

effort and time consumed in their solution. Numerical solutions to these problems frequently tax the computational and storage capacities of available computers [Ref. 3:p. 367].

The *explicit method* offers the benefits of relatively low storage requirements and simplicity in formulation, coding, and execution. However, it is only conditionally stable and is subject to a limitation on the maximum timestep which may be employed. If the maximum timestep is exceeded, the solution experiences numerically induced oscillations which may cause the solution to diverge from the correct result [Ref. 1:p. 214]. This timestep restriction causes the explicit method to be inefficient in terms of computational effort for the analysis of lengthy transient periods.

The *implicit method* offers the advantage of being unconditionally stable and therefore exempt from timestep restrictions. However, this method calculates node temperatures by the simultaneous solution of the nodal heat balance equations. Thus, savings gained from the lack of a timestep restriction are often quickly offset by the increased computational effort associated with the required matrix inversions.

The *implicit alternating direction method* (ADI) is an unconditionally stable extension of the implicit method which reduces the number of required computations by transforming the problem into tridiagonal form. However, a direct extension of this method into three dimensions becomes only conditionally stable, is restricted to a maximum timestep in a fashion similar to the explicit method, and demonstrates similar shortcomings [Ref. 4:p. 453].

C. REQUIREMENT

A finite difference technique which is unconditionally stable in three dimensions and unencumbered by the requirement to manipulate large matrices is required for the solution of transient heat-conduction problems in three dimensions. The algorithm proposed by Brian [Ref. 3] offers such a technique.

D. OBJECTIVES

1. Adapt the algorithm proposed by Brian to the solution of three-dimensional transient heat-conduction problems in cylindrical geometries.
2. Evaluate the relative accuracy of the results obtained using Brian's Method by comparing them against those obtained using the explicit method.
3. Evaluate the speed of execution of Brian's Method relative to that of the explicit method in terms of central processing unit (CPU) seconds.
4. Illustrate the relative benefits derived from the Brian's Method by applying it to the solution of an application problem involving long-period transient behavior.

II. THEORY

A. FUNDAMENTALS

1. Material Properties

The capacity for energy storage and rate of energy transfer within a medium are functions of the material's thermophysical properties. Isotropic materials, such as metals and alloys, are uniform in nature and their properties generally demonstrate only negligible directional and temperature dependence. However, layered or stratified materials, such as glass-reinforced plastic (GRP) or laminates, may demonstrate strong dependence on both direction and temperature. While directional dependence of properties is generally linear in nature, temperature dependence is strongly nonlinear and will not be addressed.

a *Heat Capacity*

The rate of change of energy storage per unit volume within a solid, isotropic medium is determined by the product of its *density* (Kg/m^3), *specific heat* (KJ/KgK), and the time-rate of change of its *temperature*:

$$\dot{E}_s = \rho C_p \frac{\partial T}{\partial t} \tag{2.1}$$

Both density and specific heat may be temperature and/or directionally dependent properties.

b. *Thermal Conductivity*

The *thermal conductivity* (W/mK) is a *transport property* (i.e., one affecting the movement of energy) of the material and is defined in the x-coordinate direction as

$$k_x = -\frac{q''_x}{\left(\frac{\partial T}{\partial x}\right)} \tag{2.2}$$

Isotropic materials generally demonstrate relatively constant thermal conductivity over broad temperature ranges, whereas metallic laminates demonstrate a strong directional dependence within the plane of each different material. Composite materials show both strong directional and temperature dependence. Temperature-dependent material properties will not be addressed.

c. *Thermal Diffusivity*

The *thermal diffusivity* (m^2/s) is a grouping of the material density, specific heat, and thermal conductivity and represents the controlling transport property for transient heat conduction [Ref. 1:p. 41]

$$\alpha = \frac{k}{\rho C_p} \tag{2.3}$$

d. *Convection Heat Transfer Coefficient*

The *convection heat transfer coefficient* (W/m^2K) encompasses all of the effects that influence the convection mode and is dependent upon boundary conditions, surface geometry, the nature

of the fluid motion, and a number of other fluid thermodynamic and transport properties [Ref. 1:p. 8].

e. Properties Affecting Radiation

Various surface properties affect the radiative characteristics of a solid material. Directionally integrated or hemispherical values are often adequate although these properties are still strongly dependent upon the wavelength of the radiant energy. Where appropriate, consideration of the wavelength dependence will be indicated by a subscripted Greek letter lamda (λ).

(1) Emissivity. The *emissivity* (ε) is a surface property representing the ratio of total energy emitted by an actual surface and an identical black surface at the same temperature. The emissivity may be altered by one or a combination of mechanical processes (e.g., sandblasting or metal-spraying) or by application of various films and/or coatings, either by design or through service conditions.

(2) Surface Properties. *Absorptivity* (α) and *reflectivity* (ρ) are surface properties representing the fractions of total incident energy which are respectively absorbed and reflected by the surface. For opaque surfaces, the sum of the absorptivity and the reflectivity equals one

$$\alpha + \rho = 1 \tag{2.4}$$

In addition to absorption and reflection, translucent materials allow a fraction of incident radiation to be transmitted through the material.

Consequently, an additional property, *transmissivity* (τ), must be incorporated into Equation 2.4, yielding

$$\alpha + \rho + \tau = 1 \tag{2.5}$$

(3) Radiation Heat Transfer Coefficient. The *radiation heat transfer coefficient* (W/m^2K^4) is a collection of terms resulting from the linearization of the radiation heat transfer calculation from a gray, diffuse surface in extensive surroundings and is represented as

$$h_r = \varepsilon \sigma (T_s + T_{sur})(T_s^2 + T_{sur}^2) \tag{2.6}$$

where σ is the *Stefan-Boltzman Constant* (5.67×10^{-8} W/m^2K) and T_s and T_{sur} are the absolute temperatures of the surface and the surroundings, respectively. As is apparent from Equation 2.6, this term is strongly temperature dependent and non-linear in nature.

2. Heat Transfer Rate

The *heat flux* (W/m^2) is rate of heat transfer per unit surface area and may be represented by

$$q'' = \frac{\dot{E}}{A} \tag{2.7}$$

The *heat transfer rate* (W/m^2s) is thus the product of the heat flux and the surface area normal to the direction of heat flow [Ref. 1:p. 4].

3. Rate Equations

The three principle modes of heat transfer are *conduction*, *convection*, and *radiation*. The rate of heat transfer attributed to each mode is described by the appropriate *rate equation*.

a Conduction Equation

The governing equation for conduction is *Fourier's Law*, given for the x-direction by

$$q''_x = - k_x \frac{\partial T}{\partial x} \tag{2.8}$$

b. Convection Equation

The relation governing convection is known as *Newton's Law of Cooling* and is given by by

$$q''_c = h_c (T_s - T_{sur}) \tag{2.9}$$

where T_S is the surface temperature and T_∞ is the temperature of the surrounding fluid.

c. Radiation Equation

The *maximum* flux at which radiation may be emitted by a surface to an infinite, black surrounding is determined by the *Stefan-Boltzman Law*

$$q''_r = h_r (T_s - T_{sur}) \tag{2.10}$$

where T_S is the surface temperature and T_{sur} is the temperature of the surrounding enclosure [Ref. 1:p. 9]. In reality, objects *exchange*

energy with surrounding enclosure boundaries in accordance with the relation

$$q'' = \sum_i F_i A_i h_r (T_{s_i} - T_{sur_i}) \qquad (2.11)$$

where F is the *view factor* representing the fraction of the radiation emitted by the surface of interest that is intercepted by the i^{th} surface of the confining enclosure. Implicit in Equation 2.11 is the assumption that all surfaces of the surrounding enclosures are black.

4. Heat Balance—The First Law of Thermodynamics

The temperature at a given point may be determined from establishment of an arbitrary control volume around the point (Figure 1) followed by application of the first law of thermodynamics (the *law of conservation of energy*). Excluding heat which is generated within the confines of the control volume (internal generation), this law may be simply stated as

> The rate at which thermal and mechanical energy enters a control volume minus the rate at which this energy leaves the control volume must equal the rate at which this energy is stored in the control volume. [Ref. 1:p. 12]

With the addition of any heat genetrated internal to the control volume, the first law may represented in the form of the *heat balance equation*:

$$\dot{E}_{st} = \dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} \qquad (2.12)$$

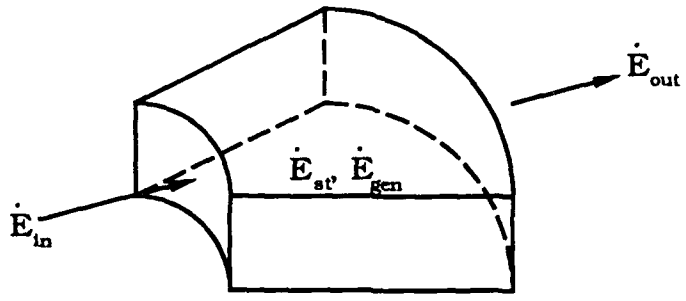


Figure 1. **Control Volume**

where

$\dot{E}_{st}$ = Rate of energy storage

$\dot{E}_{in}$ = Rate of energy gain

$\dot{E}_{out}$ = Rate of energy loss

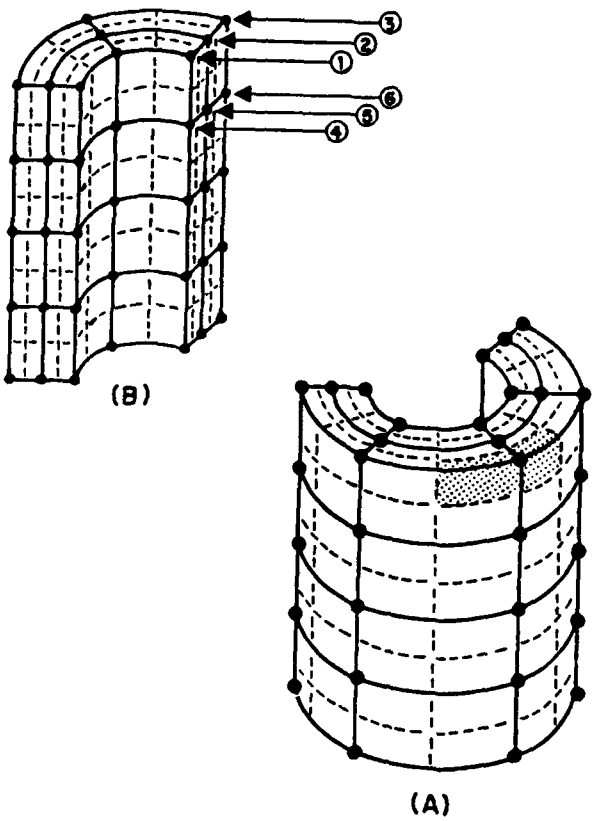
$\dot{E}_{gen}$ = Rate of energy generation.

B. FINITE DIFFERENCE APPROXIMATIONS

1. General Approach

The numerical solution of transient, heat-conduction problems by digital computer requires the transformation of the heat conduction equation into a form which allows differentiation to be approximated by finite differences [Ref. 2:p. 472]. This may be accomplished by dividing the problem into a number of discrete points, or nodes, each surrounded by an associated control volume (Figure 2(A)). Conducting an energy balance on each control volume yields a system of linear algebraic equations which describes the temperature field. In the absence of round-off error, the solutions obtained from finite

difference approximations tend to the analytic solution as the step size and grid spacings tend to zero [Ref. 4:p. 449].



Type 1—End-layer, inner-surface

Type 2—End-layer, mid-radius

Type 3—End-layer, outer-surface

Type 4—Mid-layer, inner-surface

Type 5—Mid-layer, mid-radius

Type 6—Mid-layer, outer-surface

Figure 2. (A) **Reference Grid**; (B) **Node Types**

a. Geometry

Figure 2(B) depicts a right-circular cylinder to which has been applied a polar-cylindrical coordinate system and an arbitrary grid which divides the cylinder into a number of nodes, each surrounded by an associated control volume. Nodes may be characterized as belonging to one of six types and are located at the grid intersection points. Each node may be associated with a specified layer, ring, and ray.

(1) Layers. Layers lie parallel to the x-y plane and are numbered sequentially beginning in the x-y plane and proceeding in the direction of the positive z-axis. Layers are spaced a distance Δz apart. Note that the end layers are in the plane containing the ends of the cylinders.

(2) Rings. Each layer is divided into an arbitrary number of concentric rings spaced a distance Δr apart. The node at the three o'clock position when viewed from above has been arbitrarily designated as node one. Other nodes are numbered sequentially in a counter-clockwise direction from node one.

(3) Rays. Rays emanate outward from the origin in the plane of the layers. Nodes lie along the rays at intervals of Δr and are numbered sequentially from the inner surface outward.

b. Subscripting Convention

To simplify notation in the development of equations, the designation of nodal grid coordinates will follow the convention contained in Table 1.

TABLE 1

NODAL SUBSCRIPT CONVENTION

Subscript	Location	Grid Coordinates*
N	Northern (upper) neighbor	[r,ϕ,(z+1)]
S	Southern (lower) neighbor	[r,ϕ,(z-1)]
E	Eastern (right) neighbor	[r,(ϕ-1),z]
W	Western (left) neighbor	[r,(ϕ+1),z]
I	Inner neighbor	[(r-1),ϕ,z]
O	Outer neighbor	[(r+1),ϕ,z]
e	Edge	(varies)

* All grid coordinates are relative to the node of interest when viewed from the inside of the cylinder.

In view of the inverse symmetric relationship existing between equivalent nodes at the upper and lower surfaces, the subscript *e* will be used to designate temperatures, fluxes, and properties associated with the end layers, thus reducing the number of required equations by half. The reader must substitute the appropriate subscript (i.e., *N* or *S*) depending upon the location or application of the equation. This issue will be further elaborated upon in the equation development example.

c. Superscripting Convention

Superscripts, as contained in Table 2, will be employed to indicate at which timestep a value is considered.

TABLE 2

NODAL SUPERSCRIPT CONVENTION

Superscript	Timestep	Time
n	Current	t
n+1	Next	$t + \Delta t$
n+1/2	Intermediate	$t + \Delta t/2$

d. Example

Consider the one-dimensional heat conduction problem represented in Figure 3 wherein $T_E = T_W$, heat is being conducted in the radial direction only, and the control volume is of unit thickness.

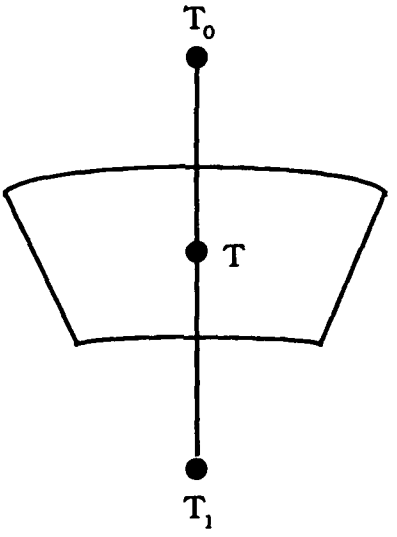


Figure 3. **One-Dimensional Heat Conduction Problem**

The appropriate heat balance equation is

$$\dot{E_s} = \dot{E_I} - \dot{E_O} \qquad (2.13)$$

The time-dependent storage term ($\dot{E}_s$) may be represented by its finite difference approximation

$$\dot{E}_s = \rho C_p V \frac{(T^{n+1} - T^n)}{\Delta t} \tag{2.14}$$

where Δt is the step size, T^n is the node temperature at the *old* (n) timestep, T^{n+1} is the node temperature at the *new* (n+1) timestep, and the volume is approximated as

$$V \equiv \frac{r\Delta\, r\Delta\phi}{2} \tag{2.15}$$

The heat rate in the radial direcrion at the inner boundary ($\dot{E}_I$) of the control volume may be approximated as

$$\dot{E}_I = - k_r A_I \frac{(T_I^n - T^{n+1})}{\Delta t} \tag{2.16}$$

where

$$A_I = (r - \frac{\Delta r}{2})\Delta\phi \tag{2.17}$$

is the area of the control volume in the direction of heat flow. Similarly, the heat rate at the outer boundary may be approximated as

$$\dot{E}_O = - k_r A_O \frac{(T_O^n - T^{n+1})}{\Delta r} \tag{2.18}$$

where

$$A_O = (r + \frac{\Delta r}{2})\Delta\, r\Delta\phi \tag{2.19}$$

Substituting these into Equation 2.13 yields the finite difference form of the heat equation

$$\frac{\rho C_p r \Delta r \Delta\phi}{2} \frac{(T^{n+1} - T^n)}{\Delta t} = \frac{k_r (r - \frac{\Delta r}{2})\Delta\phi}{\Delta r} (T_I^n - T^n) \qquad (2.20)$$

$$+ \frac{k_r (r + \Delta r)\Delta\phi}{\Delta r} (T_O^n - T^n)$$

The steps leading up to and including the formation of Equation 2.20 are common to both the explicit and Brian's methods.

Similar equations may be obtained for other nodes and boundary conditions.

2. The Explicit Method

The explicit method is a direct application of the general approach to finite difference analysis detailed in the preceding section. Equations describing the temperature at the new time level are formed for each node by conducting a heat balance on the associated control volume. Temperatures at the new time level may then be approximated explicitly by substituting temperatures at the old time level into the appropriate node equation. The explicit form of the one-dimensional heat conduction problem of the preceding section may be obtained from Equation 2.20 by dividing by the lead coefficient on the left-hand side, substituting for the grid Fourier number, and solving for the new temperature, yielding

$$T^{n+1} = T^n + Fo \left[\frac{(r - \frac{\Delta r}{2})}{\Delta r} (T_I^n - T^n) + \frac{(r + \frac{\Delta r}{2})}{\Delta r} (T_O^n - T^n) \right] \qquad (2.21)$$

where

$$Fo = \frac{\alpha \Delta t}{\Delta r^2} \tag{2.22}$$

Defining the constants

$$c_1 = \frac{(r - \frac{\Delta r}{2})}{\Delta r} \tag{2.23}$$

and

$$c_2 = \frac{(r + \frac{\Delta r}{2})}{\Delta r} \tag{2.24}$$

and grouping the T^n terms outside of the the parentheses yields

$$T^{n+1} = Fo(c_1 T_I^n + c_2 T_O^n) + [1 - Fo(c_1 + c_2)]T^n \tag{2.25}$$

from which the stability criterion on the timestep may be determined from the relation

$$[1 - Fo(c_1 + c_2)] \geq 0 \tag{2.26}$$

[Ref. 1:p. 215]

3. Brian's Method

In Brian's Method, the node types are the same as depicted in Figure 2. However, the temperature at the new time level is formed from a combination of values taken from the old time level and three

directionally dependent temperature arrays formed at an intermediate time

$$t' = t + \frac{\Delta t}{2} \tag{2.27}$$

Although each of these arrays requires a full complement of supporting node equations, the equations are quite similar and enjoy a common derivation with the explicit method for the initial stages of their development.

Each intermediate array is associated with one arbitrarily assigned coordinate direction and has its temperatures denoted by the superscripting of one, two, or three asterisks. *Calculation precedence must follow the order of star level, two-star level, three-star level, and new temperature arrays!*

Within each intermediate calculation process, temperatures are determined by solution of a tridiagonal matrix of coefficients of nodes lying along the specified coordinate axis. For example, equations for the r-direction nodes at the star level in a one-dimensional heat conduction problem are of the form

$$AT_I^* + BT^* + CT_O^* = D(T^*) \tag{2.23}$$

Although Brian's method is conceptually no more difficult than the explicit method, it involves considerably more algebra and is best illustrated in the form of the code development model contained in the following chapter.

III. DERIVATION OF EQUATIONS

A. MODEL FOR CODE DEVELOPMENT

1. General

In this chapter, detailed descriptions of the explicit and Brian's methods will be presented in the form of the model employed for the development of code.

2. Problem Description

Selection of a model with simplicity and symmetry of geometry and boundary conditions was made to facilitate error checking and the comparison of results during code development. The preliminary model consisted of a right-circular cylinder of uniform properties with adiabatic ends and specified initial temperature distribution (initial condition) and constant inner-surface temperature of zero degrees Celsius. An arbitrary grid consisting of five layers in the z-direction, each with three and four nodes respectively in the radial and phi-directions, was applied to the cylinder in a fashion similar to that depicted in Figure 4. At time t = 0, a uniform, constant heat flux was applied to the outer surface. Subsequent modifications to the model added radiative and convective boundary conditions at all surfaces as well as directionally-dependent thermal conductivities. Temperatures are in terms of the rise above the initial temperature (*excess* temperature) [Ref. 1:p. 100].

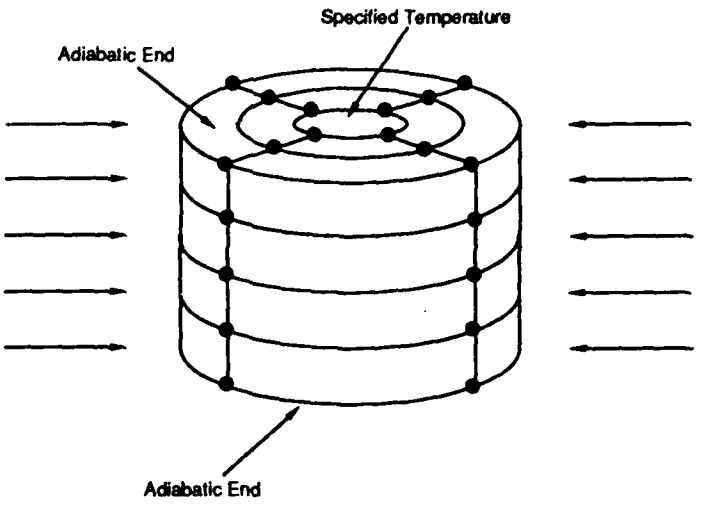
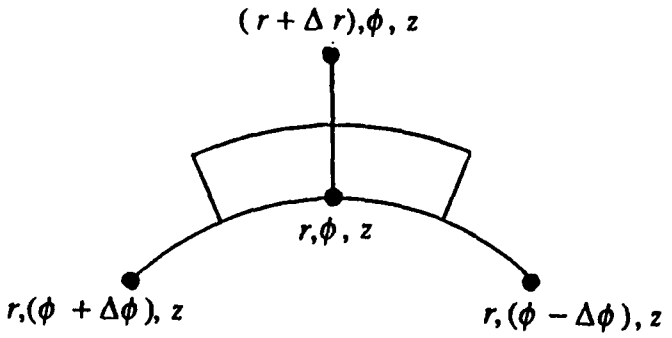


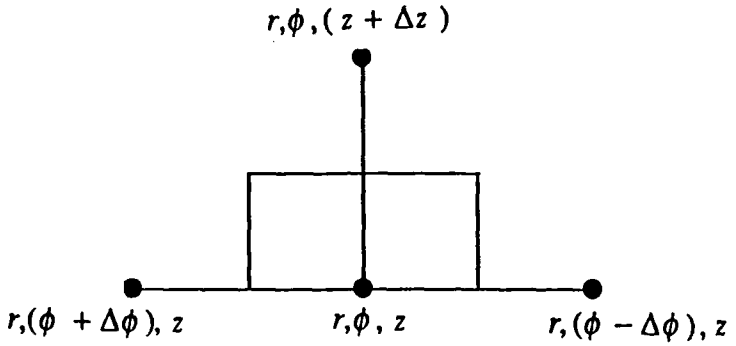
Figure 4. **Model for Code Development**

B. EXPLICIT METHOD EQUATIONS

The process for development of equations using the explicit method is similar for all node types. For illustrative purposes, we will consider a Type 1 node located at the inner edge of the bottom surface of the cylinder as depicted in Figure 5. Complete equations for all node types are included as Appendix A to this report. Directionally dependent thermal conductivities and radiative and convective boundary conditions are included in this development. Heat flow into and out of the associated control volume is represented in Figure 6.



A. Plan View



B. Frontal View

Figure 5. **Bottom Layer, Inner Surface Node Configuration**

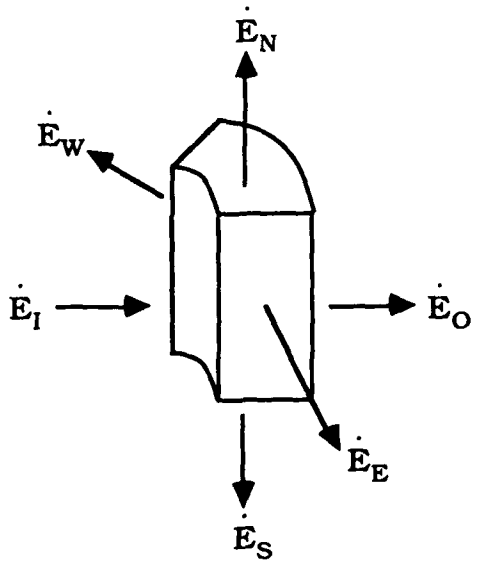


Figure 6. **Heat Balance**

The heat balance equation is

$$\dot{E_s} = \dot{E_I} - \dot{E_N} - \dot{E_S} - \dot{E_E} - \dot{E_W} - \dot{E_O} \tag{3.1}$$

Application of appropriate finite difference approximations results in

$$\rho C_p V \frac{(T^{n+1} - T^n)}{\Delta t} = k_z A_z \frac{(T_N^n - T^n)}{\Delta z} + k_\phi A_\phi \frac{(T_E^n + T_W^n - 2T^n)}{r\Delta\phi} \tag{3.2}$$

$$+ k_r A_r \frac{(T_O^n - T^n)}{\Delta r} - \dot{E_S} + \dot{E_I}$$

where

$$V = \frac{(r + \frac{\Delta r}{4})\Delta r \Delta\phi \Delta z}{4} \tag{3.3}$$

$$A_o = \frac{(r + \frac{\Delta r}{2})\Delta\phi\,\Delta z}{2} \tag{3.4}$$

$$A_\phi = \frac{\Delta r \Delta z}{4} \tag{3.5}$$

$$A_z = \frac{(r + \frac{\Delta r}{4})\Delta r \Delta\phi}{2} \tag{3.6}$$

We note in Equation 3.2 that the conduction terms are evaluated at time level n in order to allow explicit solution of the temperatures. Evaluation of $\dot{E_i}$ and $\dot{E_j}$ reveals that these terms are comprised of the sum of the incident surface heat flux and the terms associated with heat loss (or gain) by convection and radiation. The convection term is linear and may be approximated as

$$q''_c = h_c(T^{n+1} - T_\infty) \tag{3.7}$$

where h_c is the convection coefficient at the appropriate surface and T_∞ is the temperature of the surrounding fluid. The radiation term is non-linear and cannot be directly applied as a function of T^{n+1}. A linearization of the radiation equation may be accomplished by defining the radiation coefficient at the previous (n) timestep

$$h_r = \varepsilon\,\sigma\,(T^n + T_{sur})\left[(T^n)^2 + T_{sur}^2)\right] \tag{3.8}$$

where

$$q''_r = h_r(T^{n+1} - T_{sur}) \tag{3.9}$$

which reduces the approximation of the radiation term to the first power of T^{n+1} and T_{sur} is the temperature of the surrounding environment to which it radiates. Thus, the boundary conditions may be represented as

$$\dot{E_s} = A_z\left[q''_s - h_{c_s}(T^{n+1} - T_\infty) - h_{r_s}(T^{n+1} - T_{sur})\right] \qquad (3.10)$$

and

$$\dot{E_l} = A_l\left[q''_l - h_{c_l}(T^{n+1} - T_\infty) - h_{r_l}(T^{n+1} - T_{sur})\right] \qquad (3.11)$$

where

$$A_l = \frac{r\Delta\phi\,\Delta z}{2} \qquad (3.12)$$

Substituting for $\dot{E_s}$ and $\dot{E_l}$ into Equation 3.2 yields

$$\rho C_p V \frac{(T^{n+1} - T^n)}{\Delta t} = k_z A_z \frac{(T_N^n - T^n)}{\Delta z} + k_\phi A_\phi \frac{(T_E^n + T_W^n - 2T^n)}{r\Delta\phi} \qquad (3.13)$$

$$+ k_r A_O \frac{(T_O^n - T^n)}{\Delta r} + A_z\left[q''_s + h_{c_s}(T_\infty - T^{n+1}) + h_{r_s}(T_{sur} - T^{n+1})\right]$$

$$+ A_l\left[q''_l + h_{c_l}(T_\infty - T^{n+1}) + h_{r_l}(T_{sur} - T^{n+1})\right]$$

Substituting for the volume and area terms yields

$$\rho C_p \frac{(r+\frac{\Delta r}{2})\Delta r\Delta\phi\,\Delta z}{4}\frac{(T^{n+1}-T^n)}{\Delta t} = \frac{k_z(r+\frac{\Delta r}{4})\Delta r\Delta\phi}{2}\frac{(T_N^n-T^n)}{\Delta z} \qquad (3.14)$$

$$\frac{k_\phi\,\Delta r\Delta z}{4}\frac{(T_E^n+T_W^n-2T^n)}{r\Delta\phi} + \frac{k_r(r+\frac{\Delta r}{2})\Delta\phi\,\Delta z}{2}\frac{(T_O^n-T^n)}{\Delta r}$$

$$+\frac{(r+\frac{\Delta r}{4})\Delta r\Delta\phi}{2}[q''_S + h_{c_S}(T_\infty - T^{n+1}) + h_{r_S}(T_{sur}-T^{n+1})]$$

$$+\frac{r\Delta\phi\,\Delta z}{2}\left[q''_I + h_{c_I}(T_\infty - T^{n+1}) + h_{r_I}(T_{sur}-T^{n+1})\right]$$

By defining the directional thermal diffusivity

$$\alpha_z = \frac{k_z}{\rho C_p} \qquad (3.15)$$

and grid Fourier number

$$Fo = \frac{\alpha\Delta t}{\Delta z^2} \qquad (3.16)$$

we may simplify by arbitrarily dividing by the lead coefficient of the z-direction difference approximation to obtain

$$\frac{1}{2Fo}(T^{n+1}-T^{n})=(T_N^{n}-T^{n})+\frac{k_\phi \Delta z^2}{2k_z r(r+\frac{\Delta r}{4})\Delta\phi^2}(T_E^{n}+T_W^{n}-2T^{n}) \tag{3.17}$$

$$+\frac{k_r(r+\frac{\Delta r}{2})\Delta z^2}{k_z(r+\frac{\Delta r}{4})\Delta r^2}(T_O^{n}-T^{n})$$

$$+\frac{\Delta z}{k_z}[q''_S+h_{c_S}(T_\infty-T^{n+1})+h_{r_S}(T_{sur}-T^{n+1})]$$

$$+\frac{r\Delta z^2}{k_z(r+\frac{\Delta r}{4})\Delta r}[q''_I+h_{c_I}(T_\infty-T^{n+1})+h_{r_I}(T_{sur}-T^{n+1})]$$

Further simplification may be realized by defining the constants

$$c_1=\frac{k_\phi \Delta z^2}{2k_z r(r+\frac{\Delta r}{4})\Delta\phi^2} \tag{3.18}$$

$$c_2=\frac{k_r(r+\frac{\Delta r}{2})\Delta z^2}{k_z(r+\frac{\Delta r}{4})\Delta r^2} \tag{3.19}$$

$$c_3=\frac{\Delta z}{k_z} \tag{3.19}$$

$$c_4=\frac{r\Delta z^2}{k_z(r+\frac{\Delta r}{4})\Delta r} \tag{3.20}$$

Substituting into Equation 3.16 and rearranging to yield

$$\frac{1}{2\,Fo}(T^{n+1} - T^{n}) = (T_N^{n} - T^{n}) + c_1 (T_E^{n} + T_W^{n} - 2T^{n}) + c_2(T_O^{n} - T^{n}) \quad (3.21)$$

$$+ c_3[q''_S - h_{c_S}(T_\infty - T^{n+1}) - h_{r_S}(T_{sur} - T^{n+1})]$$

$$+ c_4[q''_I - h_{c_I}(T_\infty - T^{n+1}) - h_{r_I}(T_{sur} - T^{n+1})]$$

Expansion and regrouping of terms yields the final form of the node equation for the temperature at the (n+1) timestep

$$T^{n+1} = \frac{1}{c_5}\{T^{n} + 2Fo\left[(T_N^{n} - T^{n}) + c_1(T_E^{n} + T_W^{n} - 2T^{n}) + c_2(T_O^{n} - T^{n})\right. \quad (3.22)$$

$$+ c_3(q''_S + h_{c_S}T_\infty + h_{r_S}T_{sur})$$

$$\left. + c_4(q''_I + h_{c_I}T_\infty + h_{r_I}T_{sur})\right]\}$$

where

$$c_5 = 1 + 2\,Fo\left[c_3(h_{c_S} + h_{r_S}) + c_4(h_{c_I} + h_{r_I})\right] \quad (3.23)$$

Thus, the temperature for each node at the successive time step may be solved for explicitly in terms of node temperatures at the previous timestep.

C. BRIAN'S METHOD

1. Step One (z-Direction)

This section addresses procedures governing the development of equations associated with the first intermediate, or *star-level*,

temperature array to which the z-coordinate direction has been arbitrarily assigned. Equation 3.21 may be applied to step one by substituting asterisk-superscripted variables for all n+1 superscripted variables and those n-superscripted variables which lie along the z-coordinate direction. Because the intermediate time step is half the overall value, the effective grid Fourier Number is half that employed in the explicit method. Thus, for step one of Brian's Method, Equation 3.21 becomes

$$\frac{1}{Fo}(T^* - T^n) = (T^*_N - T^*) + c_1(T^n_E + T^n_W - 2T^n) + c_2(T^n_O - T^n) \quad (3.24)$$

$$+ c_3[q''_S - h_{c_S}(T^* - T_\infty) - h_{r_S}(T^* - T_{sur})]$$

$$+ c_4[q''_I - h_{c_I}(T^* - T_\infty) - h_{r_I}(T^* - T_{sur})]$$

Expanding and grouping like terms of starred quantities on the left-hand side of the equation yields

$$AT^*_S + BT^* + CT^*_N = D \quad (3.25)$$

where

$$A = 0 \quad (3.26)$$

$$B = 1 + Fo\left[1 + c_3(h_{c_S} + h_{r_S}) + c_4(h_{c_I} + h_{r_I})\right] \quad (3.27)$$

$$C = -Fo \quad (3.28)$$

$$D = T^{n} + Fo\left[c_1(T_E^{n} + T_W^{n} - 2T^{n}) + c_2(T_O^{n} - T^{n})\right. \tag{3.29}$$

$$+ c_3(q''_S + h_{c_S} T_\infty + h_{r_S} T_{sur})$$

$$\left. + c_4(q''_I + h_{c_I} T_\infty + h_{r_I} T_{sur})\right]$$

This process is repeated until similar equations are derived for each node type.

2. Step Two (ϕ-Direction)

Derivation of step two equations is an extension of the procedures employed in the preceding section with the phi-axis now arbitrarily designated the *two-star* coordinate direction. In this step, beginning with Equation 3.24, which already contains the initial step one substitutions, we add further modifications by substituting two-star subscripted variables in the same manner as before. This yields

$$\frac{1}{Fo}(T^{**} - T^{n}) = (T_N^{*} - T^{*}) + c_1(T_E^{**} + T_W^{**} - 2T^{**}) + c_2(T_O^{n} - T^{n}) \tag{3.30}$$

$$+ c_3\left[q''_S - h_{c_S}(T^{**} - T_\infty) - h_{r_S}(T^{**} - T_{sur})\right]$$

$$+ c_4\left[q''_I - h_{c_I}(T^{**} - T_\infty) - h_{r_I}(T^{**} - T_{sur})\right]$$

Again, expanding and grouping like terms of two-starred quantities on the left-hand side of the equation yields

$$AT_E^{**} + BT^{**} + CT_W^{**} = D \tag{3.31}$$

where

$$A = - c_1 Fo \tag{3.31}$$

$$B = 1 + Fo\left[2c_1 + c_2(h_{c_S} + h_{r_S}) + c_3(h_{c_I} + h_{r_I})\right] \tag{3.32}$$

$$C = - c_1 Fo \tag{3.33}$$

$$D = T^n + Fo\left[(T^*_N - T^*) + c_2(T^n_O - T^n)\right. \tag{3.34}$$

$$+ c_3 [q''_S + h_{c_S} T_\infty + h_{r_S} T_{sur}]$$

$$\left. + c_4 (q''_I + h_{c_I} T_\infty + h_{r_I} T_{sur})\right]$$

This process is again repeated until similar equations are derived for each node type.

3. Step Three (r-Direction)

Derivation of step three equations is an identical extension of the procedures employed in the preceding section, with the r-axis now arbitrarily designated the *three-star* coordinate direction. In this step, we begin with Equation 3.30 and further modify by substituting three-star subscripted variables. This yields

$$\frac{1}{Fo}(T^{***} - T^{*}) = (T_N^{*} - T^{*}) + c_1(T_E^{**} + T_W^{**} - 2T^{**}) + c_2(T_O^{***} - T^{***}) \quad (3.35)$$

$$+ c_3[q''_e - h_{c_e}(T^{***} - T_\infty) - h_{r_e}(T^{***} - T_{sur})]$$

$$+ c_4[q''_I - h_{c_I}(T^{***} - T_\infty) - h_{r_I}(T^{***} - T_{sur})]$$

Again, expanding and grouping like terms of two-starred quantities on the left-hand side of the equation yields

$$AT_E^{***} + BT^{***} + CT_W^{***} = D \quad (3.36)$$

where

$$A = 0 \quad (3.37)$$

$$B = 1 + Fo\left[c_2 + c_3(h_{c_S} + h_{r_S}) + c_4(h_{c_I} + h_{r_I})\right] \quad (3.38)$$

$$C = -c_2 Fo \quad (3.39)$$

$$D = T^{*} + Fo\left[(T_N^{*} - T^{*}) + c_1(T_E^{**} + T_W^{**} - 2T^{**}) \right. \quad (3.40)$$

$$+ c_3(q''_S + h_{c_S}T_\infty + h_{r_S}T_{sur})$$

$$\left. + c_4(q''_I + h_{c_I}T_\infty + h_{r_I}T_{sur})\right]$$

This process is repeated until similar equations are derived for each node type.

4. Step Four (New Temperatures)

Step four is a direct extension of the procedures employed in the explicit method. In this step, the n+1 timestep equations are derived from the explicit equations by substituting starred values for the associated node temperatures for node under consideration. Continuing with our example, we substitute into Equation 3.22 to obtain

$$T^{n+1} = \frac{1}{c_5}\{T^n + 2Fo\left[(T_N^* - T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**}) + c_2(T_O^{***} - T^{***})\right. \quad (3.41)$$

$$+ c_3(q''_S + h_{c_S} T_\infty + h_{r_S} T_{sur})$$

$$\left. + c_4(q''_I - h_{c_I} T_\infty - h_{r_I} T_{sur})\right]$$

where

$$c_5 = 1 + 2\,Fo[c_3(h_{c_S} + h_{r_S}) + c_4(h_{c_I} + h_{r_I})] \quad (3.42)$$

Step four equations for each of the six types of nodes are derived in an identical fashion.

Complete node equations in the polar-cylindrical form of Brian's method are included in Appendix B for each of the six node types identified in Figure 2.

IV. CODE DEVELOPMENT

A. ANALYTIC SOLUTION

Analytic solutions to the model described in Chapter III, paragraph A.2, were obtained for the range of zero to 300 seconds in five-second increments. The calculations of eigenvalues and the resulting solutions were made using MATHCAD V2.0 on an INTEL 80386-based personal computer. The calculated eigenvalues were verified with tabular results [Ref. 5] and those obtained by an appropriate IMSL subroutine [Refs. 6-10]. Equations describing the complete solution are contained in Appendix C.

B. EXPLICIT METHOD

1. Requirements

The coding of the explicit method is relatively straightforward and subject to only two notable requirements:

- That the selected timestep must not exceed the theoretical maximum, and
- That each node be considered for temperature calculation once per timestep.

2. General Procedure

a. Timestep Restriction

The maximum allowable timestep varies with control volume dimensions. Exceeding the allowable timestep for *any one node* will result in the eventual loss of stability of the entire system. The rapidity with which this occurs is a function of the affected node's

significance in the path of the dominating heat flux. Consequently, a maximum timestep must be determined for each node type (Figure 2) and the most restrictive taken as the limiting value. This was effected through use of the subroutine *STABIL*, which calculated maximum allowable timestep for each node-type and returned the most restrictive as the argument *DTMAX*. A listing of equations defining stability requirements for each node type is contained as Appendix D.

b. Indexing

The method by which incrementation through the grid system is implemented is entirely arbitrary as long as the temperature is calculated once per timestep for each node on the grid. In systems with a dominant heat flux, convergence will occur more quickly if the dominant direction is incremented first. Appendix E contains the explicit method code as developed for this study. A polar-cylindrical coordinate system was arbitrarily applied with the z-axis aligned with the central axis of the cylinder. Indexing commenced at the inner edge of the bottom surface and proceeded up the z-coordinate to the upper surface. The grid was then indexed in ϕ, followed by r, until the entire grid had been covered, at which time the old temperature matrix was replaced by the new temperature matrix, the time counter indexed by Δt, and the process repeated until the end of the desired period of evaluation was reached.

C. BRIAN'S METHOD

1. Requirements

a. Timestep

The timestep may be selected as befits the duration of the analysis and the expected temperature gradients. However, the discretization error is a function of the square of the timestep. On the other hand, the property of unconditional stability allows the flexibility to initially choose a large timestep for macro-analysis followed by application of a smaller timestep for a detailed look at regions of interest (e.g., an area where a predefined threshold is exceeded).

b. Indexing

A separate temperature matrix must be developed for each coordinate direction at the level of the intermediate timestep. Due to this directional nature, the indexing requirements are more complex and restrictive. Once assignment of a coordinate axis to an intermediate matrix (e.g., star-level) has been made, the convention must be adhered to for the duration of the timestep. One acceptable method for indexing is outlined below and the companion code is included as the *STEP(#)* subroutines to the Brian's Method code contained in Appendix F.

(1) Star Level (z-Direction). The z-coordinate direction was arbitrarily assigned to the star-level intermediate matrix. Consequently, equations were developed for nodes lying along the z-axis in the fashion described in Section C of Chapter III. Appropriate indexing was accomplished by designating an arbitrary node on the

inner ring of the bottom surface of the cylinder (Figure 7) as a node of reference (node 1). An equation similar to Equation 3.25 was developed for this first node and the values of the A, B, C, and D coefficients calculated and stored in four associated vectors. The pointer was then indexed in the z-direction to select the node immediately above node 1 (i.e., the northern neighbor when developing the equation for node 1). The process was repeated for the second node and all other nodes lying on the selected z-axis

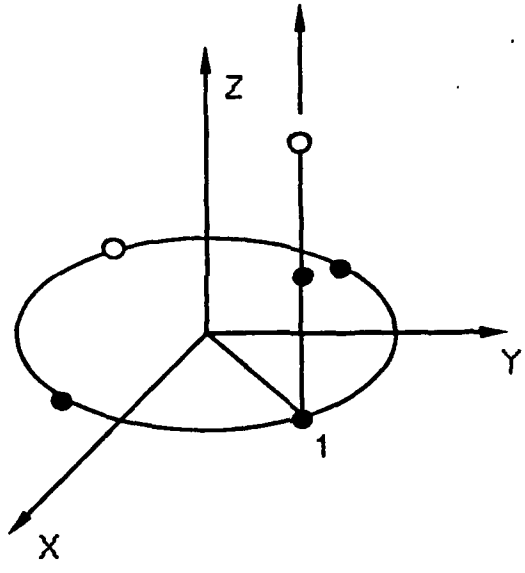


Figure 7. **Star-Level Indexing Direction**

When the values of the coefficients for the last node had been calculated and stored in the appropriate vectors, a tridiagonal matrix solver was called to simultaneously solve the system of equations. The resultant node temperatures were then saved in the star-level intermediate matrix and the.pointer indexed to the next adjacent node in the

counter-clockwise ϕ-direction. Note that at this point, indexing in either the radial or the ϕ-direction would have been equally appropriate. However, once chosen, the specified direction must be adhered to throughout the immediate timestep.

(2) Two-Star Level (ϕ-Direction). Because the rings associated with the ϕ-direction have neither a beginning nor an end, one node must be arbitrarily defined as a reference node. Using node 1 from the one-star level and working in a counter-clockwise direction (Figure 8), the values of the coefficients for each node along the chosen ring were calculated and temporarily stored in suitable vectors. The resultant system of equations for nodes in the ϕ-direction was of the form

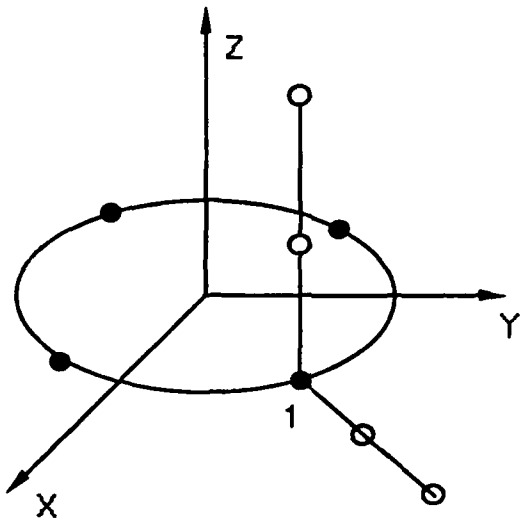


Figure 8. **Two-Star Level Indexing**

$$\begin{bmatrix} b_1 & c_1 & 0 & a_1 \\ a_2 & b_2 & c_2 & 0 \\ 0 & a_3 & b_3 & c_3 \\ c_3 & 0 & a_3 & b_4 \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} \tag{4.1}$$

which is not tridiagonal. To achieve tridiagonality, the ring was cut at the reference node. The resulting coefficient matrix was then of the form

$$\begin{bmatrix} (b_1 - a_1) & c_1 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 \\ 0 & a_3 & b_3 & c_3 \\ 0 & 0 & a_4 & (b_4 - c_4) \end{bmatrix} \tag{4.2}$$

Node temperatures were then obtained by applying a suitable tridiagonal matrix solver [Ref. 4:p. 441] and stored in the two-star intermediate level matrix. The pointer was then incremented to the node adjacent to node 1 in the r-direction and the process repeated. Upon completion of the first layer calculations, the pointer was indexed in the positive z-direction and the process repeated until all two-star level temperatures have been calculated.

(3) Three-Star Level (r-Direction). Construction of the three-star temperature matrix was accomplished in a fashion similar to the one- and two-star level procedures (Figure 9). Beginning at the reference node, the values of the node coefficients were calculated and temporarily stored in suitable vectors. When the coefficients for the last node on the selected ray had been caluclated and stored, the tridiagonal matrix solver was applied to the system of equations, the

resultant node temperatures written to the three-star temperature array, the pointer indexed to the next adjacent ray in the counter-clockwise direction, and the process repeated until all node temperatures were calculated for the layer. The pointer was then indexed in the positive z-direction until all three-star level temperatures had been calculated.

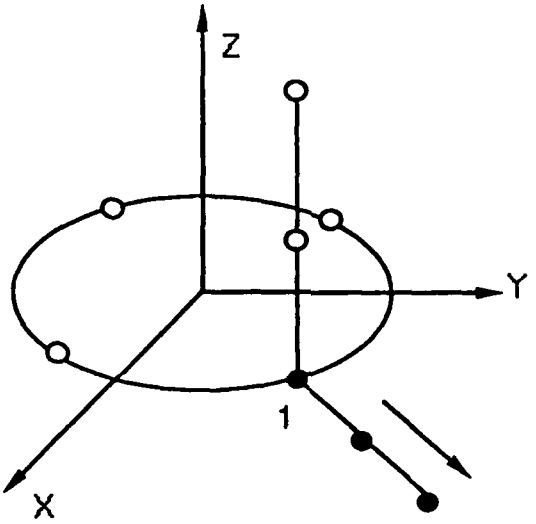


Figure 9. **Three-Star Indexing**

(4) New Temperature Array (n+1 Level). When all intermediate matrices had been developed, new node temperatures were calculated in terms of the three intermediate temperature arrays and the array of temperatures at the previous (old) timestep in accordance with the discussion in paragraph 4 of Chapter III. The choice of indexing direction at this point was arbitrary, so the method applied to the star level was repeated. It must be noted that *the new node*

temperatures must be stored in a separate (new-temperature) array until all node temperatures have been calculated. Upon completion of the calculations, the new temperature array was printed or stored (when appropriate), copied onto the old temperature array, the time counter indexed, and the entire process repeated until the limit of the period of evaluation was reached.

D. EXECUTION TIMEKEEPING

The codes for both the explicit method and Brian's Method were of common origin, had nearly identical main codes, and shared common input/output and utility subroutines. The codes were organized such that the calculations unique to the specific method employed were segregated into a single, cohesive block in the form of one or more subroutines. A timing utility which allowed placement within the code of pointers to initiate and stop CPU timekeeping was employed to measure the execution times of each method for each example or application tested. Among other places, pointers were placed at the entry to and the exit from the respective method-specific subroutine blocks. Benchmark runs exclusive of I/O calls were conducted for each method during the validation process. Subsequent runs involving printed or stored output were structured such that required I/O handling instructions were common to each code to allow accurate comparison of relative speeds.

V. VALIDATION

A. INITIAL CODE

1. General Procedure

To simplify code development, initial forms of the codes for both the explicit method and Brian's Method were developed without including convection or radiation. Validation for both methods was accomplished by solving for the temperature distribution within the cylinder described in paragraph A.2 of Chapter III, comparing results with the analytic solution and calculating the percentage error. Upon achievement of satisfactory results, the initial codes were expanded to encompass the effects of both convection and radiation. Validation of the core of this revised code was accomplished by setting appropriate convective and radiative coefficients to zero and repeating the initial validation procedure with the revised code.

2. Time Incrementation

Initial runs were conducted using a fixed time step of 0.1 second. Following initial data collection, additional runs were conducted to determine operating limits for both methods. During this phase, Brian's method was tested and validated using constant time increments in the range from 0.01 to 100 seconds and included a logarithmically increasing time step arrangement summarized in Table 3.

TABLE 3

VARIABLE TIME STEP EMPLOYMENT

RANGE (SEC)	TIME STEP (SEC)
0.1–1.0	0.1
1.0–10.0	1.0
10.0–100	10.0
100–300	100.0

Additional runs were conducted using a varying time step in which the final step size was limited to 20 seconds.

B. APPLICATION CODE

No analytic solution was available for a cylinder with convective and radiative boundary conditions. Consequently, the application codes may not be considered fully validated. A comparison of results was obtained by solving the application example with both methods and comparing the results point-for-point against each other in the form of a percentage disagreement. All combinations of heat flux and boundary conditions considered are summarized in Table 4.

TABLE 4

SUMMARY OF APPLIED BOUNDARY CONDITIONS

Boundary Class	Inner	Outer	Upper	Lower
I	Q	R	R	R
II	Q	R,Q	R	R
III	Q	R,Q	R,Q	R
IV	Q	R,Q	R	R,Q
V	Q	R,Q	R,Q	R,Q
VI	Q	R,Q,H	R	R
VII	Q	R,Q	R,H	R
VIII	Q	R,Q	R	R,H
IX	Q	R,Q	R,H	R,H
X	Q	R,Q,V1	R	R
XI	Q	R,Q	R,V2	R
XII	Q	R,Q	R	R,V3
XIII	Q	R,Q	R,V2	R,V3
XIV	Q	R,Q,V1	R,V2	R,V3
XV	Q	R,Q	R,V2	R,V2
XVI	Q	R,Q	R,V3	R,V3

Where:

Q = Constant, uniform surface heat flux

R = Radiation to extensive, black surroundings

H = Convection from surface

V1* = Periodic, uniform surface heat flux 1

V2* = Periodic, uniform surface heat flux 2

V3* = Periodic, uniform surface heat flux 3

* As described in Appendix G.

VI. RESULTS

A. DEFINITIONS

1. Convergence

On validation runs, calculated values were considered to have converged when the net remaining change in the value comprized less than one percent of the analytic solution for the given time step. For comparison runs, values were considered to have converged when their difference comprised less than one percent of the value of the smaller of the two numbers.

2. Steady-State

A condition of steady-state was considered to have been reached when the net remaining change in a temperature comprized less than one percent of the final steady-state result for single transient evaluation.

B. VALIDATION

1. Analytic Solution

The analytic solution converged to steady-state at 300 seconds. Consequently, all validation runs were limited to 300 seconds duration to economize on analysis time.

2. Explicit Method

The theoretical limit on the maximum time step allowed for the geometry selected was approximately 0.395 seconds, which did not lend itself to convenient comparisons with the analytic results.

The explicit method was validated using a numerically convenient time step of 0.1000 second and converged with the analytic result within four seconds of the start of the transient cycle (Figure 10). Total execution time was 40.37 seconds for a ten (radial) by four (ϕ) by five (z) grid. The observed error at the end of the 300-second evaluation period was on the order of 0.009 percent. For processing speed comparison runs, the maximum theoretical timestep was employed and demonstrated a minimum execution time of 10.53 CPU seconds. Due to the odd step size, no comparison of relative accuracy was made.

3. Brian's Method

Brian's method was initially limited to a fixed timestep of 0.1 second for the validation process to allow a relative speed comparison with the explicit method. While convergence times and observed errors were comparable (Figures 10, 11, and 12), the explicit method demonstrated a significant speed advantage over Brian's Method for equivalent timesteps (Table 5). This situation was dramatically reversed when the logarithmically increasing timestep arrangement (Table 3) was employed and resulted in an execution speed of 2.53 CPU seconds, which comprised a 410 percent advantage over the optimum result of the explicit method (Table 6).

C. FOLLOW-ON TESTING

Upon conclusion of the validation runs and speed comparisons, a variety of boundary conditions with easily predictable effects were applied to allow observation of system response (Table 4). The following general observations were made.

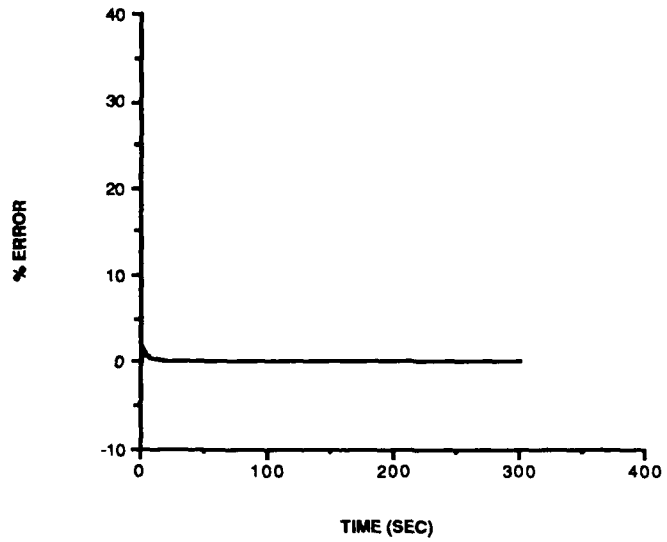


Figure 10. **Percentage Error of Explicit Code as Compared to Analytic Results and Plotted Versus Time**

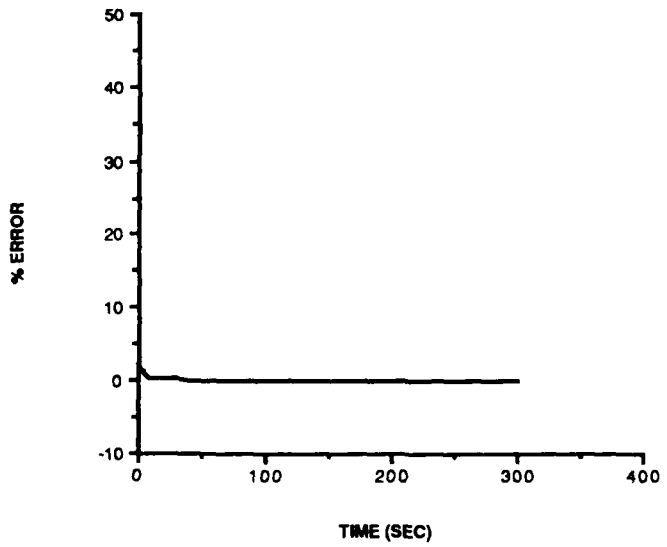


Figure 11. **Percentage Error of Brian's Code as Compared to Analytic Results and Plotted Versus Time**

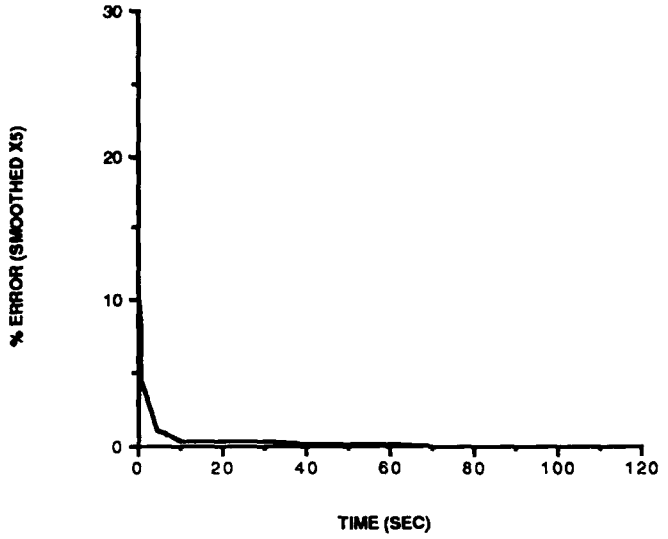


Figure 12. **Comparison of Brian vs. Explicit Validation Results Expressed as a Percentage Difference**

TABLE 5

VALIDATION RESULTS—FIXED 0.1 SEC. TIMESTEP

10 x 4 x 5 GRID
300 SEC TRANSIENT

RESULT	**EXPLICIT**	**BRIAN'S**
STEP SIZE	0.1 SEC	0.1 SEC
CPU TIME	40.37 SEC	223.24 SEC
ERROR AFTER		
0.1%	15.188%	20.308%
1%	1.247%	1.668%
2%	0.612%	0.814%
10%	0.110%	0.183%
100%	0.009%	0.009%
CONVERGENCE (<1 % ERROR)	3–4 SEC	4–5 SEC

SPEED ADVANTAGE: EXPLICIT (553 %)

TABLE 6

VALIDATION RESULTS—LOG-VARIABLE TIMESTEP

BRIAN'S METHOD
VARIABLE STEP SIZE

TIME (SEC)	**STEP SIZE (SEC)**
0.1–1.0	0.1
1.0–10	1.0
10–100	10
100–300	20

% TRANSIENT	**% ERROR**	**% IMPROVEMENT***
0.1	20.308	0.00
1.0	1.446	13.3
10.	-0.057	68.8
100.	0.01	-22.2

CPU TIME: 2.53 SEC

SPEED ADVANTAGE : BRIAN'S (416 %)

* Explicit method with optimum time step.

1. Symmetric Boundary Conditions

For both radially and axi-symmetric boundary conditions, the results of both methods rapidly converged with each other within the first three percent of the transient cycle.

2. Asymmetric Boundary Conditions

When asymmetric boundary conditions were applied, large differences were observed at surfaces nearest the surface of application. These differences were eliminated by apllication of a finer grid

size in the direction of principal flux and are thus attributed to discretization errors due to the generally coarse nature of the applied grid.

D. APPLICATION EXAMPLE

A satellite thermal analysis problem (Appendix G) was selected as offering a practical example of a system subjected to long-period steady-state transient behavior. Figure 13 presents a graphical comparison of the results of both methods for the initial 3000 seconds of the first transient cycle. Figure 14 exhibits the thermal response of selected nodes as calculated by Brian's Method for the initial 10 transient cycles of the application example.

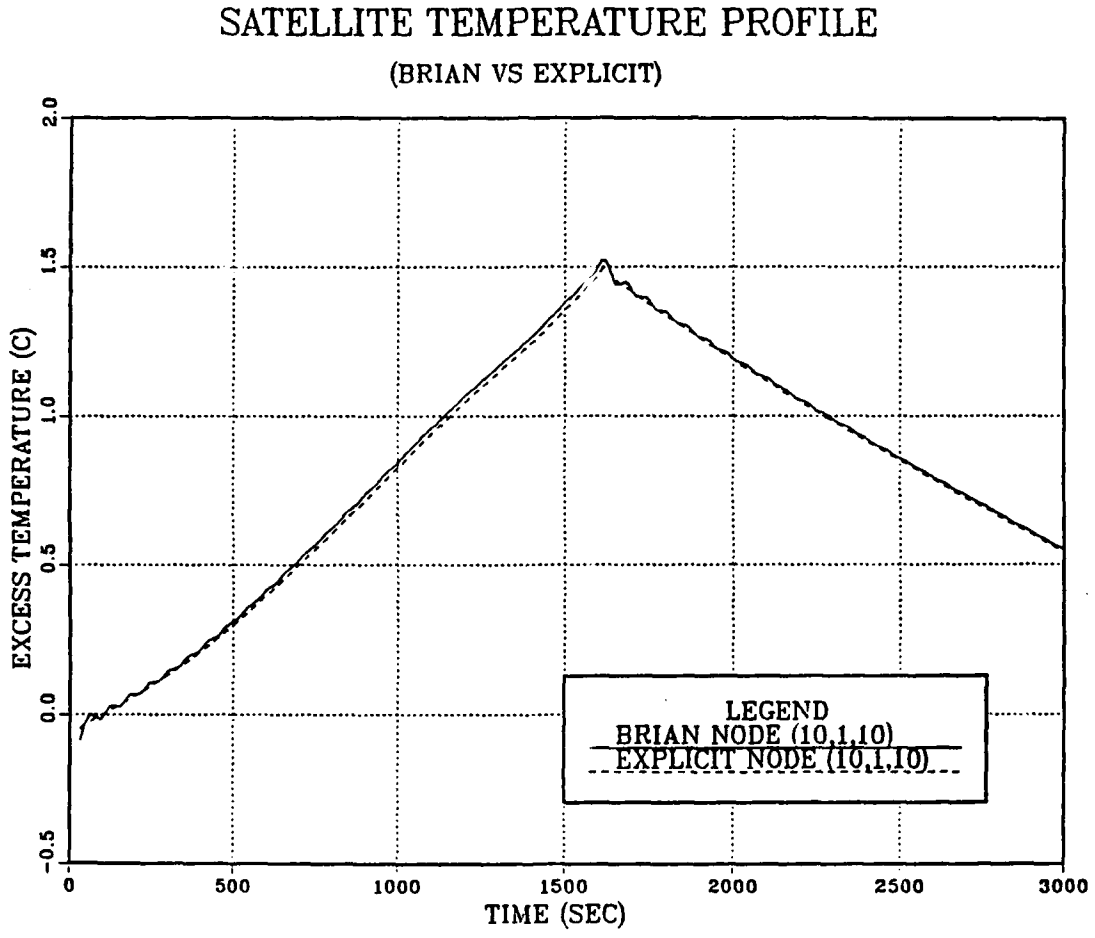


Figure 13. **Graphical Comparison of Brian's Method and the Explicit Method for the Initial 3,000 Seconds of the First Transient Cycle of the Application Problem**

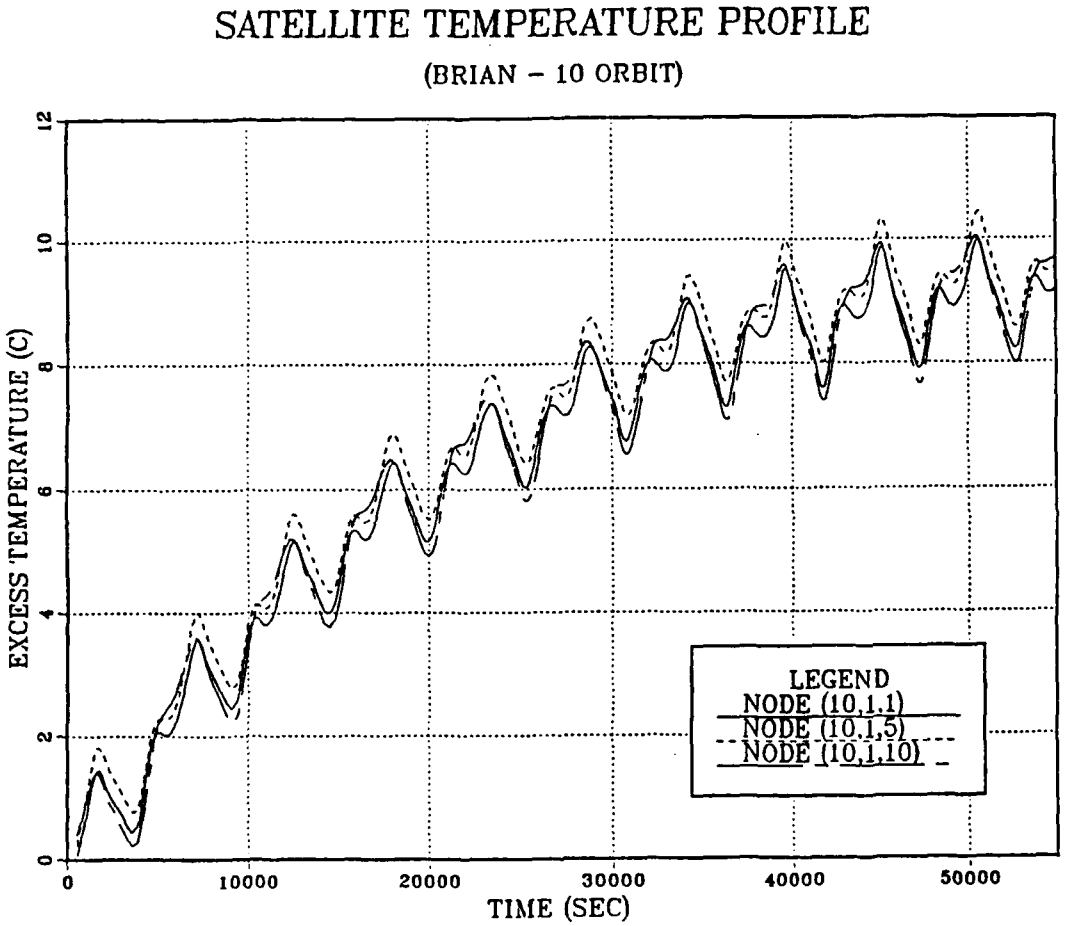


Figure 14. **Thermal Response of Selected Nodes as Calculated by Brian's Method for the Initial 10 Transient Cycles of the Application Problem**

VII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

1. Adaptability

Brian's Method is readily adaptable to cylindrical geometries. However, compensation must be made for the lack of tridiagonality of the coefficient matrix when calculating intermediate temperatures in the phi-direction.

2. Relative Accuracy

For symmetric boundary conditions, Brian's Method delivers accuracy comparable to the explicit method. Convergence is negligibly slower than that observed for the explicit method.

3. Relative Execution Speed

Brian's Method offers distinct advantages over the explicit method for systems demonstrating periodic steady-state temperature response, and for those systems with extended single transient cycles whose geometry makes the explicit timestep limitation overly restrictive. For some geometries, Brian's method is slower than the explicit method and is not recommended.

4. Relative Benefits

The principal benefit of Brian's Method is its unconditional stability and associated freedom from timestep restrictions. This allows the user to apply a coarse grid and timestep to allow rapid coarse analysis with the ability to adjust the step size downward as the

situation warrants. However, the relative advantages of the unrestricted timestep must be carefully considered against the substantial increase in algebra and code requirements.

B. RECOMMENDATIONS

1. Adaptation of Brian's Method FORTRAN Code to C Programming Language

a. Background

The large storage and calculation requirements associated with finite difference approximations of transient heat conduction problems have historically relegated their analysis to the realm of the mainframe computer. However, recent developments in microprocessor technology have dramatically improved the storage capability and execution speeds of personal computers (PCs). Consequently, increasing amounts of engineering analysis and design are being conducted on these devices at significant cost reductions. In research being conducted under the auspices of the Electrical and Computer Science Department at the Naval Postgraduate School, FORTRAN codes involving large numbers of repetetive calculations or iterations have been demonstrated to experience significant improvements in execution speed when converted to the C programming language prior to execution on a personal computer [Ref. 11].

b. Recommendation

It is strongly recommended that research be conducted to investigate the feasibility of and limitations on the conduct of transient heat conduction analysis on personal computers employing a C

programming language adaptation of the Brian's Method FORTRAN code.

2. Incorporation of the Analysis of Directional and Time-Dependent Properties in Brian's Method

a. Background

Composites demonstrate strong directional and temperature dependent thermal properties and are being rapidly assimilated into many engineering applications.

b. Recommendation

It is recommended that additional research be conducted to develop procedures for the incorporation of directional and time-dependent thermal properties into Brian's Method.

APPENDIX A

NODE EQUATIONS FOR EXPLICIT METHOD

A. TYPE 1 NODES—END LAYERS, INNER SURFACE

$$T^{n+1} = \frac{1}{c_5}\left\{T^n + 2Fo\left[(T_e^n + T^n) + c_1(T_E^n + T_W^n - 2T^n)\right.\right.$$

$$+ c_2(T_O^n - T^n)$$

$$+ c_3(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})$$

$$\left.\left. + c_4(q''_I + h_{c_I} T_\infty + h_{r_I} T_{sur})\right]\right\}$$

where

$$c_1 = \frac{k_\phi \Delta z^2}{2 k_z r(r + \frac{\Delta r}{4})\Delta\phi^2}$$

$$c_2 = \frac{k_r(r + \frac{\Delta r}{2})\Delta z^2}{k_z(r + \frac{\Delta r}{4})\Delta r^2}$$

$$c_3 = \frac{\Delta z}{k_z}$$

$$c_4 = \frac{r\Delta z^2}{k_z(r + \frac{\Delta r}{4})\Delta r}$$

$$c_5 = 1 + 2Fo\left[c_3(h_{c_e} + h_{r_e}) + c_4(h_{c_I} + h_{r_I})\right]$$

$$h_{r_e} = \varepsilon_e \sigma (T^n + T_{sur})\left[(T^n)^2 + T_{sur}^2\right]$$

$$h_{r_I} = \varepsilon_I \sigma\ (T^n + T_{sur})\left[(T^n)^2 + T_{sur}^2\right]$$

B. TYPE 2 NODES—END LAYERS, MID-RADII

$$T^{n+1} = \frac{1}{c_5}\left\{T^n + 2\,Fo\left[(T_e^n - T^n) + c_1(T_E^n + T_W^n - 2T^n)\right.\right.$$

$$+\ c_2(T_I^n - T^n)$$

$$+ c_3(T_O^n - T^n)$$

$$\left.\left. + c_4(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})\right]\right\}$$

where

$$c_1 = \frac{k_\phi \Delta z^2}{2\,k_z\, r^2 \phi^2}$$

$$c_2 = \frac{k_r(r - \frac{\Delta r}{2})\Delta z^2}{2\,k_z\, r \Delta r^2}$$

$$c_3 = \frac{k_r(r + \frac{\Delta r}{2})\Delta z^2}{2\,k_z\, r \Delta r^2}$$

$$c_4 = \frac{\Delta z}{k_z}$$

$$c_5 = 1 + 2Fo\left[c_4(h_{c_e} + h_{r_e}) \right]$$

$$h_{r_e} = \varepsilon_e \sigma (T^n + T_{sur})\left[(T^n)^2 + T_{sur}^2 \right]$$

C. TYPE 3 NODES—END LAYERS, OUTER SURFACE

$$T^{n+1} = \frac{1}{c_5}\left\{ T^n + 2Fo\left[(T_e^n - T^n) + c_1 (T_E^n + T_W^n - 2T^n) \right.\right.$$

$$+ c_2 (T_I^n - T^n)$$

$$+ c_3 (q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})$$

$$\left.\left. + c_4 (q''_O + h_{c_O} T_\infty + h_{r_O} T_{sur}) \right]\right\}$$

where

$$c_1 = \frac{k_\phi \Delta z^2}{2 k_z (r - \frac{\Delta r}{4}) r \Delta \phi^2}$$

$$c_2 = \frac{k_r (r - \frac{\Delta r}{2}) \Delta z^2}{k_z (r - \frac{\Delta r}{4}) \Delta r^2}$$

$$c_3 = \frac{\Delta z}{k_z}$$

$$c_4 = \frac{r\Delta z^2}{k_z(r - \frac{\Delta r}{4})\Delta r}$$

$$c_5 = 1 + 2Fo\left[c_3(h_{c_e} + h_{r_e}) + c_4(h_{c_O} + h_{r_O})\right]$$

$$h_{r_e} = \varepsilon_e \sigma\ (T^n + T_{sur})\left[(T^n)^2 + T_{sur}^{\ 2}\right]$$

$$h_{r_O} = \varepsilon_O \sigma\ (T^n + T_{sur})\left[(T^n)^2 + T_{sur}^{\ 2}\right]$$

D. TYPE 4 NODES—MID-LAYERS, INNER SURFACE

$$T^{n+1} = \frac{1}{c_4}\left\{T^n + Fo\left[(T_N^n + T_S^n - 2T^n) + c_1(T_E^n + T_W^n - 2T^n)\right.\right.$$

$$+ c_2(T_O^n - T^n)$$

$$\left.\left. + c_3(q''_I + h_{c_I} T_\infty + h_{r_I} T_{sur})\right]\right\}$$

where

$$c_1 = \frac{k_\phi \Delta z^2}{k_z\, r(r + \frac{\Delta r}{4})\Delta\phi^2}$$

$$c_2 = \frac{2k_r(r + \frac{\Delta r}{2})\Delta z^2}{k_z(r + \frac{\Delta r}{4})\Delta r^2}$$

$$c_3 = \frac{2r\Delta z^2}{k_z(r+\frac{\Delta r}{4})\Delta r}$$

$$c_4 = 1 + Fo\left[c_3(h_{c_I} + h_{r_I})\right]$$

$$h_{r_I} = \varepsilon_I \sigma \ (T^n + T_{sur})\left[(T^n)^2 + T_{sur}^{\ 2}\right]$$

E. TYPE 5 NODES—MID-LAYERS, MID-RADII

$$T^{n+1} = T^n + Fo\Big[(T_N^n + T_S^n - 2T^n) + c_1(T_E^n + T_W^n - 2T^n)$$

$$+ c_2(T_I^n - T^n)$$

$$+ c_3(T_O^n - T^n)\Big]$$

where

$$c_1 = \frac{k_\phi \Delta z^2}{k_z r^2 \Delta\phi^2}$$

$$c_2 = \frac{k_r(r+\frac{\Delta r}{2})\Delta z^2}{k_z r \Delta r^2}$$

$$c_3 = \frac{(r+\frac{\Delta r}{2})\Delta z^2}{k_z \Delta r^2}$$

F. TYPE 6 NODES—MID-LAYERS, OUTER SURFACE

$$T^{n+1} = \frac{1}{c_4}\left\{T^n + Fo\left[(T_N^n + T_S^n - 2T^n) + c_1(T_E^n + T_W^n - 2T^n)\right.\right.$$

$$+ c_2(T_I^n - T^n)$$

$$\left.\left. + c_3(q''_O + h_{c_O} T_\infty + h_{r_O} T_{sur})\right]\right\}$$

where

$$c_1 = \frac{k_\phi \, \Delta z^2}{k_z \, r\,(r - \frac{\Delta r}{4})\Delta\phi^2}$$

$$c_2 = \frac{2k_r(r - \frac{\Delta r}{4})\Delta z^2}{k_z(r - \frac{\Delta r}{4})\Delta r^2}$$

$$c_3 = \frac{2r\,\Delta z^2}{k_z(r - \frac{\Delta r}{4})\Delta r}$$

$$c_4 = 1 + Fo\left[c_3(h_{c_O} + h_{r_O})\right]$$

$$h_{r_O} = \varepsilon_O \sigma \, (T^n + T_{sur})\left[(T^n)^2 + T_{sur}^{\,2}\right]$$

APPENDIX B

NODE EQUATIONS FOR BRIAN'S METHOD

A. TYPE 1 NODES—END LAYERS, INNER SURFACE

$$c_1 = \frac{k_\phi \Delta z^2}{2k_z r (r + \frac{\Delta r}{4}) \Delta \phi^2}$$

$$c_2 = \frac{k_r (r + \frac{\Delta r}{2}) \Delta z^2}{k_z (r + \frac{\Delta r}{4}) \Delta r^2}$$

$$c_3 = \frac{\Delta z}{k_z}$$

$$c_4 = \frac{r \Delta z^2}{k_z (r + \frac{\Delta r}{4}) \Delta r}$$

$$c_5 = 1 + 2Fo\left[c_3 (h_{c_e} + h_{r_e}) + c_4 (h_{c_I} + h_{r_I}) \right]$$

$$h_{r_e} = \varepsilon_e \sigma (T^n + T_{swr}) \left[(T^n)^2 + T_{swr}^2 \right]$$

$$h_{r_I} = \varepsilon_I \sigma (T^n + T_{swr}) \left[(T^n)^2 + T_{swr}^2 \right]$$

1. Step 1 (z-Direction)

$$AT_S^* + BT^* + CT_N^* = D$$

where

$$A = \begin{Bmatrix} 0, & Bottom\ layer\ nodes \\ -\ Fo, & Top\ layer\ nodes \end{Bmatrix}$$

$$B = 1 + Fo\left[1 + c_2(h_{c_e} + h_{r_e}) + c_3(h_{c_I} + h_{r_I})\right]$$

$$C = \begin{Bmatrix} -\ Fo, & Top\ layer\ nodes \\ 0, & Bottom\ layer\ nodes \end{Bmatrix}$$

$$D = T^n + Fo\left[c_1(T_E^n + T_W^n - 2T^n) + c_2(T_O^n - T^n)\right.$$

$$+ c_3(q''_e + h_{c_e}T_\infty + h_{r_e}T_{sur})$$

$$\left. + c_4(q''_I + h_{c_I}T_\infty + h_{r_I}T_{sur})\right]$$

2. Step 2 (ϕ-Direction)

$$AT_E^{**} + BT^{**} + CT_W^{**} = D$$

where

$$A = -\ c_1 Fo$$

$$B = 1 + Fo\left[2c_1 + c_3(h_{c_e} + h_{r_e}) + c_4(h_{c_I} + h_{r_I})\right]$$

$$C = -\,c_1 Fo$$

$$D = T + Fo\left[(T_e^n - T^*) + c_4(T_O^n - T^n)\right.$$

$$+ c_3(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})$$

$$\left. + c_4(q''_I + h_{c_I} T_\infty + h_{r_I} T_{sur})\right]$$

3. Step 3 (r-Direction)

$$AT_I^{***} + BT^{***} + CT_O^{***} = D$$

where

$$A = 0$$

$$B = 1 + Fo\left[c_2 + c_3(h_{c_e} + h_{r_e}) + c_4(h_{c_I} + h_{r_I})\right]$$

$$C = -\,c_2 Fo$$

$$D = T^n + Fo\left[(T_e^* - T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**})\right.$$

$$+ c_3(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})$$

$$+ c_4(q''_I + h_{c_I} T_\infty + h_{r_I} T_{sur})\Big]$$

4. Step 4 (ϕ-Direction)

$$T^{n+1} = \frac{1}{c_5}\Big\{T^n + 2Fo\Big[(T_e^* + T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**})$$

$$+ c_2(T_O^{***} - T^{***})$$

$$+ c_3(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})$$

$$+ c_4(q''_I - h_{c_I} T_\infty - h_{r_I} T_{sur})\Big]\Big\}$$

B. TYPE 2 NODES—END LAYERS, MID-RADII

$$c_1 = \frac{k_\phi \Delta z^2}{2k_z r^2 \phi^2}$$

$$c_2 = \frac{k_r(r - \frac{\Delta r}{2})\Delta z^2}{2k_z r \Delta r^2}$$

$$c_3 = \frac{k_r(r + \frac{\Delta r}{2})\Delta z^2}{2k_z r \Delta r^2}$$

$$c_4 = \frac{\Delta z}{k_z}$$

$$c_5 = 1 + 2Fo\left[c_4(h_{c_e} + h_{r_e})\right]$$

$$h_{r_e} = \varepsilon_e \sigma \ (T^n + T_{sur}) \left[(T^n)^2 + T_{sur}^2 \right]$$

1. Step 1 (z-Direction)

$$AT_S^* + BT^* + CT_N^*$$

where

$$A = \begin{Bmatrix} 0, & Bottom\ layer\ nodes \\ -Fo, & Top\ layer\ nodes \end{Bmatrix}$$

$$B = 1 + Fo\left[1 + c_4(h_{c_e} + h_{r_e})\right]$$

$$C = \begin{Bmatrix} -Fo, & Top\ layer\ nodes \\ 0, & Bottom\ layer\ nodes \end{Bmatrix}$$

$$D = T^n + Fo \left[c_1(T_E^n + T_W^n - 2T^n) + c_2(T_I^n - T^n) \right.$$

$$+ c_3(T_O^n - T^n)$$

$$\left. + c_4(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur}) \right]$$

2. Step 2 (ϕ-Direction)

$$AT_E^{**} + BT^{**} + CT_W^{**} = D$$

where

$$A = -c_1 Fo$$

$$B = 1 + Fo\left[2c_1 + c_4(h_{c_e} + h_{r_e})\right]$$

$$C = -\,c_1 Fo$$

$$D = T^{n} + Fo\left[(T_e^{*} - T^{*}) + c_2(T_I^{n} - T^{n}) + c_3(T_O^{n} - T^{n})\right.$$

$$\left. + c_4(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})\right]$$

3. Step 3 (r-Direction)

$$AT_I^{***} + BT^{***} + CT_O^{***} = D$$

where

$$A = -\,c_2 Fo$$

$$B = 1 + Fo\left[c_2 + c_3 + c_4(h_{c_e} + h_{r_e})\right]$$

$$C = -\,c_3 Fo$$

$$D = T^{n} + Fo\left[(T_e^{*} - T^{*}) + c_1(T_E^{**} + T_W^{**} - 2T^{**})\right.$$

$$\left. + c_4(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})\right]$$

4. Step 4 (New Temperatures)

$$T^{n+1} = \frac{1}{c_5}\Big\{T^n + 2Fo\,[(T_e^* - T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**})$$

$$+ c_2(T_I^{**} - T^{***}) + c_3(T_O^{***} - T^{***})$$

$$+ c_4(q''_e + h_{c_e}T_\infty + h_{r_e}T_{sur})\Big]\Big\}$$

C. TYPE 3 NODES—END LAYERS, OUTER SURFACE

$$c_1 = \frac{k_\phi \Delta z^2}{2k_z(r - \frac{\Delta r}{4})r\Delta\phi^2}$$

$$c_2 = \frac{k_r(r - \frac{\Delta r}{2})\Delta z^2}{k_z(r - \frac{\Delta r}{4})\Delta r^2}$$

$$c_3 = \frac{\Delta z}{k_z}$$

$$c_4 = \frac{r\Delta z^2}{k_z(r - \frac{\Delta r}{4})\Delta r}$$

$$c_5 = 1 + 2Fo\Big[c_3(h_{c_e} + h_{r_e}) + c_4(h_{c_O} + h_{r_O})\Big]$$

$$h_{r_e} = \varepsilon_e\sigma(T^n + T_{sur})\Big[(T^n)^2 + T_{sur}^2\Big]$$

$$h_{r_O} = \varepsilon_O \sigma \ (T^n + T_{sur}) \left[(T^n)^2 + T_{sur}^2 \right]$$

1. Step 1 (z-Direction)

$$AT_S^* + BT^* + CT_N^* = D$$

where

$$A = \begin{Bmatrix} 0, & Bottom\ layer\ nodes \\ -Fo, & Top\ layer\ nodes \end{Bmatrix}$$

$$B = 1 + Fo \left[1 + c_3 (h_{c_e} + h_{r_e}) + c_4 (h_{c_O} + h_{r_O}) \right]$$

$$C = \begin{Bmatrix} -Fo, & Top\ layer\ nodes \\ 0, & Bottom\ layer\ nodes \end{Bmatrix}$$

$$D = T^n + Fo \left[c_1 (T_E^n + T_W^n - 2T^n) + c_2 (T_I^n - T^n) \right.$$

$$+ c_3 (q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})$$

$$\left. + c_4 (q''_O - h_{c_O} T_\infty - h_{r_O} T_{sur}) \right]$$

2. Step 2 (ϕ-Direction)

$$AT_E^{**} + BT^{**} + CT_W^{**} = D$$

where

$$A = -c_1 Fo$$

$$B = 1 + Fo\left[2c_1 + c_3(h_{c_e} + h_{r_e}) + c_4(h_{c_O} + h_{r_O})\right]$$

$$C = -\,c_1 Fo$$

$$D = T^n + Fo\left[(T_e^* - T^*) + c_2(T_I^n - T^n)\right.$$

$$+ c_3(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})$$

$$\left. + c_4(q''_O + h_{c_O} T_\infty + h_{r_O} T_{sur})\right]$$

3. Step 3 (r-Direction)

$$AT_I^{***} + BT^{***} + CT_O^{***} = D$$

where

$$A = -\,c_2 Fo$$

$$B = 1 + Fo\left[c_2 + c_3(h_{c_e} + h_{r_e}) + c_4(h_{c_O} + h_{r_O})\right]$$

$$C = 0$$

$$D = T^n + Fo\left[(T_e^* - T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**})\right.$$

$$+ c_3(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})$$

$$+ c_4(q''_O + h_{c_O} T_\infty + h_{r_O} T_{sur})\Big]$$

4. Step 4 (New Temperatures)

$$T^{n+1} = \frac{1}{c_5}\Big\{T^n + 2Fo\Big[(T_e^* - T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**})$$

$$+ c_2(T_I^{***} - T^{***})$$

$$+ c_3(q''_e + h_{c_e} T_\infty + h_{r_e} T_{sur})$$

$$+ c_4(q''_e + h_{c_O} T_\infty + h_{r_O} T_{sur})\Big]\Big\}$$

D. TYPE 4 NODES—MID-LAYERS, INNER SURFACE

$$c_1 = \frac{k_\phi \Delta z^2}{k_z r(r + \frac{\Delta r}{4})\Delta\phi^2}$$

$$c_2 = \frac{2k_r(r + \frac{\Delta r}{2})\Delta z^2}{k_z(r + \frac{\Delta r}{4})\Delta r^2}$$

$$c_3 = \frac{2r\Delta z^2}{k_z(r + \frac{\Delta r}{4})\Delta r}$$

$$c_6 = 1 + Fo\,[c_3(h_{c_I} + h_{r_I})]$$

$$h_{r_I} = \varepsilon_I \sigma\,(T^n + T_{sur})\Big[(T^n)^2 + T_{sur}^2\Big]$$

1. Step 1 (z-Direction)

$$AT_S^* + BT^* + CT_N^* = D$$

where

$$A = -\frac{Fo}{2}$$

$$B = 1 + \frac{Fo}{2}\left[2 + c_3(h_{c_I} + h_{r_I})\right]$$

$$C = -\frac{Fo}{2}$$

$$D = T^n + \frac{Fo}{2}\left[c_1(T_E^n + T_W^n - 2T^n) + c_2(T_O^n - T^n)\right.$$

$$\left. + c_3(q''_I + h_{c_I}T_\infty + h_{r_I}T_{sur})\right]$$

2. Step 2 (ϕ-direction)

$$AT_E^{**} + BT^{**} + CT_W^{**} = D$$

where

$$A = -\frac{c_1 Fo}{2}$$

$$B = 1 + \frac{Fo}{2}\left[2c_1 + c_3(h_{c_I} + h_{r_I})\right]$$

$$C = -\frac{c_1 Fo}{2}$$

$$D = T^n + \frac{Fo}{2}\left[(T_N^* + T_S^* - 2T^*) + c_2(T_O^n - T^n)\right.$$

$$\left. + c_3(q''_I + h_{c_I} T_\infty + h_{r_I} T_{sur})\right]$$

3. Step 3 (r-Direction)

$$AT_I^{***} + BT^{***} + CT_O^{***} = D$$

where

$$A = 0$$

$$B = 1 + \frac{Fo}{2}\left[c_2 + c_3(h_{c_I} + h_{r_I})\right]$$

$$C = -\frac{c_2 Fo}{2}$$

$$D = T + \frac{Fo}{2}\left[(T_N^* + T_S^* - 2T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**})\right.$$

$$\left. + c_3(q''_I + h_{c_I} T_\infty + h_{r_I} T_{sur})\right]$$

4. Step 4 (New Temperatures)

$$T^{n+1} = \frac{1}{c_6}\left\{T^n + Fo\left[(T_N^* + T_S^* - 2T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**})\right.\right.$$

$$+ c_2(T_o^{***} - T^{***})$$

$$+ c_3(q''_1 + h_{c_1} T_\infty + h_{r_1} T_{sur})\Big]\Big\}$$

E. TYPE 4 NODES—MID-LAYERS, MID-RADII

$$c_1 = \frac{k_\phi \Delta z^2}{k_z r^2 \Delta\phi^2}$$

$$c_2 = \frac{k_r (r + \frac{\Delta r}{2}) \Delta z^2}{k_z r \Delta r^2}$$

$$c_3 = \frac{(r + \frac{\Delta r}{2}) \Delta z^2}{k_z \Delta r^2}$$

1. Step 1 (z-Direction)

$$AT_S^* + BT^* + CT_N^* = D$$

where

$$A = -\frac{Fo}{2}$$

$$B = 1 + Fo$$

$$C = -\frac{Fo}{2}$$

$$D = T + \frac{Fo}{2}\left[c_1(T_E^n + T_W^n - 2T^n) + c_2(T_I^n - T^n) + c_3(T_O^n - T^n)\right]$$

2. Step 2 (ϕ-Direction)

$$AT_E^{**} + BT^{**} + CT_W^{**} = D$$

where

$$A = -\frac{c_1 Fo}{2}$$

$$B = 1 + c_1 Fo$$

$$C = -\frac{c_1 Fo}{2}$$

$$D = T + \frac{Fo}{2}\left[(T_N^* + T_S^* - 2T^*) + c_2(T_I^n - T^n) + c_3(T_O^n - T^n)\right]$$

3. Step 3 (r-Direction)

$$AT_I^{***} + BT^{***} + CT_O^{***} = D$$

where

$$A = -\frac{c_2 Fo}{2}$$

$$B = 1 + \frac{Fo}{2}(c_2 + c_3)$$

$$C = -\frac{c_3 Fo}{2}$$

$$D = T^n + \frac{Fo}{2}\left[(T_N^* + T_S^* - 2T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**})\right]$$

4. Step 4 (New Temperatures)

$$T^{n+1} = T^n + Fo\left[(T_N^* + T_S^* - 2T^*) + c_1(T_E^{**} + T_W^{**} - 2T^{**}) + c_2(T_I^{***} - T^{***})\right.$$

$$\left. + c_3(T_O^{***} - T^{***})\right]$$

F. TYPE 4 NODES—MID-LAYERS, OUTER SURFACE

$$c_1 = \frac{k_\phi \Delta z^2}{k_z r(r - \frac{\Delta r}{4})\Delta\phi^2}$$

$$c_2 = \frac{2k_r(r - \frac{\Delta r}{4})\Delta z^2}{k_z(r - \frac{\Delta r}{4})\Delta r^2}$$

$$c_3 = \frac{2r\Delta z^2}{k_z(r - \frac{\Delta r}{4})\Delta r}$$

$$c_4 = 1 + Fo\left[c_3(h_{c_O} + h_{r_O})\right]$$

$$h_{r_O} = \varepsilon_O \sigma\ (T^n + T_{sur})\left[(T^n)^2 + T_{sur}^2\right]$$

1. Step 1 (z-Direction)

$$AT_S^* + BT^* + CT_N^* = D$$

where

$$A = -\frac{Fo}{2}$$

$$B = 1 + \frac{Fo}{2}\left[2 + c_3(h_{c_O} + h_{r_O})\right]$$

$$C = -\frac{Fo}{2}$$

$$+ c_3(q''_O + h_{c_O} T_\infty + h_{r_O} T_{sur})\Big]$$

2. Step 2 (ϕ-direction)

$$AT_E^{**} + BT^{**} + CT_W^{**} = D$$

where

$$A = -\frac{c_1 Fo}{2}$$

$$B = 1 + \frac{Fo}{2} + 2c_1 + c_3(h_{c_O} + h_{r_O})$$

$$C = -\frac{c_1 Fo}{2}$$

$$D = T^{n} + \frac{Fo}{2}\left[(T_N^{*} + T_S^{*} - 2T^{*}) + c_2(T_I^{n} - T^{n})\right.$$

$$\left. + c_3(q''_O + h_{c_O} T_\infty + h_{r_O} T_{sur})\right]$$

3. Step 3 (r-Direction)

$$AT_I^{***} + BT^{***} + CT_O^{***} = D$$

where

$$A = -\frac{c_2 Fo}{2}$$

$$B = 1 + \frac{Fo}{2} c_2 + c_3(h_{c_O} + h_{r_O})$$

$$C = 0$$

$$D = T^{n} + \frac{Fo}{2}\left[(T_N^{*} + T_S^{*} - 2T^{*}) + c_1 (T_E^{n} + T_W^{n} - 2T^{n})\right.$$

$$\left. + c_3(q''_O + h_{c_O} T_\infty + h_{r_O} T_{sur})\right]$$

4. Step 4 (New Temperatures)

$$T^{n+1} = \frac{1}{c_4}\left\{T^{n} + Fo\left[(T_N^{*} + T_S^{*} - 2T^{*}) + c_1 (T_E^{**} + T_W^{**} - 2T^{**})\right.\right.$$

$$+ c_2(T_I^{***} - T^{***})$$

$$\left.\left. + c_3(q''_O + h_{c_O} T_\infty + h_{r_O} T_{sur})\right]\right\}$$

APPENDIX C

VALIDATION MODEL

A. PROBLEM SPECIFICATION

Given a right circular cylinder with adiabatic ends and specified initial temperature distribution and constant inner-surface temperature of zero degrees Celsius. At time t = 0, a uniform constant heat flux is applied to the outer surface of the cylinder. Assume constant properties and no losses due to convection or radiation.

B. ANALYTIC SOLUTION

Fourier Equation

$$\nabla^2 T = \frac{1}{\alpha}\frac{\delta T}{\delta t}$$

Boundary Conditions:

$$r_o = a \qquad T = 0$$

$$r_o = b \qquad \frac{\partial T}{\partial t} = \frac{q''}{k}$$

Initial Condition:

$$t = 0 \qquad T = 0$$

Let: $T = \Psi(r_o, t) + \Phi(r_o)$

Auxiliary Problem

$$\nabla^2 \Psi(r_o, t) + \nabla^2 \Phi(r_o) = \frac{1}{\alpha} \frac{\partial \Psi(r_o, t)}{\partial t}$$

$r_o = a$ $\qquad \Psi(r_o, t) + \nabla^2 \Phi(r_o) = \frac{1}{\alpha} \frac{\partial \Psi}{\partial t}$

$r_o = b$ $\qquad \frac{\partial}{\partial r} \Psi(r_o, t) + \frac{d\Phi}{dr_o} = \frac{q''}{k}$

$t = 0$ $\qquad \Psi(r_o, 0) + \Phi(r_o) = 0$

Separating the Auxiliary Problem

Equation 1:

$$\frac{d^2\Phi}{dr_o^2} + \frac{1}{r} \cdot \frac{d\Phi}{dr_o} = 0$$

$r_o = a$ $\qquad \Phi(a) = 0$

$r_o = b$ $\qquad \frac{d\Phi}{dr_o} = \frac{q''}{k}$

Let: $y = \frac{d\Phi}{dr_o}$

Then:

$$\frac{dy}{dr_o} = -\frac{y}{r_o}$$

$$\frac{dy}{y} = -\frac{dr_o}{r_o}$$

$$\ln(y) = -\ln(r_o) + c_1$$

$$y = \frac{c_1}{r_o}$$

And:

$$\frac{d\Phi}{dr_o} = \frac{c_1}{r_o}$$

$$d\Phi = c_1 \frac{dr_o}{r_o}$$

Yielding:

$$\Phi = c_1 \ln(r_o) + c_2$$

Evaluating Constants

$$r = a \quad \rightarrow \quad \Phi = 0 \quad \Rightarrow \quad 0 = c_1 \ln(a) + c_2$$

$$c_2 = -\frac{b\,q''}{k} \ln(a)$$

$$r = b \quad \rightarrow \quad \frac{d\Phi}{dr_o} = \frac{q''}{k} \quad \Rightarrow \quad c_1 = \frac{b \cdot q''}{k}$$

Thus: $\Phi(r_o) = \frac{b\,q''}{k} \ln\left(\frac{r_o}{a}\right)$

Transforming the Variable

Let: $r = \frac{r_o}{b}$

And: $\kappa = \frac{a}{b}$

Then: $\Phi(r) = \frac{b\,q''}{k} \ln(\frac{r}{\kappa})$

Converting Boundary Conditions

$$r_o = a \qquad \Rightarrow \qquad r = \frac{r_o}{b} = \kappa$$

$$r_o = b \qquad \Rightarrow \qquad r = 1$$

The Transformed Problem Becomes

$$\Phi(r) = \frac{b\,q''}{k} \ln(\frac{r}{\kappa})$$

$$r = \kappa \qquad \phi = 0$$

$$r = 1 \qquad \frac{d\phi(r)}{dr} = \frac{bq''}{k}$$

Equation 2:

$$\frac{\partial^2 \Psi}{\partial r_o^{\,2}} + \frac{1}{r_o}\frac{\partial \Psi}{\partial r_o} = \frac{1}{\alpha}\frac{\partial \Psi}{\partial t}$$

$$r_o = a \qquad \Psi(a) = 0$$

$$r_o = b \qquad \frac{\partial \Psi}{\partial r_o} = 0$$

$$t = 0 \qquad \Psi\,(r_o,0) = -\,\Phi\,(r_o)$$

Let: $\Psi\,(r_o,t) = R(r_o)\,\Gamma\,(t)$

Then: $R''\Gamma + \frac{1}{r}R'\Gamma = \frac{1}{\alpha}R\Gamma'$

Which reduces to

$$\frac{R''}{R} + \frac{1}{r_o}\frac{R'}{R} = \frac{1}{\alpha}\frac{\Gamma'}{\Gamma} = -\beta^2$$

Separating the two equalities, we obtain

Equation 2.a

$$\frac{1}{\alpha}\cdot\frac{\Gamma'}{\Gamma} = -\beta^2$$

$$\ln(\Gamma) = -\alpha\beta^2 t$$

$$\Gamma\,(t) = e^{-\alpha\beta^2 t}$$

Equation 2.b

$$r_o = a \qquad R = 0$$

$$r_o = b \qquad \frac{dR}{dr_o} = 0$$

From Table 3-2, Case B of *Özisik*

$$R(\beta^{*}, r_o) = J_o(\beta^{*} r_o) \cdot Y'_o(\beta^{*} b) - J'_o(\beta^{*} b) \cdot Y_o(\beta^{*} r_o)$$

the inverse of the norm is given by

$$\frac{1}{N(\beta^{*})} = \frac{\frac{\pi^2}{2} \beta^{*2} J_o^2(\beta^{*} a)}{J_o^2(\beta^{*} a) - J'^2_o(\beta^{*} b)}$$

and the eigenvalues are the roots of

$$J_o(\beta^{*} a) \cdot Y'_o(\beta^{*} b) - J'_o(\beta^{*} b) \cdot Y_o(\beta^{*} a) = 0$$

From the recursion relations we know

$$J'_o(x) = -J_1(x)$$

and similarly

$$Y'_o(x) = -Y_1(x)$$

Substituting, multiplying by -1, and rearranging terms yields

$$Y_1(\beta^{*} b) \cdot J_o(\beta^{*} a) - J_1(\beta^{*} b) \cdot Y_o(\beta^{*} a) = 0$$

Transforming Variables:

Letting: $\beta = \beta^{*} b$

and recalling that: $\kappa = \frac{a}{b}$

we obtain by substitution the common tabular form:

$$Y_1(\beta) \cdot J_o(\kappa \beta) - J_1(\beta) \cdot Y_o(\kappa \beta) = 0$$

Performing similar variable transformations on the transient solution yields:

$$R(\beta, r) = J_1(\beta) \cdot Y(r\beta) - J_o(r\beta) \cdot Y_1(\beta)$$

with the norm

$$N(\beta) = \frac{2}{\pi^2} \frac{J_o^2(\kappa\beta) - J_1^2(\beta)}{\frac{\beta^2}{b^2} J_o^2(\kappa\beta)}$$

Total Solution:

$$T(r,t) = \frac{bq}{k} \ln\left(\frac{r}{k}\right) \sum_i \{\exp\left(\frac{-\alpha \beta_i^2 t}{b^2}\right) \frac{1}{N_i(\beta)} R_i(r,\beta)$$

$$\cdot \int_\kappa^1 [\rho R_i(r,\beta) \frac{b \cdot q}{\kappa} \ln\left(\frac{\rho}{\kappa}\right)] d\rho \}$$

APPENDIX D

EXPLICIT METHOD STABILITY REQUIREMENTS

The following equations represent the limiting time step size for their respective node types.

A. TYPE 1 NODES—END LAYER, INNER SURFACE

$$\Delta t_1 \leq 1 + \frac{k_\phi \Delta z^2}{k_z r (r + \frac{\Delta r}{4}) \Delta \phi^2} + \frac{k_r (r + \frac{\Delta r}{2}) \Delta z^2}{k_z (r + \frac{\Delta r}{4}) \Delta r^2}$$

B. TYPE 2 NODES—END LAYERS, MID-RADII

$$\Delta t_2 \leq 1 + \frac{k_\phi \Delta z^2}{k_z r^2 \Delta \phi^2} + \frac{k_r (r - \frac{\Delta r}{2}) \Delta z^2}{2 k_z r \Delta r^2}$$

C. TYPE 3 NODES—END LAYERS, OUTER SURFACE

$$\Delta t_3 \leq 1 + \frac{k_\phi \Delta z^2}{k_z r (r + \frac{\Delta r}{4}) \Delta \phi^2} + \frac{k_r (r + \frac{\Delta r}{2}) \Delta z^2}{k_z (r + \frac{\Delta r}{4}) \Delta r^2}$$

D. TYPE 4 NODES—MID-LAYERS, INNER SURFACE

$$\Delta t_4 \leq 2 + \frac{2 k_\phi \Delta z^2}{k_z (r - \frac{\Delta r}{4}) \Delta \phi^2} + \frac{2 k_\phi (r + \frac{\Delta r}{4})}{\Delta r^2}$$

E. TYPE 5 NODES—MID-LAYERS, MID-RADII

$$\Delta t_5 \leq 2 + \frac{2k_\phi \Delta z^2}{k_z r^2 \Delta \phi^2} + \frac{k_r(r - \frac{\Delta r}{2})}{k_z r \Delta r^2} + \frac{k_r(r + \frac{\Delta r}{2})}{k_z r \Delta r^2}$$

F. TYPE 6 NODES—MID-LAYERS, OUTER-SURFACE

$$\Delta t_6 \leq \frac{2k_\phi \Delta z^2}{k_z r (r - \frac{\Delta r}{4}) \Delta \phi^2} + \frac{2k_r(r - \frac{\Delta r}{2}) \Delta z^2}{k_z (r - \frac{\Delta r}{4}) \Delta r^2}$$

APPENDIX E

EXPLICIT METHOD CODE

```
C     ECR FORTRAN A - (REV. 11/23/88)
C     SATELLITE MODEL - VALIDATED 11/26/88
C     ************************************************************
C     * ECR - PROGRAM TO COMPUTE NODE TEMPERATURES FOR A RIGHT   *
C     *       CIRCULAR CYLINDER SUBJECTED TO CONVECTIVE AND/OR   *
C     *       RADIATIVE BOUNDARY CONDITIONS USING THE EXPLICIT   *
C     *       METHOD OF FINITE DIFFERENCE ANALYSIS.              *
C     ************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION TOLD(15,15,15),TNEW(15,15,15)
      DIMENSION DMAT(15,15,15)

      COMMON /ARRSIZ/ III,JJJ,KKK
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
      COMMON /TIMES/  DELTIM,DTMAX,FINTIM,LIMIT
C     START TIMER
      CALL STIME
      OPEN (UNIT=11,FILE='ECRLNG')
      OPEN (UNIT=14,FILE='ECRDAT')
      OPEN (UNIT=15,FILE='ECRCPU')
C********************** USER DEFINED VARIABLES ************************
      DELTIM = 0.30D+00
      FINTIM = 3000.0D+00
      IPRINT = 100
C     ACTUAL MATRIX DIMENSIONS
      NODES  = 10
      NPHI   = 4
      NLAYER = 5
C     NODE SPECIFICATION FOR FILE ESHORT
      MRAD   = 10
      MPHI   = 2
      MLAYER = 3
C     DECLARED ARRAY DIMENSIONS
      III = 15
      JJJ = 15
      KKK = 15
C     ABSORPTIVITY COEFFICIENTS
      ALFAN = 0.4D+00
      ALFAS = 0.4D+00
```

```
      ALFAO = 0.4D+00
C     ALBEDO COEFFICIENT
      ALBEDO = 0.30D+00
C     THERMAL CONDUCTIVITIES
      DKR = 177.D+00
      DKP = 177.D+00
      DKZ = 177.D+00
C     EMISSIVITIES
      EPSUBN = 0.9D+00
      EPSUBS = 0.9D+00
      EPSUBI = 0.D+00
      EPSUBO = 0.9D+00
C     CONVECTION COEFFICIENTS
      HSUBN = 0.D+00
      HSUBS = 0.D+00
      HSUBI = 0.D+00
      HSUBO = 0.D+00
C     HEAT FLUXES
      SOLAR = 1353.D+00
      EIR   = 238.0D+00
      GEN = 1000.
      QPSUBI = 0.D+00
      QPSUBO = 0.D+00
      QPSUBN = 0.D+00
      QPSUBS = 0.D+00
C     GEOMETRY
      RMIN = 0.15244D+00
      RMAX = 0.22866D+00
      ZMAX = 1.D+00
C     VIEW FACTORS WITH THE EARTH
      VERTHN = 0.D+00
      VERTHS = 1.D+00
      VERTHI = 0.D+00
      VERTH0 = 0.D+00
C     VIEW FACTORS WITH SPACE
      VSPCN = 1.D+00
      VSPCS = 0.D+00
      VSPCI = 0.D+00
      VSPCO = 1.D+00
C     TEMPERATURES
      TABS = 273.D+00
      TINIT = 293.D+00
      TINF  = 0.D+00
      TSUR  = 4.D+00
C     MATERIAL PROPERTIES
      CSUBP = 875.0D+00
      RHO   = 2270.0D+00
C     INITIALIZE OTHER VARIABLES
      ICOUNT = 0
      ITIME  = 0
      ORBTIM = 0.D+00
      PI     = 4.0D+00*DTAN(1.D+00)
      RAD    = RMIN
      SIGMA  = 5.67D-08
      TIME   = 0.D+00
```

```
      T1     = 0.D+00
C     INITIALIZE MATRICES
      CALL BMINIT (TOLD,TINIT)
      CALL BMINIT (TNEW,TINIT)
C     SURFACE AREA CALCULATIONS
      SAREAN = PI*(RMAX**2 - RMIN**2)
      SAREAS = PI*(RMAX**2 - RMIN**2)
      SAREAI = PI*(2*RMIN)*ZMAX
      SAREAO = PI*(2*RMAX)*ZMAX
C     INITIAL CALCULATIONS
      ALFA = DKZ/(RHO*CSUBP)
      DPHI = 2.*PI/NPHI
      DR = (RMAX-RMIN)/(NODES-1)
      DZ = ZMAX/(NLAYER-1)
      LPHI = 360/NPHI
      QPSUBI = GEN/SAREAI
C     ORBITAL PARAMETERS
      PERIOD = 5440.D+00
      ECLIPS = 2181.9D+00
      OMEGA = 1.155D-03
      TAU = 0.01*PERIOD/4
      TIME1 = 0.D+00
      TIME2 = PERIOD/4.D+00
      TIME3 = 107.8D+00/360 * PERIOD
      TIME4 = (270 - 17.8)/360 * PERIOD - 4*TAU
      TIME5 = (270 - 17.8)/360 * PERIOD
      TIME6 = 270/360 * PERIOD
C     ESTABLISH STABILITY REQUIREMENT
      CALL STABIL (DTMAX)
      FO = ALFA*DELTIM/(DZ**2)
      LIMIT = FINTIM/DELTIM + 500
C     WRITE MATRIX DIMENSIONS TO FILE 14 (ECRDAT)
      WRITE (14,*) NODES,NPHI,NLAYER
C
C     *** PRINT OPTION 3 - SAVES DESIGNATED NODE TEMPS AT SPECIFIED
C                          PRINT INTERVAL.  USE FOR LONG PERIOD EVALS.
      WRITE (11,*) 'ECRSAT'
      WRITE (11,*)
      WRITE (11,888)
888   FORMAT (T6,'TIME',T18,'DELTIM',T30,'LAYER1',T45,'LAYER3',T60,'LAYE
      1R')
      WRITE (11,*)
C
CMMMMMMMMMMMMMMMMMMM BEGIN TEMPERATURE COMPUTATIONS MMMMMMMMMMMMMMMMMMM
C     GET TIME ENTERING LOOP (MAINFRAME VERSIONS OF CODE ONLY)
      WRITE (11,*) 'ENTERING LOOP TIME'
      CALL GTIME(1,11)
      DO 100 I=1,LIMIT
           ICOUNT=ICOUNT+1
           TIME=TIME+DELTIM
           ORBTIM = ORBTIM + DELTIM
           IF (ORBTIM.GE.PERIOD) ORBTIM = ORBTIM - PERIOD
C     SURFACE FLUX CALCULATIONS (TIME DEPENDENT)
           IF ((ORBTIM.GE.0).AND.(ORBTIM.LE.TIME2)) THEN
                QPSUBN = ALFAN*SOLAR*DCOS(OMEGA*ORBTIM)
```

```
                QPSUBS = ALFAS*(EIR + ALBEDO*SOLAR*DCOS(OMEGA*ORBTIM))
                QPSUBO = 0.5*ALFAO*SOLAR*DSIN(OMEGA*ORBTIM)
           ELSEIF ((ORBTIM.GT.TIME2).AND.(ORBTIM.LE.TIME3))THEN
                QPSUBN = 0.
                QPSUBS = ALFAS*(EIR + (ALBEDO+DCOS(OMEGA*ORBTIM))*SOLAR)
                QPSUBO = 0.5*ALFAO*SOLAR*DSIN(OMEGA*ORBTIM)
           ELSEIF ((ORBTIM.GT.TIME3).AND.(ORBTIM.LE.TIME4)) THEN
                QPSUBN = 0.
                QPSUBS = ALFAS*EIR
                QPSUBO = 0.D+00
           ELSEIF ((ORBTIM.GT.TIME4).AND.(ORBTIM.LE.TIME5)) THEN
                QPSUBN = 0.D+00
                QPSUBS = ALFAS*(EIR
          1                + DABS(ALBEDO*SOLAR*DSIN(OMEGA*ORBTIM))
          1                *(1-DEXP(-(ORBTIM-TIME4)/TAU)))
                QPSUBO = 0.5*ALFAO*DABS(ALBEDO*SOLAR
          1                + DSIN(OMEGA*ORBTIM)
          1                *(1-DEXP(-(ORBTIM-TIME4))/TAU))
           ELSEIF ((ORBTIM.GT.TIME5).AND.(ORBTIM.LT.TIME6)) THEN
                QPSUBN = 0.D+00
                QPSUBS = ALFAS*(EIR
          1                + DABS(ALBEDO*SOLAR*DSIN(OMEGA*ORBTIM)))
                QPSUBO = 0.5*ALFAO*DABS(SOLAR*DSIN(OMEGA*ORBTIM))
           ELSEIF (ORBTIM.GT.TIME6) THEN
                QPSUBN = ALFAN*DABS(SOLAR*DCOS(OMEGA*ORBTIM))
                QPSUBS = ALFAS*(EIR
          1                + DABS(ALBEDO*SOLAR*DSIN(OMEGA*ORBTIM)))
                QPSUBO = 0.5*ALFAO*DABS(SOLAR*DSIN(OMEGA*ORBTIM))
           ENDIF
C     CALL MAIN SUBROUTINE
           CALL NUTEMP (TOLD,TNEW,DELTIM)
C     MAP NEW TEMPERATURE MATRIX ONTO OLD TEMPERATURE MATRIX
           CALL MAP122 (TNEW,TOLD)
C     SAVE TIME, DELTIM AND TEMPERATURE MATRIX TO DATA FILE 14 (ECRDAT)
           IF (ICOUNT.EQ.IPRINT) THEN
                ICOUNT = 0
                WRITE (14,*) TIME,DELTIM
C
C     **** CONVERT NEW TEMPERATURES TO EXCESS FOR PRINTOUT ****
      DO 200 LAYER=1,NLAYER
        DO 300 INCRAD=1,NODES
          DO 400 IPHI=1,NPHI
            TNEW(INCRAD,IPHI,LAYER) = TNEW(INCRAD,IPHI,LAYER 1) - TINIT
C
C     **** PRINT OPTION 1 - USE WITH CMPSHT,COMPAR,LNGFRM,SHTFRM,BCRERR
C                           SHORT EVALUATION TIMES ONLY (<400 SEC)
C                         WRITE (14,*) TNEW(INCRAD,IPHI,LAYER)
400                   CONTINUE
300                 CONTINUE
200               CONTINUE
C
C     *** PRINT OPTION 2 - SAVES COMPLETE MATRIX AT SPECIFIED PRINT
C                          INTERVAL - SWITCH OFF ALL SUBROUTINES LISTED
C                                     UNDER PRINT OPTION 1 FIRST.
C               CALL PRTMAT (TIME,DTIME,TNEW,NODES,NPHI,NLAYER)
```

```
C
C     *** PRINT OPTION 3 - SAVES DESIGNATED NODE TEMPS AT SPECIFIED
C                          PRINT INTERVAL.  USE FOR LONG PERIOD EVALS.
      WRITE (11,889) TIME,DELTIM,TNEW(5,1,1),TNEW(5,1,3),TNEW(5,1,5)
889   FORMAT (T1,F9.3,T15,F9.3,T25,F12.6,T40,F12.6,T55,F12.6)
C
C     ******** RE-SPECIFY PRINT INTERVALS ********
C               IF (TIME.GE.99.9) THEN
C                    IPRINT = 200
C               ELSEIF (TIME.GE.9.9) THEN
C                    IPRINT = 100
C               ELSEIF (TIME.GE.0.99) THEN
C                    IPRINT = 10
C               ENDIF
      ENDIF
C     TIME INTERVAL DELIMITER
          IF (TIME.GE.FINTIM) GOTO 101
100   CONTINUE
C     STOP CLOCK TIMER (MAINFRAME VERSIONS OF CODE ONLY)
101   CONTINUE
C     GET TIME EXITING LOOP
      CALL GTIME (1,11)
      CLOSE (14)
      CLOSE (15)
C     WRITE APPROPRIATE DATA FILES
C     CALL ERROR (MRAD,MPHI,MLAYER)
C     CALL SHTFRM (MRAD,MPHI,MLAYER)
C     CALL LNGFRM
      CALL GTIME(1,11)
999   STOP
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BMINIT FORTRAN A - (REV. 9/28/88)
C     ************************************************************
C     * BMINIT - SUBROUTINE TO INITIALIZE TEMPERATURE MATRIX TO A *
C     *          SPECIFIED VALUE (BRIAN'S METHOD ARRAY FORMAT)    *
C     ************************************************************
C     PROGRAM LINE 213
      SUBROUTINE BMINIT (DMAT,TINIT)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /ARRSIZ/ III,JJJ,KKK
      DIMENSION DMAT(III,JJJ,KKK)
      DO 100 LAYER=1,KKK
          DO 200 IPHI=1,JJJ
               DO 300 INCRAD=1,III
                    DMAT(INCRAD,IPHI,LAYER) = TINIT
300            CONTINUE
200       CONTINUE
100   CONTINUE
998   RETURN
      END
C
```

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CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     STABIL FORTRAN A (REV. 11/14/88)
C     ****************************************************************
C     * STABIL - SUBROUTINE TO COMPUTE THEORETICAL MAXIMUM TIME      *
C     *         INCREMENT ALLOWED TO MAINTAIN STABILITY IN EXPLICIT  *
C     *         FINITE DIFFERENCE ANALYSIS OF A CYLINDER IN 3-D      *
C     *         POLAR CYLINDRICAL COORDINATES.                       *
C     ****************************************************************
C     PROGRAM LINE 237
      SUBROUTINE STABIL (DTMAX)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION DT(6)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
C                       *** BODY ***
      DO 50 I=1,6
          DT(I) = 0.D+00
50    CONTINUE
      DUMMY = (DZ**2)/ALFA
C     END LAYERS, INNER SURFACE NODES
          DCREI = 1 + (2*DKP*DZ**2)/(2*DKZ*RAD*(RAD+DR/4)*DPHI**2)
     1+ (DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
          DT(1) = DUMMY/(2*DCREI)
          DTMAX = DT(1)
C     END LAYERS, MID-RADII NODES
          DCREM = 1 + 2*(DKP*DZ**2)/(2*DKZ*RAD**2*DPHI**2)
     1+ (DKR*(RAD-DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
     2+ (DKR*(RAD+DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
          DT(2) = DUMMY/(2*DCREM)
C     END LAYERS, OUTER SURFACE NODES
          DCREO = 1 + 2*(DKP*DZ**2)/(2*DKZ*(RAD-DR/4)*RAD*DPHI**2)
     1+ (DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
          DT(3) = DUMMY/(2*DCREO)
C     MID-LAYERS, INNER SURFACE NODES
          DCRMI = 2 + 2*(DKP*DZ**2)/(DKZ*RAD*(RAD+DR/4)*DPHI**2)
     1+ (2*DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
          DT(4) = DUMMY/DCRMI
C     MID-LAYERS, MID-RADII NODES
          DCRMM = 2 + 2*(DKP*DZ**2)/(DKZ*RAD**2*DPHI**2)
     1+ (DKR*(RAD-DR/2)*DZ**2)/(DKZ*RAD*DR**2)
     2+ (DKR*(RAD+DR/2)*DZ**2)/(DKZ*RAD*DR**2)
          DT(5) = DUMMY/DCRMM
C     MID-LAYERS, OUTER SURFACE NODES
          DCRMO = 2 + 2*(DKP*DZ**2)/(DKZ*RAD*(RAD-DR/4)*DPHI**2)
     1+ (2*DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
          DT(6) = DUMMY/DCRMO
      DO 100 I=1,6
          IF ((DT(I).GT.0).AND.(DTMAX.GT.DT(I))) DTMAX = DT(I)
      WRITE (*,*) 'FROM STABIL, I,DT(I)',I,DT(I)
100   CONTINUE
      IF (DTMAX.LE.0) WRITE (*,*) 'STABIL - DTMAX LE ZERO'
```

```
998   RETURN
      END
C
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     NUTEMP FORTRAN A - (REV. 11/14/88)
C     ********************************************************
C     * NUTEMP - THIS SUBROUTINE COMPUTES THE MATRIX OF NODE *
C     *          TEMPERATURES AT THE N+1 TIMESTEP FOR THE    *
C     *          EXPLICIT METHOD OF FINITE DIFFERENCE        *
C     *          ANALYSIS FOR A RIGHT CIRCULAR CYLINDER WITH *
C     *          CONSTANT SURFACE HEAT FLUX AND A SPECIFIED  *
C     *          INNER SURFACE TEMPERATURE.                  *
C     ********************************************************
C     PROGRAM LINE 298
      SUBROUTINE NUTEMP (TOLD,TNEW,DELTIM)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /ARRSIZ/ III,JJJ,KKK
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
      DIMENSION TOLD(III,JJJ,KKK),TNEW(III,JJJ,KKK)
CBBBBBBBBBBBBBBBBBBBBBBBBBBBB BOTTOM LAYER BBBBBBBBBBBBBBBBBBBBBBBBBBB
      LAYER  = 1
C                **** BOTTOM LAYER, INNER RADIUS ****
      INCRAD = 1
      RAD = RMIN
C     ******** BOTTOM LAYER, INNER RADIUS, FIRST PHI ********
          IPHI =1
               T1=TOLD(INCRAD,IPHI,LAYER)
               T2=TOLD(INCRAD,IPHI,(LAYER+1))
               T3=TOLD(INCRAD,NPHI,LAYER)
               T4=TOLD(INCRAD,(IPHI+1),LAYER)
               T5=TOLD((INCRAD+1),IPHI,LAYER)
               CALL ECREI (TEMP)
               TNEW(INCRAD,IPHI,LAYER)=TEMP
C     ******** BOTTOM LAYER, INNER RADIUS, MID-PHIS ********
          DO 100 IPHI=2,(NPHI-1)
               T1=TOLD(INCRAD,IPHI,LAYER)
               T2=TOLD(INCRAD,IPHI,(LAYER+1))
               T3=TOLD(INCRAD,(IPHI-1),LAYER)
               T4=TOLD(INCRAD,(IPHI+1),LAYER)
               T5=TOLD((INCRAD+1),IPHI,LAYER)
               CALL ECREI (TEMP)
               TNEW(INCRAD,IPHI,LAYER)=TEMP
100       CONTINUE
C     ******** BOTTOM LAYER, INNER RADIUS, LAST PHI ********
          IPHI=NPHI
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```
                T1=TOLD(INCRAD,IPHI,LAYER)
                T2=TOLD(INCRAD,IPHI,(LAYER+1))
                T3=TOLD(INCRAD,(IPHI-1),LAYER)
                T4=TOLD(INCRAD,1,LAYER)
                T5=TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREI (TEMP)
                TNEW(INCRAD,IPHI,LAYER)=TEMP
C               ******** BOTTOM LAYER, MID-RADII ********
      DO 200 INCRAD=2,(NODES-1)
           RAD = RMIN + (INCRAD-1)*DR
C          ******** BOTTOM LAYER, MID-RADII, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,NPHI,LAYER)
                T4 = TOLD(INCRAD,(IPHI+1),LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                T6 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREM (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C           ******** BOTTOM LAYER, MID RADII, MID-PHIS ********
      DO 300 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,(IPHI-1),LAYER)
                T4 = TOLD(INCRAD,(IPHI+1),LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                T6 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREM (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
300             CONTINUE
C           ******** BOTTOM LAYER, MID RADII, LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,(IPHI-1),LAYER)
                T4 = TOLD(INCRAD,1,LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                T6 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREM (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
200        CONTINUE
C                          **** OUTER RADIUS ****
      INCRAD = NODES
      RAD = RMAX
C        ******** BOTTOM LAYER, OUTER RADIUS, FIRST PHI ********
           IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,NPHI,LAYER)
                T4 = TOLD(INCRAD,(IPHI+1),LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL ECREO (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C           ******** FIRST LAYER, OUTER RADIUS, MID-PHIS ********
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      DO 400 IPHI=2,(NPHI-1)
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,IPHI,(LAYER+1))
               T3 = TOLD(INCRAD,(IPHI-1),LAYER)
               T4 = TOLD(INCRAD,(IPHI+1),LAYER)
               T5 = TOLD((INCRAD-1),IPHI,LAYER)
               CALL ECREO (TEMP)
               TNEW(INCRAD,IPHI,LAYER) = TEMP
400       CONTINUE
C        ******** BOTTOM LAYER, OUTER RADIUS, LAST PHI ********
      IPHI = NPHI
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,IPHI,(LAYER+1))
               T3 = TOLD(INCRAD,(IPHI-1),LAYER)
               T4 = TOLD(INCRAD,1,LAYER)
               T5 = TOLD((INCRAD-1),IPHI,LAYER)
               CALL ECREO (TEMP)
               TNEW(INCRAD,IPHI,LAYER) = TEMP
CMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM MID-LAYERS MMMMMMMMMMMMMMMMMMMMMMMMMMMMM
      DO 500 LAYER=2,(NLAYER-1)
C                  **** MID-LAYERS, INNER RADIUS ****
      INCRAD = 1
           RAD = RMIN
C       ******** MID-LAYERS, INNER RADIUS, FIRST PHI ********
      IPHI=1
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,IPHI,(LAYER+1))
               T3 = TOLD(INCRAD,IPHI,(LAYER-1))
               T4 = TOLD(INCRAD,(NPHI),LAYER)
               T5 = TOLD(INCRAD,(IPHI+1),LAYER)
               T6 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL ECRMI (TEMP)
               TNEW(INCRAD,IPHI,LAYER) = TEMP
C        ******** MID-LAYERS,INNER RADIUS, MID-PHIS ********
      DO 600 IPHI=2,NPHI
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,IPHI,(LAYER+1))
               T3 = TOLD(INCRAD,IPHI,(LAYER-1))
               T4 = TOLD(INCRAD,(IPHI-1),LAYER)
               T5 = TOLD(INCRAD,(IPHI+1),LAYER)
               T6 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL ECRMI (TEMP)
               TNEW(INCRAD,IPHI,LAYER) = TEMP
600            CONTINUE
C        ******** MID-LAYERS,INNER RADIUS, LAST PHI ********
      IPHI=NPHI
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,IPHI,(LAYER+1))
               T3 = TOLD(INCRAD,IPHI,(LAYER-1))
               T4 = TOLD(INCRAD,(IPHI-1),LAYER)
               T5 = TOLD(INCRAD,1,LAYER)
               T6 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL ECRMI (TEMP)
               TNEW(INCRAD,IPHI,LAYER) = TEMP
C               ******** MID-LAYERS, MID-RADII ********
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      DO 700 INCRAD=2,(NODES-1)
           RAD = RMIN + (INCRAD-1)*DR
C          ******** MID-LAYERS, MID-RADII, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,IPHI,(LAYER-1))
                T4 = TOLD(INCRAD,NPHI,LAYER)
                T5 = TOLD(INCRAD,(IPHI+1),LAYER)
                T6 = TOLD((INCRAD-1),IPHI,LAYER)
                T7 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECRMM (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C          ******** MID-LAYERS, MID-RADII, MID-PHIS ********
      DO 800 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,IPHI,(LAYER-1))
                T4 = TOLD(INCRAD,(IPHI-1),LAYER)
                T5 = TOLD(INCRAD,(IPHI+1),LAYER)
                T6 = TOLD((INCRAD-1),IPHI,LAYER)
                T7 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECRMM (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
800             CONTINUE
C          ******** MID-LAYERS, MID-RADII, LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,IPHI,(LAYER-1))
                T4 = TOLD(INCRAD,(IPHI-1),LAYER)
                T5 = TOLD(INCRAD,1,LAYER)
                T6 = TOLD((INCRAD-1),IPHI,LAYER)
                T7 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECRMM (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
700       CONTINUE
C                    **** MID-LAYERS, OUTER RADII ****
      INCRAD = NODES
           RAD = RMAX
C          ******** MID-LAYERS, OUTER RADIUS, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,IPHI,(LAYER-1))
                T4 = TOLD(INCRAD,NPHI,LAYER)
                T5 = TOLD(INCRAD,(IPHI+1),LAYER)
                T6 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL ECRMO (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C          ******** MID-LAYERS, OUTER RADIUS, MID-PHIS ********
      DO 900 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,IPHI,(LAYER-1))
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                T4 = TOLD(INCRAD,(IPHI-1),LAYER)
                T5 = TOLD(INCRAD,(IPHI+1),LAYER)
                T6 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL ECRMO (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
900             CONTINUE
C           ******** MID-LAYERS, OUTER RADIUS,LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER+1))
                T3 = TOLD(INCRAD,IPHI,(LAYER-1))
                T4 = TOLD(INCRAD,(IPHI-1),LAYER)
                T5 = TOLD(INCRAD,1,LAYER)
                T6 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL ECRMO (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
500         CONTINUE
CTTTTTTTTTTTTTTTTTTTTTTTTTTTT TOP LAYER TTTTTTTTTTTTTTTTTTTTTTTTTTTTT
LAYER = NLAYER
C                   **** TOP LAYER, INNER RADIUS ****
      INCRAD = 1
           RAD = RMIN
C      ******** TOP LAYER, INNER RADIUS, FIRST PHI ********
      IPHI=1
                T1=TOLD(INCRAD,IPHI,LAYER)
                T2=TOLD(INCRAD,IPHI,(LAYER-1))
                T3=TOLD(INCRAD,NPHI,LAYER)
                T4=TOLD(INCRAD,(IPHI+1),LAYER)
                T5=TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREI (TEMP)
                TNEW(INCRAD,IPHI,LAYER)=TEMP
C       ******** TOP LAYER, INNER RADIUS, MID-PHIS ********
      DO 1000 IPHI=2,(NPHI-1)
                T1=TOLD(INCRAD,IPHI,LAYER)
                T2=TOLD(INCRAD,IPHI,(LAYER-1))
                T3=TOLD(INCRAD,(IPHI-1),LAYER)
                T4=TOLD(INCRAD,(IPHI+1),LAYER)
                T5=TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREI (TEMP)
                TNEW(INCRAD,IPHI,LAYER)=TEMP
1000            CONTINUE
C       ******** TOP LAYER, INNER RADIUS, LAST PHI ********
           IPHI=NPHI
                T1=TOLD(INCRAD,IPHI,LAYER)
                T2=TOLD(INCRAD,IPHI,(LAYER-1))
                T3=TOLD(INCRAD,(IPHI-1),LAYER)
                T4=TOLD(INCRAD,1,LAYER)
                T5=TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREI (TEMP)
                TNEW(INCRAD,IPHI,LAYER)=TEMP
C                     **** TOP LAYER, MID RADII ****
           DO 1100 INCRAD=2,(NODES-1)
           RAD = RMIN + (INCRAD-1)*DR
C           ******** TOP LAYER, MID-RADII, FIRST PHI ********
           IPHI = 1
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                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER-1))
                T3 = TOLD(INCRAD,NPHI,LAYER)
                T4 = TOLD(INCRAD,(IPHI+1),LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                T6 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREM (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C             ******** TOP LAYER, MID RADII, MID-PHIS ********
              DO 1200 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER-1))
                T3 = TOLD(INCRAD,(IPHI-1),LAYER)
                T4 = TOLD(INCRAD,(IPHI+1),LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                T6 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREM (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
1200            CONTINUE
C             ******** TOP LAYER, MID RADII, LAST PHI ********
              IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER-1))
                T3 = TOLD(INCRAD,(IPHI-1),LAYER)
                T4 = TOLD(INCRAD,1,LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                T6 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL ECREM (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
1100          CONTINUE
C                       **** TOP LAYER, OUTER RADIUS ****
              INCRAD = NODES
              RAD = RMAX
C             ******** TOP LAYER, OUTER RADIUS, FIRST PHI ********
              IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER-1))
                T3 = TOLD(INCRAD,NPHI,LAYER)
                T4 = TOLD(INCRAD,(IPHI+1),LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL ECREO (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C             ******** TOP LAYER, OUTER RADIUS, MID-PHIS ********
              DO 1300 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,IPHI,(LAYER-1))
                T3 = TOLD(INCRAD,(IPHI-1),LAYER)
                T4 = TOLD(INCRAD,(IPHI+1),LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL ECREO (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
1300          CONTINUE
C             ******** TOP LAYER, OUTER RADIUS, LAST PHI ********
              IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
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                T2 = TOLD(INCRAD,IPHI,(LAYER-1))
           T3 = TOLD(INCRAD,(IPHI-1),LAYER)
           T4 = TOLD(INCRAD,1,LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
           CALL ECREO (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
998    RETURN
       END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C      ECREI FORTRAN A - (REV. 11/14/88)
C      ***************************************************************
C      * ECREI - THIS SUBROUTINE COMPUTES THE NODAL TEMPERATURES AT     *
C      *         THE INNER SURFACE OF THE END LAYERS (NORTH, SOUTH)     *
C      *         OF A RIGHT-CIRCULAR CYLINDER USING THE EXPLICIT METHOD *
C      *         OF FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL *
C      *         COORDINATES.                                           *
C      ***************************************************************
C      PROGRAM LINE 631
       SUBROUTINE ECREI (TNEW)
       IMPLICIT DOUBLE PRECISION (A-H,O-Z)
       COMMON /CONDUC/ DKR,DKP,DKZ
       COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
       COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
       COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
       COMMON /MATSIZ/ NLAYER,NODES,NPHI
       COMMON /NPOSIT/ INCRAD,IPHI,LAYER
       COMMON /PARAMS/ CSUBP,RHO
       COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
       COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                            *** BODY ***
       IF (LAYER.EQ.1) THEN
            Q = QPSUBS
            HE = HSUBS
            EPSE = EPSUBS
       ELSEIF (LAYER.EQ.NLAYER) THEN
            Q = QPSUBN
            HE = HSUBN
            EPSE = EPSUBN
       ENDIF
       D1 = DKP*DZ**2/(2*DKZ*RAD*(RAD+DR/4)*DPHI**2)
       D2 = DKR*(RAD+DR/2)*DZ**2/(DKZ*(RAD+DR/4)*DR**2)
       D3 = DZ/DKZ
       D4 = (RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
       D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
       D6 = 1 + 2*FO*(D3*(HE + D5*EPSE) + D4*(HSUBI + D5*EPSUBI))
C
       TNEW = 1/D6*(T1+2*FO*((T2-T1) + D1*(T3+T4-2*T1) + D2*(T5-T1)
       1+ D3*(Q + HE*TINF + D5*EPSE*TSUR)
       2+ D4*(QPSUBI + HSUBI*TINF + D5*EPSUBI*TSUR)))
C
998    RETURN
       END
C
C
```

```
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     ECREM FORTRAN A - (REV. 11/14/88)
C     ***************************************************************
C     * ECREM - THIS SUBROUTINE COMPUTES THE TEMPERATURES OF THE MID-  *
C     *         RADIAL NODES FOR THE END LAYERS (NORTH, SOUTH)         *
C     *         OF A RIGHT-CIRCULAR CYLINDER USING THE EXPLICIT METHOD *
C     *         OF FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL *
C     *         COORDINATES.                                           *
C     ***************************************************************
C     PROGRAM LINE 678
      SUBROUTINE ECREM (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
      C                              *** BODY ***
      IF (LAYER.EQ.1) THEN
           Q = QPSUBS
           EPSE = EPSUBS
           HE = HSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
           Q = QPSUBN
           EPSE = EPSUBN
           HE = HSUBN
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*RAD**2*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      D3 = (DKR*(RAD+DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      D4 = DZ/DKZ
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      D6 = 1 + 2*FO*D4*(HE+D5*EPSE)
      TNEW = 1/D6*(T1 + 2*FO*((T2-T1) + D1*(T3+T4-2*T1) + D2*(T5-T1)
     1+ D3*(T6-T1) + D4*(Q + HE*TINF + D5*EPSE*TSUR)))
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     ECREO FORTRAN A - (REV. 11/14/88)
C     ***************************************************************
C     * ECREO - THIS SUBROUTINE COMPUTES THE NODE TEMPERATURES AT      *
C     *         THE OUTER SURFACE OF THE END LAYERS (NORTH, SOUTH)     *
C     *         OF A RIGHT-CIRCULAR CYLINDER USING THE EXPLICIT METHOD *
C     *         OF FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL *
C     *         COORDINATES.                                           *
C     ***************************************************************
C     PROGRAM LINE 721
      SUBROUTINE ECREO (TNEW)
```

```
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      IF (LAYER.EQ.1) THEN
          Q = QPSUBS
          EPSE = EPSUBS
          HE = HSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
          Q = QPSUBN
          EPSE = EPSUBN
          HE = HSUBN
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*(RAD-DR/4)*RAD*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = DZ/DKZ
      D4 = (RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      D6 = 1 + 2*FO*( D3*(HE+D5*EPSE) + D4*(HSUBO+D5*EPSUBO))
C
      TNEW = 1/D6*(T1 + 2*FO*((T2-T1) + (T3+T4-2*T1) + D2*(T5-T1)
     1+ D3*(Q + HE*TINF + D5*EPSE*TSUR)
     2+ D4*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR)))
C
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     ECRMI FORTRAN A - (REV. 11/14/88)
C     ******************************************************************
C     * ECRMI - THIS SUBROUTINE COMPUTES THE NODE TEMPERATURES AT      *
C     *         THE INNER SURFACE OF THE MID-LAYERS (2 THRU NLAYER-1)  *
C     *         OF A RIGHT-CIRCULAR CYLINDER USING THE EXPLICIT METHOD *
C     *         OF FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL *
C     *         COORDINATES.                                           *
C     ******************************************************************
C     PROGRAM LINE 767
      SUBROUTINE ECRMI (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/ T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
```

```
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD+DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      D6 = 1 + FO*D3*(HSUBI+D5*EPSUBI)
C
      TNEW = 1/D6*(T1 + FO*((T2+T3-2*T1) + D1*(T4+T5-2*T1) + D2*(T6-T1)
      1+ D3*(QPSUBI + HSUBI*TINF + D5*EPSUBI*TSUR)))
C
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     ECRMM FORTRAN A - (REV. 11/14/88)
C     ********************************************************************
C     * ECRMM - THIS SUBROUTINE COMPUTES THE NODAL TEMPERATURES AT      *
C     *         THE MID-RADII OF THE  MID-LAYERS (2 THRU NLAYER-1)      *
C     *         OF A RIGHT-CIRCULAR CYLINDER USING THE EXPLICIT METHOD  *
C     *         OF FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL  *
C     *         COORDINATES.                                            *
C     ********************************************************************
C     PROGRAM LINE 801
      SUBROUTINE ECRMM (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD**2*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(DKZ*RAD*DR**2)
      D3 = (DKR*(RAD+DR/2)*DZ**2)/(DKZ*RAD*DR**2)
C     *** NO D4, D5 OR D6 REQUIRED ***
      TNEW = T1 + FO*((T2+T3-2*T1) + D1*(T4+T5-2*T1) + D2*(T6-T1)
      1+ D3*(T7-T1))
C
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     ECRMO FORTRAN A - (REV. 11/14/88)
C     ********************************************************************
C     * ECRMO - THIS SUBROUTINE COMPUTES THE NODAL TEMPERATURES AT      *
C     *         THE OUTER SURFACE OF THE MID-LAYERS (NORTH, SOUTH)      *
C     *         OF A RIGHT-CIRCULAR CYLINDER USING BRIAN'S METHOD OF    *
C     *         FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL     *
C     *         COORDINATES.                                            *
C     ********************************************************************
C     PROGRAM LINE 832
```

```
      SUBROUTINE ECRMO (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD-DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      D6 = 1 + FO*D3*(HSUBO + D5*EPSUBO)
      TNEW = 1/D6*(T1 + FO*((T2+T3-2*T1) + D1*(T4+T5-2*T1) + D2*(T6-T1)
      1+ D3*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR)))
C
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     MAP122 FORTRAN A - (REV. 10/04/88)
C     ********************************************************
C     * MAP122 - SUBROUTINE TO MAP MATRIX ONE ONTO MATRIX TWO *
C     ********************************************************
C     PROGRAM LINE 861
      SUBROUTINE MAP122 (DMAT1,DMAT2)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /ARRSIZ/ III,JJJ,KKK
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      DIMENSION DMAT1(III,JJJ,KKK),DMAT2(III,JJJ,KKK)
      DO 100 LAYER=1,NLAYER
          DO 200 IPHI=1,NPHI
              DO 300 INCRAD=1,NODES
                  DMAT2(INCRAD,IPHI,LAYER) = DMAT1(INCRAD,IPHI,LAYER)
300           CONTINUE
200       CONTINUE
100   CONTINUE
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     SHTFRM FORTRAN A - (REV. 11/14/88)
C     ************************************************************
C     * SHTFRM - SUBROUTINE TO PRINT THE TEMPERATURES OF A DESIG-  *
C     *          DESIGNATED NODE FROM A DATA FILE.                 *
C     ************************************************************
C     PROGRAM LINE 884
      SUBROUTINE SHTFRM (L,M,N)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
```

```
      COMMON /HEAT/ ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO COMMON /TIMES/
      DELTIM,DTMAX,FINTIM,LIMIT
      DIMENSION TEMP(15,15,15)
      OPEN(UNIT=13,FILE='ANALYT')
      OPEN(UNIT=14,FILE='ECRDAT')
      OPEN(UNIT=15,FILE='ECRSHT')
C     READ IN MATRIX SIZE
      READ (14,*) NODES,NPHI,NLAYER
C     WRITE HEADERS TO DATA FILE 15 (ERROR)
      WRITE (15,*) 'EXPLICIT - WITH CONDUCTION, CONVECTION AND RADIATION
      1'
      WRITE (15,*)
      WRITE (15,*) 'DATA SAVED AS FILE:'
      WRITE (15,*)
      WRITE (15,*) 'CPU TIME:'
      WRITE (15,*)
      WRITE (15,*)'GRID SIZE (RADIAL,PHI,Z DIRECTIONS):',NODES,NPHI,NLAY
      1ER
      WRITE (15,*)
      WRITE (15,*) 'SELECTED NODE FOR PRINTOUT:',L,M,N WRITE (15,*)
      WRITE (15,*) 'THEORETICAL MAX TIMESTEP (EXPLICIT):',DTMAX WRITE
      (15,*)
      WRITE (15,*) 'SELECTED TIMESTEP (SEC):',DELTIM WRITE (15,*)
      WRITE (15,*) 'GRID FOURIER NUMBER (Z,DELTIM DEPENDENT): ',FO WRITE
      (15,*)
      WRITE (15,*) 'OUTER SURFACE HEAT FLUX (W/M**2):',QPSUBO WRITE
      (15,*)
      WRITE (15,*)
      WRITE (15,15)
15    FORMAT (T33,'NODE')
      WRITE (15,25)
25    FORMAT (T5,'TIME',T17,'DELTIM',T30,'TEMPERATURE')
C     READ TIME, DELTIM AND TEMPERATURE MATRICES FROM FILE 14 (*DATA)
2     READ (14,*,END=998) TIME,DELTIM
      DO 100 K=1,NLAYER
           DO 200 I=1,NODES
                DO 300 J=1,NPHI
                     READ (14,*,END=997) TEMP(I,J,K)
300             CONTINUE
200        CONTINUE
100   CONTINUE
      WRITE (15,55) TIME,DELTIM,TEMP(L,M,N)
55        FORMAT (T1,F9.2,T15,F8.4,T26,F14.8)
      GOTO 2
997   WRITE (*,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 864'
      WRITE (15,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 864'
998   CLOSE (13)
      CLOSE (14)
      CLOSE (15)
      RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     LNGFRM FORTRAN A - (REV. 11/05/88)
```

```
C     ************************************************************
C     * LNGFRM - SUBROUTINE TO PRINT FULL MATRIX OF TEMPERATURES *
C     *          FROM DATA FILES.                                *
C     ************************************************************
C     PROGRAM LINE 948
      SUBROUTINE LNGFRM
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /HEAT/ ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /TIMES/ DELTIM,DTMAX,FINTIM,LIMIT
      DIMENSION TEMP(15,15,15)
      OPEN (UNIT=11,FILE='ECRLNG')
      OPEN (UNIT=14,FILE='ECRDAT')
      READ (14,*) NODES,NPHI,NLAYER
      WRITE (15,*) 'EXPLICIT - WITH CONDUCTION, CONVECTION AND RADIATION
      1'
      WRITE (15,*)
      WRITE (15,*) 'DATA SAVED AS FILE:'
      WRITE (15,*)
      WRITE (15,*) 'CPU TIME:'
      WRITE (15,*)
      WRITE (11,*) 'GRID SIZE (RADIAL,PHI,Z DIRECTIONS):',NODES,NPHI,NLA
      1YER
      WRITE (11,*)
      WRITE (11,*) 'THEORETICAL MAX TIMESTEP (EXPLICIT):',DTMAX WRITE
      (11,*)
      WRITE (11,*) 'SELECTED TIMESTEP (SEC):',DELTIM WRITE (11,*)
      WRITE (11,*) 'GRID FOURIER NUMBER (Z,DELTIM DEPENDENT): ',FO WRITE
      (11,*)
      WRITE (11,*) 'SURFACE HEAT FLUX (W/M**2):',QPSUBO WRITE (11,*)
      WRITE (11,*)
1     READ (14,*,END=998) TIME,DTIME
      DO 100 K=1,NLAYER
          DO 200 I=1,NODES
              DO 300 J=1,NPHI
                  READ (14,*,END=997) TEMP(I,J,K)
300           CONTINUE
200       CONTINUE
100   CONTINUE
      CALL PRTMAT (TIME,DTIME,TEMP,NODES,NPHI,NLAYER)
      GOTO 1
997   WRITE(11,*) 'LNGFRM - EOF READ ERROR IN PROGRAM LINE 712'
      WRITE(*,*) 'LNGFRM - EOF READ ERROR IN PROGRAM LINE 712'
998   REWIND 14
      CLOSE (11)
      RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     PRTMAT FORTRAN A - (REV. 11/04/88)
C     ************************************************************
C     * PRTMAT - SUBROUTINE TO PRINT THREE DIMENDIONAL MATRIX    *
C     *          TO FILE 11 (*LONG).                             *
C     ************************************************************
C     PROGRAM LINE 1000
```

```
      SUBROUTINE PRTMAT (TIME,DTIME,DMAT,NODES,NPHI,NLAYER) IMPLICIT
      DOUBLE PRECISION (A-H,O-Z)
      DIMENSION DMAT(15,15,15)
      WRITE (11,15) TIME,DTIME
15    FORMAT (T1,'TIME = ',F9.3,' SEC',T30,'TIME INCREMENT = ',F9.3,' SE
      1C')
      DO 100 K=1,NLAYER
      WRITE(11,10) K
10       FORMAT (//T1,'NODE:',T45,'LAYER NUMBER: ',I4/) DO 200 I=1,NODES
      WRITE (11,25) I,(DMAT(I,J,K),J=1,NPHI)
25          FORMAT (T1,I3,T10,4(D16.10,1X))
200      CONTINUE
100   CONTINUE
      WRITE (11,*)
      WRITE (11,*)
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     PRTARR FORTRAN A - (REV. 11/04/88)
C     ***********************************************************
C     * PRTARR - SUBROUTINE TO PRINT THREE DIMENDIONAL MATRIX   *
C     *          TO FILE 11 (*LONG).                            *
C     ***********************************************************
C     PROGRAM LINE 1027
      SUBROUTINE PRTARR (TIME,DTIME,DMAT,NODES,NPHI,NLAYER)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION DMAT(15,15,15)
      WRITE (11,15) TIME,DTIME
15    FORMAT (T1,'TIME = ',F9.3,' SEC',T30,'TIME INCREMENT = ',F9.3,' SE
      1C')
      DO 100 K=1,NLAYER
      WRITE(11,10) K
10       FORMAT (//T1,'NODE:',T45,'LAYER NUMBER: ',I4/) DO 200 I=1,NODES
      WRITE (11,25) I,(DMAT(I,J,K),J=1,NPHI)
25          FORMAT (T1,I3,T10,4(D16.10,1X))
200      CONTINUE
100   CONTINUE
      WRITE (11,*)
      WRITE (11,*)
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     ERROR FORTRAN A - (REV. 11/05/88)
C     ***************************************************************
C     * ERROR - SUBROUTINE TO COMPUTE ERROR FOR AND PRINT TEMPERA-  *
C     *         TURES OF A DESIGNATED NODE FROM A VALIDATION DATA   *
C     *         FILE.                                               *
C     ***************************************************************
C     PROGRAM LINE 1025
      SUBROUTINE ERROR (L,M,N)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
```

```
      COMMON /HEAT/ ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /TIMES/ DELTIM,DTMAX,FINTIM,LIMIT
      DIMENSION TEMP(15,15,15)
      OPEN(UNIT=13,FILE='ANALYT')
      OPEN(UNIT=14,FILE='ECRDAT')
      OPEN(UNIT=15,FILE='ECRERR')
      IF (((QPSUBO.GT.1352).AND.(QPSUBO.LT.1354)).OR.((QPSUBO.GT.14999)
     1.AND.(QPSUBO.LT.15001))) THEN
      CONTINUE
      ELSE
      GOTO 998
      ENDIF
C     READ IN MATRIX SIZE
      READ (14,*) NODES,NPHI,NLAYER
C     WRITE HEADERS TO DATA FILE 15 (ERROR)
      WRITE (15,*) 'EXPLICIT - WITH CONDUCTION, CONVECTION AND RADIATION
     1'
      WRITE (15,*)
      WRITE (15,*) 'DATA SAVED AS FILE:'
      WRITE (15,*)
      WRITE (15,*) 'CPU TIME:'
      WRITE (15,*)
      WRITE (15,*)'GRID SIZE (RADIAL,PHI,Z DIRECTIONS):',NODES,NPHI,NLAY
     1ER
      WRITE (15,*)
      WRITE (15,*) 'SELECTED NODE FOR COMPARISON:',L,M,N WRITE (15,*)
      WRITE (15,*) 'THEORETICAL MAX TIMESTEP (EXPLICIT):',DTMAX WRITE
     (15,*)
      WRITE (15,*) 'SELECTED TIMESTEP (SEC):',DELTIM WRITE (15,*)
      WRITE (15,*) 'GRID FOURIER NUMBER (Z,DELTIM DEPENDENT): ',FO WRITE
     (15,*)
      WRITE (15,*) 'OUTER SURFACE HEAT FLUX (W/M**2):',QPSUBO WRITE
     (15,*)
      WRITE (15,*)
      WRITE (15,15)
15    FORMAT (T33,'NODE',T48,'ANALYTIC',T64,'PERCENT') WRITE (15,25)
25    FORMAT (T5,'TIME',T17,'DELTIM',T30,'TEMPERATURE',T46,'TEMPERATURE'
     1,T65,'ERROR'/)
C     READ TIME, DELTIM AND TEMPERATURE MATRICES FROM FILE 14 (*DATA) 2
      READ (14,*,END=998) TIME,DELTIM
      DO 100 K=1,NLAYER
      DO 200 I=1,NODES
      DO 300 J=1,NPHI
      READ (14,*,END=995) TEMP(I,J,K)
300           CONTINUE
200       CONTINUE
100   CONTINUE
C     READ IN APPROPRIATE ANALYTIC SOLUTIONS FROM FILE 13 (ANALYT)
3         IF (QPSUBO.LT.1400.) THEN
              READ (13,35,END=996) EXACT
35            FORMAT (T19,F14.8)
      ELSE
              READ (13,45,END=997) EXACT
45            FORMAT (T39,F14.8)
      ENDIF
```

```
C     COMPUTE PERCENT ERROR AND WRITE TO FILE 15 (ERROR)
           PE = (EXACT-TEMP(L,M,N))/EXACT * 100.
           WRITE (15,55) TIME,DELTIM,TEMP(L,M,N),EXACT,PE
55         FORMAT (T1,F9.2,T15,F8.4,T26,F14.8,T41,F14.8,T61,F9.3)
      GOTO 2
995   WRITE (*,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 652'
      WRITE (15,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 652'
996   WRITE (*,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 658'
      WRITE (15,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 658'
997   WRITE (*,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 661'
      WRITE (15,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 661'
998   REWIND 14
      CLOSE (13)
      CLOSE (15)
      RETURN
      END
```

APPENDIX F

BRIAN'S METHOD CODE

```
C     BCR FORTRAN A - (REV. 12/08/88)
C     SATELLITE MODEL - VALIDATED 12/08/88
C     FOR FORTVS APPLICATION ONLY
C     TO EXECUTE TYPE 'LOAD BCR TIMER (START'
C     ********************************************************
C     * BCR - PROGRAM TO COMPUTE NODE TEMPERATURES FOR A RIGHT *
C     *       CIRCULAR CYLINDER SUBJECTED TO CONVECTIVE AND/OR *
C     *       RADIATIVE BOUNDARY CONDITIONS USING BRIAN'S      *
C     *       METHOD OF FINITE DIFFERENCE ANALYSIS.            *
C     ********************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION TOLD(15,15,15),TNEW(15,15,15)
      DIMENSION TSTAR(15,15,15),T2STAR(15,15,15)
      DIMENSION T3STAR(15,15,15)
      COMMON /ARRSIZ/ III,JJJ,KKK
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
      COMMON /TIMES/  DELTIM,DTMAX,FINTIM,LIMIT
C     START TIMER
      CALL STIME
      OPEN (UNIT=11,FILE='BCRLNG')
      OPEN (UNIT=14,FILE='BCRDAT')
      OPEN (UNIT=15,FILE='BCRCPU')
C********************** USER DEFINED VARIABLES ************************
      DELTIM = 120.0D+00
      FINTIM = 3000.D+00
      IPRINT = 1
C     ACTUAL MATRIX DIMENSIONS
      NODES  = 10
      NPHI   = 4
      NLAYER = 5
C     NODE SPECIFICATION FOR FILE ESHORT
      MRAD   = 10
      MPHI   = 2
      MLAYER = 3
C     DECLARED ARRAY DIMENSIONS
      III = 15
      JJJ = 15
      KKK = 15
C     ABSORPTIVITY COEFFICIENTS
```

```
      ALFAN = 0.4D+00
      ALFAS = 0.4D+00
      ALFAO = 0.4D+00
C     ALBEDO COEFFICIENT
      ALBEDO = 0.30D+00
C     THERMAL CONDUCTIVITIES
      DKR = 177.D+00
      DKP = 177.D+00
      DKZ = 177.D+00
C     EMISSIVITIES
      EPSUBN = 0.9D+00
      EPSUBS = 0.9D+00
      EPSUBI = 0.D+00
      EPSUBO = 0.9D+00
C     CONVECTION COEFFICIENTS
      HSUBN = 0.D+00
      HSUBS = 0.D+00
      HSUBI = 0.D+00
      HSUBO = 0.D+00
C     HEAT FLUXES
      SOLAR = 1353.D+00
      EIR   = 238.0D+00
      GEN = 1000.
      QPSUBI = 0.D+00
      QPSUBO = 0.D+00
      QPSUBN = 0.D+00
      QPSUBS = 0.D+00
C     GEOMETRY
      RMIN = 0.15244D+00
      RMAX = 0.22866D+00
      ZMAX = 1.D+00
C     VIEW FACTORS WITH THE EARTH
      VERTHN = 0.D+00
      VERTHS = 1.D+00
      VERTHI = 0.D+00
      VERTH0 = 0.D+00
C     VIEW FACTORS WITH SPACE
      VSPCN = 1.D+00
      VSPCS = 0.D+00
      VSPCI = 0.D+00
      VSPCO = 1.D+00
C     TEMPERATURES
      TABS  = 273.D+00
      TINIT = 293.D+00
      TINF  = 0.D+00
      TSUR  = 4.D+00
C     MATERIAL PROPERTIES
      CSUBP = 875.0D+00
      RHO   = 2270.0D+00
C     INITIALIZE OTHER VARIABLES
      ICOUNT = 0
      ITIME  = 0
      ORBTIM = 0.D+00
      PI     = 4.0D+00*DTAN(1.D+00)
      RAD    = RMIN
```

```
      SIGMA  = 5.67D-08
      TIME   = 0.D+00
      T1     = 0.D+00
C     INITIALIZE MATRICES
      CALL BMINIT (TOLD,TINIT)
      CALL BMINIT (TSTAR,TINIT)
      CALL BMINIT (T2STAR,TINIT)
      CALL BMINIT (T3STAR,TINIT)
      CALL BMINIT (TNEW,TINIT)
C     SURFACE AREA CALCULATIONS
      SAREAN = PI*(RMAX**2 - RMIN**2)
      SAREAS = PI*(RMAX**2 - RMIN**2)
      SAREAI = PI*(2*RMIN)*ZMAX
      SAREAO = PI*(2*RMAX)*ZMAX
C     INITIAL CALCULATIONS
      ALFA = DKZ/(RHO*CSUBP)
      DPHI = 2.*PI/NPHI
      DR = (RMAX-RMIN)/(NODES-1)
      DZ = ZMAX/(NLAYER-1)
      LPHI = 360/NPHI
      QPSUBI = GEN/SAREAI
C     ORBITAL PARAMETERS
      PERIOD = 5440.D+00
      ECLIPS = 2181.9D+00
      OMEGA = 1.155D-03
      TAU = 0.01*PERIOD/4
      TIME1 = 0.D+00
      TIME2 = PERIOD/4.D+00
      TIME3 = 107.8D+00/360 * PERIOD
      TIME4 = (270 - 17.8)/360 * PERIOD - 4*TAU
      TIME5 = (270 - 17.8)/360 * PERIOD
      TIME6 = 270/360 * PERIOD
C     ESTABLISH EXPLICIT STABILITY REQUIREMENT (FOR REFERENCE ONLY)
      CALL STABIL (DTMAX)
      FO = ALFA*DELTIM/(DZ**2)
      LIMIT = FINTIM/DELTIM + 500
C     WRITE MATRIX DIMENSIONS TO FILE 14 (BCRDAT)
      WRITE (14,*) NODES,NPHI,NLAYER
C
C     *** PRINT OPTION 3 - SAVES DESIGNATED NODE TEMPS AT SPECIFIED
C                          PRINT INTERVAL.  USE FOR LONG PERIOD EVALS.
      WRITE (11,*) 'BCRSAT'
      WRITE (11,*)
      WRITE (11,888)
888   FORMAT (T1,'TIME',T15,'DELTIM',T25,'LAYER1',T40,'LAYER3',T55,'LAYE
      1R')
      WRITE (11,*)
C
CMMMMMMMMMMMMMMMMMMM BEGIN TEMPERATURE COMPUTATIONS MMMMMMMMMMMMMMMMMMM
C     START CLOCK TIMER (MAINFRAME VERSIONS OF CODE ONLY)
      WRITE (11,*) 'ENTERING LOOP TIME'
      CALL GTIME (1,11)
      DO 100 I=1,LIMIT
           ICOUNT=ICOUNT+1
           TIME=TIME+DELTIM
```

```
            ORBTIM = ORBTIM + DELTIM
            IF (ORBTIM.GE.PERIOD) ORBTIM = ORBTIM - PERIOD
C     SURFACE FLUX CALCULATIONS (TIME DEPENDENT)
            IF ((ORBTIM.GE.0).AND.(ORBTIM.LE.TIME2)) THEN
                QPSUBN = ALFAN*SOLAR*DCOS(OMEGA*ORBTIM)
                QPSUBS = ALFAS*(EIR + ALBEDO*SOLAR*DCOS(OMEGA*ORBTIM))
                QPSUBO = 0.5*ALFAO*SOLAR*DSIN(OMEGA*ORBTIM)
            ELSEIF ((ORBTIM.GT.TIME2).AND.(ORBTIM.LE.TIME3))THEN
                QPSUBN = 0.
                QPSUBS = ALFAS*(EIR + (ALBEDO+DCOS(OMEGA*ORBTIM))*SOLAR)
                QPSUBO = 0.5*ALFAO*SOLAR*DSIN(OMEGA*ORBTIM)
            ELSEIF ((ORBTIM.GT.TIME3).AND.(ORBTIM.LE.TIME4)) THEN
                QPSUBN = 0.
                QPSUBS = ALFAS*EIR
                QPSUBO = 0.D+00
            ELSEIF ((ORBTIM.GT.TIME4).AND.(ORBTIM.LE.TIME5)) THEN
                QPSUBN = 0.D+00
                QPSUBS = ALFAS*(EIR
     1                    + DABS(ALBEDO*SOLAR*DSIN(OMEGA*ORBTIM))
     1                    *(1-DEXP(-(ORBTIM-TIME4)/TAU)))
                QPSUBO = 0.5*ALFAO*DABS(ALBEDO*SOLAR
     1                    + DSIN(OMEGA*ORBTIM)
     1                    *(1-DEXP(-(ORBTIM-TIME4))/TAU))
            ELSEIF ((ORBTIM.GT.TIME5).AND.(ORBTIM.LT.TIME6)) THEN
                QPSUBN = 0.D+00
                QPSUBS = ALFAS*(EIR
     1                    + DABS(ALBEDO*SOLAR*DSIN(OMEGA*ORBTIM)))
                QPSUBO = 0.5*ALFAO*DABS(SOLAR*DSIN(OMEGA*ORBTIM))
            ELSEIF (ORBTIM.GT.TIME6) THEN
                QPSUBN = ALFAN*DABS(SOLAR*DCOS(OMEGA*ORBTIM))
                QPSUBS = ALFAS*(EIR
     1                    + DABS(ALBEDO*SOLAR*DSIN(OMEGA*ORBTIM)))
                QPSUBO = 0.5*ALFAO*DABS(SOLAR*DSIN(OMEGA*ORBTIM))
            ENDIF
C     CALL MAIN SUBROUTINES
            CALL STEP1 (TOLD,TSTAR)
C               WRITE (11,*) 'TSTAR'
C               CALL PRTMAT (TIME,DELTIM,TSTAR,NODES,NPHI,NLAYER)
            CALL STEP2 (TOLD,TSTAR,T2STAR)
C               WRITE (11,*) 'T2STAR'
C               CALL PRTMAT (TIME,DELTIM,T2STAR,NODES,NPHI,NLAYER)
            CALL STEP3 (TOLD,TSTAR,T2STAR,T3STAR)
C               WRITE (11,*) 'T3STAR'
C               CALL PRTMAT (TIME,DELTIM,T3STAR,NODES,NPHI,NLAYER)
            CALL STEP4 (TOLD,TSTAR,T2STAR,T3STAR,TNEW)
C               WRITE (11,*) 'TNEW'
C               CALL PRTMAT (TIME,DELTIM,TNEW,NODES,NPHI,NLAYER)
C     MAP NEW TEMPERATURE MATRIX ONTO OLD TEMPERATURE MATRIX
            CALL MAP122 (TNEW,TOLD)
C
C     **** GENERATE PRINTOUT ****
      IF (ICOUNT.GE.IPRINT)  THEN
            ICOUNT = 0
            WRITE (14,*) TIME,DELTIM
C     **** CONVERT TNEW TO EXCESS TEMPERATURE PRIOR TO PRINTING ****
```

```
            DO 200 LAYER=1,NLAYER
            DO 300 INCRAD=1,NODES
            DO 400 IPHI=1,NPHI
              TNEW(INCRAD,IPHI,LAYER) = TNEW(INCRAD,IPHI,LAYER 1) - TINIT
C     **** PRINT OPTION 1 - USE WITH CMPSHT,COMPAR,LNGFRM,SHTFRM,BCRERR
C                           SHORT EVALUATION TIMES ONLY (<400 SEC)
C                        WRITE (14,*) TNEW(INCRAD,IPHI,LAYER)
400                   CONTINUE
300                CONTINUE
200             CONTINUE
C
C     *** PRINT OPTION 2 - SAVES COMPLETE MATRIX AT SPECIFIED PRINT
C                          INTERVAL - SWITCH OFF ALL SUBROUTINES LISTED
C                                     UNDER PRINT OPTION 1 FIRST.
C               WRITE (11,*) 'BCR SATELLITE MODEL'
C               WRITE (*,*)
C               CALL PRTMAT (TIME,DELTIM,TNEW,NODES,NPHI,NLAYER)
C
C     *** PRINT OPTION 3 - SAVES DESIGNATED NODE TEMPS AT SPECIFIED
C                          PRINT INTERVAL.  USE FOR LONG PERIOD EVALS.
      WRITE (11,889) TIME,DELTIM,TNEW(5,1,1),TNEW(5,1,3),TNEW(5,1,5)
889   FORMAT (T1,F9.3,T15,F9.3,T25,F12.6,T40,F12.6,T55,F12.6)
C
C     ******** RE-SPECIFY PRINT INTERVALS ********
C               IF (TIME.GE.99.9) THEN
C                    IPRINT = 200
C                    DELTIM = 20.
C                    FO = ALFA*DELTIM/(DZ**2)
C               ELSEIF (TIME.GE.9.9) THEN
C                    IPRINT = 100
C                    DELTIM = 10.
C                    FO = ALFA*DELTIM/(DZ**2)
C               ELSEIF (TIME.GE.0.99) THEN
C                    IPRINT = 10
C                    DELTIM = 1.
C                    FO = ALFA*DELTIM/(DZ**2)
C               ENDIF
            ENDIF
C     TIME INTERVAL DELIMITER
            IF (TIME.GE.FINTIM) GOTO 101
100   CONTINUE
C     STOP CLOCK TIMER (MAINFRAME VERSIONS OF CODE ONLY)
101   CONTINUE
CALL GTIME (1,11)
WRITE (11,*) 'EXITED LOOP'
CLOSE (14)
CLOSE (15)
C     WRITE APPROPRIATE DATA FILES
C     CALL CMPSHT
C     CALL SHTFRM (MRAD,MPHI,MLAYER)
C     CALL COMPAR
C     CALL ERROR (MRAD,MPHI,MLAYER)
C     CALL LNGFRM
CALL GTIME(1,11)
999   STOP
```

```
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BMINIT FORTRAN A - (REV. 9/28/88)
C     ************************************************************
C     * BMINIT - SUBROUTINE TO INITIALIZE TEMPERATURE MATRIX TO A *
C     *          SPECIFIED VALUE (BRIAN'S METHOD ARRAY FORMAT)    *
C     ************************************************************
C     PROGRAM LINE 172
      SUBROUTINE BMINIT (DMAT,TINIT)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /ARRSIZ/ III,JJJ,KKK
      DIMENSION DMAT(III,JJJ,KKK)
      DO 100 LAYER=1,KKK
           DO 200 IPHI=1,JJJ
                DO 300 INCRAD=1,III
                     DMAT(INCRAD,IPHI,LAYER) = TINIT
300             CONTINUE
200        CONTINUE
100   CONTINUE
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     STABIL FORTRAN A (REV. 11/14/88)
C     ************************************************************
C     * STABIL - SUBROUTINE TO COMPUTE THEORETICAL MAXIMUM TIME    *
C     *          INCREMENT ALLOWED TO MAINTAIN STABILITY IN EXPLICIT *
C     *          FINITE DIFFERENCE ANALYSIS OF A CYLINDER IN 3-D     *
C     *          POLAR CYLINDRICAL COORDINATES.                      *
C     ************************************************************
C     PROGRAM LINE 196
      SUBROUTINE STABIL (DTMAX)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION DT(6)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI COMMON
      /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO COMMON /RADN/
      EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
C                          *** BODY ***
      DO 50 I=1,6
      DT(I) = 0.D+00
50    CONTINUE
      DUMMY = (DZ**2)/ALFA
C     END LAYERS, INNER SURFACE NODES
           DCREI = 1 + (2*DKP*DZ**2)/(2*DKZ*RAD*(RAD+DR/4)*DPHI**2)
      1+ (DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
           DT(1) = DUMMY/(2*DCREI)
           DTMAX = DT(1)
C     END LAYERS, MID-RADII NODES
           DCREM = 1 + 2*(DKP*DZ**2)/(2*DKZ*RAD**2*DPHI**2)
      1+ (DKR*(RAD-DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
```

```
     2+ (DKR*(RAD+DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      DT(2) = DUMMY/(2*DCREM)
C     END LAYERS, OUTER SURFACE NODES
      DCREO = 1 + 2*(DKP*DZ**2)/(2*DKZ*(RAD-DR/4)*RAD*DPHI**2)
     1+ (DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      DT(3) = DUMMY/(2*DCREO)
C     MID-LAYERS, INNER SURFACE NODES
      DCRMI = 2 + 2*(DKP*DZ**2)/(DKZ*RAD*(RAD+DR/4)*DPHI**2)
     1+ (2*DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
      DT(4) = DUMMY/DCRMI
C     MID-LAYERS, MID-RADII NODES
      DCRMM = 2 + 2*(DKP*DZ**2)/(DKZ*RAD**2*DPHI**2)
     1+ (DKR*(RAD-DR/2)*DZ**2)/(DKZ*RAD*DR**2)
     2+ (DKR*(RAD+DR/2)*DZ**2)/(DKZ*RAD*DR**2)
      DT(5) = DUMMY/DCRMM
C     MID-LAYERS, OUTER SURFACE NODES
      DCRMO = 2 + 2*(DKP*DZ**2)/(DKZ*RAD*(RAD-DR/4)*DPHI**2)
     1+ (2*DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      DT(6) = DUMMY/DCRMO
      DO 100 I=1,6
       IF ((DT(I).GT.0).AND.(DTMAX.GT.DT(I))) DTMAX = DT(I)
      WRITE (*,*) 'FROM STABIL, I,DT(I)',I,DT(I)
100   CONTINUE
      IF (DTMAX.LE.0) WRITE (*,*) 'STABIL - DTMAX LE ZERO'
      WRITE (*,*) 'FROM STABIL, DTMAX =',DTMAX
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     STEP1 FORTRAN A - (REV. 10/12/88)

C     ****************************************************************
C     * STEP1 - THIS SUBROUTINE CONSTRUCTS THE TEMPERATURE MATRIX    *
C     *         FOR THE FIRST INTERMEDIATE STEP (STAR LEVEL) IN      *
C     *         THE Z DIRECTION OF A 3-D, POLAR-CYLINDRICAL ADAP-    *
C     *         TATION OF BRIAN'S METHOD OF FINITE DIFFERENCE        *
C     *         ANALYSIS FOR A RIGHT CIRCULAR CYLINDER WITH A        *
C     *         SPECIFIED INNER SURFACE TEMPERATURE AND CONSTANT     *
C     *         OUTER SURFACE HEAT FLUX.                             *
C     ****************************************************************
C     PROGRAM LINE 259
      SUBROUTINE STEP1 (TOLD,TSTAR)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /ARRSIZ/ III,JJJ,KKK
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/    T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR D
      DIMENSION TOLD(15,15,15),TSTAR(15,15,15)
```

```
      DIMENSION AA(15),BB(15),CC(15),DD(15),TEMP(15)
CIIIIIIIIIIIIIIIIIIIIIII INNER SURFACE NODES IIIIIIIIIIIIIIIIIIIIIIII
      INCRAD = 1
      RAD = RMIN
C
C             ******** INNER SURFACE, FIRST PHI ********
          IPHI = 1
C       ******** INNER SURFACE, FIRST PHI, BOTTOM LAYER ********
          LAYER = 1
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,NPHI,LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREI1(AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER)) C
C        ******** INNER SURFACE, FIRST PHI, MID-LAYERS ********
          DO 100 LAYER=2,(NLAYER-1)
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,NPHI,LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCRMI1(AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
100   CONTINUE
C
C        ******** INNER SURFACE, FIRST PHI, TOP LAYER ********
      LAYER = NLAYER
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,NPHI,LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREI1(AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NLAYER,AA,BB,CC,DD,TEMP)
      DO 110 LAYER=1,NLAYER
      TSTAR(INCRAD,IPHI,LAYER) = TEMP(LAYER)
110            CONTINUE
C
C             ******** INNER SURFACE,MID-PHIS ********
      DO 200 IPHI=2,(NPHI-1)
C       ******** INNER SURFACE, MID-PHIS, BOTTOM LAYER ********
      LAYER = 1
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,(IPHI-1),LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREI1(AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C        ******** INNER SURFACE, MID-PHIS, MID-LAYERS ********
      DO 210 LAYER=2,(NLAYER-1)
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,(IPHI-1),LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCRMI1(AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
210   CONTINUE
```

```
C
C        ******** INNER SURFACE, MID-PHIS, TOP LAYER ********
      LAYER = NLAYER
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,(IPHI-1),LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREI1(AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NLAYER,AA,BB,CC,DD,TEMP)
      DO 230 LAYER=1,NLAYER
      TSTAR(INCRAD,IPHI,LAYER) = TEMP(LAYER)
230            CONTINUE
C
200   CONTINUE
C
C                ******** INNER SURFACE, LAST PHI ********
      IPHI = NPHI
C
C       ******** INNER SURFACE, LAST PHI, BOTTOM LAYER ********
      LAYER = 1
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,(IPHI-1),LAYER)
               T3 = TOLD(INCRAD,1,LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREI1(AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C        ******** INNER SURFACE, LAST PHI, MID-LAYERS ********
      DO 300 LAYER=2,(NLAYER-1)
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,(IPHI-1),LAYER)
               T3 = TOLD(INCRAD,1,LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCRMI1(AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
300   CONTINUE
C
C        ******** INNER SURFACE, LAST PHI, TOP LAYER ********
      LAYER = NLAYER
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,(IPHI-1),LAYER)
               T3 = TOLD(INCRAD,1,LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREI1(AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NLAYER,AA,BB,CC,DD,TEMP)
      DO 350 LAYER=1,NLAYER
      TSTAR(INCRAD,IPHI,LAYER) = TEMP(LAYER)
350            CONTINUE
C
C
CMMMMMMMMMMMMMMMMMMMMMMMMMM MID-RADIUS NODES MMMMMMMMMMMMMMMMMMMMMMMMMM
      DO 400 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
```

```
C                ******** MID-RADII, FIRST PHI ********
      IPHI = 1
C
C          ******** MID-RADII, FIRST PHI, BOTTOM LAYER ********
      LAYER = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,NPHI,LAYER)
                T3 = TOLD(INCRAD,(IPHI+1),LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCREM1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER)) C
C          ******** MID-RADII, FIRST PHI, MID-LAYERS ********
      DO 500 LAYER=2,(NLAYER-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,NPHI,LAYER)
                T3 = TOLD(INCRAD,(IPHI+1),LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCRMM1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
500        CONTINUE
C
C          ******** MID-RADII, FIRST PHI, TOP LAYER ********
      LAYER = NLAYER
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,NPHI,LAYER)
                T3 = TOLD(INCRAD,(IPHI+1),LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCREM1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NLAYER,AA,BB,CC,DD,TEMP)
      DO 475 LAYER=1,NLAYER
      TSTAR(INCRAD,IPHI,LAYER) = TEMP(LAYER)
475             CONTINUE
C
C                ******** MID-RADII, MID-PHIS ********
      DO 600, IPHI=2,(NPHI-1)
C
C          ******** MID-RADII, MID-PHIS, FIRST LAYER ********
      LAYER = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,(IPHI+1),LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCREM1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C          ******** MID-RADII, MID-PHIS, MID-LAYERS ********
      DO 700 LAYER=2,(NLAYER-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,(IPHI+1),LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
```

```
                CALL BCRMM1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
700         CONTINUE
C
C             ******** MID-RADII, MID-PHIS, TOP LAYER ********
      LAYER = NLAYER
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,(IPHI+1),LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCREM1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NLAYER,AA,BB,CC,DD,TEMP)
      DO 575 LAYER=1,NLAYER
      TSTAR(INCRAD,IPHI,LAYER) = TEMP(LAYER)
575             CONTINUE
600         CONTINUE
C
C                 ******** MID-RADII, LAST PHI ********
      IPHI = NPHI
C
C           ******** MID-RADII, LAST PHI, FIRST LAYER ********
      LAYER = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,1,LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCREM1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C           ******** MID-RADII, LAST PHI, MID-LAYERS ********
      DO 800 LAYER=2,(NLAYER-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,1,LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCRMM1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
800         CONTINUE
C
C             ******** MID-RADII, LAST PHI, TOP LAYER ********
      LAYER = NLAYER
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,1,LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCREM1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))

C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NLAYER,AA,BB,CC,DD,TEMP)
      DO 675 LAYER=1,NLAYER
      TSTAR(INCRAD,IPHI,LAYER) = TEMP(LAYER)
675             CONTINUE
```

```
400   CONTINUE
C
COOOOOOOOOOOOOOOOOOOOOOO OUTER SURFACE NODES OOOOOOOOOOOOOOOOOOOOOOOOO
INCRAD = NODES
RAD = RMAX
C
C              ******** OUTER RADIUS, FIRST PHI ********
      IPHI = 1
C
C         ******** OUTER RADIUS, FIRST PHI, BOTTOM LAYER ********
      LAYER = 1
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,NPHI,LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD-1),IPHI,LAYER)
               CALL BCREO1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C         ******** OUTER RADIUS, FIRST PHI, MID-LAYERS ********
      DO 900 LAYER=2,(NLAYER-1)
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,NPHI,LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD-1),IPHI,LAYER)
               CALL BCRMO1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
900       CONTINUE
C
C          ******** OUTER RADIUS, FIRST PHI, TOP LAYER ********
      LAYER = NLAYER
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,NPHI,LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD-1),IPHI,LAYER)
               CALL BCREO1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C                    **** COMPUTE NODE TEMPERATURES ***
      CALL TRIDAG (1,NLAYER,AA,BB,CC,DD,TEMP)
      DO 775 LAYER=1,NLAYER
      TSTAR(INCRAD,IPHI,LAYER) = TEMP(LAYER)
775            CONTINUE
C
C              ******** OUTER SURFACE, MID-PHIS ********
      DO 1000 IPHI=2,(NPHI-1)
C
C         ******** OUTER RADIUS, MID-PHIS, FIRST LAYER ********
      LAYER = 1
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,(IPHI-1),LAYER)
               T3 = TOLD(INCRAD,(IPHI+1),LAYER)
               T4 = TOLD((INCRAD-1),IPHI,LAYER)
               CALL BCREO1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C          ******** OUTER RADIUS, MID-PHIS, MID-LAYERS ********
      DO 1100 LAYER=2,(NLAYER-1)
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TOLD(INCRAD,(IPHI-1),LAYER)
```

```
                T3 = TOLD(INCRAD,(IPHI+1),LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL BCRMO1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
1100        CONTINUE
C
C           ******** OUTER RADIUS, MID-PHIS, TOP LAYER ********
        LAYER = NLAYER
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,(IPHI+1),LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL BCREO1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C                   **** COMPUTE NODE TEMPERATURES ****
        CALL TRIDAG (1,NLAYER,AA,BB,CC,DD,TEMP)
        DO 875 LAYER=1,NLAYER
        TSTAR(INCRAD,IPHI,LAYER) = TEMP(LAYER)
875             CONTINUE
1000        CONTINUE
C
C                ******** OUTER SURFACE, LAST PHI ********
        IPHI = NPHI
C
C           ******** OUTER RADIUS, LAST PHI, BOTTOM LAYER ********
        LAYER = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,1,LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL BCREO1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C           ******** OUTER RADIUS, LAST PHI, MID-LAYERS ********
        DO 1200 LAYER=2,(NLAYER-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,1,LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL BCRMO1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
1200        CONTINUE
C
C           ******** OUTER RADIUS, LAST PHI, TOP LAYER *********
        LAYER = NLAYER
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TOLD(INCRAD,(IPHI-1),LAYER)
                T3 = TOLD(INCRAD,1,LAYER)
                T4 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL BCREO1 (AA(LAYER),BB(LAYER),CC(LAYER),DD(LAYER))
C
C                   **** COMPUTE NODE TEMPERATURES ****
        CALL TRIDAG (1,NLAYER,AA,BB,CC,DD,TEMP)
        DO 975 LAYER=1,NLAYER
        TSTAR(INCRAD,IPHI,LAYER) = TEMP(LAYER)
975             CONTINUE
998     RETURN
        END
```

```
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     STEP2 FORTRAN A - (REV. 10/18/88)
C     ********************************************************
C     * STEP2 - SECOND INTERMEDIATE STEP (PHI DIRECTION)    *
C     *         OF BRIAN'S METHOD OF FINITE DIFFERENCE      *
C     *         ANALYSIS FOR A RIGHT CIRCULAR CYLINDER.     *
C     ********************************************************
C     PROGRAM LINE 614
      SUBROUTINE STEP2 (TOLD,TSTAR,T2STAR)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /ARRSIZ/ III,JJJ,KKK
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
      DIMENSION TOLD(15,15,15),TSTAR(15,15,15),T2STAR(15,15,15)
      DIMENSION AA(15),BB(15),CC(15),DD(15),TEMP(15)

C
C1111111111111111111111111111 FIRST LAYER 1111111111111111111111111111
LAYER  = 1
C
C            ******** FIRST LAYER, INNER RADIUS ********
      INCRAD = 1
      RAD = RMIN
      DO 100 IPHI=1,NPHI
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREI2 (AA(IPHI),BB(IPHI),CC(IPHI),DD(IPHI))
100   CONTINUE
C
C                 **** CONVERT TO TRIDIAGONAL FORM ****
      BB(1) = BB(1)+AA(1)
      AA(1) = 0.D+00
      BB(NPHI) = BB(NPHI)+CC(NPHI)
      CC(NPHI) = 0.D+00
C
C               **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NPHI,AA,BB,CC,DD,TEMP)
      DO 150 IPHI=1,NPHI
      T2STAR(INCRAD,IPHI,LAYER) = TEMP(IPHI)
150       CONTINUE
C
C               ******** FIRST LAYER, MID-RADII ********
      DO 200 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
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```
      DO 300 IPHI=1,NPHI
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = TOLD((INCRAD-1),IPHI,LAYER)
               T5 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREM2 (AA(IPHI),BB(IPHI),CC(IPHI),DD(IPHI))
300       CONTINUE
C
C                **** CONVERT TO TRIDIAGONAL FORM ****
      BB(1) = BB(1)+AA(1)
      AA(1) = 0.D+00
      BB(NPHI) = BB(NPHI)+CC(NPHI)
      CC(NPHI) = 0.D+00
C
C               **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NPHI,AA,BB,CC,DD,TEMP)
      DO 375 IPHI=1,NPHI
      T2STAR(INCRAD,IPHI,LAYER) = TEMP(IPHI)
375       CONTINUE
200   CONTINUE
C
C             ******** FIRST LAYER, OUTER RADIUS ********
      INCRAD = NODES
      RAD = RMAX
      DO 400 IPHI=1,NPHI
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = TOLD((INCRAD-1),IPHI,LAYER)
               CALL BCREO2 (AA(IPHI),BB(IPHI),CC(IPHI),DD(IPHI))
400       CONTINUE
C
C                **** CONVERT TO TRIDIAGONAL FORM ****
      BB(1) = BB(1)+AA(1)
      AA(1) = 0.D+00
      BB(NPHI) = BB(NPHI)+CC(NPHI)
      CC(NPHI) = 0.D+00
C
C                 **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NPHI,AA,BB,CC,DD,TEMP)
      DO 475 IPHI=1,NPHI
      T2STAR(INCRAD,IPHI,LAYER) = TEMP(IPHI)
475       CONTINUE
C
CMMMMMMMMMMMMMMMMMMMMMMMMMMMMM MID-LAYERS MMMMMMMMMMMMMMMMMMMMMMMMMMMM
DO 500 LAYER=2,(NLAYER-1)
C
C              ******** MID LAYERS, INNER RADIUS ********
      INCRAD = 1
      RAD = RMIN
      DO 600 IPHI=1,NPHI
               T1 = TOLD (INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
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                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCRMI2 (AA(IPHI),BB(IPHI),CC(IPHI),DD(IPHI))
600     CONTINUE
C
C                  **** CONVERT TO TRIDIAGONAL FORM ****
        BB(1) = BB(1)+AA(1)
        AA(1) = 0.D+00
        BB(NPHI) = BB(NPHI)+CC(NPHI)
        CC(NPHI) = 0.D+00
C
C                   **** COMPUTE NODE TEMPERATURES ****
        CALL TRIDAG (1,NPHI,AA,BB,CC,DD,TEMP)
        DO 675 IPHI=1,NPHI
        T2STAR(INCRAD,IPHI,LAYER) = TEMP(IPHI)
675         CONTINUE
C
C                 ******** MID-LAYERS, MID-RADII ********
        DO 700 INCRAD=2,(NODES-1)
        RAD=RMIN+(INCRAD-1)*DR
        DO 800 IPHI=1,NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                T6 = TOLD((INCRAD+1),IPHI,LAYER)
                CALL BCRMM2 (AA(IPHI),BB(IPHI),CC(IPHI),DD(IPHI))
800         CONTINUE
C
C                  **** CONVERT TO TRIDIAGONAL FORM ****
        BB(1) = BB(1)+AA(1)
        AA(1) = 0.D+00
        BB(NPHI) = BB(NPHI)+CC(NPHI)
        CC(NPHI) = 0.D+00
C
C                   **** COMPUTE NODE TEMPERATURES ****
        CALL TRIDAG (1,NPHI,AA,BB,CC,DD,TEMP)
        DO 875 IPHI=1,NPHI
        T2STAR(INCRAD,IPHI,LAYER) = TEMP(IPHI)
875         CONTINUE
700     CONTINUE
C
C                 ******** MID-LAYERS, OUTER RADIUS ********
        INCRAD = NODES
        RAD = RMAX
        DO 900 IPHI=1,NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = TOLD((INCRAD-1),IPHI,LAYER)
                CALL BCRMO2 (AA(IPHI),BB(IPHI),CC(IPHI),DD(IPHI))
900         CONTINUE
C
```

```
C                **** CONVERT TO TRIDIAGONAL FORM ****
      BB(1) = BB(1)+AA(1)
      AA(1) = 0.D+00
      BB(NPHI) = BB(NPHI)+CC(NPHI)
      CC(NPHI) = 0.D+00
C                 **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NPHI,AA,BB,CC,DD,TEMP)
      DO 975 IPHI=1,NPHI
      T2STAR(INCRAD,IPHI,LAYER) = TEMP(IPHI)
975       CONTINUE
500   CONTINUE
C
CUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUU TOP LAYER UUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUU
LAYER = NLAYER
C
C             ******** TOP LAYER, INNER RADIUS ********
      INCRAD = 1
      RAD = RMIN
      DO 1000 IPHI=1,NPHI
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREI2 (AA(IPHI),BB(IPHI),CC(IPHI),DD(IPHI))
1000  CONTINUE
C
C                **** CONVERT TO TRIDIAGONAL FORM ****
C
      BB(1) = BB(1)+AA(1)
      AA(1) = 0.D+00
      BB(NPHI) = BB(NPHI)+CC(NPHI)
      CC(NPHI) = 0.D+00
C
C                 **** COMPUTE NODE TEMPERATURES ****
C
      CALL TRIDAG (1,NPHI,AA,BB,CC,DD,TEMP)
      DO 1075 IPHI=1,NPHI
      T2STAR(INCRAD,IPHI,LAYER) = TEMP(IPHI)
1075       CONTINUE
C
C              ******** TOP LAYER, MID-RADII ********
      DO 1100 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
      DO 1200 IPHI=1,NPHI
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = TOLD((INCRAD-1),IPHI,LAYER)
               T5 = TOLD((INCRAD+1),IPHI,LAYER)
               CALL BCREM2 (AA(IPHI),BB(IPHI),CC(IPHI),DD(IPHI))
1200       CONTINUE
C
C                **** CONVERT TO TRIDIAGONAL FORM ****
      BB(1) = BB(1)+AA(1)
      AA(1) = 0.D+00
```

```
      BB(NPHI) = BB(NPHI)+CC(NPHI)
      CC(NPHI) = 0.D+00
C
C     **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NPHI,AA,BB,CC,DD,TEMP)
      DO 1275 IPHI=1,NPHI
      T2STAR(INCRAD,IPHI,LAYER) = TEMP(IPHI)
1275       CONTINUE
1100  CONTINUE
C
C             ******** TOP LAYER, OUTER RADIUS ********
      INCRAD = NODES
      RAD = RMAX
      DO 1300 IPHI=1,NPHI
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = TOLD((INCRAD-1),IPHI,LAYER)
               CALL BCREO2 (AA(IPHI),BB(IPHI),CC(IPHI),DD(IPHI))
1300       CONTINUE
C
C                **** CONVERT TO TRIDIAGONAL FORM ****
      BB(1) = BB(1)+AA(1)
      AA(1) = 0.D+00
      BB(NPHI) = BB(NPHI)+CC(NPHI)
      CC(NPHI) = 0.D+00
C
C                 **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NPHI,AA,BB,CC,DD,TEMP)
      DO 1375 IPHI=1,NPHI
      T2STAR(INCRAD,IPHI,LAYER) = TEMP(IPHI)
1375      CONTINUE
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     STEP3 FORTRAN A - (REV. 10/17/88)
C     ****************************************************
C     * STEP3 - THIRD INTERMEDIATE STEP (RADIAL DIRECTION) *
C     *         OF BRIAN'S METHOD (LONG FORM) OF FINITE    *
C     *         DIFFERENCE ANALYSIS FOR A RIGHT CIRCULAR   *
C     *         CYLINDER WITH CONSTANT SURFACE HEAT FLUX   *
C     *         AND SPECIFIED INNER SURFACE TEMPERATURE.   *
C     ****************************************************
C     PROGRAM LINE 870
      SUBROUTINE STEP3 (TOLD,TSTAR,T2STAR,T3STAR)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /ARRSIZ/ III,JJJ,KKK
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
```

```
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
      DIMENSION TOLD(15,15,15),TSTAR(15,15,15),T2STAR(15,15,15)
      DIMENSION T3STAR(15,15,15)
      DIMENSION AA(15),BB(15),CC(15),DD(15),TEMP(15)
C
C1111111111111111111111111111 BOTTOM LAYER 1111111111111111111111111111
      LAYER = 1
C
C             ******** BOTTOM LAYER, FIRST RAY ********
      IPHI   = 1
C
C        ******** BOTTOM LAYER, FIRST RAY, INNER NODE ********
      INCRAD = 1
      RAD = RMIN
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,NPHI,LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREI3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C         ******** BOTTOM LAYER, FIRST RAY, MID-NODES
      ******** DO 100 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,NPHI,LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREM3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
100        CONTINUE
C
C        ******** BOTTOM LAYER, FIRST RAY, OUTER NODE ********
      RAD = RMAX
      INCRAD = NODES
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,NPHI,LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREO3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NODES,AA,BB,CC,DD,TEMP)
      DO 175 INCRAD=1,NODES
      T3STAR(INCRAD,IPHI,LAYER) = TEMP(INCRAD)
175            CONTINUE
C
C               ******** BOTTOM LAYER, MID-RAYS ********
      DO 200 IPHI=2,(NPHI-1)
```

```
C
C          ******** BOTTOM LAYER, MID-RAYS, INNER NODE ********
      INCRAD = 1
      RAD    = RMIN
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREI3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C          ******** BOTTOM LAYER, MID-RAYS, MID-NODES ********
      DO 300 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREM3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
300       CONTINUE
C
C          ******** BOTTOM LAYER, MID-RAYS, OUTER NODE ********
      RAD = RMAX
      INCRAD = NODES
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREO3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NODES,AA,BB,CC,DD,TEMP)
      DO 375 INCRAD=1,NODES
      T3STAR(INCRAD,IPHI,LAYER) = TEMP(INCRAD)
375            CONTINUE
200   CONTINUE
C
C                ******** BOTTOM LAYER, LAST RAY ********
      IPHI   = NPHI
C
C          ******** BOTTOM LAYER, LAST RAY, INNER NODE ********
      INCRAD = 1
      RAD    = RMIN
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
               T5 = T2STAR(INCRAD,1,LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREI3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
```

```
C
C          ******** BOTTOM LAYER, LAST RAY, MID-NODES ********
      DO 400 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCREM3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
400        CONTINUE
C
C        ******** BOTTOM LAYER, LAST RAY, OUTER NODE ********
      RAD = RMAX
      INCRAD = NODES
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCREO3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C                   **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NODES,AA,BB,CC,DD,TEMP)
      DO 475 INCRAD=1,NODES
      T3STAR(INCRAD,IPHI,LAYER) = TEMP(INCRAD)
475             CONTINUE
C
CIIIIIIIIIIIIIIIIIIIIII INTERMEDIATE LAYERS IIIIIIIIIIIIIIIIIIIIII
      DO 500 LAYER=2,(NLAYER-1)
C
C           ******** INTERMEDIATE LAYERS, FIRST RAY ********
      IPHI   = 1
C
C    ******** INTERMEDIATE LAYERS, FIRST RAY, FIRST NODE ********
      INCRAD = 1
      RAD    = RMIN
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,NPHI,LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCRMI3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C     ******** INTERMEDIATE LAYERS, FIRST RAY, MID-NODES ********
      DO 600 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
```

```
                T5 = T2STAR(INCRAD,NPHI,LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCRMM3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
600         CONTINUE
C
C       ******** INTERMEDIATE LAYERS, FIRST RAY, OUTER NODE ********
        INCRAD = NODES
        RAD = RMAX
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,NPHI,LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCRMO3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C                   **** COMPUTE NODE TEMPERATURES ****
        CALL TRIDAG (1,NODES,AA,BB,CC,DD,TEMP)
        DO 675 INCRAD=1,NODES
        T3STAR(INCRAD,IPHI,LAYER) = TEMP(INCRAD)
675             CONTINUE
C
C           ******** INTERMEDIATE LAYERS, MID-RAYS ********
        DO 700 IPHI=2,(NPHI-1)
C
C       ******** INTERMEDIATE LAYERS, MID-RAYS, INNER NODE ********
        INCRAD = 1
        RAD    = RMIN
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCRMI3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C        ******** INTERMEDIATE LAYERS, MID-RAYS, MID-NODES ********
        DO 800 INCRAD=2,(NODES-1)
                RAD = RMIN + (INCRAD-1)*DR
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCRMM3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
800         CONTINUE
C
C       ******** INTERMEDIATE LAYERS, MID-RAYS, OUTER NODE ********
        INCRAD = NODES
        RAD = RMAX
```

```
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCRMO3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NODES,AA,BB,CC,DD,TEMP)
      DO 875 INCRAD=1,NODES
      T3STAR(INCRAD,IPHI,LAYER) = TEMP(INCRAD)
875             CONTINUE
700   CONTINUE
C
C           ******** INTERMEDIATE LAYERS, LAST RAY ********
      IPHI   = NPHI
C
C     ******** INTERMEDIATE LAYERS, LAST RAY, INNER NODE ********
      INCRAD = 1
      RAD    = RMIN
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,1,LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCRMI3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C      ******** INTERMEDIATE LAYERS, LAST RAY, MID-NODES ********
      DO 900 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,1,LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCRMM3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
900        CONTINUE
C
C     ******** INTERMEDIATE LAYERS, LAST RAY, OUTER NODE ********
      INCRAD = NODES
      RAD = RMAX
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,1,LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCRMO3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
```

```
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NODES,AA,BB,CC,DD,TEMP)
      DO 975 INCRAD=1,NODES
      T3STAR(INCRAD,IPHI,LAYER) = TEMP(INCRAD)
975             CONTINUE
500   CONTINUE
C
CTTTTTTTTTTTTTTTTTTTTTTTTTTTT TOP LAYER TTTTTTTTTTTTTTTTTTTTTTTTTTTT
LAYER = NLAYER
C
C                ******** TOP LAYER, FIRST RAY ********
      IPHI   = 1
C
C          ******** TOP LAYER, FIRST RAY, INNER NODE ********
      INCRAD = 1
      RAD    = RMIN
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,NPHI,LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
      CALL BCREI3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C          ******** TOP LAYER, FIRST RAY, MID-NODES ********
      DO 1000 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,NPHI,LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREM3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
1000       CONTINUE
C
C          ******** TOP LAYER, FIRST RAY, OUTER NODE ********
      INCRAD = NODES
      RAD = RMAX
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,NPHI,LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREO3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NODES,AA,BB,CC,DD,TEMP)
      DO 1075 INCRAD=1,NODES
T3STAR(INCRAD,IPHI,LAYER) = TEMP(INCRAD)
1075            CONTINUE
C
C                ******** TOP LAYER, MID-RAYS ********
```

```
      DO 1100 IPHI=2,(NPHI-1)
C
C          ******** TOP LAYER, MID-RAYS, INNER NODE ********
      INCRAD = 1
      RAD    = RMIN
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREI3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C          ******** TOP LAYER, MID-RAYS, MID-NODES ********
      DO 1200 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREM3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
1200       CONTINUE
C
C          ******** TOP LAYER, MID-RAYS, OUTER NODE ********
      INCRAD = NODES
      RAD = RMAX
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
               T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
               CALL BCREO3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C                  **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NODES,AA,BB,CC,DD,TEMP)
      DO 1275 INCRAD=1,NODES
      T3STAR(INCRAD,IPHI,LAYER) = TEMP(INCRAD)
1275           CONTINUE
1100   CONTINUE
C
C                ******** TOP LAYER, LAST RAY ********
      IPHI   = NPHI
C
C          ******** TOP LAYER, LAST RAY, INNER NODE ********
      INCRAD = 1
      RAD    = RMIN
               T1 = TOLD(INCRAD,IPHI,LAYER)
               T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
               T3 = TSTAR(INCRAD,IPHI,LAYER)
               T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
               T5 = T2STAR(INCRAD,1,LAYER)
               T6 = T2STAR(INCRAD,IPHI,LAYER)
```

```
                CALL BCREI3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C           ******** TOP LAYER, LAST RAY, MID-NODES ********
      DO 1300 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCREM3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
1300        CONTINUE
C
C           ******** TOP LAYER, LAST RAY, OUTER NODE ********
      INCRAD = NODES
      RAD = RMAX
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                CALL BCREO3 (AA(INCRAD),BB(INCRAD),CC(INCRAD),DD(INCRAD))
C
C                   **** COMPUTE NODE TEMPERATURES ****
      CALL TRIDAG (1,NODES,AA,BB,CC,DD,TEMP)
      DO 1375 INCRAD=1,NODES
      T3STAR(INCRAD,IPHI,LAYER) = TEMP(INCRAD)
1375            CONTINUE
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     STEP4 FORTRAN A - (REV. 10/25/88)
C     ******************************************************
C     * STEP 4 - FOURTH INTERMEDIATE STEP (TEMPERATURE AT   *
C     *          N+1 TIME STEP) FOR BRAIN'S METHOD OF       *
C     *          FINITE DIFFERENCE ANALYSIS FOR A RIGHT     *
C     *          CIRCULAR CYLINDER WITH CONSTANT SURFACE    *
C     *          HEAT FLUX AND SPECIFIED INNER SURFACE      *
C     *          TEMPERATURE.                               *
C     ******************************************************
C     PROGRAM LINE 1310
      SUBROUTINE STEP4 (TOLD,TSTAR,T2STAR,T3STAR,TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /ARRSIZ/ III,JJJ,KKK
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
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```
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
      DIMENSION TOLD(15,15,15),TSTAR(15,15,15),T2STAR(15,15,15)
      DIMENSION T3STAR(15,15,15),TNEW(15,15,15)
C
CBBBBBBBBBBBBBBBBBBBBBBBBBBB BOTTOM LAYER BBBBBBBBBBBBBBBBBBBBBBBBBBB
      LAYER  = 1
C
C                 **** BOTTOM LAYER, INNER RADIUS ****
      INCRAD = 1
      RAD = RMIN
C
C ******** BOTTOM LAYER, INNER RADIUS, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,NPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD+1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREI4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C         ******** BOTTOM LAYER, INNER RADIUS, MID-PHIS ********
      DO 100 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD+1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREI4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
100   CONTINUE
C
C          ******** BOTTOM LAYER, INNER RADIUS, LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD+1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREI4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C               ******** BOTTOM LAYER, MID-RADII ********
      DO 200 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
```

```
C
C          ******** BOTTOM LAYER, MID-RADII, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,NPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR((INCRAD+1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREM4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C          ******** BOTTOM LAYER, MID RADII, MID-PHIS ********
      DO 300 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
           T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR((INCRAD+1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREM4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
300             CONTINUE
C
C          ******** BOTTOM LAYER, MID RADII, LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR((INCRAD+1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREM4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
200       CONTINUE
C
C                         **** OUTER RADIUS ****
INCRAD = NODES
RAD = RMAX
C
C          ******** BOTTOM LAYER, OUTER RADIUS, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,NPHI,LAYER)
```

```
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREO4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C          ******** FIRST LAYER, OUTER RADIUS, MID-PHIS ********
      DO 400 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)

CALL BCREO4 (TEMP)
TNEW(INCRAD,IPHI,LAYER) = TEMP
400       CONTINUE
C
C         ******** BOTTOM LAYER, OUTER RADIUS, LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREO4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
CMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM MID-LAYERS MMMMMMMMMMMMMMMMMMMMMMMMMMMMM
DO 500 LAYER=2,(NLAYER-1)
C
C                    **** MID-LAYERS, INNER RADIUS ****
      INCRAD = 1
      RAD = RMIN
C
C          ******** MID-LAYERS, INNER RADIUS, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,NPHI,LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                T8 = T3STAR((INCRAD+1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCRMI4 (TEMP)
                     TNEW(INCRAD,IPHI,LAYER) = TEMP
```

```
C
C       ******** MID-LAYERS, INNER RADIUS, MID-PHIS ********
      DO 600 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                T8 = T3STAR((INCRAD+1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCRMI4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
600       CONTINUE
C
C   ******** MID-LAYERS, INNER RADIUS, LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,1,LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                T8 = T3STAR((INCRAD+1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCRMI4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C                ******** MID-LAYERS, MID-RADII ********
      DO 700 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
C
C          ******** MID-LAYERS, MID-RADII, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,NPHI,LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                T8 = T3STAR((INCRAD-1),IPHI,LAYER)
                T9 = T3STAR((INCRAD+1),IPHI,LAYER)
                T10 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCRMM4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C          ******** MID-LAYERS, MID-RADII, MID-PHIS ********
      DO 800 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
```

```
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                T8 = T3STAR((INCRAD-1),IPHI,LAYER)
                T9 = T3STAR((INCRAD+1),IPHI,LAYER)
                T10 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCRMM4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
800             CONTINUE
C
C          ******** MID-LAYERS, MID-RADII, LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,1,LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                T8 = T3STAR((INCRAD-1),IPHI,LAYER)
                T9 = T3STAR((INCRAD+1),IPHI,LAYER)
                T10 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCRMM4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
700         CONTINUE
C
C                    **** MID-LAYERS, OUTER RADII ****
      INCRAD = NODES
      RAD = RMAX
C
C          ******** MID-LAYERS, OUTER RADII, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,NPHI,LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                T8 = T3STAR((INCRAD-1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCRMO4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C          ******** MID-LAYERS, OUTER RADII, MID-PHIS ********
      DO 900 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                T8 = T3STAR((INCRAD-1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
```

```
                CALL BCRMO4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
900             CONTINUE
C
C          ******** MID-LAYERS, OUTER RADII, LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER+1))
                T3 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T4 = TSTAR(INCRAD,IPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T6 = T2STAR(INCRAD,1,LAYER)
                T7 = T2STAR(INCRAD,IPHI,LAYER)
                T8 = T3STAR((INCRAD-1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCRMO4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
500        CONTINUE
C
CTTTTTTTTTTTTTTTTTTTTTTTTTTTTT TOP LAYER TTTTTTTTTTTTTTTTTTTTTTTTTTTTT
LAYER = NLAYER
C
C                    **** TOP LAYER, INNER RADIUS ****
      INCRAD = 1
      RAD = RMIN
C
C         ******** TOP LAYER, INNER RADIUS, FIRST PHI********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,NPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD+1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREI4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C         ******** TOP LAYER, INNER RADIUS, MID-PHIS ********
      DO 1000 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD+1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREI4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
1000  CONTINUE
C
C         ******** TOP LAYER, INNER RADIUS, LAST PHI ********
      IPHI = NPHI
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```
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD+1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREI4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C                    **** TOP LAYER, MID RADII ****
      DO 1100 INCRAD=2,(NODES-1)
      RAD = RMIN + (INCRAD-1)*DR
C
C          ******** TOP LAYER, MID-RADII, FIRST PHI ********
      IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,NPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR((INCRAD+1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREM4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C           ******** TOP LAYER, MID RADII, MID-PHIS ********
      DO 1200 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR((INCRAD+1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREM4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
1200            CONTINUE
C
C           ******** TOP LAYER, MID RADII, LAST PHI ********
      IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR((INCRAD+1),IPHI,LAYER)
                T9 = T3STAR(INCRAD,IPHI,LAYER)
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```
                CALL BCREM4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
1100        CONTINUE
C
C                   **** TOP LAYER, OUTER RADIUS ****
        INCRAD = NODES
        RAD = RMAX
C
C           ******** TOP LAYER, OUTER RADIUS, FIRST PHI ********
        IPHI = 1
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,NPHI,LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREO4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
C
C           ******** TOP LAYER, OUTER RADIUS, MID-PHIS ********
        DO 1300 IPHI=2,(NPHI-1)
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,(IPHI+1),LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREO4 (TEMP)

1300        CONTINUE
C
C           ******** TOP LAYER, OUTER RADIUS, LAST PHI ********
IPHI = NPHI
                T1 = TOLD(INCRAD,IPHI,LAYER)
                T2 = TSTAR(INCRAD,IPHI,(LAYER-1))
                T3 = TSTAR(INCRAD,IPHI,LAYER)
                T4 = T2STAR(INCRAD,(IPHI-1),LAYER)
                T5 = T2STAR(INCRAD,1,LAYER)
                T6 = T2STAR(INCRAD,IPHI,LAYER)
                T7 = T3STAR((INCRAD-1),IPHI,LAYER)
                T8 = T3STAR(INCRAD,IPHI,LAYER)
                CALL BCREO4 (TEMP)
                TNEW(INCRAD,IPHI,LAYER) = TEMP
998     RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C       BCREI1 FORTRAN A - (REV. 11/16/88)
C       ***************************************************************
C       * BCREI1 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
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C     *          THE INNER SURFACE OF THE END LAYERS (NORTH, SOUTH)     *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE FIRST INTERME-    *
C     *          DIATE STEP IN Z DIRECTION OF A 3-D POLAR-CYLINDRICAL   *
C     *          COORDINATE ADAPTATION OF BRIAN'S METHOD OF FINITE      *
C     *          DIFFERENCE ANALYSIS.                                   *
C     ***********************************************************************
C     PROGRAM LINE 1767
      SUBROUTINE BCREI1 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                            *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      HE = HSUBS
      EPSE = EPSUBS
      A = 0.D+00
      C = -FO
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      HE = HSUBN
      EPSE = EPSUBN
      A = -FO
      C = 0.D+00
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*RAD*(RAD+DR/4)*DPHI**2)
      D2 = (DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
      D3 = DZ/DKZ
      D4 = (RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
C
      B = 1 + FO*(1 + D3*(HE+D5*EPSE) + D4*(HSUBI+D5*EPSUBI))
C
      D = T1 + FO*(D1*(T2+T3-2*T1) + D2*(T4-T1)
     1          + D3*(Q+HE*TINF+D5*EPSE*TSUR)
     2          + D4*(QPSUBI+HSUBI*TINF+D5*EPSUBI*TSUR)) 998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREI2 FORTRAN A - (REV. 11/16/88)
C     ***********************************************************************
C     * BCREI2 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE INNER SURFACE OF THE END LAYERS (NORTH, SOUTH)     *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE SECOND INTERME-   *
C     *          DIATE STEP (PHI DIRECTION) IN BRIAN'S METHOD OF        *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL    *
C     *          COORDINATES.                                           *
```

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C     ******************************************************************
C     PROGRAM LINE 1818
      SUBROUTINE BCREI2 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      HE = HSUBS
      EPSE = EPSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      HE = HSUBN
      EPSE = EPSUBN
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*RAD*(RAD+DR/4)*DPHI**2)
      D2 = (DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
      D3 = DZ/DKZ
      D4 = (RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -D1*FO
      B = 1 + FO*(2*D1 + D3*(HE+D5*EPSE) + D4*(HSUBI+D5*EPSUBI))
C = -D1*FO
C
      D = T1 + FO*((T2-T3) + D2*(T4-T1) + D3*(Q-HE*TINF-D5*EPSE*TSUR)
1           + D4*(QPSUBI+HSUBI*TINF+D5*EPSUBI*TSUR))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREI3 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCREI3 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE INNER SURFACE OF THE END LAYERS (NORTH, SOUTH)    *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE THIRD INTERME-   *
C     *          DIATE STEP (Z DIRECTION) IN BRIAN'S METHOD OF         *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL   *
C     *          COORDINATES.                                          *
C     ******************************************************************
C     PROGRAM LINE 1865
      SUBROUTINE BCREI3 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
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```
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                             *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      HE = HSUBS
      EPSE = EPSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      HE = HSUBN
      EPSE = EPSUBN
      ENDIF
      D1 = DKP*DZ**2/(2*DKZ*RAD*(RAD+DR/4)*DPHI**2)
      D2 = DKR*(RAD+DR/2)*DZ**2/(DKZ*(RAD+DR/4)*DR**2)
      D3 = DZ/DKZ
      D4 = (RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = 0.D+00
      B = 1 + FO*(D2 + D3*(HE+D5*EPSE) + D4*(HSUBI+D5*EPSUBI))
      C = -D2*FO
C
        D = T1 + FO*((T2-T3) + D1*(T4+T5-2*T6)
1      + D3*(Q + HE*TINF + D5*EPSE*TSUR)
2      + D4*(QPSUBI + HSUBI*TINF + D5*EPSUBI*TSUR))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREI4 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCREI4 - THIS SUBROUTINE COMPUTES THE NODAL TEMPERATURES AT    *
C     *          THE INNER SURFACE OF THE END LAYERS (NORTH, SOUTH)    *
C     *          OF A RIGHT-CIRCULAR CYLINDER USING BRIAN'S METHOD OF  *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL   *
C     *          COORDINATES.                                          *
C     ******************************************************************
C     PROGRAM LINE 1978
      SUBROUTINE BCREI4 (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                             *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
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```
      HE = HSUBS
      EPSE = EPSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      HE = HSUBN
      EPSE = EPSUBN
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*RAD*(RAD+DR/4)*DPHI**2)
      D2 = (DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
      D3 = DZ/DKZ
      D4 = (RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      D6 = 1 + 2*FO*(D3*(HE+D5*EPSE) + D4*(HSUBI+D5*EPSUBI))
C
      TNEW = 1/D6*(T1+2*FO*((T2-T3) + D1*(T4+T5-2*T6) + D2*(T7-T8)
     1+ D3*(Q + HE*TINF + D5*EPSE*TSUR)
     2+ D4*(QPSUBI + HSUBI*TINF + D5*EPSUBI*TSUR)))

C
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREM1 FORTRAN A - (REV. 11/16/88)
C     ************************************************************
C     * BCREM1 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR   *
C     *          NODES AT THE MID-RADII OF THE END LAYERS        *
C     *          (NORTH, SOUTH FACES) OF A RIGHT-CIRCULAR CYL-   *
C     *          INDER FOR THE FIRST INTERMEDIATE TIMESTEP       *
C     *          (STAR LEVEL) OF A 3-D, POLAR-CYLINDRICAL        *
C     *          COORDINATE ADAPTATION OF BRIAN'S METHOD OF      *
C     *          FINITE DIFFERENCE ANALYSIS.                     *
C     ************************************************************
C     PROGRAM LINE 2027
      SUBROUTINE BCREM1 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      EPSE = EPSUBS
      HE = HSUBS
      A = 0.D+00
      C = -FO
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
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      EPSE = EPSUBN
      HE = HSUBN
      A = -FO
      C = 0.D+00
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*RAD**2*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      D3 = (DKR*(RAD+DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      D4 = DZ/DKZ
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      B = 1 + FO*(1 + D4*(HE + D5*EPSE))
C
      D = T1 + FO*(D1*(T2+T3-2*T1) + D2*(T4-T1) + D3*(T5-T1)
     1+ D4*(Q + HE*TINF + D5*EPSE*TSUR))
C
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREM2 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCREM2 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE MID-RADIAL NODES OF THE END LAYERS (NORTH, SOUTH)  *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE SECOND INTERMED-  *
C     *          IATE STEP (PHI DIRECTION) IN BRIAN'S METHOD OF         *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL    *
C     *          COORDINATES.                                           *
C     ******************************************************************
C     PROGRAM LINE 2077
      SUBROUTINE BCREM2 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C
C                           *** BODY ***
C
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      EPSE = EPSUBS
      HE = HSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      EPSE = EPSUBN
      HE = HSUBN
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*RAD**2*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
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      D3 = (DKR*(RAD+DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      D4 = DZ/DKZ
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -D1*FO
      B = 1 + FO*(2*D1 + D4*(HE + D5*EPSE))
      C = -D1*FO
      D = T1 + FO*((T2-T3) + D2*(T4-T1) + D3*(T5-T1)
      1            + D4*(Q + HE*TINF + D5*EPSE*TSUR))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREM3 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCREM3 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE MID-RADIAL NODES OF THE END LAYERS (NORTH, SOUTH)  *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE THIRD INTERMED-   *
C     *          IATE STEP (Z DIRECTION) IN BRIAN'S METHOD OF           *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL    *
C     *          COORDINATES.                                           *
C     ******************************************************************
C     PROGRAM LINE 2059
      SUBROUTINE BCREM3 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/ T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C
C                           *** BODY ***
C
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      EPSE = EPSUBS
      HE = HSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      EPSE = EPSUBN
      HE = HSUBN
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*RAD**2*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      D3 = (DKR*(RAD+DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      D4 = DZ/DKZ
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -D2*FO
      B = 1 + FO*(D2 + D3 +D4*(HE + D5*EPSE))
      C = -D3*FO
      D = T1 + FO*((T2-T3) + D1*(T4+T5-2*T6)
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     1             + D4*(Q + HE*TINF + D5*EPSE*TSUR))
998  RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREM4 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCREM4 - THIS SUBROUTINE COMPUTES THE TEMPERATURES OF THE MID- *
C     *          RADIAL NODES FOR THE END LAYERS (NORTH, SOUTH)        *
C     *          OF A RIGHT-CIRCULAR CYLINDER USING BRIAN'S METHOD OF  *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL   *
C     *          COORDINATES.                                          *
C     ******************************************************************
C     PROGRAM LINE 2106
      SUBROUTINE BCREM4 (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      EPSE = EPSUBS
      HE = HSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      EPSE = EPSUBN
      HE = HSUBN
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*RAD**2*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      D3 = (DKR*(RAD+DR/2)*DZ**2)/(2*DKZ*RAD*DR**2)
      D4 = DZ/DKZ
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      D6 = 1 + 2*FO*D4*(HE + D5*EPSE)
      TNEW = 1/D6*(T1 + 2*FO*((T2-T3) + D1*(T4+T5-2*T6) + D2*(T7-T9)
     1+ D3*(T8-T9) + D4*(Q + HE*TINF + D5*EPSE*TSUR)))
998  RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREO1 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCREO1 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE OUTER SURFACE OF THE END LAYERS (NORTH, SOUTH)     *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE FIRST INTERMEDIATE*
C     *          STEP (Z DIRECTION) IN BRIAN'S METHOD OF FINITE         *
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C     *         DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL          *
C     *         COORDINATES.                                          *
C     ****************************************************************
C     PROGRAM LINE 2216
      SUBROUTINE BCREO1 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      EPSE = EPSUBS
      HE = HSUBS
      A = 0.D+00
      C = -FO
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      EPSE = EPSUBN
      HE = HSUBN
      A = -FO
      C = 0.D+00
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*(RAD-DR/4)*RAD*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = DZ/DKZ
      D4 = (RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      B = 1 + FO*(1 + D3*(HE+D5*EPSE) + D4*(HSUBO+D5*EPSUBO))
      D = T1 + FO*(D1*(T2+T3-2*T1) + D2*(T4-T1)
     1             + D3*(Q + HE*TINF + D5*EPSE*TSUR)
     2             + D4*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREO2 FORTRAN A - (REV. 11/16/88)
C     ****************************************************************
C     * BCREO2 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE OUTER SURFACE OF THE END LAYERS (NORTH, SOUTH)    *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE SECOND INTERMED- *
C     *          IATE STEP (PHI DIRECTION) IN BRIAN'S METHOD OF        *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL   *
C     *          COORDINATES.                                          *
C     ****************************************************************
C     PROGRAM LINE 2265
      SUBROUTINE BCREO2 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
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      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      EPSE = EPSUBS
      HE = HSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      EPSE = EPSUBN
      HE = HSUBN
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*(RAD-DR/4)*RAD*DPHI**2)
      D2 = (DKR*(RADDR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = DZ/DKZ
      D4 = (RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -D1*FO
      B = 1 + FO*(2*D1 + D3*(HE+D5*EPSE) + D4*(HSUBO+D5*EPSUBO))
      C = -D1*FO
      D = T1+FO*((T2-T3) + D2*(T4-T1) + D3*(Q + HE*TINF + D5*EPSE*TSUR)
     1+           D4*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREO3 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCREO3 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE OUTER SURFACE OF THE END LAYERS (NORTH, SOUTH)     *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE THIRD INTERMED-   *
C     *          IATE STEP (Z DIRECTION) IN BRIAN'S METHOD OF           *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL    *
C     *          COORDINATES.                                           *
C     ******************************************************************
C     PROGRAM LINE 2311
      SUBROUTINE BCREO3 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
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```
C                                  *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      EPSE = EPSUBS
      HE = HSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      EPSE = EPSUBN
      HE = HSUBN
      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*(RAD-DR/4)*RAD*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = DZ/DKZ
      D4 = (RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -D2*FO
      B = 1 + FO*(D2 + D3*(HE+D5*EPSE) + D4*(HSUBO+D5*EPSUBO))
      C = 0.D+00
      D = T1 + FO*((T2-T3) + D1*(T4+T5-2*T6)
     1             + D3*(Q + HE*TINF + D5*EPSE*TSUR)
     2             + D4*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCREO4 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCREO4 - THIS SUBROUTINE COMPUTES THE NODAL TEMPERATURES AT    *
C     *          THE OUTER SURFACE OF THE END LAYERS (NORTH, SOUTH)     *
C     *          OF A RIGHT-CIRCULAR CYLINDER USING BRIAN'S METHOD OF   *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL    *
C     *          COORDINATES.                                           *
C     ******************************************************************
C     PROGRAM LINE 2357
      SUBROUTINE BCREO4 (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /MATSIZ/ NODES,NPHI,NLAYER
      COMMON /NPOSIT/ INCRAD,IPHI,LAYER
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      IF (LAYER.EQ.1) THEN
      Q = QPSUBS
      EPSE = EPSUBS
      HE = HSUBS
      ELSEIF (LAYER.EQ.NLAYER) THEN
      Q = QPSUBN
      EPSE = EPSUBN
      HE = HSUBN
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      ENDIF
      D1 = (DKP*DZ**2)/(2*DKZ*(RAD-DR/4)*RAD*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = DZ/DKZ
      D4 = (RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      D6 = 1 + 2*FO*( D3*(HE+D5*EPSE) + D4*(HSUBO+D5*EPSUBO))
C
      TNEW = 1/D6*(T1 + 2*FO*((T2-T3) + D1*(T4+T5-2*T6) + D2*(T7-T8)
     1                + D3*(Q + HE*TINF + D5*EPSE*TSUR)
     2                + D4*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR)))
C
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMI1 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCRMI1 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE INNER SURFACE OF THE MID-LAYERS (2 THRU NLAYERS-1) *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE FIRST INTERMEDIATE*
C     *          STEP (RADIAL DIRECTION) IN BRIAN'S METHOD OF FINITE    *
C     *          DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL           *
C     *          COORDINATES.                                           *
C     ******************************************************************
C     PROGRAM LINE 2404
      SUBROUTINE BCRMI1 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/ T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD+DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -FO/2
      B = 1 + FO/2*(2 + D3*(HSUBI + D5*EPSUBI))
      C = -FO/2
      D = T1 + FO/2*(D1*(T2+T3-2*T1) + D2*(T4-T1)
     1               + D3*(QPSUBI + HSUBI*TINF + D5*EPSUBI*TSUR))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMI2 FORTRAN A - (REV. 09/30/88)
C     ******************************************************************
C     * BCRMI2 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
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C     *          THE INNER SURFACE OF THE MID-LAYERS (2 THRU NLAYERS-1) *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE SECOND INTERMED-  *
C     *          IATE STEP (PHI DIRECTION) IN BRIAN'S METHOD OF         *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL    *
C     *          COORDINATES.                                           *
C     ***********************************************************************
C     PROGRAM LINE 2373
      SUBROUTINE BCRMI2 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/ T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                              *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD+DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -D1*FO/2
      B = 1 + FO/2*(2*D1 + D3*(HSUBI + D5*EPSUBI))
      C = -D1*FO/2
      D = T1 + FO/2*((T2+T3-2*T4) + D2*(T5-T1)
     1               + D3*(QPSUBI + HSUBI*TINF + D5*EPSUBI*TSUR))
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMI3 FORTRAN A - (REV. 11/16/88)
C     ***********************************************************************
C     * BCRMI3 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE INNER SURFACE OF THE MID-LAYERS (2 THRU NLAYERS-1) *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE THIRD INTERMED-   *
C     *          IATE STEP (Z DIRECTION) IN BRIAN'S METHOD OF           *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL    *
C     *          COORDINATES.                                           *
C     ***********************************************************************
C     PROGRAM LINE 2474
      SUBROUTINE BCRMI3 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/ T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                              *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD+DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
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C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = 0.D+00
      B = 1 + FO/2*(D2 + D3*(HSUBI + D5*EPSUBI))
      C = -D2*FO/2
      D = T1 + FO/2*((T2+T3-2*T4) + D1*(T5+T6-2*T7)
     1               + D3*(QPSUBI + HSUBI*TINF + D5*EPSUBI*TSUR))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMI4 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCRMI4 - THIS SUBROUTINE COMPUTES THE NODAL TEMPERATURES AT    *
C     *          THE INNER SURFACE OF THE MID-LAYERS (2 THRU NLAYER-1) *
C     *          OF A RIGHT-CIRCULAR CYLINDER USING BRIAN'S METHOD OF  *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL   *
C     *          COORDINATES.                                          *
C     ******************************************************************
C     PROGRAM LINE 2508
      SUBROUTINE BCRMI4 (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                            *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD+DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD+DR/2)*DZ**2)/(DKZ*(RAD+DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD+DR/4)*DR)
C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      D6 = 1 + FO*D3*(HSUBI + D5*EPSUBI)
C
      TNEW = 1/D6*(T1 + FO*((T2+T3-2*T4) + D1*(T5+T6-2*T7)
     1   + D2*(T8-T9) + D3*(QPSUBI + HSUBI*TINF + D5*EPSUBI*TSUR)))
C
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMM1 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCRMM1 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE MID-RADII OF THE MID-LAYERS (2 THRU NLAYERS-1)    *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE FIRST INTERMEDIATE*
C     *          STEP (RADIAL DIRECTION) IN BRIAN'S METHOD OF FINITE   *
C     *          DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL          *
C     *          COORDINATES.                                          *
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C     ******************************************************************
C     PROGRAM LINE 2543
      SUBROUTINE BCRMM1 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD**2*DPHI**2
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(DKZ*RAD*DR**2)
      D3 = (DKR*(RAD+DR/2)*DZ**2)/(DKZ*RAD*DR**2)
      A = -D1*FO/2
      B = 1 + D1*FO
      C = -D1*FO/2
      D = T1 + FO/2*(D1*(T2+T3-2*T1) + D2*(T4-T1) + D3*(T5-T1))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMM2 FORTRAN A - (REV. 10/12/88)
C     ******************************************************************
C     * BCRMM2 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE MID-RADII OF THE MID-LAYERS (2 THRU NLAYERS-1)     *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE SECOND INTERMED-  *
C     *          IATE STEP (PHI DIRECTION) IN BRIAN'S METHOD OF FINITE  *
C     *          DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL           *
C     *          COORDINATES.                                           *
C     ******************************************************************
C     PROGRAM LINE 2575
      SUBROUTINE BCRMM2 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD**2*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(DKZ*RAD*DR**2)
      D3 = (DKR*(RAD+DR/2)*DZ**2)/(DKZ*RAD*DR**2)
      A = -D1*FO/2
      B = 1 + D1*FO
      C = -D1*FO/2
      D = T1 + FO/2*((T2+T3-2*T4) + D2*(T5-T1) + D3*(T6-T1))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
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C
C     BCRMM3 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCRMM3 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE MID-RADII OF THE MID-LAYERS (2 THRU NLAYERS-1)     *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE THIRD INTERMED-   *
C     *          IATE STEP (Z DIRECTION) IN BRIAN'S METHOD OF FINITE    *
C     *          DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL           *
C     *          COORDINATES.                                           *
C     ******************************************************************
C     PROGRAM LINE 2607
      SUBROUTINE BCRMM3 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
      C                              *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD**2*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(DKZ*RAD*DR**2)
      D3 = (DKR*(RAD+DR/2)*DZ**2)/(DKZ*RAD*DR**2)
      A = -D2*FO/2
      B = 1 + FO/2*(D2+D3)
      C = -D3*FO/2
      D = T1 + FO/2*((T2+T3-2*T4) + D1*(T5+T6-2*T7))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMM4 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCRMM4 - THIS SUBROUTINE COMPUTES THE NODAL TEMPERATURES AT     *
C     *          THE MID-RADII OF THE  MID-LAYERS (2 THRU NLAYER-1)     *
C     *          OF A RIGHT-CIRCULAR CYLINDER USING BRIAN'S METHOD OF   *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL    *
C     *          COORDINATES.                                           *
C     ******************************************************************
C     PROGRAM LINE 2572
      SUBROUTINE BCRMM4 (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
      C                              *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD**2*DPHI**2)
      D2 = (DKR*(RAD-DR/2)*DZ**2)/(DKZ*RAD*DR**2)
      D3 = (DKR*(RAD+DR/2)*DZ**2)/(DKZ*RAD*DR**2)
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C     *** NO D4, D5 OR D6 REQUIRED ***
      TNEW = T1 + FO*((T2+T3-2*T4) + D1*(T5+T6-2*T7) + D2*(T8-T10)
     1                + D3*(T9-T10))
C
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMO1 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCRMO1 - THIS SUBROUTINE COMPUTES THE COEFFECIENTS FOR NODES AT*
C     *          THE OUTER SURFACE OF THE MID-LAYERS                   *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE FIRST INTERMEDIATE*
C     *          STEP (RADIAL DIRECTION) IN BRIAN'S METHOD OF FINITE   *
C     *          DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL          *
C     *          COORDINATES.                                          *
C     ******************************************************************
C     PROGRAM LINE 2604
      SUBROUTINE BCRMO1 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD-DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -FO/2
      B = 1 + FO/2*(2 + D3*(HSUBO+D5*EPSUBO))
      C = -FO/2
      D = T1 + FO/2*(D1*(T2+T3-2*T1) + D2*(T4-T1)
      1               + D3*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR))
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMO2 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCRMO2 - THIS SUBROUTINE COMPUTES THE COEFFICIENTS FOR NODES AT*
C     *          THE OUTER SURFACE OF THE MID-LAYERS                   *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE SECOND INTERMED- *
C     *          IATE STEP (PHI DIRECTION) IN BRIAN'S METHOD OF        *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL   *
C     *          COORDINATES.                                          *
C     ******************************************************************
C     PROGRAM LINE 2639
      SUBROUTINE BCRMO2 (A,B,C,D)
```

```
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD-DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -D1*FO/2
      B = 1 + FO/2*(2*D1 + D3*(HSUBO + D5*EPSUBO))
      C = -D1*FO/2
      D = T1 + FO/2*((T2+T3-2*T4) + D2*(T5-T1)
     1               + D3*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR))
998   RETURN
      END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMO3 FORTRAN A - (REV. 11/16/88)
C     ******************************************************************
C     * BCRMO3 - THIS SUBROUTINE COMPUTES THE COEFFICIENTS FOR NODES AT*
C     *          THE OUTER SURFACE OF THE MID-LAYERS                   *
C     *          OF A RIGHT-CIRCULAR CYLINDER FOR THE THIRD INTERMED-  *
C     *          IATE STEP (RADIAL DIRECTION) IN BRIAN'S METHOD OF     *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL   *
C     *          COORDINATES.                                          *
C     ******************************************************************
C     PROGRAM LINE 2674
      SUBROUTINE BCRMO3 (A,B,C,D)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD-DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      A = -D2*FO/2
      B = 1 + FO/2*(D2 + D3*(HSUBO + D5*EPSUBO))
      C = 0.D+00
      D = T1 + FO/2*((T2+T3-2*T4) + D1*(T5+T6-2*T7)
     1               + D3*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR))
998   RETURN
```

```
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     BCRMO4 FORTRAN A - (REV. 11/16/88)
C     ********************************************************************
C     * BCRMO4 - THIS SUBROUTINE COMPUTES THE NODAL TEMPERATURES AT      *
C     *          THE OUTER SURFACE OF THE MID-LAYERS                     *
C     *          OF A RIGHT-CIRCULAR CYLINDER USING BRIAN'S METHOD OF    *
C     *          FINITE DIFFERENCE ANALYSIS IN 3-D POLAR-CYLINDRICAL     *
C     *          COORDINATES.                                            *
C     ********************************************************************
C     PROGRAM LINE 2708
      SUBROUTINE BCRMO4 (TNEW)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /CONDUC/ DKR,DKP,DKZ
      COMMON /CONVEC/ HSUBN,HSUBS,HSUBI,HSUBO
      COMMON /GEOMET/ DPHI,DR,DZ,RAD,RMAX,RMIN,THICK,ZMAX,LPHI
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /PARAMS/ CSUBP,RHO
      COMMON /RADN/   EPSUBN,EPSUBS,EPSUBI,EPSUBO,SIGMA
      COMMON /TEMPS/  T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,TINF,TSUR
C                           *** BODY ***
      D1 = (DKP*DZ**2)/(DKZ*RAD*(RAD-DR/4)*DPHI**2)
      D2 = (2*DKR*(RAD-DR/2)*DZ**2)/(DKZ*(RAD-DR/4)*DR**2)
      D3 = (2*RAD*DZ**2)/(DKZ*(RAD-DR/4)*DR)
C     *** NO D4 REQUIRED ***
      D5 = SIGMA*(T1+TSUR)*(T1**2+TSUR**2)
      D6 = 1 + FO*D3*(HSUBO + D5*EPSUBO)
      TNEW = 1/D6*(T1 + FO*((T2+T3-2*T4) + D1*(T5+T6-2*T7)
1       + D2*(T8-T9) + D3*(QPSUBO + HSUBO*TINF + D5*EPSUBO*TSUR)))
C
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     MAP122 FORTRAN A - (REV. 10/04/88)
C     ********************************************************
C     * MAP122 - SUBROUTINE TO MAP MATRIX ONE ONTO MATRIX TWO *
C     ********************************************************
C     PROGRAM LINE 2737
      SUBROUTINE MAP122 (DMAT1,DMAT2)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /ARRSIZ/ III,JJJ,KKK
      COMMON /MATSIZ/ NLAYER,NODES,NPHI
      DIMENSION DMAT1(III,JJJ,KKK),DMAT2(III,JJJ,KKK) DO 100 LAYER=1,KKK
      DO 200 IPHI=1,JJJ
      DO 300 INCRAD=1,III
      DMAT2(INCRAD,IPHI,LAYER) = DMAT1(INCRAD,IPHI,LAYER)
300           CONTINUE
200       CONTINUE
100   CONTINUE
998   RETURN
END
```

```
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     TRIDAG FORTRAN A (REV. 11/07/88)
C     ************************************************************
C     * TRIDAG - SUBROUTINE FOR SOLVING A SYSTEM OF LINEAR,      *
C     *          SIMULTANEOUS EQUATIONS HAVING A TRIDIAGONAL     *
C     *          COEFFICIENT MATRIX.  THE EQUATIONS ARE NUM-     *
C     *          BERED FROM IF THROUGH L, AND THEIR SUBDIAGONAL, *
C     *          DIAGONAL, AND SUPER-DIAGONAL COEFFICIENTS ARE   *
C     *          STORED IN THE VECTORS A, B, AND C.  THE COM-    *
C     *          PUTED SOLUTION VECTOR V(IF)...V(L) IS STORED    *
C     *          IN THE VECTOR V.                                *
C     ************************************************************
C     PROGRAM LINE 2766
      SUBROUTINE TRIDAG (IF,L,A,B,C,D,V)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION A(L),B(L),C(L),D(L),V(L)
      DIMENSION BETA(100),GAMMA(100)
C
C     *** COMPUTE INTERMEDIATE ARRAYS BETA AND GAMMA ***
C
      BETA(IF) = B(IF)
      GAMMA(IF) = D(IF)/BETA(IF)
      IFP1 = IF + 1
      DO 100 I=IFP1,L
      BETA(I) = B(I) - A(I)*C(I-1)/BETA(I-1)
      100       GAMMA(I) = (D(I) - A(I)*GAMMA(I-1))/BETA(I)
C
C     *** COMPUTE FINAL SOLUTION VECTOR V ***
C
      V(L) = GAMMA(L)
      LAST = L - IF
      DO 200 K=1,LAST
      I = L - K
      200       V(I) = GAMMA(I) - C(I)*V(I+1)/BETA(I)
RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     COMPARE FORTRAN A - (REV. 11/05/88)
C     ************************************************************
C     * COMPAR - SUBROUTINE TO COMPARE THE RESULTS OF BRIAN'S    *
C     *          METHOD AGAINST THE RESULTS OF THE EXPLICIT      *
C     *          METHOD FOR A GIVEN GEOMETRY AND BOUNDARY        *
C     *          CONDITIONS.  CAN BE CALLED FROM EVALID ONLY.    *
C     ************************************************************
C     PROGRAM LINE 2800
      SUBROUTINE COMPAR
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION PE(15,15,15)
      REAL TIME
      OPEN (UNIT=11,FILE='EBCMP')
      OPEN (UNIT=14,FILE='BCRDAT')
```

```
      OPEN (UNIT=15,FILE='ECRDAT')
      ICOUNT = 0
      JCOUNT = 0
      READ (14,*) NODES,NPHI,NLAYER
      READ (15,*) LL,MM,NN
      WRITE (*,*) 'MATRIX 1 SIZE:',NODES,NPHI,NLAYER WRITE (*,*) 'MATRIX
      2 SIZE:',LL,MM,NN
      WRITE (*,*)
      IF ((NODES.NE.LL).OR.(NPHI.NE.MM).OR.(NLAYER.NE.NN)) THEN WRITE
      (*,*) '*** ABORTED COMPARE - ARRAY SIZES OF UNEQUAL LENG 1TH ***'
GOTO 998
ENDIF
      1     WRITE (11,5)
      5     FORMAT (T1,'COMPARISON OF BRIAN''S METHOD TEMPERATURES
      AGAINST EXP 1LICIT METHOD TEMPERATURES'/,T1,'EXPRESSED AS A
      PERCENTAGE DIFFEREN 2CE'//)
      WRITE (11,*) 'BCRSAT'
      WRITE (11,*)
      WRITE (11,*) '*** 999.0 INDICATES ZERO VALUE AS DIVISOR ***'
      WRITE (11,*)
      WRITE (11,*)
      2     READ (14,*,END=998) BTIME,BDTIME
      ICOUNT=ICOUNT+1
      ITIME=BTIME
      READ (15,*,END=998) ETIME,EDTIME
      JCOUNT=JCOUNT+1
      JTIME=ETIME
      IF (ITIME.NE.JTIME) THEN
      WRITE (*,*) '*** ABORTED COMPARE - TIME MISMATCH ***' WRITE (*,*)
      ITIME,EDTIME
      WRITE (*,*) JTIME,BDTIME
      GOTO 998
      ENDIF
      DO 100 K=1,NLAYER
      DO 200 I=1,NODES
      DO 300 J=1,NPHI
      READ (14,*) DMAT1
      READ (15,*) DMAT2
      IF (DMAT2.NE.0) THEN
      PE(I,J,K) = (DMAT1-DMAT2)/DMAT2 * 100 ELSE
      PE(I,J,K) = 999.
      ENDIF
300         CONTINUE
200      CONTINUE
100   CONTINUE
      CALL PRTARR (BTIME,BDTIME,PE,NODES,NPHI,NLAYER) GOTO 2
998   IF (ICOUNT.NE.JCOUNT) THEN
      WRITE (*,*) 'COMPAR - EOF ERROR ON PROGRAM LINE 2431 OR 2433'
      WRITE(11,*) 'COMPAR - EOF ERROR ON PROGRAM LINE 2431 OR 2433'
      ENDIF
      CLOSE (11)
      CLOSE (14)
      CLOSE (15)
      RETURN
END
```

```
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     SHTFRM FORTRAN A - (REV. 11/14/88)
C     ****************************************************************
C     * SHTFRM - SUBROUTINE TO PRINT THE TEMPERATURES OF A DESIG-  *
C     *          DESIGNATED NODE FROM A DATA FILE.                 *
C     ****************************************************************
C     PROGRAM LINE 2871
      SUBROUTINE SHTFRM (L,M,N)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /HEAT/ ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO COMMON /TIMES/
      DELTIM,DTMAX,FINTIM,LIMIT
      DIMENSION TEMP(15,15,15)
      OPEN(UNIT=13,FILE='ANALYT')
      OPEN(UNIT=14,FILE='BCRDAT')
      OPEN(UNIT=15,FILE='BCRSHT')
C     READ IN MATRIX SIZE
      READ (14,*) NODES,NPHI,NLAYER
C     WRITE HEADERS TO DATA FILE 15 (ERROR)
      WRITE (15,*) 'BCRSAT - WITH CONDUCTION, CONVECTION AND RADIATION'
      WRITE (15,*)
      WRITE (15,*) 'DATA SAVED AS FILE:'
      WRITE (15,*)
      WRITE (15,*) 'CPU TIME:'
      WRITE (15,*)
      WRITE (15,*)'GRID SIZE (RADIAL,PHI,Z DIRECTIONS):',NODES,NPHI,NLAY
      1ER
      WRITE (15,*)
      WRITE (15,*) 'SELECTED NODE FOR PRINTOUT:',L,M,N WRITE (15,*)
      WRITE (15,*) 'THEORETICAL MAX TIMESTEP (EXPLICIT):',DTMAX WRITE
      (15,*)
      WRITE (15,*) 'SELECTED TIMESTEP (SEC):',DELTIM WRITE (15,*)
      WRITE (15,*) 'GRID FOURIER NUMBER (Z,DELTIM DEPENDENT): ',FO WRITE
      (15,*)
      WRITE (15,*) 'OUTER SURFACE HEAT FLUX (W/M**2):',QPSUBO WRITE
      (15,*)
      WRITE (15,*)
      WRITE (15,15)
      15    FORMAT (T33,'NODE')
      WRITE (15,25)
      25    FORMAT (T5,'TIME',T17,'DELTIM',T30,'TEMPERATURE')
C     READ TIME, DELTIM AND TEMPERATURE MATRICES FROM FILE 14 (*DATA) 2
      READ (14,*,END=998) TIME,DELTIM
      DO 100 K=1,NLAYER
      DO 200 I=1,NODES
      DO 300 J=1,NPHI
      READ (14,*,END=997) TEMP(I,J,K)
300           CONTINUE
200       CONTINUE
100   CONTINUE
      WRITE (15,55) TIME,DELTIM,TEMP(L,M,N)
55        FORMAT (T1,F9.2,T15,F8.4,T26,F14.8)
GOTO 2
```

```
997   WRITE (*,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 864' WRITE
(15,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 864' 998   CLOSE (13)
CLOSE (14)
CLOSE (15)
RETURN
END
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     LNGFRM FORTRAN A - (REV. 11/05/88)
C     ************************************************************
C     * LNGFRM - SUBROUTINE TO PRINT FULL MATRIX OF TEMPERATURES *
C     *          FROM DATA FILES.                                *
C     ************************************************************
C     PROGRAM LINE 2933
      SUBROUTINE LNGFRM
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /HEAT/   ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /TIMES/ DELTIM,DTMAX,FINTIM,LIMIT
      DIMENSION TEMP(15,15,15)
      OPEN (UNIT=11,FILE='BCRLNG')
      OPEN (UNIT=14,FILE='BCRDAT')
      READ (14,*) NODES,NPHI,NLAYER
      WRITE (11,*) 'BCRSAT - WITH CONDUCTION, CONVECTION AND RADIATION'
      WRITE (11,*)
      WRITE (11,*) 'DATA SAVED AS FILE:'
      WRITE (11,*)
      WRITE (11,*) 'CPU TIME:'
      WRITE (11,*)
      WRITE (11,*) 'GRID SIZE (RADIAL,PHI,Z DIRECTIONS):',NODES,NPHI,NLA
      1YER
      WRITE (11,*)
      WRITE (11,*) 'THEORETICAL MAX TIMESTEP (EXPLICIT):',DTMAX WRITE
      (11,*)
      WRITE (11,*) 'SELECTED TIMESTEP (SEC):',DELTIM WRITE (11,*)
      WRITE (11,*) 'GRID FOURIER NUMBER (Z,DELTIM DEPENDENT): ',FO WRITE
      (11,*)
      WRITE (11,*) 'SURFACE HEAT FLUX (W/M**2):',QPSUBO WRITE (11,*)
      WRITE (11,*)
      1     READ (14,*,END=998) TIME,DTIME
      DO 100 K=1,NLAYER
      DO 200 I=1,NODES
      DO 300 J=1,NPHI
      READ (14,*,END=997) TEMP(I,J,K)
300           CONTINUE
200       CONTINUE
100   CONTINUE
      CALL PRTMAT (TIME,DTIME,TEMP,NODES,NPHI,NLAYER) GOTO 1
997   WRITE (6,996) I,J,K
996   FORMAT (1X,'LNGFRM: READ BEYOND DATA IN *DATA (I,J,K)',3I4) 998
REWIND 14
CLOSE (11)
RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
```

```
C
C     PRTMAT FORTRAN A - (REV. 11/04/88)
C     ************************************************************
C     * PRTMAT - SUBROUTINE TO PRINT THREE DIMENDIONAL MATRIX    *
C     *          TO FILE 11 (*LONG).                             *
C     ************************************************************
C     PROGRAM LINE 2984
      SUBROUTINE PRTMAT (TIME,DTIME,DMAT,NODES,NPHI,NLAYER) IMPLICIT
      DOUBLE PRECISION (A-H,O-Z)
      DIMENSION DMAT(15,15,15)
      WRITE (11,15) TIME,DTIME
15    FORMAT (T1,'TIME = ',F9.3,' SEC',T30,'TIME INCREMENT = ',F9.3,' SE
      1C')
      DO 100 K=1,NLAYER
      WRITE(11,10) K
10       FORMAT (//T1,'NODE:',T45,'LAYER NUMBER: ',I4/) DO 200 I=1,NODES
      WRITE (11,25) I,(DMAT(I,J,K),J=1,NPHI)
25          FORMAT (T1,I3,T10,4(D16.10,1X))
200      CONTINUE
100   CONTINUE
      WRITE (11,*)
      WRITE (11,*)
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     PRTARR FORTRAN A - (REV. 11/04/88)
C     ************************************************************
C     * PRTARR - SUBROUTINE TO PRINT THREE DIMENDIONAL MATRIX    *
C     *          TO FILE 11 (*LONG).                             *
C     ************************************************************
C     PROGRAM LINE 3011
      SUBROUTINE PRTARR (TIME,DTIME,DMAT,NODES,NPHI,NLAYER) IMPLICIT
      DOUBLE PRECISION (A-H,O-Z)
      DIMENSION DMAT(15,15,15)
      WRITE (11,15) TIME,DTIME
15    FORMAT (T1,'TIME = ',F9.3,' SEC',T30,'TIME INCREMENT = ',F9.3,' SE
      1C')
      DO 100 K=1,NLAYER
      WRITE(11,10) K
10       FORMAT (//T1,'NODE:',T45,'LAYER NUMBER: ',I4/) DO 200 I=1,NODES
      WRITE (11,25) I,(DMAT(I,J,K),J=1,NPHI)
25          FORMAT (T1,I3,T10,4(F9.3,1X))
200      CONTINUE
100   CONTINUE

      WRITE (11,*)
      WRITE (11,*)
998   RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     ERROR FORTRAN A - (REV. 11/05/88)
```

```
C     ****************************************************************
C     * ERROR - SUBROUTINE TO COMPUTE ERROR FOR AND PRINT TEMPERA-   *
C     *         TURES OF A DESIGNATED NODE FROM A VALIDATION DATA    *
C     *         FILE.                                                *
C     ****************************************************************
C     PROGRAM LINE 3039
      SUBROUTINE ERROR (L,M,N)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /HEAT/  ALFA,FO,QPSUBN,QPSUBS,QPSUBI,QPSUBO
      COMMON /TIMES/ DELTIM,DTMAX,FINTIM,LIMIT
      DIMENSION TEMP(15,15,15)
      REAL TIME
      OPEN(UNIT=13,FILE='ANALYT')
      OPEN(UNIT=14,FILE='BCRDAT')
      OPEN(UNIT=15,FILE='BCRERV')
      IF (((QPSUBO.GT.1352).AND.(QPSUBO.LT.1354)).OR.((QPSUBO.GT.14999).
     1AND.(QPSUBO.LT.15001))) THEN
      CONTINUE
ELSE
GOTO 998
ENDIF
C     READ IN MATRIX SIZE
1     READ (14,*) NODES,NPHI,NLAYER
C     WRITE HEADERS TO FILE 15 (*ERROR)
      WRITE (15,*) 'BCRSAT - WITH CONDUCTION, CONVECTION AND RADIATION'
      WRITE (15,*)
      WRITE (15,*) 'DATA SAVED AS FILE:'
      WRITE (15,*)
      WRITE (15,*) 'CPU TIME:'
      WRITE (15,*)
      WRITE (15,*)'GRID SIZE (RADIAL,PHI,Z DIRECTIONS):',NODES,NPHI,NLAY
     1ER
      WRITE (15,*)
      WRITE (15,*)'SELECTED GRID POINT :',L,M,N
      WRITE (15,*)
      WRITE (15,*) 'THEORETICAL MAX TIMESTEP (EXPLICIT):',DTMAX WRITE
     (15,*)
      WRITE (15,*) 'SELECTED TIMESTEP (SEC):',DELTIM WRITE (15,*)
      WRITE (15,*) 'GRID FOURIER NUMBER (Z,DELTIM DEPENDENT): ',FO WRITE
     (15,*)
      WRITE (15,*) 'OUTER SURFACE HEAT FLUX (W/M**2):',QPSUBO WRITE
     (15,*)
      WRITE (15,*)
      WRITE (15,15)
15    FORMAT (T33,'NODE',T48,'ANALYTIC',T64,'PERCENT') WRITE (15,25)
25    FORMAT (T5,'TIME',T17,'DELTIM',T30,'TEMPERATURE',T46,'TEMPERATURE'
     1,T65,'ERROR'/)
C     READ TIME, DELTIM AND TEMPERATURE MATRICES FROM FILE 14 (*DATA)
2     READ (14,*,END=998) TIME,DELTIM
      DO 100 K=1,NLAYER
      DO 200 I=1,NODES
      DO 300 J=1,NPHI
      READ (14,*,END=995) TEMP(I,J,K)
300           CONTINUE
200       CONTINUE
```

```
100   CONTINUE
C     READ IN APPROPRIATE ANALYTIC SOLUTIONS FROM FILE 13 (ANALYT)
      IF (QPSUBO.LT.1400) THEN
      READ (13,35,END=996) EXACT
35            FORMAT (T19,F14.8)
      ELSE
      READ (13,45,END=997) EXACT
45            FORMAT (T39,F14.8)
      ENDIF
C     COMPUTE PERCENT ERROR AND WRITE TO FILE 15 (ERROR)
      PE = (EXACT-TEMP(L,M,N))/EXACT * 100.
      WRITE (15,55)
      TIME,DELTIM,TEMP(L,M,N),EXACT,PE 55        FORMAT
      (T1,F9.2,T15,F8.4,T26,F14.8,T41,F14.8,T61,F9.3) GOTO 2
995   WRITE (*,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 2515'
       WRITE (15,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 2525'
996   WRITE (*,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 2521'
      WRITE (15,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 2521'
997   WRITE (*,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 2514'
      WRITE (15,*) 'ERROR - EOF READ ERROR ON PROGRAM LINE 2524'
998   CLOSE (13)
CLOSE (14)
CLOSE (15)
RETURN
END
C
CSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
C
C     CMPSHT FORTRAN A - (REV. 11/22/88)
C     ***********************************************************
C     * CMPSHT - SUBROUTINE TO COMPARE THE RESULTS OF BRIAN'S *
C     *          METHOD AGAINST THE RESULTS OF THE EXPLICIT   *
C     *          METHOD FOR A GIVEN GEOMETRY AND BOUNDARY     *
C     *          CONDITIONS.  DATA SAVED IN SHORT FORMAT.     *
C     *          NOTE - CALLED FROM BCRV ONLY.                *
C     ***********************************************************
C     PROGRAM LINE 3132
      SUBROUTINE CMPSHT
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      REAL TIME
      OPEN (UNIT=14,FILE='BCRDAT')
      OPEN (UNIT=15,FILE='ECRDAT')
      OPEN (UNIT=17,FILE='CMPSHT')
      ICOUNT = 0
      JCOUNT = 0
      READ (14,*) NODES,NPHI,NLAYER
      READ (15,*) LL,MM,NN
      WRITE (*,*) 'MATRIX 1 SIZE:',NODES,NPHI,NLAYER WRITE (*,*) 'MATRIX
      2 SIZE:',LL,MM,NN
      WRITE (*,*)
      IF ((NODES.NE.LL).OR.(NPHI.NE.MM).OR.(NLAYER.NE.NN)) THEN
      WRITE (*,*) '*** ABORTED COMPARE - ARRAY SIZES OF UNEQUAL LENGTH
***'
GOTO 998
      ENDIF
```

```
1     WRITE (17,15)
15    FORMAT (T1,'COMPARISON OF BRIANS METHOD TEMPERATURES AGAINST EXPLI
               1CIT METHOD TEMPERATURES'/,T1,'EXPRESSED AS A PERCENTAGE
               DIFFERENCE 2.'/)
               WRITE (17,*) 'BCRSAT SHORT FORMAT'
               WRITE (17,*)
               WRITE (17,*) 'SAVED AS FILE:'
               WRITE (17,*)
               WRITE (17,*) 'CPU TIME:'
               WRITE (17,*)
               WRITE (17,*) '*** 999.0 INDICATES ZERO VALUE AS DIVISOR
***' WRITE (17,*)
      WRITE (17,*)
      WRITE (17,25)
25    FORMAT (T33,'EXPLICIT',T48,'BRIANS',T64,'PERCENT') WRITE (17,35)
35    FORMAT (T5,'TIME',T30,'TEMPERATURE',T46,'TEMPERATURE',T62,'DIFFERE
1NCE'/)
2     READ (14,*,END=998) BTIME,BDTIME
      ICOUNT=ICOUNT+1
      ITIME=BTIME
      READ (15,*,END=998) ETIME,EDTIME
      JCOUNT=JCOUNT+1
      JTIME=ETIME
      IF (ITIME.NE.JTIME) THEN
      WRITE (*,*) '*** WARNING - TIME MISMATCH IN CMPSHT ***'
      WRITE (*,*) ITIME,BTIME
      WRITE (*,*) JTIME,ETIME
      ENDIF
      DO 100 K=1,NLAYER
      DO 200 I=1,NODES
      DO 300 J=1,NPHI
      READ (14,*) DMAT1
      READ (15,*) DMAT2
      IF ((I.EQ.NODES).AND.(J.EQ.2).AND.(K.EQ.NLAYER).AND.
      1(DMAT2.NE.0)) THEN
      PE = (DMAT1-DMAT2)/DMAT2 * 100
      WRITE (17,45) BTIME,DMAT2,DMAT1,PE 45                    FORMAT
      (T1,F9.2,T26,F14.8,T41,F14.8,T61,F9.3) ELSE
      PE = 999.
      ENDIF
300         CONTINUE
200      CONTINUE
100   CONTINUE
GOTO 2
998   IF (ICOUNT.NE.JCOUNT) THEN
      WRITE (*,*) 'CMPSHT - EOF ERROR ON PROGRAM LINE 2431 OR 2433'
      WRITE(17,*) 'CMPSHT - EOF ERROR ON PROGRAM LINE 2431 OR 2433'
      ENDIF
CLOSE (14)
CLOSE (15)
CLOSE (17)
RETURN
END
```

APPENDIX G

APPLICATION MODEL CHARACTERISTICS

A. ASSUMPTIONS*

1. Satellite

- Three-axis stabilized
- Orbit lies within and z-axis normal to the plane of the ecliptic

2. Earth

- Atmosphere is a diffuse scatterer of solar radiation
- Albedo coefficient is 0.30 (nominal)
- Emitted infrared heat flux is 237 W/m^2
- Shadow appears cylindrical from low orbits

3. Sun

- Diffuse, blackbody emitter of radiation

B. OTHER PARAMETERS*

Internal Heat Loading:	q_{int} = 1000 W (avg)
Solar flux:	q''_{solar} = 1353 W/m^2
Earth's IR:	q''_{IR} = 237 W/m^2
Albedo coefficient:	albedo = 0.30
Temperature of deep space:	T_{sur} = 4 K
Geometric shape factor: (flat plate, 320 km)	F = 0.31
Surface absorptivity: (white paint)	α = 0.4 (nominal)

Surface emissivity: (white paint) $\varepsilon = 0.9$

Operating Limits

Non-op/turn-on	Operating
40/+40 K	-15/+40 K

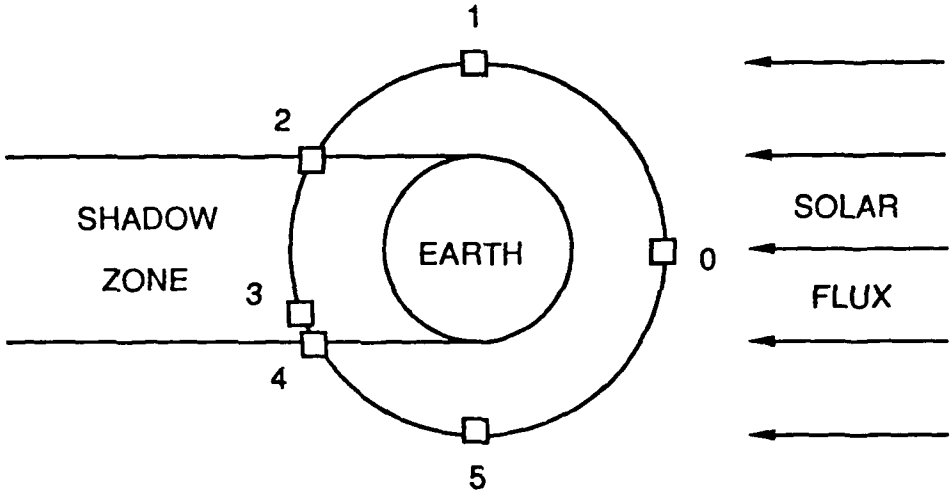

Figure 15. **Orbit Time Numbering Sequence**

FLUX CONDITIONS

TIME	FLUX CONDITION
$0 \le t' < t_1$	$q''_N = \alpha_N q''_{sun} \cos(\omega t')$
	$q''_S = \alpha_s \lfloor q''_{Eir} + a\, q''_{sun} \cos(\omega t') \rfloor$
	$q''_O = 0.5 \alpha_O q''_S \sin(\omega t')$
$t_1 \le t' < t_2$	$q''_N = 0$
	$q''_S = \alpha_S \{ q''_{Eir} + \lfloor a + \cos(\omega t') \rfloor q''_{sun} \}$
	$q''_O = 0.5 \alpha_O q_O \sin(\omega t')$
$t_2 \le t' < t_3$	$q''_N = 0$
	$q''_S = \alpha_S q''_{Eir}$

$$q''_O = 0$$

$t_3 \le t' < t_4$

$$q''_N = 0$$

$$q''_S = \alpha_S \left\{ q''_{Eir} + \left| a\, q''_{am} \sin(\omega t') \cdot \left[1 - \exp\left(\frac{t_4 - t'}{\tau} \right) \right] \right| \right\}$$

$$q''_O = 0.5\alpha_O \left| \left[a + q''_{am} \sin(\omega t') \right] \cdot \left\{ 1 - \exp \frac{(t_4 - t')}{\tau} \right\} \right|$$

$t_4 \le t' < t_5$

$$q''_N = 0$$

$$q''_S = \alpha_S \left\{ q''_{Eir} + \left| a\, q''_{am} \sin \omega t' \right| \right\}$$

$$q''_O = 0.5\alpha_O\, q''_{am} \left| \sin(\omega t') \right|$$

where times in seconds are:

$$t_1 = 0$$

$$t_2 = 1360$$

$$t_3 = 1629$$

$$t_4 = 3757$$

$$t_5 = 3811$$

$$t_6 = 4080$$

* Taken from Agrawal [Ref. 12]

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