

INTERACTION OF LARGE AMPLITUDE STRESS WAVES IN
ELASTIC - PLASTIC MATERIALS

FINAL REPORT

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1. FORWARD

The aim of this study has been to develop mathematical techniques that can be used to analyze the responses of materials whose transmitting properties vary rapidly in space and time, either because of inherent inhomogeneities in the material or because of inhomogeneities induced by nonlinearity

The dynamic response of a material whose physical properties vary is usually very difficult to analyze. Even when the material response is elastic and linear, with elastic moduli varying in a known way from point to point, it is only possible to analyze the simplest of problems: either the moduli must be piecewise constant, as in laminates, or they must be slowly varying. Similarly, if the transmitting properties of the material are strongly temperature dependent, and the temperature is varying rapidly in space and time, only the simplest of problems can be analyzed. Such problems become even more complex when the material response is also nonlinear. Then, essentially, the nonlinearity induces inhomogeneities in space and time that are not known a priori but must be determined as part of the overall solution to the problem.

In the course of this research we have developed mathematical procedures that can be used to analyze a wide variety of physical processes in strongly inhomogeneous materials. Although the main emphasis has been on problems involving wave propagation in diverse materials, the techniques have also been used to study the problem of heat conduction in strongly stratified materials, and also static (elliptic) problems associated with inhomogeneous materials.



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2. SUMMARY OF COMPLETED RESEARCH

2.1 Elastic waves in materials whose properties vary in space.

In its simplest form, the equation governing uni-directional stretching, or shear, waves in an inhomogeneous elastic material can be written as

$$\frac{\partial}{\partial x}(E(x)\frac{\partial w}{\partial x}) = \rho(x)\frac{\partial^2 w}{\partial t^2}. \quad (1)$$

$w(x,t)$ denotes the displacement of the particle x at time t , $E(x)$ denotes the dynamic modulus of the material, and $\rho(x)$ denotes the density of the material in its undeformed state. Until recently, the only types of problems that could be easily analyzed using equations such as (1) were those involving laminated materials, for which $E(x)$ and $\rho(x)$ are piecewise constant, and those for which $E(x)$ and $\rho(x)$ are varying slowly.

During the course of this research we have developed general procedures that can be used to obtain a representation for solutions to equations such as (1) without having to restrict the forms of the coefficients $E(x)$ and $\rho(x)$. Briefly, the technique amounts to expanding these coefficients in terms of a series of functions. These are chosen so that if the approximating series are terminated at *any* stage then the general solution to the resulting equation can be written in terms of the solution to the wave equation with constant coefficients. The representations obtained can be used to analyze many technically important deformations. The results have been generalised so that they can be used to construct solutions to equations governing processes in anisotropic, inhomogeneous materials. The processes are governed by an equation for a function $w(x_1, x_2, \dots, t)$ which can be written in the form

$$\begin{aligned} & \left[\alpha_1(x_1) \frac{\partial^2 w}{\partial x_1^2} + \beta_1(x_1) \frac{\partial w}{\partial x_1} + \gamma_1(x_1) w \right] + \left[\alpha_2(x_2) \frac{\partial^2 w}{\partial x_2^2} + \beta_2(x_2) \frac{\partial w}{\partial x_2} + \gamma_2(x_2) w \right] + \dots \\ & = a_0 w + a_1 \frac{\partial w}{\partial t} + a_2 \frac{\partial^2 w}{\partial t^2} + \dots \end{aligned} \quad (2)$$

This work was described in [1].

2.2 Elastic waves in materials whose properties vary with temperature.

In many technically important problems stress waves are produced by rapidly heating the material. In its simplest form the mathematical problem reduces to solving an equation of the form

$$\frac{\partial}{\partial x}(E(x, \theta) \frac{\partial w}{\partial x}) - \rho(x) \frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial x}(\beta(x) \theta), \quad (3)$$

where the temperature $\theta(x, t)$ satisfies the diffusion equation

$$\frac{\partial}{\partial x}(k(x) \frac{\partial \theta}{\partial x}) = c(x) \frac{\partial \theta}{\partial t}. \quad (4)$$

Equation (4), which is of the form (2), can be analyzed by the techniques described in [1]. The resulting equation for $w(x, t)$ can be written as

$$\frac{\partial}{\partial x}(a(x, t) \frac{\partial w}{\partial x}) - \rho(x) \frac{\partial^2 w}{\partial t^2} = S(x, t). \quad (5)$$

In [2] we have generalised some of the techniques described in [1] to analyze such equations.

2.3 The propagation of large amplitude shock waves and elastic-plastic boundaries in rate independent materials.

In a series of papers ([5] - [9]) we have developed mathematical techniques that can be used to analyze the diverse nonlinear wave interactions that can occur when a slab of elastic-plastic material is finitely deformed by the passage of plane waves propagating in directions normal to the material interfaces that bound the slab. Typically, the slab could be contained between two other different elastic-plastic materials which, essentially, are of semi-infinite extent in the direction in which the wave propagates. Alternatively, the slab could be just one layer in a multi-layered material. In the course of this research we have extended the results described in [5] - [9] to account for the presence of strong shock waves and elastic - plastic boundaries.

In its simplest form, the equation governing the variation in stress, $\sigma(x,t)$, in a nonlinear but rate independent material is the nonlinear wave equation

$$\frac{\partial}{\partial t} \left[S(\sigma) \frac{\partial \sigma}{\partial t} \right] = \frac{\partial^2 \sigma}{\partial x^2}. \quad (6)$$

$S(\sigma)$ is a material function which is different during loading and unloading. By using the hodograph techniques together with the techniques described in [1], we have been able to construct efficient procedures for analyzing deformations governed by equations such as (6) even when strong shock waves and elastic plastic boundaries are present. For example, if a shock is formed by impacting one end of a slab and then moves into an undisturbed region we are able to track its path and calculate its strength by solving *linear ordinary differential equations*. The coefficients in this equation depend on the material function $S(\sigma)$ and also on the variation in σ at the surface of the slab. The corresponding problem has also been solved when the propagating surface of discontinuity is an elastic - plastic boundary. A paper on this aspect of our work is being prepared.

2.4 Waves in visco - plastic materials.

The dynamic responses of many materials are strongly rate dependent - even in deformations that are produced by impact. In [3] and [4] we have analyzed wave motions in visco - plastic materials for which the stress $\sigma(x,t)$ and material velocity $u(x,t)$ satisfied equations of the form

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial u}{\partial t}, \text{ and } \frac{\partial u}{\partial x} = S(\sigma) \frac{\partial \sigma}{\partial t} + \omega(\sigma). \quad (7)$$

ρ is a constant while $S(\sigma)$ and $\omega(\sigma)$ are material functions. It was shown that for certain special forms of S and ω , which correspond to physically reasonable nonlinear behavior, any solution to (7) can be expressed in terms of a corresponding solution to the *linear* telegraph equation. This fact has been used to investigate the combined effects of nonlinearity and viscosity on a wide variety of technically important deformations.

2.5 Adiabatic shear.

Adiabatic shear banding is a form of material instability that occurs during the rapid deformation of a wide variety of thermo - plastic materials. When a slab of such a material is subjected to a pure homogeneous simple shear at a high constant rate of shear, the shear force needed to produce the deformation at first increases with increasing time. However, it may happen that with continuing deformation the shear stress reaches a peak and thereafter continues to decrease. This phenomena occurs when the effect of thermal softening dominates that of work hardening. During this stage the pure homogenous deformation is unstable: some initially small perturbation to the deformation can grow by extracting energy from the ambient deformation. The form of the instability can be spectacular: energy is focused toward isolated zones where the temperature soars.

In collaboration with Dr. Timothy Wright at A.R.O a theory has been developed which describes the main features of these shear band instabilities.

2.6 A method for solving singular integro - differential equations

During the course of the research on layered materials we were required many times to solve certain singular integro - differential equations. Such equations occur throughout the study of problems involving interfaces - especially in static problems involving composites. In [10] we described a general procedure for solving such equations.

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