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Zonal modeling is a particularly useful approach for a large range of engineering problems. Four flow situations were studied. Namely, strained homogeneous flows, free shear layers, backward facing step and boundary layers. Zonal modeling can provide a single model as accurate as the known data. Nearly any common flow field can be constructed from something like 10 zones. Zonal modeling is a promising avenue for constructing predictive models of turbulent flows wherever the importance of the problem warrants construction of a model.

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**STANFORD UNIVERSITY**

MECHANICAL ENGINEERING DEPARTMENT  
STANFORD, CALIFORNIA

94305

**AFOSR-TR- 89 - 0225**

January 9, 1989

Dr. James McMichael  
AFOSR/NA  
Bolling Air Force Base  
Washington, DC 20332-6448

Dear Dr. McMichael,

This letter, with its attachments, constitutes a final report for our work on Zonal Modeling under AFOSR Contract AF F49620-86-K-0008.

Attachment 1 is a summary of the results of the three studies we have done by the writer. It sets zonal modeling into perspective as a tool for engineering work.

Attachment 2 is a section of the dissertation of Sergio Bordalo which provides a taxonomy of the turbulence modeling situation. Attachment 2 is not new material, but is a particularly good and concise statement of the situation. You may find it useful as background to attachment 3.

Attachment 3, also from Bordalo's dissertation, gives a description of how we now understand and how we proceed in zonal modeling. It is a particularly clear, concise and complete summary of the situation. Thus for your purposes Attachments 1 and 3 together may provide a compact and relatively complete summary of zonal modeling.

Regarding the work completed, we have sent to your office copies of the dissertations of Tzuoo and Avva. A summary of the project (by Kline, Ferziger, Tzuoo, Avva and Bordalo) will appear in the Proceedings of the Zaric Memorial Conference on Near Wall Turbulence held in Dubrovnik 16-20 May 1988. I am currently editing this volume for publication by Hemisphere. The completed volume should appear in 1989. Separate, more detailed, journal articles are also under preparation by Tzuoo and by Avva for later publication.

As of this date, Mr. Bordalo's dissertation is unfortunately still not complete. His descriptions of the nature of the work are outstanding. He has a remarkable talent for clear explanation. However, a review of the work in early December 1988 by his Reading Committee (Profs. J. H. Ferziger, S. J. Kline and W. M. Kays) concluded that Mr. Bordalo's work is not yet sufficient (either in original contribution or in coverage of available cases relating to his problem of modeling turbulent boundary layers on flat and curved surfaces). Mr. Bordalo has accepted a teaching post at a new engineering school near Sao Paulo, Brazil and will continue the work there. Given the exigencies of a new teaching post, we cannot estimate reliably when Mr. Bordalo's work will be complete and ready for journal publication.

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102

Fortunately, the finished work of Tzuoo and of Avva together with results in hand from Bendalo's work are quite sufficient to provide a good picture of what zonal modeling entails. That is reported in attachment 1.

If you have any further questions, or lack any documents, please let me know.

Sincerely,



S. J. Kline  
Principal Supervisor

encl.

cc: Sponsored Projects Office (letter only), J. Ferziger  
SJK/cmm

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## ATTACHMENT 1

### A SUMMARY OF ZONAL MODELING

#### I. THE CENTRAL RESULT

Overall, the work under AFOSR Contract 49620-86-K-0008 has established that zonal modeling is a particularly useful approach for a large range of engineering problems. In fact, the results of the project show the zonal approach works even better than suggested in the Opinion by Kline in the Proceedings of the 1980-81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows which was the underlying basis for this contract. The background ideas and the specific results which lead to these conclusions are given in the sections which follow.

#### II. THE CONCEPT OF ZONAL MODELING--WHY IT IS NEEDED

The community of fluid mechanics researchers, with no significant exceptions, believes that the full, unaveraged Navier-Stokes equations are a necessary and sufficient set of governing equations for incompressible, continuum flows with Newtonian viscosity, in a detailed sense.

This belief is not arbitrary, it is founded on nearly two centuries of detailed checks in numerous flow situations, many detailed studies and reviews, and a lack of failure of the equations in all these examples coupled with ability to predict fine grained details of many flows wherever complete analytic or complete full turbulent simulations have become available.

The words "necessary and sufficient" and also the words "in a detailed sense" in the underlined phrase above are important to understanding the need for zonal modeling. Together they imply not only that the Navier-Stokes equations are an adequate (sufficient) model, but also that there are no terms in the equations that are not necessary in the sense that they describe behaviors of real fluid motions and DO NOT DESCRIBE BEHAVIORS THAT DO NOT OCCUR.

If the Navier-Stokes equations are necessary and sufficient in this sense, then any simplification of the equations, as for example by taking time or ensemble mean averages, must necessarily be missing information that is needed for the complete solution in at least some flow situations. It follows that, any model equations using these less than complete equations cannot contain all the information needed for solution of all flow fields. This does not, however, imply that such model equations cannot be adequate for a narrower domain of applications, only that they probably cannot be adequate for the full range of flows governed by the Navier-Stokes equations.

Given this reasoning, the results of the 1980-81 conference, as cited by Mr. Bordalo in Attachment 3, are not surprising. Indeed they must be expected.

Given those results, as Mr. Bordalo states, if we want models that are both fast-running and accurate, then we are forced to use some form of zonal modeling (since no model less complex than the full Navier-Stokes equations will do everywhere, and the N-S equations are by no means fast-running even in the largest computers available or foreseen in the near future). The appropriate question thus becomes, "What form of zonal modeling?"

As Bordalo also states in attachment 3, zonal modeling is not new. Prandtl's 1904 paper deriving the boundary layer equations, which is widely taken to be the genesis of modern fluid mechanics, succeeds precisely because it uses a zonal approach. What the work under AFOSR Contract 49620-86-K-0008 then does is to explore the zonal modeling concept further in the context of modern models employing large digital computers. In the next section, we summarize what this exploration has shown.

### III. RESULTS OF THE PROJECT

The work under Contract AF F49620-86-K-0008 studied four flow situations: (i) strained homogeneous flows; (ii) free-shear-layers (jets, wakes, mixing layers); (iii) the backward facing step; (iv) boundary layers. Study of categories (i), (ii), and (iii) are completed. The work on boundary layers is sufficiently advanced to provide clear understanding of what zonal modeling can do in this problem.

The procedures which evolved during the work under AFOSR Contract 49620-86-K-0008 for appropriate and effective zonal modeling are summarized in Attachment 3. The remainder of this section deals with what has been accomplished and what those accomplishments imply.

Tzuoo showed that zonal modeling, applied as described in Attachment 3, can provide a single model as accurate as the known data for the entire class of free shear flows. This includes planar and axisymmetric cases, the near field and the far field for all well documented cases of jets, wakes and mixing layers. The accuracy of the results are an order of magnitude better than that using the same base equations (k-e) without the zonal concept. Tzuoo's model is the first solution to cover all these cases accurately. It thus has many very practical applications. It is simple and fast-running.

Even more important, conceptually, are three other results established by Tzuoo.

First, it is possible to extend the domain of zonal modeling significantly by making the coefficients in the equations functions of the local and/or global non-dimensional governing parameters.

Second, when one makes the coefficients functions of the governing non-dimensional parameters, the results can generalize so that a wider range of flows can be predicted by a single fast model. This same effect shows that a single invariant model (one with the same unaltered form of both the model equations and coefficients in the model equations) may fit more than one kind of region (for example, wakes, jets and mixing layers). Thus while one makes an initial guess about what the zones are, in the end what is called a zone depends on the verified range of the invariant model.

Third, systematic modeling which removes what Attachment 3 calls "contamination" provides steady increases in knowledge about specific kinds of flow (i.e. zones), and this knowledge becomes "building blocks" which can be used for constructing complex flow fields which contain many zones.

Given these three results, and the fact that nearly any common flow field can be constructed from something like 10 "zones" (at most), zonal modeling becomes a highly practical and efficient way of modeling any particular flow situation of sufficient engineering importance to warrant a phase or two of research in constructing appropriate zonal models. These models will NOT be universal models, but will be very useful and practical for many engineering problems over well defined domains. In point of fact,

successful fast and accurate predictive computer models of the past are of this type without exceptions known to the author, although this has often not been explicitly stated.

An important corollary is that efficient engineering codes should be single purpose codes. That is, they should be designed for a single type of flow whose zones are known in advance and whose locations are easily determined by the code itself.

Avva's work shows that the construction of a complex flow field based on zonal models for the component parts of the flow field works very well for the backward facing step, a paradigmatic case for this kind of testing.

The results of Bordalo show that the same zonal concepts and procedures work for the boundary layer. More specifically, when two zones are used (for the inner and outer layers) more precise results are achievable, as one ought to expect from available knowledge of the physics. Bordalo also shows that the zonal models can be extended to handle effects of wall curvature and pressure gradient. The STAN 5 and STAN 6 programs of W. M. Kays, illustrate that the same zonal concepts work for convective heat transfer and for blowing and suction.

A surprising, and happy, result of the work of Tzuoo, Avva and Bordalo is that systematic and careful use of zonal modeling often provides additional insight into the physics of the flows studied. In Tzuoo's case the work led to the creation of a sufficient set of global non-dimensional governing parameters for the class of free shear flows. Two parameters are needed; one had been known before; the other had not. In the work of Avva the systematic zonal modeling resulted in disclosure of a small but significant three dimensional effect in flows that had been previously thought to be two-dimensional. This provides better understanding of future experiments and computation.

The same insight is NOT obtained by use of a model which is assumed to be universal since one then applies the model, sees if results are adequate or inadequate, but lacks the direct links to specific flow zones which allow the sources of troubles to be tracked down and understanding thereby increased. This remark is not only true in principle, but has played out in practice again and again. Computers have repeatedly constructed models which they assumed were universal, and then found they worked for some flows, but not others. The only recourse, inside the "universal model" framework, is then to begin again from scratch and build another assumedly "universal" model. Thus the "universal" model approach does not have the very important "building block" property of zonal modeling. In the long run, this has made the "universal model" approach inefficient.

A number of workers objected to the concept of zonal modeling when it was expressed in the OPINION by Kline on the grounds that readjustment regions would at best be troublesome, and perhaps could not be handled. Experience in the work of Tzuoo, Avva and Bordalo, indicates clearly that modeling readjustment zones is not difficult. In all cases good results have been possible, and in no case has a readjustment form more complex than a linear first order ODE been necessary.

In sum then, zonal modeling works well, even better than anticipated in the project proposal. It is a very promising avenue for constructing predictive computer models of turbulent flows wherever the importance of the problem warrants construction of a model. In the efforts thus far each model has taken 1-4 man-years of effort. However, these times are for workers without prior experience and without a backlog of "building blocks". As workers gain experience, building blocks in the form of codes

capable of dealing with various zones will become available to them, and these times will become significantly lower.

A ~~few~~ remarks follow on the status of zonal modeling in the community. The work of T. Avva, and Bordalo establish the validity and utility of the zonal modeling approach. ~~This~~ validity and utility is now voiced not only by the supervisors of this project but also by such leading workers as Peter Bradshaw and Dennis Bushnell. The basic research phase is thus largely over. There are many applications that can be profitably exploited. It is appropriate for companies to take up use of the concept. Some companies are now doing this. Boeing, for example, is using the concept and is considering funding work on some applications by Bradshaw, Johnston and perhaps also Ferziger. A more extensive list of users was sent your office a short time ago.

SJK  
January 9, 1989

Attachments: 1,2,3

## 1.1 Turbulence Modeling

The performance of many engineering systems is determined by the flow of fluids contained in the system. The prediction of fluid flows pervades design activities including selection of configuration, evaluation of components, development of devices, determination of sensitivity of performance to off-design conditions, identification of potential problems, etc. The simulation of flows by computers can be a useful tool in all these activities.

The benefits of Computational Fluid Dynamics (CFD) must be weighed against its costs. There are costs associated with the size of computer memory needed and the time it takes to run a program. The running time of a simulation can be important when designs require numerous runs. The computer program (code) must be fast enough to warrant its use. Other requirements, which depend on the specific application, are the accuracy of the output and the amount of detail to be computed.

The development of codes for flow simulation is not a easy task. Among other problems facing the code developer, most flows of importance are turbulent. The term 'turbulence' refers to a class of related fluid dynamics phenomena exhibiting a variety of behaviors and structures. The complexity of turbulence is reflected by the equations of fluid motion. Turbulence has been, for over a century, a challenging subject of research.

The Navier-Stokes (NS) equations are the best available model for predicting the flow of Newtonian fluids — a class that covers the overwhelming majority of practical cases. However, these are non-linear partial differential equations, and analytical solutions are known for only a handful of laminar flows. Matters are further complicated by the chaotic time-dependent nature of turbulent flows. With the capacity and power of present computers, direct solution of the NS equations is unjustifiable, or downright impossible, for all but a few rather simple flows; even solving the Reynolds average Navier-Stokes (RANS) equations for flows with a steady average may be beyond a cost-benefit ratio affordable in engineering practice. The engineer, designer or code developer, has to resort to the various approximate



methods that have been developed to predict turbulent flows.

These methods have been classified according to the following categories, or levels of computation (Kline 1981):

1. **Correlations.** These are well-established functional relations among flow parameters — in algebraic or graphical form, explicit or implicit — determined from experiments or by one of the higher-level methods described below. Correlations are excellent engineering tools, because they are easy to use, permit rapid calculation, and allow one to grasp the relationship among the variables involved at a glance. Each correlation usually applies only to a specific geometric configuration and may require the compilation of a considerable amount of reliable data. A correlation provides almost no details about the flow, and has a scope limited to the range for which it was established; i.e., it usually will not extrapolate to other flows.
2. **Integral methods (IMs).** These were developed mainly for the computation of thin shear layers before the advent of computers, and were later extended to some internal flows. The NS equations are reduced to ordinary differential equations (ODEs) by integration over one or more coordinates. The resulting ODEs may be exact, but they are not closed and cannot be solved without introducing approximate relations among the integral variables. An empirical expression for the velocity profile, or correlations among the integral variables are required to close the set of equations. An IM provides more details about the flow than a correlation and is more flexible. The differential equations allow the treatment of a range of flow cases, and the empirical relations employed are usually of a more general nature than those used in correlations. However, IMs are limited to rather simple geometries and by the range of validity of the empirical relations employed.
3. **One-point methods (1PMs).** These were developed to take advantage of the increasing power of computers. 1PMs pursue generality by

employing partial differential equations (PDEs) derived by Reynolds averaging the NS equations. Indeed, 1PMs can be applied, in principle, to any geometry, thanks to their formulation in terms of PDEs. In these methods, all flow variables are decomposed into mean and fluctuating parts; *e.g.*, the velocity component in the  $x$ -direction is represented as  $U = u + u'$ , where  $u$  and  $u'$  are the mean and fluctuating parts, respectively. The NS equations are Reynolds-averaged to avoid the need to compute the turbulent fluctuations of the flow field. The resulting PDEs — known as the *Reynolds-averaged Navier-Stokes* (RANS) equations — describe the “mean” flow. The RANS equations require less computational capability than the complete NS equations while retaining most of the NS equations fundamental character. However, some physical information is necessarily lost in the process of averaging. Due to the non-linearity of the NS equations, terms that represent the interaction between the mean and turbulent components of the flow arises in the averaging process. These terms, or “virtual stresses”, comprise the *Reynolds stress tensor*, and represent the average momentum transported by the turbulent fluctuations. For example, the  $xy$ -component of the tensor is  $R_{xy} = -\overline{u'v'}$ , where  $u'$  and  $v'$  are the turbulent velocity fluctuations in the  $x$ - and  $y$ -direction, respectively. Approximations that allow estimation of the Reynolds stresses are needed to close the set of equations. 1PMs include the following closure schemes:

**3.1 Mixing-length models.** The concept of *eddy viscosity* was proposed by Boussinesq (1877) as a model of the Reynolds stresses. He derived this model using an analogy to Newton’s model for the viscous shear stress. It postulates a linear relation between the Reynolds stresses and the mean strain rate of the flow; a “turbulent” or “eddy” viscosity ( $\nu_t$ ) is the proportionality parameter. The validity of such an assumption is questionable, and must be substantiated by experiments. Even though Boussinesq’s concept is very simple, it has proven quite useful. Dimen-

sional analysis suggests that the eddy viscosity is proportional to the product of velocity and length scales of the mean flow. The mixing-length model employs empirical algebraic formulas that prescribe the values of these quantities. The degree of generality achieved by the model depends on the range of validity of these formulas. The mixing-length model is credited to Prandtl (1925) and is based on a physical concept that is easy to grasp: the length scale in the eddy viscosity formula is the "size" of the turbulent eddies. These eddies vary in size throughout the flow field, and the size distribution varies from flow to flow. It has been possible to express these distributions generally enough that they apply to a reasonably wide range of flows, including boundary layers and free-shear layers. The mixing-length model is simple and fast; it has found great success in aerodynamic applications for which the flow is adequately described as a composite of an inviscid potential region and a turbulent boundary layer.

3.2 *One-equation models.* It is difficult to extend the mixing-length model to complex flows — three-dimensional flows, flows with separation and reattachment, etc. The difficulty is traced to the lack of universality of the formulas for the distribution of velocity and length scales. The next logical step is to seek differential equations to describe the evolution of these quantities. One-equation models use a transport equation to determine the velocity scale, but still prescribe the length scale. This is not a great improvement on the mixing-length method because the inadequacies of the modeling of the length scale are retained. Two-equation models address this problem and quickly supplanted one-equation models.

3.3 *Two-equation models.* Two-equation models retain the concept of eddy-viscosity and use transport equations for evaluation of both velocity and length scales. The  $k$ - $\epsilon$  model will be reviewed

here as an example; other two-equation models have been proposed. The velocity scale is related to  $k$ , the turbulence kinetic energy, and the length scale to both  $k$  and  $\epsilon$ , the rate of dissipation of turbulent kinetic energy. PDEs for  $k$  and  $\epsilon$  can be derived from the NS equations. The use of PDEs guarantees that the model can be applied to complex geometries; whether it can accurately describe the flow remains to be determined. The derived equations are exact but once again there is a closure problem because they contain high-order moments of the turbulence fluctuations. Approximations are introduced by modeling the "problem" terms. Each of the modeled terms in the  $k$ - $\epsilon$  equations has a model coefficient that must be determined empirically. These coefficients were assumed to be constants in the initial versions of the  $k$ - $\epsilon$  model; one of these versions will henceforth be referred to as the *standard  $k$ - $\epsilon$  model*. Experience shows that the predictions of the standard  $k$ - $\epsilon$  model are not accurate enough for many engineering purposes, although they are not far off the mark. With a little modification of the model coefficients, it may be possible to simulate a wide range of flows with acceptable accuracy. Such modifications should be based on results for flows for which accurate data is available — a process we refer to as *tuning* or calibration. One should be careful about accepting results for flows outside the calibration range.

**3.4 Algebraic stress models.** These models are offshoots of the Reynolds stress models described below. Rodi (1976) proposed a scheme in which the PDEs for the Reynolds stresses are reduced to a system of algebraic equations at each point of the flow. The equations for  $k$  and  $\epsilon$  are retained. Therefore, the algebraic model is faster and cheaper to use than the Reynolds stress model and only slightly more expensive than the  $k$ - $\epsilon$  model. In practice, it seems to perform as well as two-equation models but no better. However, algebraic stress models are still maturing

and improved versions may be available in the future.

- 3.5 *Reynolds stress models.* Reynolds stress models avoid use of the eddy viscosity concept. They compute all the components of the Reynolds stress tensor directly from PDEs derived from the NS equations. Again, one encounters terms dependent on the turbulence fluctuations that need to be modeled in order to achieve closure. Construction of these models is a demanding task because the terms requiring modeling are tensors, there are more model constants to evaluate, it is hard to compile the data needed, and computations using the model are expensive. Their development has been slow; Reynolds stress models have been applied to relatively few engineering flows. They are the most complex models that have been used for this purpose. Their effectiveness relative to simpler models is yet to be proven and their future is uncertain.
4. **Two-point methods (2PMs).** These methods are based on the Fourier-transformed Navier-Stokes equations. They have, so far, been applied only to homogeneous flows. Application of these methods for inhomogeneous flows leads to difficult and expensive computations. It is not clear whether they will prove practical for flows in complex geometries. Nevertheless, they are the subject of intense academic research, as they are helpful in the study of turbulence phenomena.
5. **Large-eddy simulation (LES).** This method calculates the large-scale structures of turbulence explicitly, while modeling the small-scale structures. It is three-dimensional and time-dependent, and provides considerable information about the flow. This makes it a valuable tool for investigating both the physics of turbulence and the models used to represent it. Its application is currently limited to simple flows and relatively low Reynolds numbers. At present, it is too expensive for engineering use because it requires the capacity and power of supercomputers.

6. **Full-turbulence simulation (FTS).** In this approach, one solves the full time dependent NS equations numerically. The set of flows that can be treated in this way is small at present. FTS requires even more computer memory and time than LES; consequently it is more expensive. It is employed as a research tool to investigate phenomena related to the small scales of turbulence, specially to evaluate the subgrid models for LES. Because of its accuracy, FTS supplements laboratory experiments in some areas of research.

The methods above are listed in order of increasing level of computational requirement. The methods at higher levels are more fundamental, in the sense that fewer approximations are made to the NS equations. They should be more general; i.e., the range of flows to which they can be applied is potentially broader. However, numerical solution for higher-level methods requires more computational work. Because the high-level methods compute more details about the flow, they also require more detailed initial and boundary condition information — often these are not known accurately. It should be kept in mind that for a particular flow, a lower-level method may be more accurate than a higher-level one due to better tuning.

It may be improper to compare these methods too generally. Engineering is a matter of trade-offs between competing factors; when selecting a method, the CFD user will certainly make compromises, pondering the costs and benefits for the job at hand. Therefore, it is advantageous to count on an arsenal of methods of varying degrees of accuracy, speed, cost, and detail of output.

All methods are in use today, as each is suited to a specific job. LES and FTS are, at present, employed mainly as research tools, while well-established correlations are used in engineering design when fast and accurate results can be obtained for flows within the limited scope of validity of the correlation.

Between these extremes lie the one-point methods. At the moment, 1PMs seem to offer a good trade-off between accuracy, speed, flexibility

and computational expense for many high-technology applications; consequently, a lot of effort has been invested in developing turbulence models for one-point methods. None of these models have, so far, been proved irrefutably superior to their competitors over a wide range of flow problems.

Consider the results of the 1980-81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows. A recognizable pattern emerged that can be described as follows:

*Statement 1* – No single turbulence model presented was accurate over the entire range of cases to which it was applied.

*Statement 2* – One model would fare better than another in some cases, and poorer in other cases.

*Statement 3* – Nearly all flows were accurately modeled by at least some method.

*Statement 4* – Higher-level models didn't always perform better than lower-level models.

Many of these models consist of a fixed set of fundamental and closure equations with a fixed set of model coefficients (the model constants) — for example, the standard  $k$ - $\epsilon$  model. The model constants were usually tuned to fit the data for some flows. Often, each constant was determined from a different class of flows — such as boundary layers, free-shear layers, etc. The values obtained were used in the simulation of all flows. Such models are sometimes referred to as “universal” models, because it is implicitly assumed that the constants are universal. This terminology is probably inappropriate, since their failure to accurately simulate some flows (ref. *statement 1*) refutes their claim to universality — computers reported success for some classes of flows and poorer performance when the model was applied to other flows without changing the model constants. We shall refer to these models as *invariant* models. The term ‘invariant’ is a reminder of the fact that in these models, the equations and coefficients are independent of the flow. At this point we wish to introduce another

concept — *contamination*. 'Contamination' refers to the loss of accuracy in predicting one class of flows caused by a model constant being determined by a different class of flows.

The cycle of investigation of invariant models typically begins with enthusiasm as a newly suggested model meets with initial success, and ends when the model is discarded after failing for more difficult problems. The search for another model is initiated and the cycle may be repeated. For the researcher seeking a universal model, the situation is frustrating.

If we accept the fact that the NS equations contain all the physics of turbulence, then any simpler invariant model has little chance of achieving universality. As discussed before, some physical information is always lost when approximations are made to the NS equations, and the closure problem is a recurrent one. Experience shows that the lost information is important. Turbulence models are, in our view, sophisticated engineering correlations with limited breath, *a limitation that is a consequence of the empiricism built into them*. Since the constants of an invariant model are tailored to specific flows, the model reflects the physics of the flows on which it is based; it is not surprising that an invariant model does a good job of predicting these flows while failing for others (ref. *statement 2*). Also, we would expect the model to work well for flows that share the physics of the flows used for tuning the model. When the physics is different, there is no guarantee of success.

It may be impossible to find a universal invariant closure model because turbulence displays different physical structures in each flow, and even among regions of a single flow field. On the other hand, experience shows that it is feasible to construct models that adequately simulate a particular region of a flow field, or a class of flows (ref. *statement 3*).

Kline (1981) suggested an alternate strategy for turbulence modeling — *zonal modeling*. In the zonal modeling approach, turbulence models are tied to the physics of individual flow zones; other zones are not allowed to contaminate the model. The zonal models are then employed in the simulation of complex flow fields; i.e., fields composed of multiple zones.



The zonal methodology is discussed in detail in the next section.

Applied to mid-level methods, zonal modeling may allow us to achieve a broad domain of applicability without loss of accuracy, at reasonable computer time and cost, thus providing the user with cost-effective tools for the simulation of complex flows.

Zonal modeling was used to extend the domain of the  $k$ - $\epsilon$  model to encompass homogeneous flows and free shear layers (Tzuoo 1986). Zonal models for the  $k$ - $\epsilon$  equations were employed to compute the flow past a backward facing step (Avva 1988).

The objective of the present work is to extend the domain of the  $k$ - $\epsilon$  model to include boundary layers subjected to pressure gradients, and to boundary layers over curved surfaces using the zonal approach.

## 1.2 The Zonal Approach

A *flow zone* is a distinct region of a flow field that can be identified by specific physical characteristics. A list of some flow zones is given in Table 1. A flow field may be decomposed into zones; for example, the zones in the flow in a diffuser is shown in Figure 1.1.

TABLE 1  
Flow zones in 2D incompressible flows<sup>†</sup>

|    |                                              |
|----|----------------------------------------------|
| 1. | Inviscid zone                                |
| 2. | Laminar boundary layer                       |
| 3. | Laminar-turbulent transition                 |
| 4. | Attached turbulent boundary layer            |
| 5. | Detaching zone                               |
| 6. | Reattaching zone                             |
| 7. | Free shear layer <sup>‡</sup>                |
|    | Plane jets, wakes and mixing layers          |
|    | Axisymmetric jets, wakes and co-flowing jets |
| 8. | Recirculation zone                           |
| 9. | Wall jets                                    |

<sup>†</sup> Adapted from Kline, 1981.

<sup>‡</sup> Zone redefined by Tzuoo, 1986.

Past experience indicates that it is difficult to construct a single invariant closure model capable of simulating all zones of a complex flow, such as the diffuser flow. Progress along this path has been very slow. The zonal methodology acknowledges the trade-off between range of application and accuracy inherent in turbulence modeling and proposes that models ought to be zone-dependent.

This point deserves emphasis; the models are tied to the local characteristics of the zone, not to the flow as a whole. A *flow* depends on the configuration — geometry plus initial and boundary conditions — and is a composite of many zones. Developing *ad hoc* models on a flow by flow basis would be bad practice, for the process would probably not close. Firstly,

such a procedure would not necessarily connect the model to the physics, so little would be learned about the latter. Secondly, it would be risky to apply such a model to flows very different from the one used in establishing the model constants. Thirdly, the number of possible flow configurations of engineering interest is limitless. The number of flow zones, on the other hand, is large but finite.

A zonal model can be very accurate because it is tailored to fit the data for the zone. Zonal modeling avoids contamination because it uses only physical information relevant to the zone under consideration. Also, zonal modeling may suggest experiments on specific zones; these are almost certainly easier to carry out than experiments on complex flows. Interaction between experiment and model for a specific zone can illuminate both the turbulence structure in the zone and the formulation of the model. We believe that this approach will provide accurate models and advance knowledge faster than the search for an universal closure model. Even before the investigation of all zones is exhausted, we will be able to compute many flows of interest with the models already developed. For example, with the nine zones listed in Table 1 one can handle a large number of flows. Additional zones could be added as needed.

When a flow field is decomposed into zones, there exist regions between the zones in which transition from one type of structure to another takes place; these regions are named *readjustment zones*. Zones may be connected in "series" or in "parallel", according to whether the fluid moves from one zone to the other, or the flow is along the zonal interface. An example of the first kind is a boundary layer leaving a surface at separation and becoming a free shear layer. An example of the second kind is the interface between a boundary layer and a potential flow. The modeling of the readjustment region must reflect the change in physics. Pragmatism suggests that we first try the simplest model for these readjustment regions. Also, to minimize the difficulty of blending the turbulence models, we should utilize the same formulation for the models in each zone; *e.g.*, the models could all be based on the two-equation  $k-\epsilon$  model. To obtain maximum advantage, this *base*

*model* should be selected from among the better existing models.

Thus, zonal models can be perceived as modifications of the base model, and the zonal methodology as a process that expands the domain of applicability of the base model. The use of a base model implies invariance of the form of the equations. This invariance can simplify the programming of a code for complex flow. In the case of the zonal  $k$ - $\epsilon$  model, the governing equations are invariant while the model coefficients are allowed to vary from zone to zone. In the readjustment regions the model coefficients may be changed (i) discontinuously; (ii) by parametric variation; (iii) through linear blending; (iv) through a first-order lag equation.

The definition of a zone must be subject to testing. We begin with an archetypical zone; i.e., a physical region of a flow with a well-defined set of characteristics that is identifiable and distinct in different flow fields — e.g., the boundary layer. As the work progresses the initial choice may need to be changed. The zone may be broken into two or more regions, each with a different model so that the zonal definition is narrowed. The zonal definition may be broadened if a model works not only for the region intended, but for other regions as well (Tzuoo 1986). Once a model that accurately simulates a zone is found, it may be employed in the computation of any flow in which that zone occurs. The usefulness of a zonal model rests on the zone being a *building block*.

The physical characteristics of a zone may depend on a number of factors — e.g., pressure gradient, curvature, blowing/suction, roughness, compressibility, etc.; the model should reflect this. To achieve some degree of generality, sound practice suggests that the relationship between model and cause be expressed via functions of non-dimensional parameters; some well-known flow parameters are the Reynolds number, Mach number, and Richardson number. The governing parameters may be *local* or *global* in nature.

The determination of the relevant governing parameters, the form of the function relationship, the modification to the base model, are parts of a process in which concepts are tested and changed as one learns more

about the physics of a zone and the ability of the model to represent them. The tools used in this investigation include semi-empirical analysis, physical insight, sensitivity analysis, numerical experimentation, and existing knowledge and experience. As experiments are the primary basis of our understanding of turbulence phenomena, reliable data are the most important guide in the development of zonal models.

To recapitulate, the construction of zonal models will follow these guidelines:

- ★ **Accuracy** – Each model should represent the zone it is designed for with sufficient accuracy. Acceptable accuracy may vary with application.
- ★ **Simplicity** – The base model should be modified only as necessary.
- ★ **Independence** – The model is altered only in the considered zone. This avoids the problem of contamination.
- ★ **Close connection to physics** – The model should reflect the dependence of the flow on its governing parameters.
- ★ **Controlled numerical error** – The uncertainty due to numerical errors must be considerably less than the experimental uncertainty so that the model can be accurately evaluated against the data.
- ★ **Controlled experimental error** – The uncertainty due to experimental error must be estimated, and adequacy of the data to represent the flow must be assessed.

The methodology of zonal modeling can be summarized as follows:

*Step 1* – Select a base model.

*Step 2* – Compute the flow with the base model. In the case of a single zone flow this is enough. In the case of a multiple zone field one should also run a simulation that includes previously established zonal models that are applicable.

*Step 3* – Compare the results with the data; identify unacceptable disagreements and propose explanations.

*Step 4* – Modify the base model in accordance with the zonal modeling guidelines set forth above.

*Step 5* – In the case of a multiple zone field, patch the zonal models together; treat the readjustment zones.

*Step 6* – Compute the flow with the proposed model; iterate back to Step 3 until satisfied with results.

Finally, it should be noted that the zonal approach is not totally new. Prandtl introduced a zonal approach that is still employed in aircraft design and many other applications. Zonal modeling has been successfully applied to integral methods to solve diffuser flows. Even in complex turbulent elliptic flows, it is a common practice to use wall functions for the region next to a solid surface and an invariant model for the rest of the field — a practice that is implicitly “zonal”.

The base model adopted for the present work, the standard  $k$ - $\epsilon$  model, is described in the next section.