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A Modified Expression for Evaluating the Steady Wave Pattern of a Ship

by

Francis Noblesse

Woei-Min Lin

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numerically. Whereas the values of both the velocity potential and its gradient at the hull are required in the usual expression for the wave-spectrum function, the new expression only involves the tangential velocity at the hull, not the potential. This new expression thus defines the wave-spectrum function in terms of the speed and the size of the ship, the hull form, and the tangential velocity at the mean hull surface.

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ABSTRACT

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ADMINISTRATIVE INFORMATION

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INTRODUCTION

Near-field potential-flow calculations about ships advancing at constant speeds in calm water are routinely required for evaluating their hydrodynamic characteristics, in calm water and in waves, and for determining the required propulsion and control devices. Calculations of far-field ship wave patterns are also important in connection with wave-resistance predictions and remote sensing of ship wakes. In particular, the latter practical application requires the ability of determining the short divergent waves in the wave spectrum having wavelengths between 5 cm and 40 cm associated with Bragg scattering of the electromagnetic waves in typical SAR systems used in remote sensing of ship wakes. No meaningful predictions of such short waves can be obtained on the basis of currently available numerical methods. More generally, numerical predictions of the steady wave pattern at large and moderate distances behind a ship are notoriously difficult and unreliable, as was recently made clear at the Workshop on Kelvin Wake Computations [1]. Ship wave-resistance calculations are also known to be difficult and unreliable.

Alternative numerical methods have been developed for evaluating near-field flow about a ship, that is, flow at the hull surface and in its vicinity. These include finite-difference methods, e.g. Coleman [2] and Miyata and Nishimura [3], and the more widely used boundary integral equation methods, also known as panel methods. The latter methods can be divided into two main groups, according to the Green function that is used. These two groups of methods are the Rankine-source method and the Neumann-Kelvin method, which are based on the simple Rankine (free-space) fundamental solution and the more complex Green function satisfying the linearized free-surface boundary condition, respectively.

The Rankine-source method was initiated by Gadd [4], Dawson [5] and Daube [6], and has since been adopted by many authors. The Neumann-Kelvin approach has a long history. A survey of recent numerical predictions obtained by a number of authors on the basis of the Neumann-Kelvin method may be found in Baar [7]. This study and that by Andrew, Baar and Price [8] also contain extensive comparisons of the authors' own Neumann-Kelvin numerical predictions with experimental data. An approximate solution, defined explicitly in terms of the value of the Froude number and the hull shape, to the Neumann-Kelvin problem was proposed in Noblesse [9]. This slender-ship approximation was recently used by Scragg et al. [10] and compared to both Neumann-Kelvin predictions and experimental data in [7] and [8] and to experimental data in [1] and [11].

The aforementioned alternative numerical methods for predicting flow in the vicinity of a ship are not all directly suitable for predicting the wave pattern of a ship at large, or even moderate, distances. More precisely, the finite-difference method and the Rankine-source panel method require truncating the flow domain at some relatively-small distance away from the ship and therefore can only be used for near-field flow calculations. (However, these near-field flow predictions can be used as input to the far-field Neumann-Kelvin flow representation considered in this study.) On the other hand, the Neumann-Kelvin theoretical framework is equally suitable for near-field and far-field flow predictions. Indeed, the far-field Neumann-Kelvin flow representation is a simplified particular case of the corresponding near-field representation.

The problem considered in this study is that of evaluating the steady wave spectrum and the wave pattern of a ship at moderate and large distances behind it in terms of the near-field flow on the hull surface. The near-field flow thus is assumed known for the purpose of the present study, which is concerned with the prediction of the steady wave spectrum and the wave potential behind a ship stern within the Neumann-Kelvin theoretical framework as was just noted.

This theory expresses the wave potential in terms of a Fourier representation, as is well known and is specifically indicated by Eq. (20) in this study. The wave-spectrum (or wave-amplitude) function in this Fourier representation is defined by the sum of an integral along the mean waterline and an integral over the mean wetted-hull surface. This expression for the wave-spectrum function, given by Eqs. (21)-(23), is quite ill suited for accurate numerical evaluation because the waterline integral and the hull integral in Eqs. (22) and (23) largely cancel out, as is shown further on in this study and is illustrated in Fig. 2. Errors in the numerical evaluation of the waterline and hull integrals cause imperfect numerical cancellations between these integrals and corresponding large errors in their sum. This fundamental difficulty was recognized in [9] and in Barnell and Noblesse [12], where attempts to remedy it were presented. However, these ad hoc approximate numerical remedies, based upon combining the waterline integral with the contribution to the hull integral stemming from the upper part of the hull surface are not satisfactory, as is attested by the fact that a very large number of panels is required for obtaining reasonably accurate numerical results [12].

A conceptually simpler and numerically more effective remedy is presented in this study, in which a modified mathematical expression for the wave-spectrum function is obtained via several applications of Stokes' theorem for combining the waterline integral and the hull integral. This new expression for the wave-spectrum function is considerably better suited than the usual expression for accurate numerical evaluation, notably for evaluating the short divergent waves of interest for remote sensing of ship wakes, because the significant numerical cancellations occurring between the waterline and hull integrals in the usual expression (see Fig. 2) are automatically and exactly accounted for, via a mathematical transformation, in the modified expression obtained in this study. The fundamental advantage of the new expression over the usual one is apparent from Figs. 8a-e.

Another interesting feature of the modified expression for the wave-spectrum function is that it only requires the tangential velocity at the hull, not the potential, whereas the usual expression requires the values of both the velocity potential and its gradient at the hull. The modified expression thus defines the wave-spectrum function in terms of the speed and the size of the ship, the hull form, and the tangential velocity at the mean hull surface. This expression is suitable for use in conjunction with a boundary-integral-equation method based on a source distribution or any other numerical method in which the velocity vector (but not the potential) is determined on the mean hull surface. It provides a coupling between a far-field Neumann-Kelvin flow representation and a near-field flow calculation method based on the use of Rankine sources or finite differences, in particular.

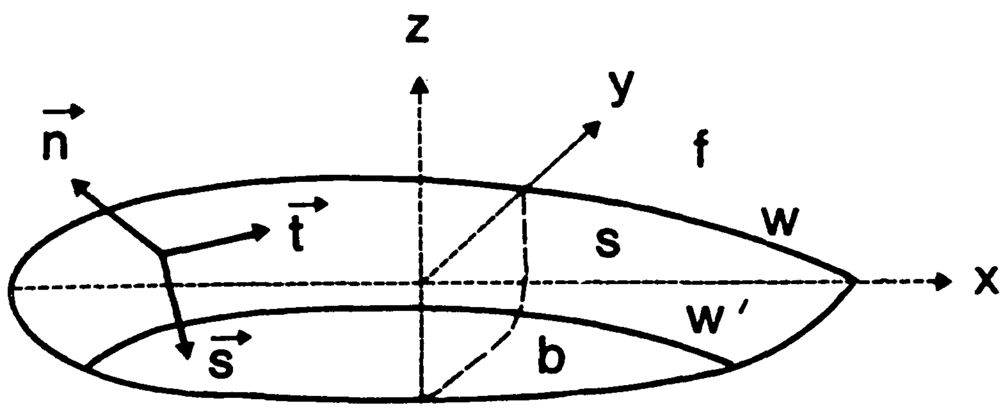

Fig. 1 Definition Sketch

NEUMANN-KELVIN REPRESENTATION FOR THE STEADY WAVE POTENTIAL OF A SHIP

As was already noted, this study considers steady potential flow about a ship advancing at constant speed in calm water of infinite depth and lateral extent. Nondimensional coordinates and flow variables are defined in terms of the length L and the speed U of the ship and the water density ρ. The undisturbed sea surface is chosen as the plane $z = 0$, with the z-axis pointing upwards, and the x-axis is taken in the ship centerplane (port- and starboard-symmetry is assumed) and pointing towards the bow, as is depicted in Fig. 1. The Froude number and its inverse are denoted by F and ν, respectively, and are given by

$$F = U/(gL)^{1/2} = 1/\nu \; , \tag{1}$$

where g is the acceleration of gravity.

Within the so-called Neumann-Kelvin theoretical framework, the velocity potential $\phi(\vec{\xi})$, at any point $\vec{\xi} = (\xi, \eta, \zeta \leq 0)$ strictly outside the ship hull surface, is defined by the following integral representation [9]:

$$\phi(\vec{\xi}) = \psi(\vec{\xi}) + \chi(\vec{\xi};\phi) \; , \tag{2}$$

$$\psi = F^2\int_w \overline{G}\, n_x^{\,2} t_y \, d\ell + \int_h \overline{G}\, n_x \, da \; , \tag{3}$$

$$\begin{aligned} \chi = {} & F^2\int_w \left[\overline{G}(t_x\phi_t + s_x\phi_s) - \phi\partial\overline{G}/\partial x\right] t_y \, d\ell \\ & - \int_h \phi\partial\overline{G}/\partial n \, da + F^2\int_f \overline{G}\, \pi(\phi)\, dxdy \; , \end{aligned} \tag{4}$$

where

$$\begin{aligned} \pi(\phi) = {} & \left[\partial\phi/\partial x - (\nabla\phi)^2/2\right]\partial(\partial\phi/\partial z + F^2\partial^2\phi/\partial x^2)/\partial z \\ & - \partial(\nabla\phi)^2/\partial x + \nabla\phi\cdot\nabla(\nabla\phi)^2/2 + O(F^2\phi^3) \; . \end{aligned} \tag{5}$$

In Eqs. (3) and (4), the symbols w, h and f represent the positive halves of the mean waterline, of the mean hull surface and of the mean free surface, respectively, as is depicted in Fig. 1 (where $h = s + b$ with s = hull side and b = hull bottom). Furthermore, $d\ell$ is the differential element of arc length of w and da the differential element of area of h. Also, $\vec{n} = (n_x, n_y, n_z)$ is the unit vector normal to h and pointing outside the ship, $\vec{t} = (t_x, t_y, t_z = 0)$ is the unit vector tangent to w and pointing towards the bow and $\vec{s} = (s_x, s_y, s_z)$ is a unit vector tangent to h and pointing downwards, as is shown in Fig. 1. In Eq. (4), ϕ_t and ϕ_s represent the components of the velocity vector $\nabla\phi$ in the directions of the tangent vectors $\vec{t}$ and $\vec{s}$ to h, respectively.

The nonlinear term $\pi(\phi)$ defined by Eq. (5) and the corresponding free-surface integral in Eq. (4) are associated with the nonlinearities in the free-surface boundary condition. This nonlinear term is usually neglected in practice.

The term $\overline{G} \equiv \overline{G}(\vec{\xi};\vec{x})$ in Eqs. (3) and (4) is the Green function for port- and starboard-symmetry defined as

$$\overline{G} = \overline{G}(\vec{\xi};\vec{x}) = G(\vec{\xi};x,y,z) + G(\vec{\xi};x,-y,z) \ , \tag{6}$$

where $G(\vec{\xi};\vec{x})$ is the Green function associated with the linearized free-surface boundary condition $\partial G/\partial\zeta + F^2\partial^2 G/\partial\xi^2 = 0$. The function $G(\vec{\xi};\vec{x})$ represents the linearized flow created at the point $\vec{\xi} = (\xi, \eta, \zeta \leq 0)$ by a unit outflow at the point $\vec{x} = (x, y, z \leq 0)$, stemming from a submerged source if $z < 0$ or from a flux across the mean free surface if $z = 0$ as is shown in Noblesse [13]. In the foregoing equations and hereafter, $\vec{\xi}$ represents the "calculation point", where the potential is evaluated, while $\vec{x}$ represents the "integration point" in the integrals on w, h and f.

The Green function may be expressed as the sum of three terms, as follows [13,9]:

$$4\pi G(\vec{\xi};\vec{x}) = S(\vec{\xi};\vec{x}) + 2\nu^2 N(\vec{X}) + 4\nu^2 H(x-\xi)W(\vec{X}) \ , \tag{7}$$

where ν is the inverse of the Froude number given by Eq. (1), and $\vec{X}$ is defined as

$$\vec{X} = (X, Y, Z \leq 0) = \left[\nu^2(\xi-x), \ \nu^2(\eta-y), \ \nu^2(\zeta+z)\right] \ , \tag{8}$$

and represents the vector of coordinates, rendered nondimensional with respect to the characteristic wavelength U^2/g instead of the ship length L, joining the free-surface mirror image (x, y, -z) of the singular point (x, y, z) to the calculation point (ξ, η, ζ).

The term $S(\vec{\xi};\vec{x})$ in Eq. (7) corresponds to a superposition of a Rankine source at the singular point (x, y, z) and a Rankine sink at its free-surface mirror image (x, y, -z), as follows:

$$S(\vec{\xi};\vec{x}) = -[(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2]^{-1/2}+[(\xi-x)^2+(\eta-y)^2+(\zeta+z)^2]^{-1/2}. \tag{9}$$

The second term $N(\vec{X})$ in Eq. (7) represents a nonoscillatory near-field (local) flow disturbance. Finally, the term $W(\vec{X})$ represents the system of Kelvin waves trailing behind the singular point (x, y, z), as is indicated explicitly by the Heaviside unit-step function $H(x-\xi)$.

By using expression (7) for the Green function in Eqs. (6) and (3)-(4) we may express the potential $\phi(\vec{\xi})$ defined by Eq. (2) as the sum of three potentials, as follows:

$$\phi(\vec{\xi}) = \phi_S(\vec{\xi}) + \phi_N(\vec{\xi}) + \phi_W(\vec{\xi}) , \tag{10}$$

which are readily defined by Eqs. (2)-(4) and (6) in which the Green function G is simply replaced by $S/(4\pi)$, $\nu^2 N/(2\pi)$ and $\nu^2 H(x-\xi)W/\pi$, respectively.

Numerical evaluation of the potential ϕ_S associated with the singular algebraic term S given by Eq.(9) is a fairly simple task (especially for points $\vec{\xi}$ strictly outside the ship since the term S then is never singular) for which extensive experience is available in both aerodynamics and hydrodynamics. In particular, only the hull-surface integrals in Eqs. (3) and (4) need be considered for the potential ϕ_S since Eq. (9) shows that we have $S \equiv 0$ on w and f, where $z = 0$.

Numerical evaluation of the nonoscillatory near-field potential ϕ_N in Eq. (10) associated with the nonoscillatory near-field term N in Eq. (7) is also a relatively simple task because the terms $N(\vec{X})$ and $\nabla N(\vec{X})$ are sufficiently well-behaved functions that can be evaluated numerically with satisfactory accuracy and efficiency. In particular, the function $N(\vec{X})$ remains finite at the origin $\vec{X} = 0$ whereas the function $\nabla N(\vec{X})$ has a relatively weak singularity, which is given in Noblesse [14]. Furthermore, this singularity does not occur for points $\vec{\xi}$ strictly outside the ship. The integrals on w, h, and f in Eqs. (3) and (4), where G is replaced by N, can then be evaluated numerically by using ordinary integration rules since the integrands in these integrals are continuous everywhere and nonoscillatory (except for the potential ϕ and the free-surface nonlinear term π, which are wavy functions with a characteristic wavelength explicitly defined in terms of the value of the Froude number). Accurate and efficient methods for numerically evaluating the terms N and ∇N have recently been developed by Newman [15] and by Telste and Noblesse [16].

Numerical evaluation of the wave potential ϕ_W in Eq. (10) associated with the wave term W in Eq. (7) is considered in this study. The wave potential ϕ_W dominates the nonoscillatory near-field potential $\phi_S + \phi_N$ at some distance behind the ship, as is well known, and ϕ_W is the most important component for practical applications to wave-resistance predictions and remote-sensing of ship wakes. The behavior of the oscillatory term W, which represents the system of Kelvin waves trailing behind the singular point $\vec{x}$ as was already noted, is considerably more complex than that of the nonoscillatory near-field term N. In particular, the functions $W(\vec{X})$ and $\nabla W(\vec{X})$ have strong and complex singularities at the origin $\vec{X} = 0$ and are ill behaved in the vicinity of the line $Y = 0 = Z$ and $X \leq 0$, as is shown in Ursell [17] and Euvrard [18]. Accurate numerical evaluation of the functions W and ∇W is difficult in the vicinity of the origin and more generally at the plane $Z = 0$. In spite of these difficulties, a method for numerically evaluating the wave terms W and ∇W has recently been developed by Baar and Price [19], Ursell [20] and Newman [21]. In principle, this method could be used for evaluating the wave potential ϕ_W in a manner analogous to that briefly explained in the foregoing for evaluating the nonoscillatory near-field potential ϕ_N. However, it is not clear a priori that the previously-noted complex behavior of the functions W and ∇W along the line $Y = 0 = Z$ and $X \leq 0$ would not cause serious numerical difficulties for evaluating the wave potential at the mean free-surface plane $z = 0$.

An alternative, indirect or Fourier-type, method for evaluating ϕ_W is used in [12] and this study. The method is based upon the following Fourier integral representation [13] of the wave term $W(\vec{X})$:

$$W(\vec{X}) = \int_{-\infty}^{\infty} \mathrm{Im}\, \exp[Z(1+t^2)+i(X+Yt)(1+t^2)^{1/2}]\, dt \ , \tag{11}$$

where Im denotes the imaginary part. By using Eq. (8) in Eq. (11) we may obtain

$$W(\vec{X}) = \int_{-\infty}^{\infty} \mathrm{Im}\, E^*(t;\vec{\xi})\, E(t;\vec{x})\, dt \ , \tag{12}$$

where the functions $E^*(t;\vec{\xi})$ and $E(t;\vec{x})$ are defined as

$$E^*(t;\vec{\xi}) = \exp[P^2\{\zeta + i(u\xi + v\eta)\}] \ , \tag{13a}$$

$$E(t;\vec{x}) = \exp[P^2\{z - i(ux + vy)\}] \ , \tag{13b}$$

In Eqs. (13a,b) we have

$$P = \nu p \quad \text{with} \quad p = (1+t^2)^{1/2} \ , \tag{14a,b}$$

$$u = 1/p \quad \text{and} \quad v = t/p \ ; \tag{15a,b}$$

we then have

$$1 \geq u \geq 0 \quad \text{and} \quad 0 \leq v \leq 1 \quad \text{for} \quad 0 \leq t \leq \infty \ , \tag{16a,b}$$

$$\text{with} \quad u^2 + v^2 = 1 \ . \tag{17}$$

Equations (12) and (13a,b) yield

$$\overline{W}(\vec{\xi};\vec{x}) = W(\vec{\xi};x,y,z) + W(\vec{\xi};x,-y,z) =$$
$$2\int_0^{\infty} \mathrm{Im}\, \exp(\nu^2\zeta p^2)\cos(\nu^2\eta pt)\exp(i\nu^2\xi p)$$
$$\exp(P^2 z)\,[E_+(t;x,y) + E_-(t;x,y)]\, dt \ , \tag{18}$$

where the functions $E_\pm(t;x,y)$ are defined as

$$E_\pm(t;x,y) = \exp[-iP^2(ux \pm vy)] \ . \tag{19}$$

The wave potential $\phi_W(\vec{\xi})$ is defined by Eqs. (2)-(4) where the Green function $\overline{G}$

is replaced by the wave term $\nu^2 H(x-\xi)\overline{W}/\pi$, as was already noted. In this study, we limit our attention to the wave pattern behind the ship stern; we then have $x \geq \xi$ and $H(x-\xi) \equiv 1$ for points (x, y, z) on the hull surface h + w. The indirect, Fourier-type method used in this study for evaluating the wave potential consists in interchanging the order of integration between the integration point $\vec{x}$ in the integrals in Eqs. (3) and (4) and the Fourier variable t in the integral (18). The wave potential ϕ_W is then expressed in the form

$$\phi_W(\vec{\xi}) = (2/\pi) \int_0^\infty \exp(\nu^2 \zeta p^2) \cos(\nu^2 \eta p t) \operatorname{Im} \exp(i\nu^2 \xi p) \left[K_+(t) + K_-(t)\right] dt \ . \tag{20}$$

In this Fourier representation, the functions $K_\pm(t)$ are given by the sum of the terms ψ and X defined by Eqs. (3) and (4), where the Green function G is replaced by the terms $\nu^2 \exp(P^2 z)\, E_\pm(t;x,y)$.

The functions $K_\pm(t)$ may then be expressed in the form

$$K_\pm(t) = K_0^{\pm}(t) + K_\phi^{\pm}(t) \ , \tag{21}$$

$$K_0^{\pm}(t) = \int_w E_\pm\, n_x{}^2 t_y \, d\ell + \nu^2 \int_h \exp(P^2 z) E_\pm\, n_x \, da \ , \tag{22}$$

$$\begin{aligned} K_\phi^{\pm}(t) = & \int_w E_\pm\, (t_x \phi_t + s_x \phi_s + i\nu^2 p\phi) t_y \, d\ell \\ & + \nu^2 P^2 \int_h \exp(P^2 z) E_\pm\, \phi n_\pm \, da + \int_{f_\xi} E_\pm\, \pi(\phi)\, dxdy \ , \end{aligned} \tag{23}$$

where Eq. (19) was used, the term $n_\pm$ is defined as

$$n_\pm = -n_z + i(un_x \pm vn_y) \ , \tag{24}$$

and f_ξ represents the portion of the mean free-surface plane upstream from the plane $x = \xi$.

The wave potential $\phi_W(\vec{\xi})$ in Eq. (20) thus is defined in terms of a familiar Fourier superposition of elementary plane waves propagating at angles θ from the x-axis given by

$$\tan\theta = \pm v/u = \pm t \ . \tag{25}$$

The amplitudes of these elementary plane-wave components are essentially given by the functions $K_\pm(t)$, which may thus be referred to as the far-field wave-amplitude functions or as the free-wave spectrum functions. These functions contain essential information directly relevant to a ship's wave resistance and signature. In particular, the wave resistance, R say, experienced by the ship is defined in terms of the wave-spectrum functions $K_\pm(t)$ by means of the well-known Havelock formula

$$\pi R/(\rho U^2 L^2) = \int_0^\infty [K_+(t) + K_-(t)]^2 p\, dt \;. \tag{26}$$

The important information contained in the functions $K_\pm(t)$ indeed represents a significant advantage of the indirect Fourier-type method over the direct integration method mentioned previously. Furthermore, the integral (20) defining the wave potential $\phi_W(\vec{\xi})$ is more amenable to numerical evaluation than the integral (11) defining the wave term $W(\vec{X})$ because the wave-spectrum function $K_+(t) + K_-(t)$ vanishes as $t \to \infty$.

Numerical evaluation of the wave integral (20) and of the functions $K_0^{\pm}(t)$ and $K_\phi^{\pm}(t)$ defined by Eqs. (22) and (23) are the two main numerical tasks which must be considered in the indirect Fourier-type approach. The second of these tasks is examined here. Numerical evaluation of the waterline, hull and free-surface integrals in Eqs. (22) and (23) is a seemingly relatively-simple task, given the value of the potential ϕ on the mean hull surface h and waterline w and the value of the free-surface nonlinear term $\pi(\phi)$; in particular, the integrands of the integrals in Eqs. (22) and (23) are continuous functions.

Nevertheless, accurate and efficient numerical evaluation of the functions $K_0^{\pm}(t)$ and $K_\phi^{\pm}(t)$ requires careful analysis because the trigonometric functions $E_\pm(x,y;t)$ defined by Eq. (19) oscillate very rapidly for large values of $P^2 = \nu^2 p^2$, as is the case for typical values of the Froude number $F = 1/\nu$ and of the Fourier variable $p^2 = 1+t^2 = \sec^2\theta$, and because the potential ϕ in the integrands of the waterline and hull integrals in Eq. (23) is multiplied by the large numbers $\nu^2 p$ and $\nu^2 P^2 = (\nu^2 p)^2$, respectively. The terms involving the potential ϕ in Eq. (23) therefore are dominant and quite large for typical values of $\nu^2 p$. More precisely, let Eq. (23) be expresed in the form

$$K_\phi = K_W + i\sigma K_W' + \sigma^2 K_H' = K_W + i\sigma(K_W' - i\sigma K_H') \ , \tag{27}$$

where the free-surface integral and the superscript ± have been ignored for simplicity and the functions K_W , K_W' and K_H' are defined as

$$K_W = \int_w E_\pm \ (t_x\phi_t + s_x\phi_s)t_y \ d\ell \ , \tag{28a}$$

$$K_W' = \int_w E_\pm \ \phi t_y \ d\ell \ , \tag{28b}$$

$$K_H' = \int_h \exp(P^2 z)E_\pm \ \phi n_\pm \ da \ , \tag{28c}$$

with σ defined by

$$\sigma = \nu^2 p = (1+t^2)^{1/2} / \ F^2 = \sec\theta \ / \ F^2 \ . \tag{29}$$

The real and imaginary parts of the sum of the port and starboard contributions to the functions K_ϕ , K_W , $i\sigma K_W'$ and $\sigma^2 K_H'$ are depicted in Fig. 2 for $0 \leq t = \tan\theta \leq 10$ (corresponding to $0 \leq \theta \leq 84°$) for a very simple case corresponding to a simple mathematically-defined hull form with an assumed simple mathematical expression for the velocity potential ϕ at the hull. More precisely, the mathematical hull considered in Fig. 2 has constant draft and rectangular framelines, with draft/length and beam/length ratios equal to 0.07 and 0.16, respectively. The hull consists of a pointed bow region $0.2 \leq x \leq 0.5$ with parabolic waterlines, a straight middle-body region $-0.3 \geq x \leq 0.2$ and a rounded stern region $-0.5 \leq x \leq -0.3$ with elliptic waterlines. The potential in Eqs. (28a,b,c) is taken as $\phi = F^2 \exp(\nu^2 z) \cos[\nu^2(x-1/2)-3\pi/8]$, which corresponds to an elementary plane wave. The foregoing simple hull form and assumed simple expression for the potential at the hull are used for the calculations presented in Fig. 2, and for all the calculations presented further on, because this simple case is adequate for the present purpose of numerically illustrating the essential properties of the alternative mathematical expressions for the wave-spectrum function examined in this study and it permits accurate calculations (the required integrations can be partially performed analytically). The results presented in Fig. 2 correspond to a value of the Froude number equal to 0.15.

Figure 2 shows that the function K_ϕ is considerably smaller than its components K_W , $i\sigma K_W'$ and $\sigma^2 K_H'$. In particular, the waterline and hull integrals $i\sigma K_W'$ and $\sigma^2 K_H'$ do not appear to vanish in the limit $\theta \to 90°$ and the waterline integral K_W vanishes appreciably more slowly than the function K_ϕ. Significant cancellations therefore occur between the functions $i\sigma K_W'$ and $\sigma^2 K_H'$ and between the sum of these two functions and the function K_W. These significant cancellations occur for all values of θ but are especially notable for large values of θ, which correspond to the short divergent waves in the spectrum. The errors which inevitably occur in the numerical evaluation of the components K_W , $i\sigma K_W'$ and $\sigma^2 K_H'$ cause imperfect numerical cancellations between these components and corresponding large errors in their sum. Numerical errors in the sum K_ϕ can be especially difficult to control because the errors associated with the numerical evaluation of the hull integral $\sigma^2 K_H'$ and the waterline integral $K_W + i\sigma K_W'$ are not necessarily comparable (due to differences in the errors associated with numerical integration over hull panels and waterline segments). The usual expression (23) for the Neumann-Kelvin correction K_ϕ in Eq. (21) thus is quite ill suited for accurate numerical evaluation. A modified mathematical expression in which the cancellations between the waterline and hull integrals depicted in Fig. 2 are automatically and exactly accounted for is presented further on in this study.

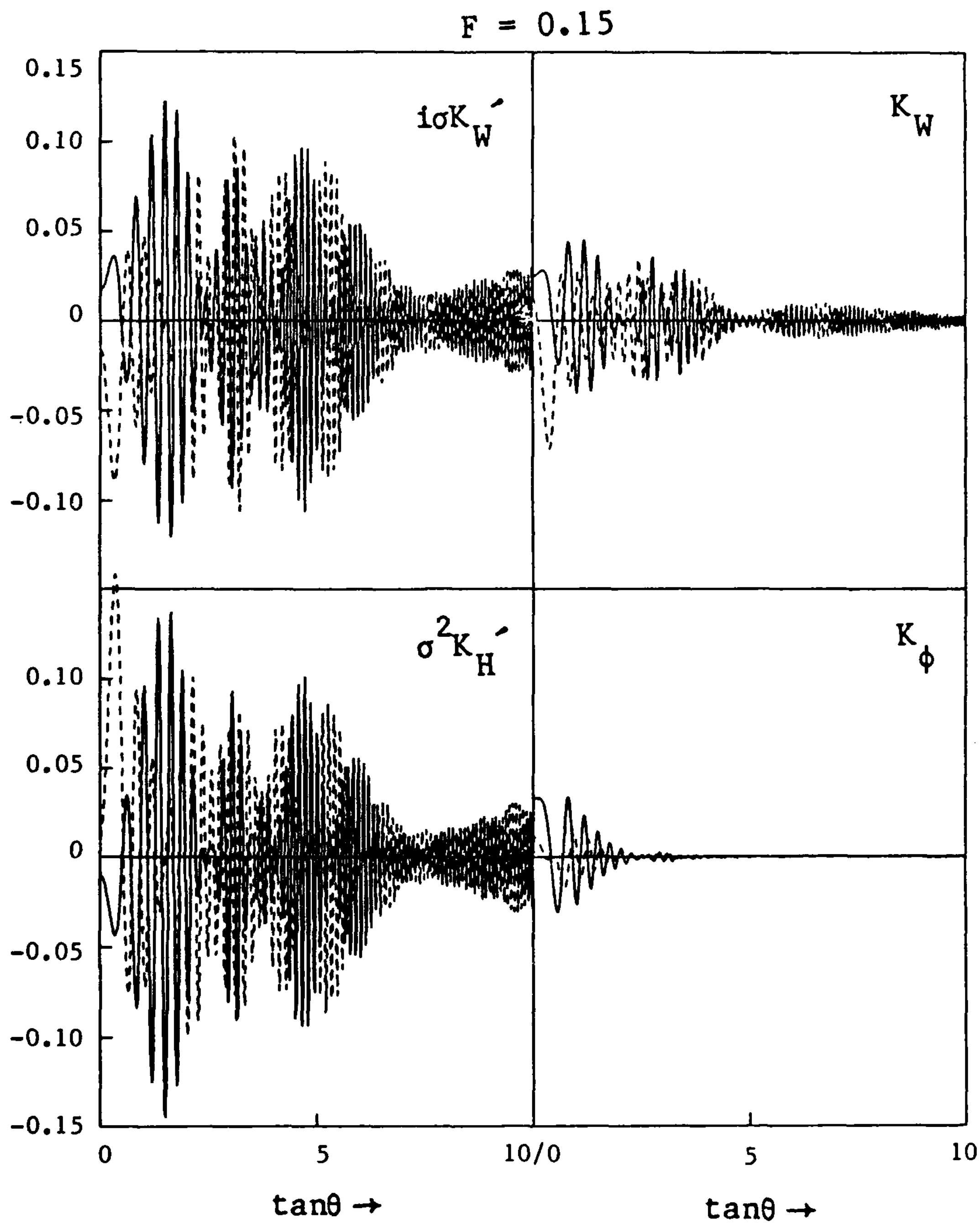

Fig. 2 The functions $i\sigma K_W'$, $\sigma^2 K_H'$, K_W and K_ϕ for $F = 0.15$.

MODIFIED EXPRESSIONS FOR THE FAR-FIELD WAVE-SPECTRUM FUNCTION

Modified expressions for the functions $K_0^{\pm}(t)$ and $K_\phi^{\pm}(t)$ can be obtained by using Stokes' theorem

$$\int_C \vec{V}\cdot\vec{t}\, d\ell = \int_S (\nabla\times\vec{V}) \cdot \vec{n}\, da \,, \tag{30}$$

where $\vec{V}$ represents a vector field, and $\vec{t}$ and $\vec{n}$ are unit vectors tangent to a closed curve C and normal to an open surface S bounded by C, respectively. In particular, we will use two special forms of Stokes' theorem corresponding to the vector fields $\vec{V} = f\, \vec{e}_y$ and $\vec{V} = f\, \vec{e}_z$, where $\vec{e}_y$ and $\vec{e}_z$ represent unit vectors along the y- and x-axes and f stands for a scalar function, namely,

$$\int_C t_y\, f\, d\ell = \int_S (n_z\, \partial f/\partial x - n_x\, \partial f/\partial z)\, da \,, \tag{31a}$$

$$\int_C t_z\, f\, d\ell = \int_S (n_x\, \partial f/\partial y - n_y\, \partial f/\partial x)\, da \,. \tag{31b}$$

The functions $K_0^{\pm}(t)$, which correspond to the so-called zeroth-order slender-ship approximation to the Neumann-Kelvin theory [9], are considered first, and a numerically-convenient modified form of Eq. (22) is obtained.

The zeroth-order slender-ship approximation

It is convenient to divide the mean hull surface h into two parts, namely the hull side, s say, and the hull bottom, b, as is depicted in Fig. 1. Equation (22) then becomes

$$K_0^{\pm}(t) = \int_w E_{\pm}\, n_x^{\,2} t_y \, d\ell + \nu^2 \int_s \exp(P^2 z) E_{\pm}\, n_x \, da + \nu^2 \int_b \exp(P^2 z) E_{\pm}\, n_x \, da \,. \tag{32}$$

The hull bottom of a typical ship is a nearly horizontal surface, so that we have $n_x \simeq 0$ on b, but n_x is usually significant on the hull side in the bow and stern regions. However, the hull side of a typical ship is a nearly vertical surface, i.e. we have $n_z \simeq 0$ on s. It is therefore convenient to express the integral on the hull side in Eq. (32) as an integral involving the source density n_z by using Stokes' theorem in the form of Eq. (31a).

More precisely, Eq. (31a), in which the open surface S and the function f are taken as the hull side s and the functions $F^2 u^2 \exp(P^2 z) E_{\pm}$, yields

$$\int_s \exp(P^2 z) E_{\pm}\, n_x \, da = -iu \int_s \exp(P^2 z) E_{\pm}\, n_z \, da - F^2 u^2 \int_w E_{\pm}\, t_y \, d\ell + F^2 u^2 \int_{w'} \exp(P^2 z) E_{\pm}\, t_y \, d\ell \,, \tag{33}$$

where Eqs.(1), (14a) and (15a) were used, and w' is the waterline-like curve separating the hull side and the hull bottom, as is shown in Fig. 1. The unit tangent vector $\vec{t} = (t_x, t_y, t_z)$ to w' is pointing towards the bow. In Eq. (33), the identity $t_y \equiv 0$ along the stem and stern lines, which lie in the ship centerplane $y = 0$, was used.

By substituting Eq. (33) into Eq. (32) we may obtain

$$K_0^{\pm}(t) = \int_w E_{\pm}\, (n_x^{\,2} - u^2) t_y \, d\ell + u^2 \int_{w'} \exp(P^2 z) E_{\pm}\, t_y \, d\ell - i\nu^2 u \int_s \exp(P^2 z) E_{\pm}\, n_z \, da + \nu^2 \int_b \exp(P^2 z) E_{\pm}\, n_x \, da \,. \tag{34}$$

Comparison of Eqs. (32) and (34) shows that, on the hull side, the source density n_x in Eq. (32) has been replaced by the density $-iun_z$ in Eq. (34). The latter density is null for a wall-sided ship and, more generally, vanishes in the limit $t \to \infty$, as may be seen from Eqs. (15a) and (14b). The hull-side integral therefore is generally less important in the modified expression (34) than in the original expression (32). In particular, the hull-side and hull-bottom integrals in Eq. (34) are null for a wall-sided ship with a flat horizontal bottom, for which Eq. (34) expresses the functions $K_0^{\pm}(t)$ as the sum of two line integrals.

The trigonometric functions $E_{\pm}$ defined by Eq. (19) are rapidly oscillatory, as was already noted. The dominant contribution to the waterline integral in Eq. (34) therefore stems from the point(s), if any, where the phases $P^2(ux \pm vy)$ of the functions $E_{\pm}$ are stationary. These points of stationary phase are defined by the conditions $udx \pm vdy = 0$, which yield the relations

$$ut_x \pm vt_y = 0 \;, \qquad t_x = v \;, \qquad t_y = \mp u \;; \tag{35a,b,c}$$

the latter two relations can be obtained from Eq. (35a) by using Eq. (17) and the identity $t_x^2 + t_y^2 = 1$.

The term u^2 in the integrand of the modified waterline integral in Eq. (34) stems from the hull-side integral in Eq. (32). We have $n_x = -t_y$ along the top waterline for a wall-sided ship. It may then be seen from Eq. (35c) that the term $n_x^2 - u^2$ in the integrand of the waterline integral in Eq. (34) vanishes at a point of stationary phase for a wall-sided ship. This result indicates that the waterline integral and the hull-side integral in the original expression (32) cancel one another in a first approximation (more precisely, within the stationary-phase approximation) for a wall-sided ship. The major contributions stemming from these two integrals thus are combined into the modified waterline integral in the alternative expression (34); and the modified hull-side integral in Eq. (34) is less important than the original hull-side integral in Eq. (32), as was already noted.

For the simple mathematical hull form defined previously we have $n_x = 0$ on the hull bottom b and $n_z = 0$ on the hull side s and Eqs. (32) and (34) then yield

$$K_0 = K_w + K_s , \tag{36a}$$

$$K_0 = K_w{}^* + K_{w'} , \tag{36b}$$

where K_w and K_s represent the waterline and hull-side integrals, respectively, in the usual expression (32) and $K_w{}^*$ and $K_{w'}$ correspond to the integrals along the top and bottom waterlines w and w′, respectively, in the modified expression (34). The real and imaginary parts of the sum of the port and starboard contributions to the functions K_w , K_s , $K_w{}^*$ and $K_{w'}$ are depicted in Figs. 3a-e for $0 \leq \tan\theta \leq 5$ (i.e. for $0 \leq \theta \leq 79°$) and for the simple hull form considered previously and five values of the Froude number, namely F = 0.1, 0.15, 0.2, 0.25 and 0.3. For F = 0.1 and 0.15, Figs. 3a,b show that the bottom-waterline integral $K_{w'}$ is quite small for all values of θ and the function K_0 is well approximated by the modified top-waterline integral $K_w{}^*$, that is we have $K_{w'} << K_w{}^* \simeq K_0$. These figures also show that the modified waterline integral $K_w{}^*$ is appreciably smaller than the waterline and hull integrals K_w and K_s in the usual expression (36a), in accordance with the previous theoretical considerations. For larger values of the Froude number, Figs. 3c-e show that the bottom-waterline integral $K_{w'}$ is significant for smaller values of θ but vanishes rapidly (exponentially) for increasing values of θ, so that we have $K_0 \simeq K_w{}^*$ for sufficiently large values of θ.

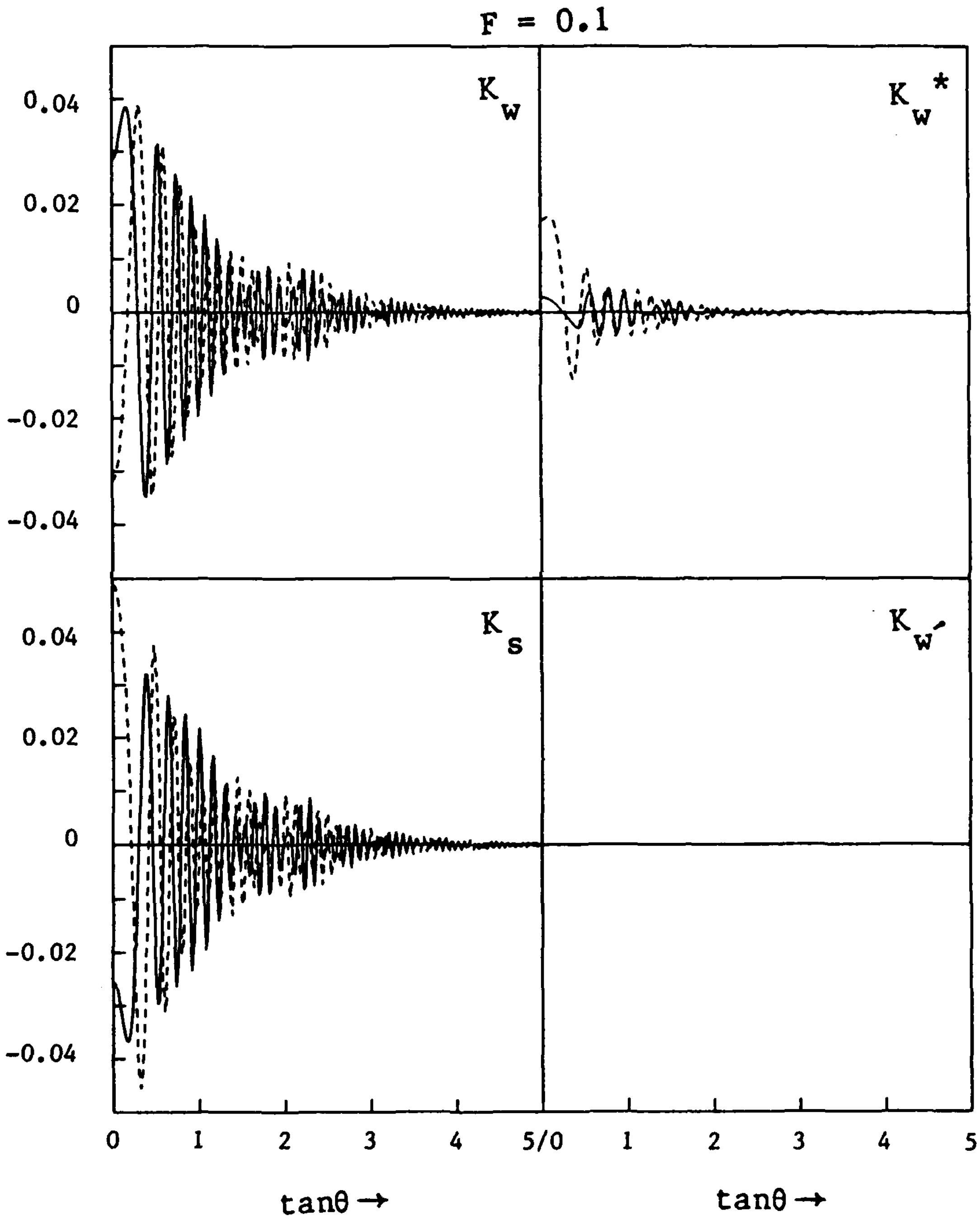

Fig. 3a The functions K_w , K_s , K_w^* and $K_{w'}$ for F = 0.1 .

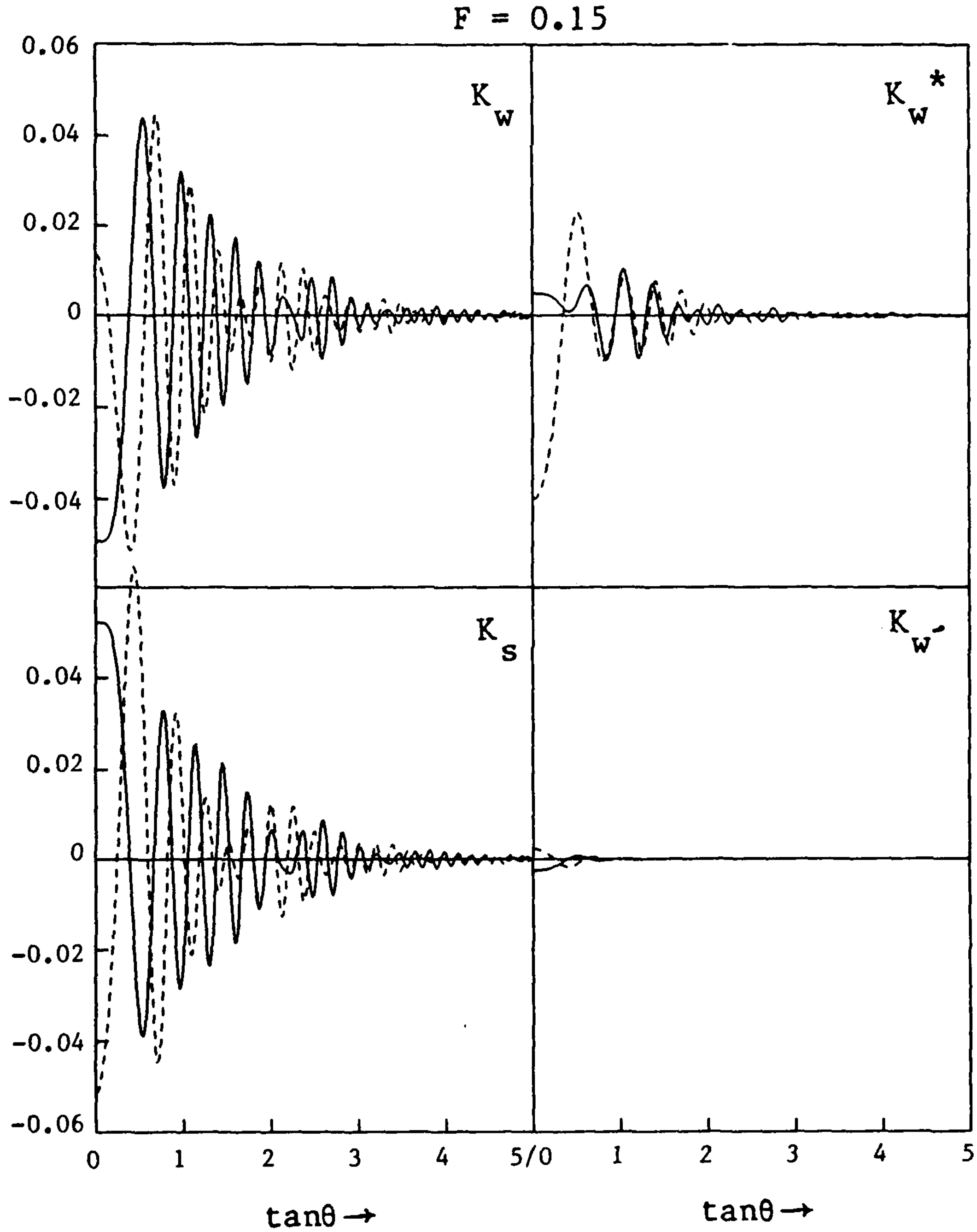

Fig. 3b The functions K_w , K_s , K_w^* and $K_{w'}$ for F = 0.15 .

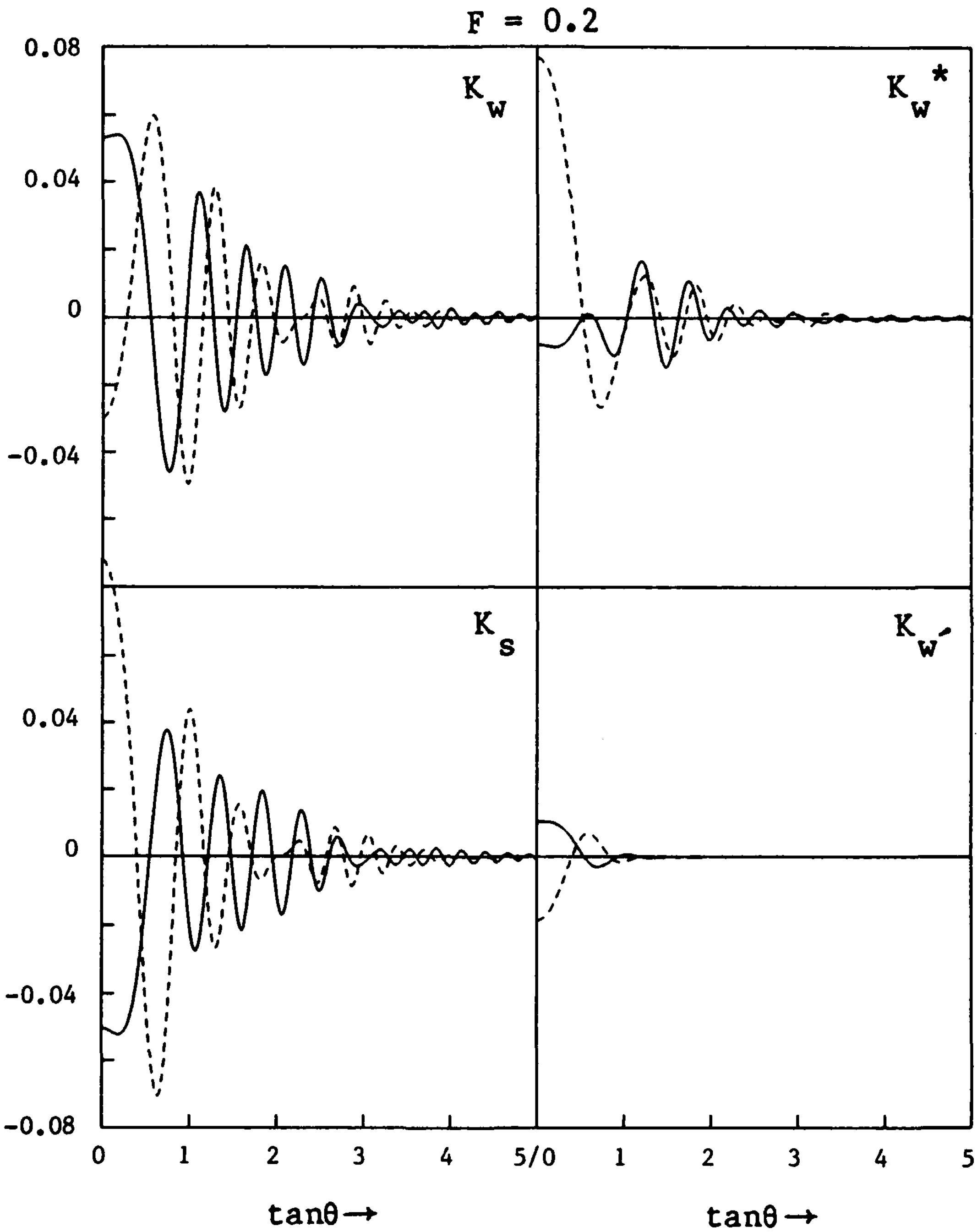

Fig. 3c The functions K_w , K_s , $K_w{}^*$ and $K_{w'}$ for F = 0.2 .

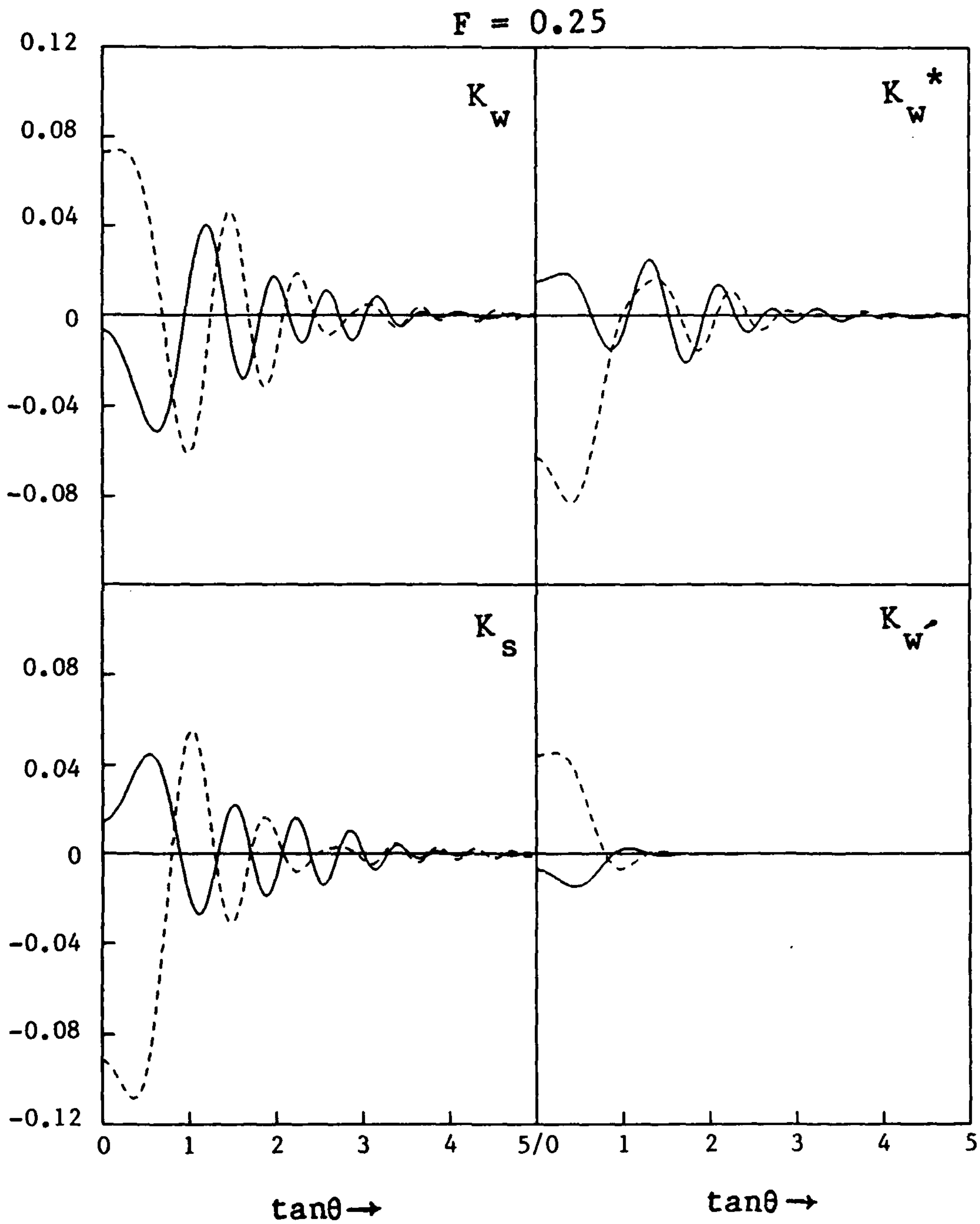

Fig. 3d The functions K_w , K_s , K_w^* and $K_{w'}$ for $F = 0.25$.

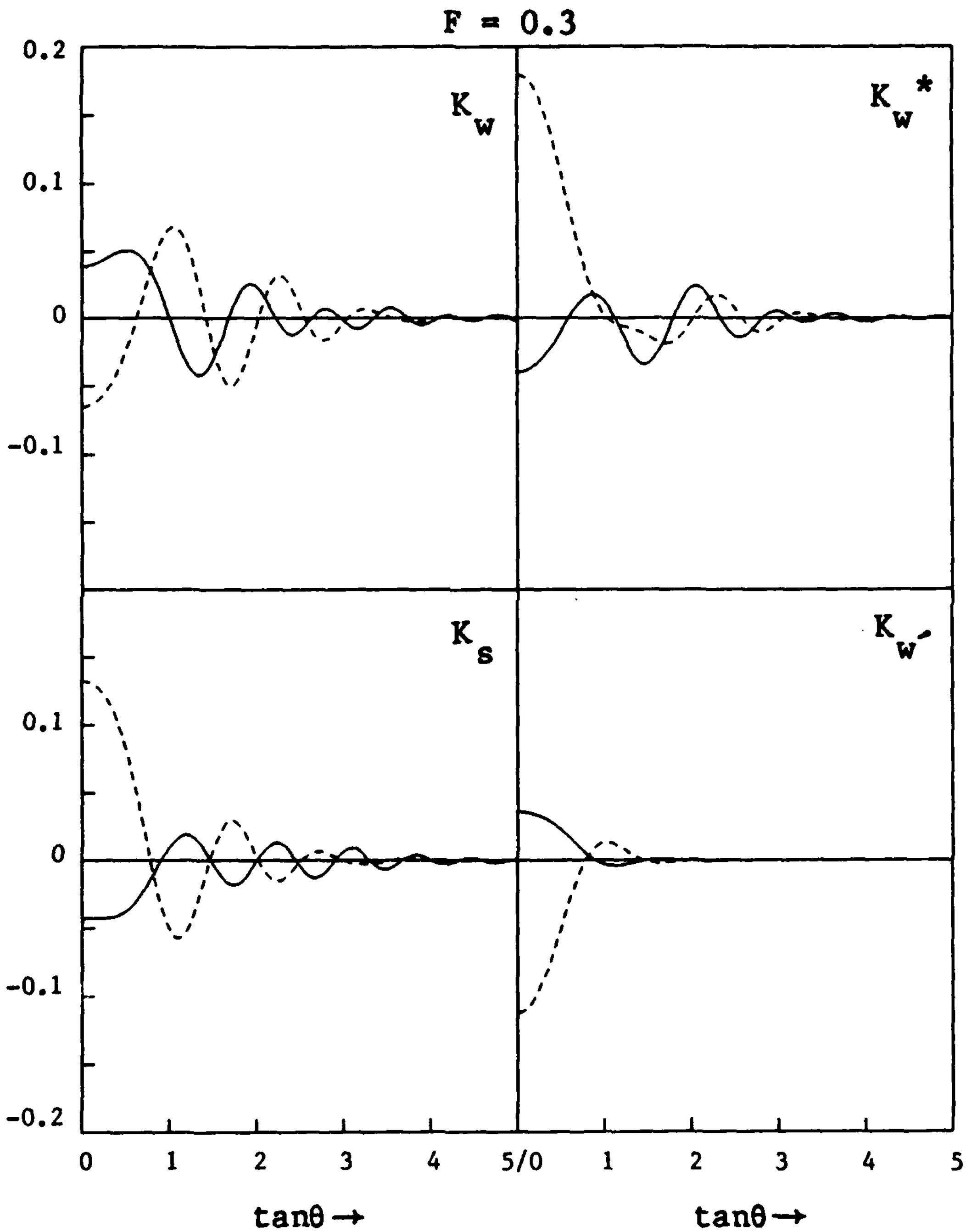

Fig. 3e The functions K_w , K_s , K_w^* and $K_{w'}$ for F = 0.3 .

The Neumann-Kelvin approximation: first transformation

The linearized Neumann-Kelvin approximation to the far-field wave-amplitude funtions $K_{\pm}(t)$ is given by the sum of the zeroth-order slender-ship approximation $K_0^{\pm}(t)$ defined by Eqs.(22) or (34) and the correction term $K_{\phi}^{\pm}(t)$ defined by Eq. (23), where the free-surface integral is neglected as is indicated in Eqs. (27), (28a-c) and (29). A modified form of Eq. (23) is now obtained. Stokes' theorem (31a), in which the surface S and the function f are taken as the hull surface h and the functions $\exp(P^2 z)E_{\pm}\phi$, yields

$$\int_w E_{\pm}\, \phi t_y\, d\ell = \int_h \left[n_z\, \partial \exp(P^2 z)E_{\pm}\phi\, /\partial x - n_x\, \partial \exp(P^2 z)E_{\pm}\phi\, /\partial z\right] da \ , \tag{37}$$

where we used the identity $t_y \equiv 0$ along the stem line, the keel line and the stern line which, together with the top waterline w, border the hull surface h. By substituting Eq. (37) into Eq. (23), and upon using Eqs. (24), (19) and (17), we may obtain

$$\begin{aligned} K_{\phi}^{\pm}(t) = & \int_w E_{\pm}\, (t_x\phi_t+s_x\phi_s)t_y\, d\ell \\ & + i\nu^2 p \int_h \exp(P^2 z)E_{\pm}\, \left[n_z\partial\phi/\partial x-n_x\partial\phi/\partial z\pm\nu^2 pt(un_y\mp vn_x)\phi\right] da \ . \end{aligned} \tag{38}$$

Equation (38) can be modified further by using Stokes' theorem (31b), where S and f are taken as h and $\exp(P^2 z)E_{\pm}\phi$, respectively. The contribution of the top waterline w is null because we have $t_z \equiv 0$ along w. The contribution of the stem line, the keel line and the stern line, which lie in the ship centerplane y = 0, to the sum $K_{\phi}^{+}(t) + K_{\phi}^{-}(t)$ can also be shown to be null. We then have

$$P^2 \int_h \exp(P^2 z)E_{\pm}\, (un_y\mp vn_x)\phi\, da = i \int_h \exp(P^2 z)E_{\pm}\, (n_x\partial\phi/\partial y-n_y\partial\phi/\partial x)\, da \ . \tag{39}$$

By substituting Eq. (39) into Eq. (38), and upon using Eqs. (14a) and (15b), we may obtain

$$K_{\phi}^{\pm}(t) = \int_w E_{\pm}\, (t_x\phi_t+s_x\phi_s)t_y\, d\ell + i\nu^2 p \int_h \exp(P^2 z)E_{\pm}\, a_{\pm}\, da \ , \tag{40}$$

where the amplitude-functions $a_{\pm}$ are defined as

$$a_{\pm} = n_z\partial\phi/\partial x - n_x\partial\phi/\partial z \pm i\nu(n_x\partial\phi/\partial y - n_y\partial\phi/\partial x) \ . \tag{41}$$

Equation (40) may be expressed in the form

$$K_\phi = K_W + i\sigma K_H \ , \tag{42}$$

where the superscript ± was ignored for simplicity, K_W represents the waterline integral defined by Eq. (28a), σ is given by Eq. (29) and K_H is the hull integral in Eq. (40), that is we have

$$K_H = \int_h \exp(P^2 z)E_\pm \ a_\pm \ da \ . \tag{43}$$

Comparison of Eqs. (27) and (42) shows that we have

$$K_H = K_W' - i\sigma K_H' \ . \tag{44}$$

Thus, the waterline integral K_W' and the hull integral $-i\sigma K_H'$ have been combined into the modified hull integral K_H via two applications of Stokes' theorem in the form given by Eqs. (31a,b). The real and imaginary parts of the sum of the port and starboard contributions to the functions $i\sigma K_W'$, $\sigma^2 K_H'$ and $i\sigma K_H$ are depicted in Figs. 4a-e for $0 \leq \tan\theta \leq 10$ (i.e. for $0 \leq \theta \leq 84°$) and for the simple hull form and the assumed simple expression for the potential at the hull considered previously in Fig. 2. Figures 4a-e correspond to the following five values of the Froude number F: 0.1, 0.15, 0.2, 0.25 and 0.3. These figures show that the waterline and hull integrals $i\sigma K_W'$ and $\sigma^2 K_H'$ are considerably larger than the modified hull integral $i\sigma K_H$. Although the latter integral is identical to the sum of the integrals $i\sigma K_W'$ and $\sigma^2 K_H'$, it clearly is preferable to evaluate $i\sigma K_H$ directly rather than the sum of the integrals $i\sigma K_W'$ and $\sigma^2 K_H'$. The modified expression for the Neumann-Kelvin correction term K_ϕ given by Eqs. (40) and (41) or (42) therefore represents a significant improvement in comparison with the usual expression given by Eqs. (23) or (27).

The cancellations between the waterline integral $i\sigma K_W'$ and the hull integral $\sigma^2 K_H'$ depicted in Figs. 4a-e can easily be explained mathematically for a wall-sided ship form. For large values of P^2, the major contribution to the hull integral K_H' stems from the upper part of the hull surface in the vicinity of the waterline, where we have $n_x = -t_y$, $n_y = t_x$ and $n_z = 0$ for a wall-sided ship. More precisely, Eqs. (28c) and (24) yield

$$K_H' \sim -i \int_W E_\pm \, \phi(ut_y \mp vt_x) \, d\ell \int_{-\infty}^{0} \exp(P^2 z) \, dz \quad \text{as } P^2 \to \infty \, .$$

We then have

$$i\sigma K_H' \sim u \int_W E_\pm \, \phi(ut_y \mp vt_x) \, d\ell \quad \text{as } P^2 \to \infty \, , \tag{45}$$

where Eqs. (29), (14a) and (15a) were used. By using Eqs. (28b) and (17) we may then obtain

$$K_W' - i\sigma K_H' \sim v \int_W E_\pm \, \phi(vt_y \pm ut_x) \, d\ell \quad \text{as } P^2 \to \infty \, . \tag{46}$$

For large values of P^2, the trigonometric functions $E_\pm$ defined by Eq. (19) are rapidly oscillatory and the dominant contributions to the waterline integrals K_W', $i\sigma K_H'$ and $K_W' - i\sigma K_H'$ defined by Eqs. (28b), (45) and (46), respectively, therefore stem from the point(s), if any, of stationary phase of the trigonometric functions $E_\pm$ defined by Eqs. (35a,b,c). At such a point of stationary phase the terms t_y, $u(ut_y \mp vt_x)$ and $v(vt_y \pm ut_x)$ in the integrands of the waterline integrals (28b), (45) and (46) take the values $\mp u$, $\mp u$ and 0, respectively, which demonstrates that the waterline integral K_W' and the hull integral $-i\sigma K_H'$ cancel out in a first approximation in the limit $P^2 \to \infty$ for a wall-sided ship.

The real and imaginary parts of the sum of the port and starboard contributions to the modified waterline and hull integrals K_W and $i\sigma K_H$, respectively, and their sum K_ϕ are depicted in Figs. 5a-e for the cases considered previously in Figs. 4a-e and Fig. 2. It may be seen from Figs. 5a-e that the waterline and hull integrals K_W and $i\sigma K_H$ are appreciably larger than their sum K_ϕ, especially for large values of $\tan\theta$. Significant cancellations therefore occur between the waterline and hull integrals in Eq. (40). Further modifications of the expression for the function K_ϕ defined by Eq. (40) are then desirable for numerical calculations. These modifications are now presented.

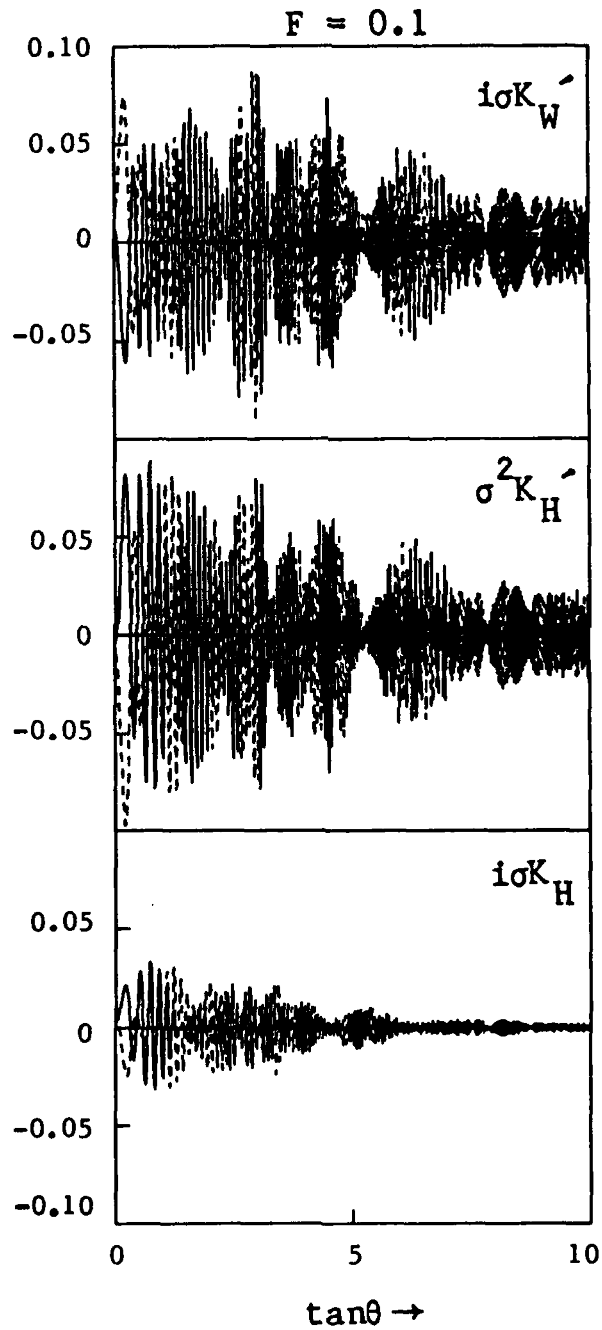

Fig. 4a The functions $i\sigma K_W'$, $\sigma^2 K_H'$ and $i\sigma K_H$ for $F = 0.1$.

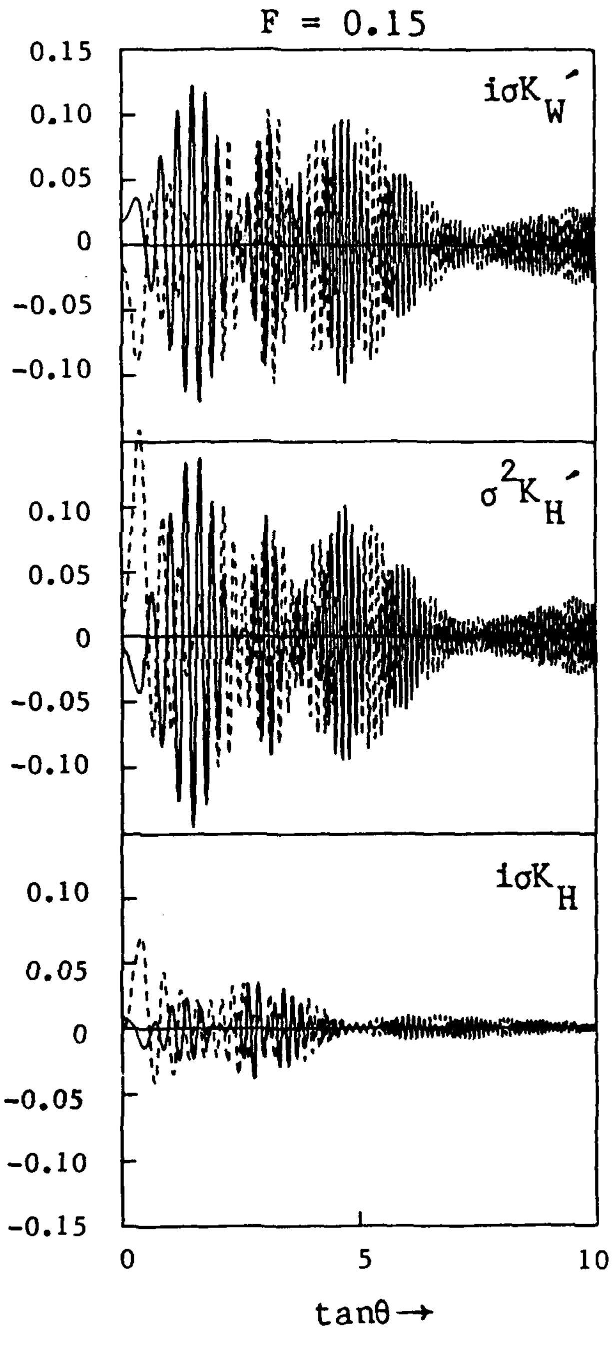

Fig. 4b The functions $i\sigma K_W'$, $\sigma^2 K_H'$ and $i\sigma K_H$ for $F = 0.15$.

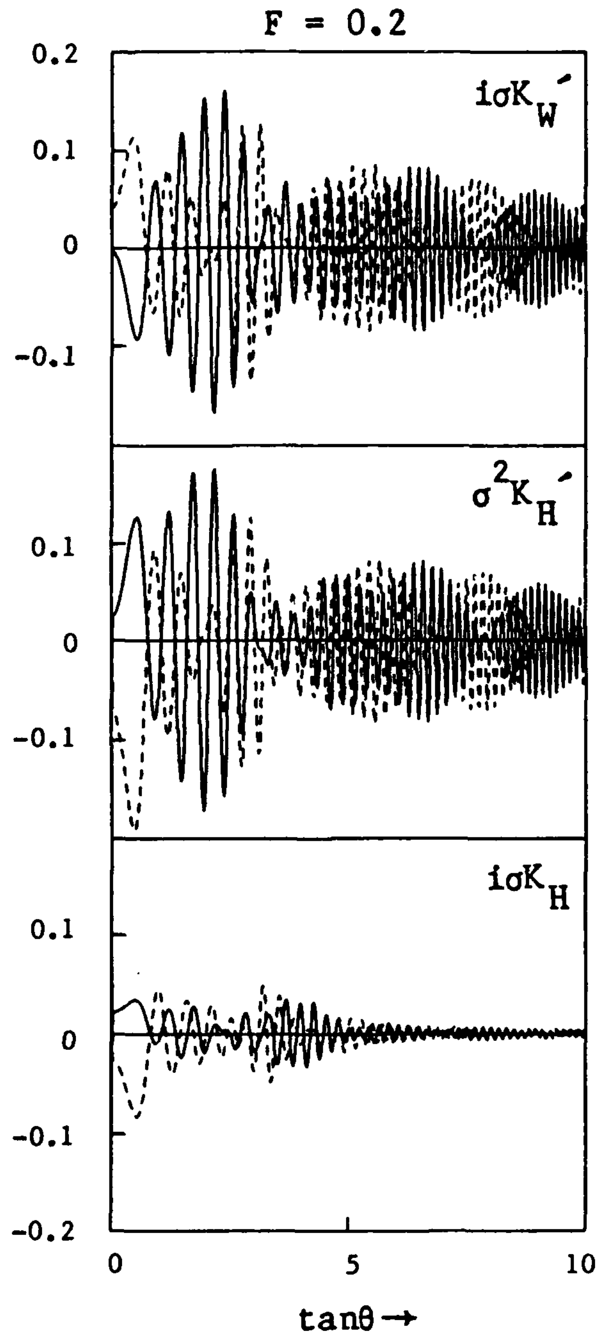

Fig. 4c The functions $i\sigma K_W'$, $\sigma^2 K_H'$ and $i\sigma K_H$ for $F = 0.2$.

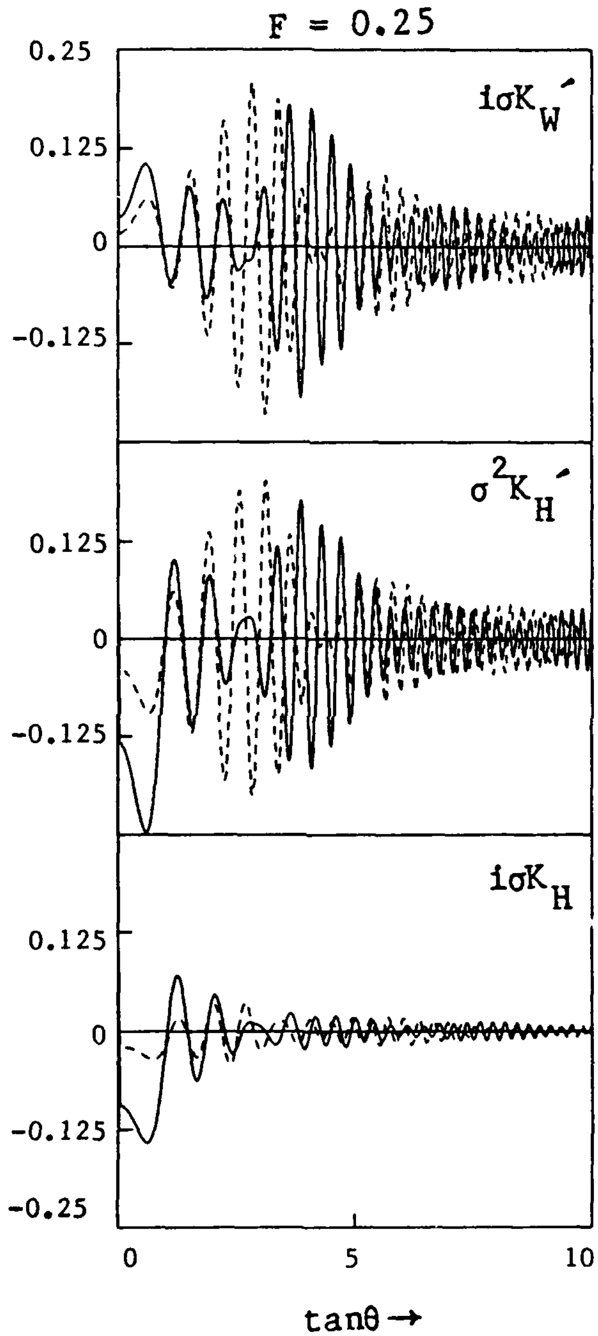

Fig. 4d The functions $i\sigma K_W'$, $\sigma^2 K_H'$ and $i\sigma K_H$ for $F = 0.25$.

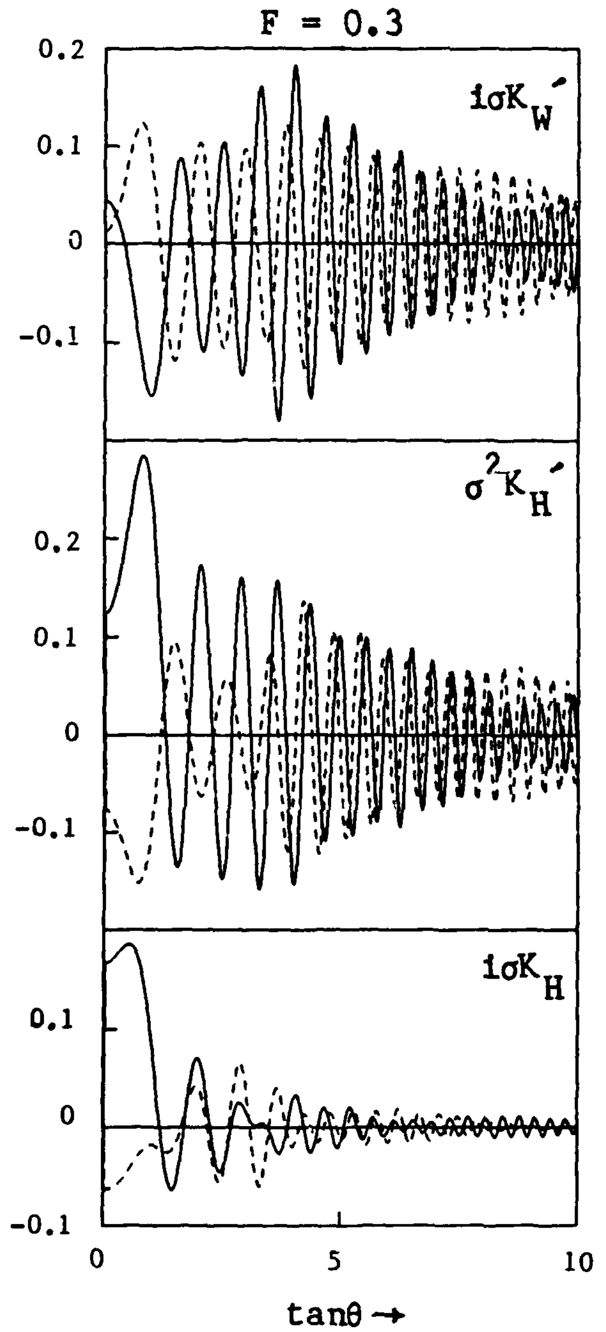

Fig. 4e The functions $i\sigma K_W'$, $\sigma^2 K_H'$ and $i\sigma K_H$ for $F = 0.3$.

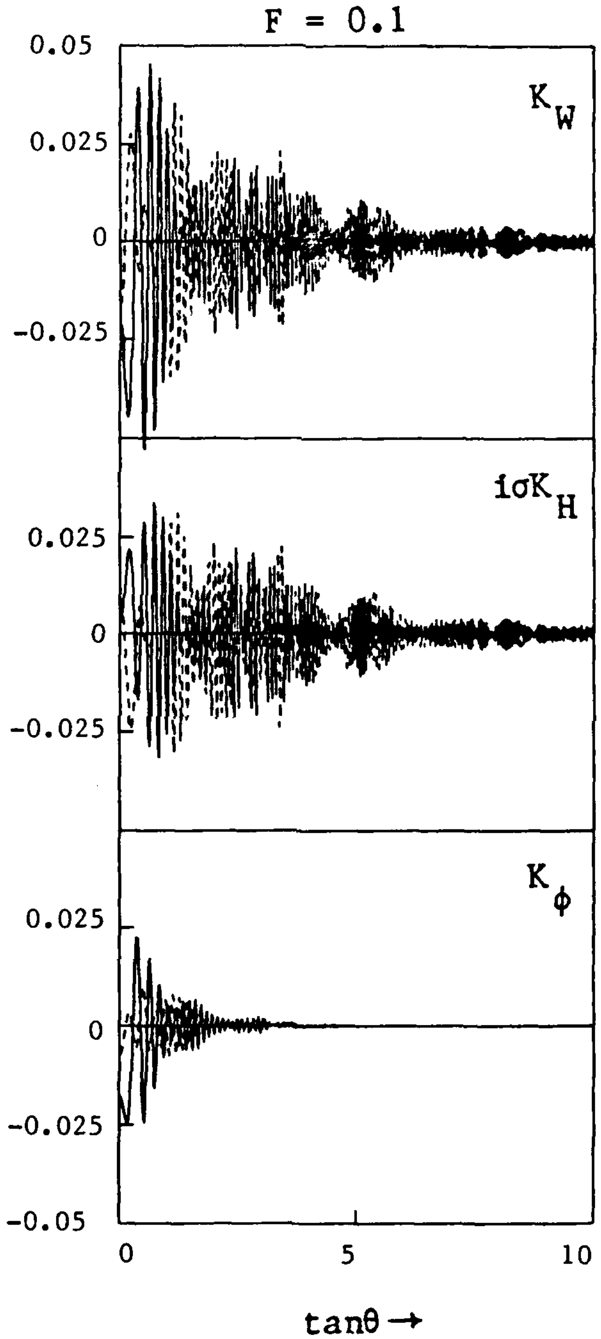

Fig. 5a The functions K_W , $i\sigma K_H$ and K_ϕ for $F = 0.1$.

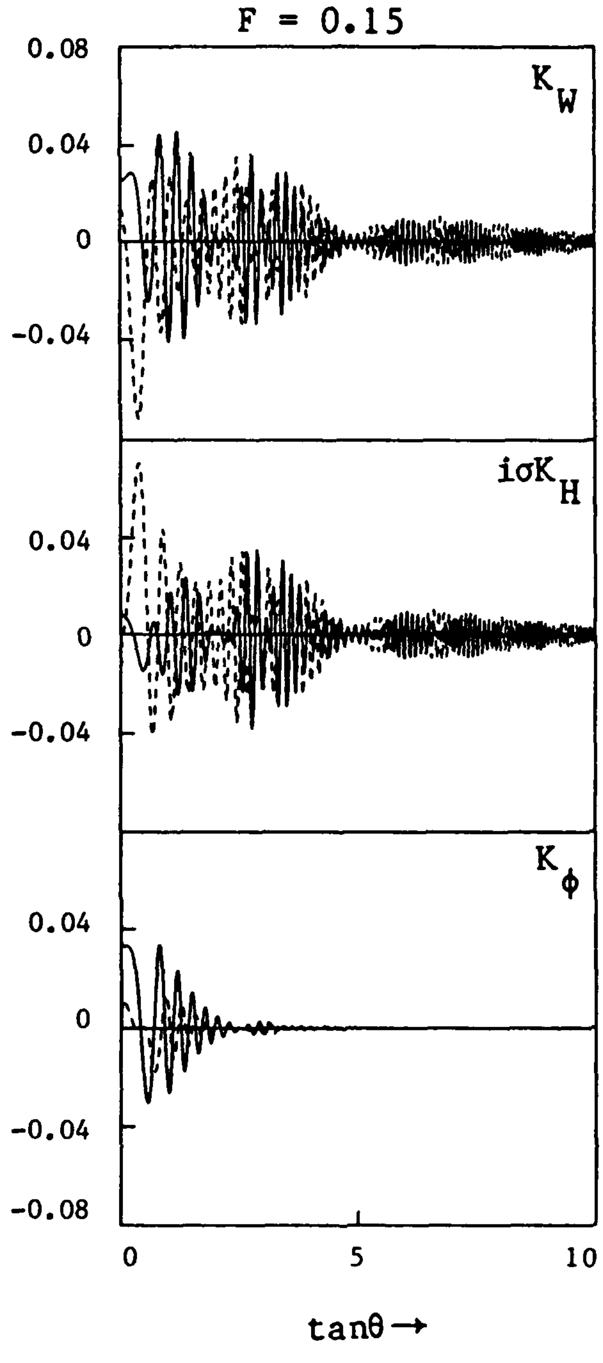

Fig. 5b The functions K_W , $i\sigma K_H$ and K_ϕ for $F = 0.15$.

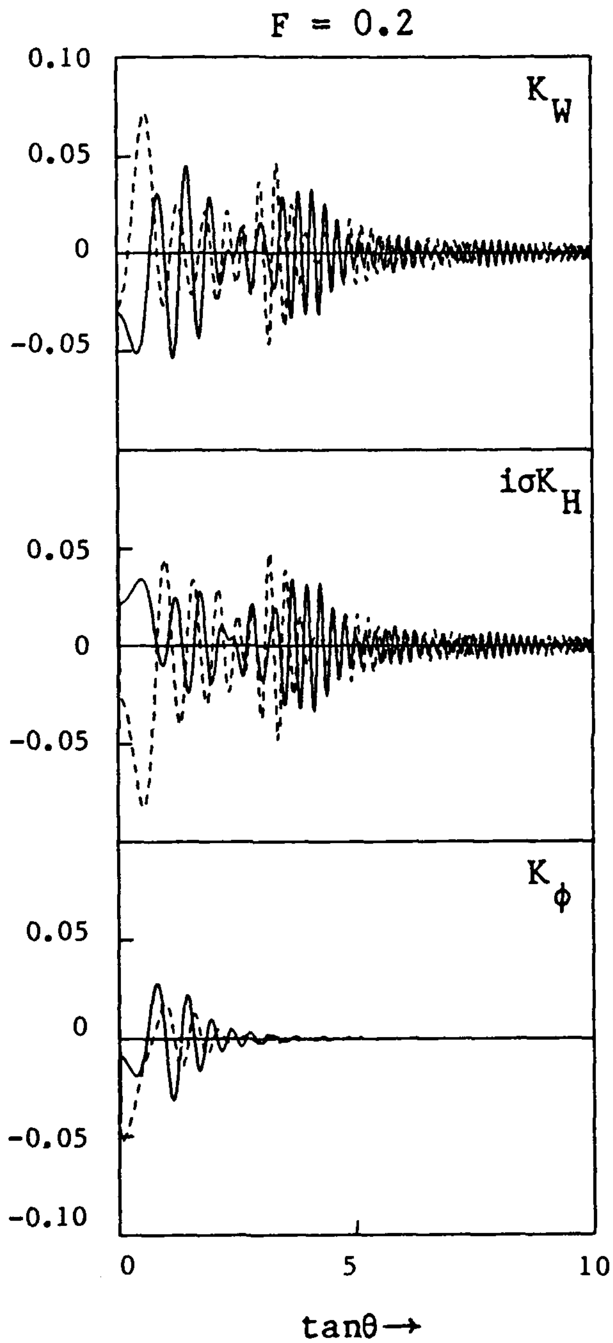

Fig. 5c The functions K_W , $i\sigma K_H$ and K_ϕ for $F = 0.2$.

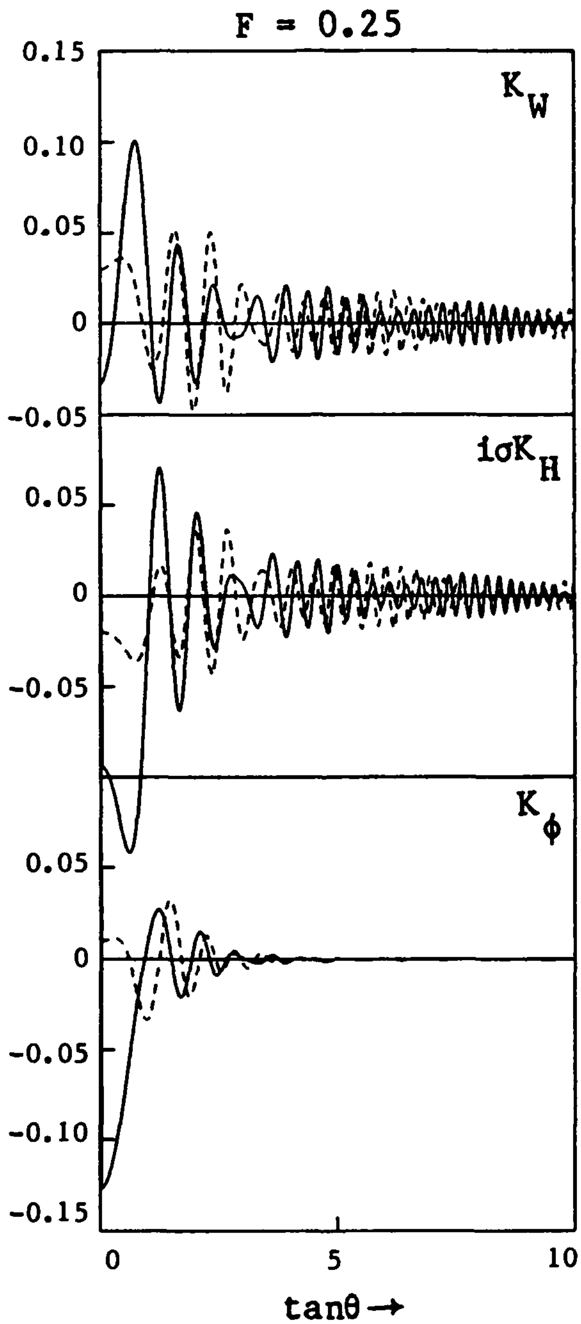

Fig. 5d The functions K_W , $i\sigma K_H$ and K_ϕ for F = 0.25 .

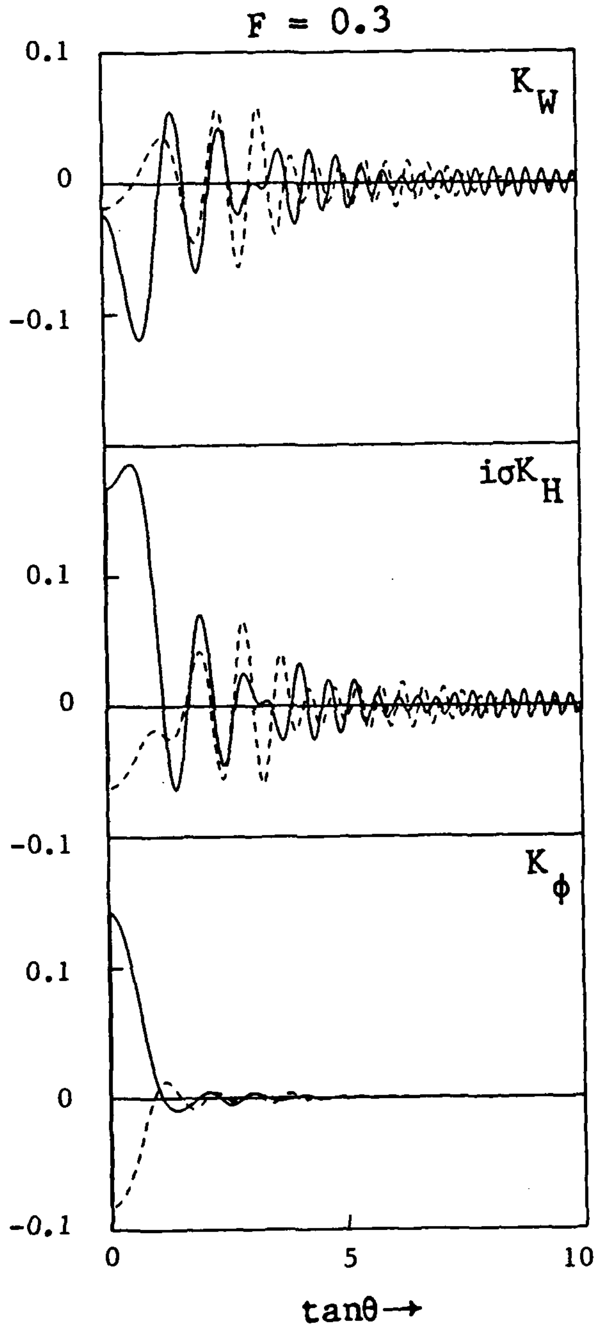

Fig. 5e The functions K_W , $i\sigma K_H$ and K_ϕ for F = 0.3 .

The Neumann-Kelvin approximation: second transformation

Let α, β, γ represent three constants and $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$ three unit vectors along the x-, y-, z-axes, respectively. We have

$$\vec{n} \cdot \nabla\times(\alpha\vec{e}_x+\beta\vec{e}_y+\gamma\vec{e}_z)\phi = \alpha(n_y\partial\phi/\partial z-n_z\partial\phi/\partial y)$$
$$+ \beta(n_z\partial\phi/\partial x-n_x\partial\phi/\partial z) + \gamma(n_x\partial\phi/\partial y-n_y\partial\phi/\partial x) \; . \tag{47}$$

Equation (47) shows that the amplitude functions $a_\pm$ defined by Eq. (41) may be expresed in the form

$$a_\pm = \vec{n} \cdot \nabla\times(\vec{e}_y\pm iv\vec{e}_z)\phi \; . \tag{48}$$

Let the functions $\exp(P^2z)E_\pm$, where $E_\pm$ are the trigonometric functions defined by Eq. (19), be denoted as $\varepsilon_\pm$. We then have

$$\varepsilon_\pm = \exp[P^2\{z-i(ux\pm vy)\}] \quad \text{and} \tag{49}$$

$$\nabla\varepsilon_\pm = -P^2\varepsilon_\pm(iu\vec{e}_x\pm iv\vec{e}_y-\vec{e}_z) \; . \tag{50}$$

We have

$$\varepsilon_\pm\nabla\times(\alpha\vec{e}_x+\beta\vec{e}_y+\gamma\vec{e}_z)\phi = \nabla\times\varepsilon_\pm(\alpha\vec{e}_x+\beta\vec{e}_y+\gamma\vec{e}_z)\phi - \nabla\varepsilon_\pm\times(\alpha\vec{e}_x+\beta\vec{e}_y+\gamma\vec{e}_z)\phi \; . \tag{51}$$

Equation (50) yields

$$-\nabla\varepsilon_\pm\times(\alpha\vec{e}_x+\beta\vec{e}_y+\gamma\vec{e}_z)\phi = P^2\varepsilon_\pm\phi\vec{m}_\pm \quad \text{with} \tag{52}$$

$$\vec{m}_\pm = (\beta\pm iv\gamma)\vec{e}_x - (\alpha+iu\gamma)\vec{e}_y + i(u\beta\mp v\alpha)\vec{e}_z \; . \tag{53}$$

Equation (53) shows that we have

$$\vec{m}_\pm \equiv 0 \quad \text{if} \quad \alpha = -iu\gamma \quad \text{and} \quad \beta = \mp iv\gamma \; , \tag{54}$$

which yields

$$\alpha\vec{e}_x+\beta\vec{e}_y+\gamma\vec{e}_z = -i\gamma(u\vec{e}_x\pm v\vec{e}_y+i\vec{e}_z) \; . \tag{55}$$

This condition merely expresses that the vector $\alpha\vec{e}_x + \beta\vec{e}_y + \gamma\vec{e}_z$ and the vector $\nabla\varepsilon_\pm$ defined by Eq. (50) are colinear. Equation (51) then becomes

$$\varepsilon_\pm\nabla\times(u\vec{e}_x\pm v\vec{e}_y+i\vec{e}_z)\phi = \nabla\times\varepsilon_\pm(u\vec{e}_x\pm v\vec{e}_y+i\vec{e}_z)\phi \; . \tag{56}$$

Equations (48) and (56) yield

$$\varepsilon_\pm a_\pm = \varepsilon_\pm\vec{n} \cdot \nabla\times(\vec{e}_y\pm iv\vec{e}_z)\phi \; , \tag{57}$$

$$\varepsilon_\pm \vec{n} \cdot \nabla\times(u\vec{e}_x \pm v\vec{e}_y + i\vec{e}_z)\phi = \vec{n} \cdot \nabla\times\varepsilon_\pm (u\vec{e}_x \pm v\vec{e}_y + i\vec{e}_z)\phi \ . \tag{58}$$

Let the amplitude functions $a_\pm$ be expressed in the form

$$a_\pm = b_\pm + c_\pm \ , \tag{59}$$

with $b_\pm$ defined as

$$b_\pm = \vec{n} \cdot \nabla\times(\alpha\vec{e}_x + \beta\vec{e}_y + \gamma\vec{e}_z)\phi \ , \tag{60}$$

where α, β, γ are constants. Equations (59), (48), and (60) yield

$$c_\pm = -\vec{n} \cdot \nabla\times[\alpha\vec{e}_x + (\beta-1)\vec{e}_y + (\gamma \mp iv)\vec{e}_z]\phi \ .$$

Equation (58) now shows that we have

$$\varepsilon_\pm c_\pm = -\Gamma_\pm \vec{n} \cdot \nabla\times\varepsilon_\pm (u\vec{e}_x \pm v\vec{e}_y + i\vec{e}_z)\phi \tag{61}$$

if the constants α, β, γ are chosen as

$$\alpha = u\Gamma_\pm \ , \quad \beta = 1 \pm v\Gamma_\pm \ , \quad \gamma = i(\Gamma_\pm \pm v) \tag{62a,b,c}$$

where $\Gamma_\pm$ is some arbitrary constant. Equations (60), (62a,b,c) and (47) then yield

$$\begin{aligned} b_\pm = {} & u\Gamma_\pm(n_y\partial\phi/\partial z - n_z\partial\phi/\partial y) + (1 \pm v\Gamma_\pm)(n_z\partial\phi/\partial x - n_x\partial\phi/\partial z) \\ & + i(\Gamma_\pm \pm v)(n_x\partial\phi/\partial y - n_y\partial\phi/\partial x) \ . \end{aligned} \tag{63}$$

Equations (59) and (61) show that we have

$$\varepsilon_\pm a_\pm = \varepsilon_\pm b_\pm - \Gamma_\pm \vec{n} \cdot \nabla\times\varepsilon_\pm (u\vec{e}_x \pm v\vec{e}_y + i\vec{e}_z)\phi \ . \tag{64}$$

By using Stokes' theorem (30) we may now obtain

$$\int_h \varepsilon_\pm a_\pm \, da = \int_h \varepsilon_\pm b_\pm \, da - \Gamma_\pm \int_c \varepsilon_\pm (ut_x \pm vt_y + it_z)\phi \, d\ell \ , \tag{65}$$

where the curve c consists of the waterline w plus the bow-keel-stern line, which lies in the centerplane $y = 0$. The hull integral in Eq. (40) has thus been expressed in Eq. (65) as the sum of a hull integral and a waterline integral involving the constants $\Gamma_\pm$, which are arbitrary and can then be selected at will. The identities (64) and (65) do not involve the term $P^2\phi$ appearing in Eqs. (52) and (51).

Along the bow-keel-stern line we have $y = 0$ and $t_y = 0$ and Eq. (49) yields

$$\varepsilon_\pm (ut_x \pm vt_y + it_z) = (ut_x + it_z) \exp[P^2(z - iux)] \ .$$

Equations (40) and (65) then show that the contribution of the bow-keel-stern line to the sum $K_\phi^+ + K_\phi^-$ can be rendered null if the constants $\Gamma_\pm$ are taken as

$$\Gamma_\pm = \mp\Gamma \ . \tag{66}$$

Equation (65) then becomes

$$\int_h \varepsilon_\pm a_\pm \ da = \int_h \varepsilon_\pm b_\pm \ da \pm \Gamma \int_w E_\pm(ut_x \pm vt_y)\phi \ d\ell \tag{67}$$

since we have $t_z = 0$ and $\varepsilon_\pm = E_\pm$ along the waterline, where $z = 0$.

Equations (19), (14a), (15a) and (1) yield

$$E_\pm(ut_x \pm vt_y)d\ell = iF^2u^2 dE_\pm$$

We then have

$$\pm\int_w E_\pm(ut_x \pm vt_y)\phi \ d\ell = \pm iF^2u^2\left[(E_\pm\phi)_{bow} - (E_\pm\phi)_{stern}\right]$$
$$\mp iF^2u^2 \int_w E_\pm(\partial\phi/\partial t) \ d\ell \ . \tag{68}$$

The bow and stern contributions to the function $K_\phi^+ + K_\phi^-$ are null because $y = 0$ and $E_+ = E_-$ at the bow and the stern. Equations (67) and (68) then yield

$$\int_h \varepsilon_\pm a_\pm \ da = \int_h \varepsilon_\pm b_\pm \ da \mp iF^2u^2\Gamma \int_w E_\pm(\partial\phi/\partial t) \ d\ell \ . \tag{69}$$

By substituting Eq. (69) into Eq. (40) we may obtain the following alternative expression for the Neumann-Kelvin correction $K_\phi^\pm$ to the spectrum function:

$$K_\phi^\pm(t) = \int_w E_\pm\left[(t_x\phi_t + s_x\phi_s)t_y \pm u\Gamma\partial\phi/\partial t\right] \ d\ell$$
$$+ \ i\nu^2 p \int_h \exp(P^2z)E_\pm \ b_\pm \ da \ , \tag{70}$$

where Eqs. (49), (19), (15a) and (1) were used.

By substituting Eq. (66) into Eq. (63) we may obtain

$$pb_\pm = \mp A_\pm \ , \tag{71}$$

where the amplitude functions $A_\pm$ are given by

$$A_\pm = \Gamma(n_y\partial\phi/\partial z - n_z\partial\phi/\partial y \pm B(n_z\partial\phi/\partial x - n_x\partial\phi/\partial z) + iC(n_x\partial\phi/\partial y - n_y\partial\phi/\partial x) \tag{72}$$

with B and C defined as

$$B = p(v\Gamma - 1) \quad \text{and} \quad C = p(\Gamma - v) \ .$$

These relations yield

$$\Gamma = Cu + v \quad \text{and} \quad B = Cv - u \ . \tag{73a,b}$$

By using Eqs. (71) , (72) and (73a,b) we may then express Eq. (70) in the form

$$K_\phi^{\pm}(t) = \int_w E_\pm\left[(t_x\phi_t+s_x\phi_s)t_y \pm u(Cu+v)\partial\phi/\partial t\right]\, d\ell$$
$$\mp i\nu^2\int_h \exp(P^2z)E_\pm\, A_\pm\, da\ , \tag{74}$$

where the amplitude functions $A_\pm$ are given by

$$A_\pm = (Cu+v)(n_y\partial\phi/\partial z-n_z\partial\phi/\partial y)$$
$$\pm(Cv-u)(n_z\partial\phi/\partial x-n_x\partial\phi/\partial z) + iC(n_x\partial\phi/\partial y-n_y\partial\phi/\partial x)\ . \tag{75}$$

The constant C in Eqs. (74) and (75) is arbitrary and may then be selected at will. More precisely, C may be chosen as an arbitrary function of t. These equations thus define a one-parameter family of alternative mathematically-equivalent expresions for the functions $K_\phi^{\pm}(t)$.

The amplitude functions in the integrands of the hull integrals in the alternative expressions (40) and (74) are given by $pa_\pm$ and $\mp A_\pm$, respectively. It may seem from Eqs. (41), (14b) and (15b) that we have $a_\pm = O(1)$ and $pa_\pm = O(t)$ as $t \to \infty$, whereas Eqs. (14b), (15a,b) and (75) show that $A_\pm = O(1)$ as $t \to \infty$ for any finite value of the limit $C(\infty)$ of the arbitrary function $C(t)$. The hull integral in Eq. (74) therefore vanishes more rapidly than the hull integral in Eq. (40) as $t \to \infty$. This result implies that the waterline integral in Eq. (74) likewise vanishes faster than the waterline integral in Eq. (40). Indeed, the term $t_xt_y\phi_t$ in the integrand of the waterline integral in Eq. (40) is replaced by the term $(t_xt_y\pm uv\pm Cu^2)\phi_t$ in Eq. (74). We have $t_xt_y\pm uv\pm Cu^2 \sim t_xt_y\pm uv$ as $t \to \infty$, with an error $O(u^2) = O(t^{-2})$, and $t_xt_y\pm uv = 0$ at a point of stationary phase of the trigonometric function $E_\pm$ defined by Eqs. (35a,b,c). The cancellations occurring for large values of t between the waterline and hull integrals in Eq. (40), as is depicted in Figs. 5a-e, may then be expected to be significantly reduced in the alternative expression (74).

For large values of t, Eqs. (14a,b) show that we have $P^2 \gg 1$. The major contribution to the hull integral in Eq. (74) therefore stems from the upper part of the hull surface in the vicinity of the waterline. For a wall-sided hull we have $n_x = -t_y$, $n_y = t_x$ and $n_z = 0$ on h in the vicinity of w, and Eq. (75) becomes

$$A_{\pm} = [vt_x \mp ut_y + C(ut_x \pm vt_y)]\partial\phi/\partial z - iC\partial\phi/\partial t \ , \tag{76}$$

where the identity $t_x\partial\phi/\partial x + t_y\partial\phi/\partial y = \partial\phi/\partial t$ was used. The hull integral in Eq. (74) can be approximated by a waterline integral for $P^2 >> 1$. Specifically, we have

$$\mp i\nu^2\int_h \exp(P^2 z)E_{\pm}\ A_{\pm}\ da \sim \int_w E_{\pm}\ A_h^{\pm}\ d\ell \ , \tag{77}$$

where $A_h^{\pm}$ is given by $A_h^{\pm} = \mp iu^2 A_{\pm}$. Equation (76) then yields

$$A_h^{\pm} = \mp iu^2[vt_x \mp ut_y + C(ut_x \pm vt_y)]\partial\phi/\partial z \mp Cu^2\partial\phi/\partial t \ . \tag{78}$$

We may choose $s_x = 0$. We then have $\phi_t = \partial\phi/\partial t$ for a wall sided hull and the amplitude function, $A_W^{\pm}$ say, in the integrand of the waterline integral in Eq. (74) becomes

$$A_w^{\pm} = (t_x t_y \pm uv \pm Cu^2)\partial\phi/\partial t \ . \tag{79}$$

The major contributions to the waterline integrals in Eqs. (74) and (77) stem from the point(s) of stationary phase of the trigonometric functions $E_{\pm}$ and from the end points, that is the bow and the stern. At a point of stationary phase, Eqs. (35a,b,c) hold and Eqs. (78) and (79) become

$$A_h^{\pm} = \mp\ iu^2\partial\phi/\partial z \mp Cu^2\partial\phi/\partial t \ , \tag{80a}$$

$$A_w^{\pm} = \pm\ Cu^2\partial\phi/\partial t \ , \tag{80b}$$

where Eq. (17) was used. These equations show that the sum of the waterline and hull amplitude functions at a point of stationary phase is independent of the constant C, as must be true in general (i.e. for all values of t, including the limit $t \to \infty$ considered here), and is given by

$$A_h^{\pm} + A_w^{\pm} = \mp iu^2\partial\phi/\partial z \ , \tag{81}$$

which stems from the hull integral in Eq. (74).

Equations (78) and (80a) show that the amplitude function $A_h^{\pm}$ is $O(u^2)$ at the bow and the stern and at a point of stationary phase. The dominant contribution to the hull integral in Eqs. (74) and (77) therefore stems from the point(s) of stationary phase and is $O(Fu^3)$. Equation (80b) shows that the stationary-phase contribution to the waterline integral in Eq. (74) likewise is $O(Fu^3)$. The bow and stern contributions to the waterline integral in Eq. (74) stem from the term

$$t_x t_y \partial\phi/\partial t \tag{82}$$

in Eq. (79), which is $O(1)$. The bow and stern contributions to the waterline integral in Eq. (74) therefore are $O(F^2u^2)$ and dominate the $O(Fu^3)$ stationary-phase contributions to the waterline and hull integrals in the limit $t \to \infty$. The waterline integral in Eq. (74) thus dominates the hull integral in this limit.

However, an exception to this general rule occurs if the amplitude function defined by Eq. (82) is null at the bow and the stern. Such would be the case for cusped ends, for which we have $t_y = 0$. A more realistic case is that of a round-ended ship form, e.g. an oil tanker, for which we have $t_x = 0$ and $\partial\phi/\partial t = 0$ at the bow and the stern. For such hull forms, the dominant contributions to the waterline and hull integrals in Eq. (74) stem from the point(s) of stationary phase and are $O(Fu^3)$, as follows from Eqs. (80a,b). These equations show that significant cancellations might then occur between the waterline and hull integrals in Eq. (74) unless the function $C(t)$ vanishes in the limit $t \to \infty$. Accordingly, we impose that the arbitrary function $C(t)$ in Eqs. (74) and (75) satisfy the condition

$$C \to 0 \quad \text{as} \quad t \to \infty \ . \tag{83}$$

An obvious choice for the function $C(t)$ satisfying Eq. (83) is

$$C = 0 \ . \tag{84}$$

The corresponding expressions for the functions $K_\phi^\pm$ are readily obtained from Eqs. (74) and (75). These equations may be expressed in the form

$$K_\phi = K_W'' + K_H'' \ , \tag{85}$$

where the superscript $\pm$ was ignored for simplicity and the functions K_W'' and K_H'' correspond to the waterline and hull integrals in Eq. (74), respectively. The real and imaginary parts of the sum of the port and starboard contributions to the functions K_W and $i\sigma K_H$ defined by Eqs. (40)-(43) and the functions K_W'' and K_H'' defined by Eqs. (74), (75) and (84), (85) are depicted in Figs. 6a-e for $0 \leq \tan\theta \leq 10$ (i.e. for $0 \leq \theta \leq 84°$) and for the cases considered previously in Figs. 2, 4a-e and 5a-e. It may be seen from Figs. 6a-e that the functions K_W'' and K_H'' vanish more rapidly than the functions K_W and $i\sigma K_H$ for increasing values of $t = \tan\theta$, in accordance with the foregoing theoretical considerations. The cancellations occurring between the waterline and hull integrals K_W and $i\sigma K_H$ for large values of $\tan\theta$ thus are significantly reduced in the alternative expression $K_W'' + K_H''$, which is therefore preferable to the expression $K_W + i\sigma K_H$ for large values of $\tan\theta$. However, the functions K_W'' and K_H'' are appreciably larger than the functions K_W and $i\sigma K_H$ for small values $\tan\theta$, especially in Figs. 6a,b,c corresponding to small values of the Froude number, and significant cancellations thus occur between the waterline and hull integrals K_W'' and K_H'' for small values of $\tan\theta$. The expression $K_W + i\sigma K_H$ therefore is preferable to the expression $K_W'' + K_H''$ for small values of $\tan\theta$, whereas the reverse holds for large values of $\tan\theta$.

The amplitude functions in the integrands of the waterline integrals in Eqs. (40) and (74) are nearly identical for small values of t if we have

$$v + Cu \ll 1 \quad \text{as} \quad t \to 0 \,. \tag{86}$$

Equations (41) and (75) show that the amplitude functions $a_\pm$ and $\mp uA_\pm$ in the integrands of the hull integrals in Eqs. (40) and (74) likewise are nearly identical as $t \to 0$ if condition (86) and the condition $1-u(u-Cv) \ll 1$ hold. By using Eq. (17), we may express the latter condition in the form $v(v+Cu) \ll 1$, which is identical to condition (86). This condition therefore ensures that the waterline and hull integrals in Eq. (74) are nearly identical to the corresponding integrals in Eq. (40) in the limit $t \to 0$.

The large- and small-t conditions (83) and (86) are satisfied if the arbitrary function $C(t)$ is selected in the form

$$C = -\lambda uv \ . \tag{87}$$

Condition (86) then becomes $v(1-\lambda u^2) \ll 1$ as $t \to 0$. Equation (17) shows that we have $v(1-\lambda u^2) \sim v^3$ as $t \to 0$ if

$$\lambda \to 1 \quad \text{as} \quad t \to 0 \ . \tag{88}$$

By substituting Eq. (87) into Eqs. (74) and (75) we may obtain

$$K_\phi^{\pm}(t) = \int_w E_\pm \, a_\pm \, d\ell \mp i\nu^2 \int_h \exp(P^2 z) E_\pm \, A_\pm \, da \ , \tag{89}$$

where the amplitude functions $a_\pm$ and $A_\pm$ are given by

$$a_\pm = (t_x \phi_t + s_x \phi_s) t_y \pm uv(1-\lambda u^2)\partial\phi/\partial t, \tag{90a}$$

$$\begin{aligned} A_\pm &= v(1-\lambda u^2)(n_y \partial\phi/\partial z - n_z \partial\phi/\partial y) \\ &\mp u(1+\lambda v^2)(n_z \partial\phi/\partial x - n_x \partial\phi/\partial z) \\ &- i\lambda uv(n_x \partial\phi/\partial y - n_y \partial\phi/\partial x) \ . \end{aligned} \tag{90b}$$

An obvious choice for the function $\lambda(t)$ satisfying condition (88) is

$$\lambda = 1 \ . \tag{91}$$

The corresponding expressions for the functions $K_\phi^{\pm}$ are readily obtained from Eqs. (89) and (90a,b), where we have $1-u^2 = v^2$ by virtue of Eq. (17). These equations may be expressed in the form

$$K_\phi = K_W^* + K_H^* \ , \tag{92}$$

where the superscript $\pm$ was ignored for simplicity and the functions K_W^* and K_H^* correspond to the waterline and hull integrals in Eqs. (89), respectively. The real and imaginary parts of the sum of the port and starboard contributions to the functions K_W and $i\sigma K_H$ defined by Eqs. (40)-(43) and the functions K_W^* and K_H^* defined by Eqs. (89)-(92) are depicted in Figs. 7a-e for the cases considered previously in Figs. 2, 4a-e, 5a-e and 6a-e. It may be seen from Figs. 7a-e that the functions K_W^* and K_H^* vanish more rapidly than the functions K_W and $i\sigma K_H$ for increasing values of $t = \tan\theta$. In this respect, the functions K_W^* and K_H^* are comparable to the functions K_W'' and K_H'' depicted in Figs. 6a-e. However, the functions K_W^* and K_H^* depicted in Figs. 7a-e and the functions K_W'' and K_H'' depicted in Figs. 6a-e are significantly different for small and moderate values of $\tan\theta$. More precisely, the functions K_W'' and K_H'' are appreciably larger than the functions K_W and $i\sigma K_H$, as was already noted, whereas the functions K_W^* and K_H^* are comparable to the functions K_W and $i\sigma K_H$. In fact, the functions K_W^* and K_H^* are somewhat smaller than the functions K_W and $i\sigma K_H$ for small and moderate values of $\tan\theta$.

Figures 5a-e and 7a-e show that the cancellations occurring between the waterline and hull integrals K_W and $i\sigma K_H$ are reduced significantly in the modified waterline and hull integrals K_W^* and K_H^*. The expression for the Neumann-Kelvin correction term $K_\phi^+ + K_\phi^-$ defined by Eqs. (89)-(91) therefore is preferable to the expression given by Eqs. (40) and (41) for numerical calculations.

The velocity components $\partial\phi/\partial x$, $\partial\phi/\partial y$ and $\partial\phi/\partial z$ in Eq. (90b) defining the amplitude functions $A_\pm$ in the hull integral in Eq. (89) can be expressed in terms of the components ϕ_t and ϕ_s of the velocity vector $\nabla\phi$ along two unit vectors

$$\vec{t} = (t_x, t_y, t_z) \quad \text{and} \quad \vec{s} = (s_x, s_y, s_z) \tag{93a,b}$$

tangent to the hull surface. More precisely, we have $\nabla\phi = (\partial\phi/\partial n)\vec{n}+\phi_t\vec{t}+\phi_s\vec{s}$, which yields

$$\partial\phi/\partial x = n_x\partial\phi/\partial n + t_x\phi_t + s_x\phi_s , \tag{94a}$$

$$\partial\phi/\partial y = n_y\partial\phi/\partial n + t_y\phi_t + s_y\phi_s , \tag{94b}$$

$$\partial\phi/\partial z = n_z\partial\phi/\partial n + t_z\phi_t + s_z\phi_s , \tag{94c}$$

where $\partial\phi/\partial n$ is the velocity component along the unit outward normal vector $\vec{n}$ to the hull surface defined as

$$\vec{n} = (\vec{t} \times \vec{s})/\left|\vec{t} \times \vec{s}\right| . \tag{95}$$

The unit vectors $\vec{t}$ and $\vec{s}$ to the ship hull are tangent to curves which approximately correspond to waterlines and framelines, respectively, and they point towards the bow and the keel, respectively. The vectors $\vec{t}$ and $\vec{s}$ thus are roughly (but not necessarily exactly) orthogonal. At the mean free surface, the vector $\vec{t}$ is tangent to the top waterline (and we thus have $t_z = 0$) in agreement with our previous definition. Equations (94a-c) yield

$$n_y\partial\phi/\partial z-n_z\partial\phi/\partial y = (n_y t_z-n_z t_y)\phi_t + (n_y s_z-n_z s_y)\phi_s \ , \tag{96a}$$

$$n_z\partial\phi/\partial x-n_x\partial\phi/\partial z = (n_z t_x-n_x t_z)\phi_t + (n_z s_x-n_x s_z)\phi_s \ , \tag{96b}$$

$$n_x\partial\phi/\partial y-n_y\partial\phi/\partial x = (n_x t_y-n_y t_x)\phi_t + (n_x s_y-n_y s_x)\phi_s \ . \tag{96c}$$

By using Eqs. (96a-c) we may then express the function A defined as

$$A = \alpha(n_y\partial\phi/\partial z-n_z\partial\phi/\partial y) + \beta(n_z\partial\phi/\partial x-n_x\partial\phi/\partial z) + \gamma(n_x\partial\phi/\partial y-n_y\partial\phi/\partial x) \tag{97}$$

in the form

$$A = T\phi_t + S\phi_s \ , \tag{98}$$

where T and S are given by

$$T = (\alpha\vec{e}_x+\beta\vec{e}_y+\gamma\vec{e}_z) \cdot \vec{n}\times\vec{t} \ , \tag{99a}$$

$$S = (\alpha\vec{e}_x+\beta\vec{e}_y+\gamma\vec{e}_z) \cdot \vec{n}\times\vec{s} \ . \tag{99b}$$

By substituting Eq. (95) into Eqs. (99a,b) we may then express Eq. (98) in the form

$$A = (T'\phi_t - S'\phi_s)/\,|\,\vec{t}\times\vec{s}\,|\ , \tag{100}$$

where T' and S' are given by

$$T' = \alpha s_x+\beta s_y+\gamma s_z - (\alpha t_x+\beta t_y+\gamma t_z)\vec{t}\cdot\vec{s} \ , \tag{101a}$$

$$S' = \alpha t_x+\beta t_y+\gamma t_z - (\alpha s_x+\beta s_y+\gamma s_z)\vec{t}\cdot\vec{s} \ . \tag{101b}$$

Equations (97), (100), and (101a,b) finally yield

$$\alpha(n_y\partial\phi/\partial z-n_z\partial\phi/\partial y) + \beta(n_z\partial\phi/\partial x-n_x\partial\phi/\partial z) + \gamma(n_x\partial\phi/\partial y-n_y\partial\phi/\partial x) =$$
$$[(\alpha s_x+\beta s_y+\gamma s_z)\partial\phi/\partial t - (\alpha t_x+\beta t_y+\gamma t_z)\partial\phi/\partial s]/\,|\,\vec{t}\times\vec{s}\,|\ , \tag{102}$$

where the relations

$$\partial\phi/\partial t = \phi_t + \vec{t}\cdot\vec{s}\ \phi_s \quad \text{and} \quad \partial\phi/\partial s = \phi_s + \vec{t}\cdot\vec{s}\ \phi_t \tag{103a,b}$$

were used. Equations (90a,b), (102) and (103a,b) show that the Neumann-Kelvin correction $K_{\phi}^{\pm}$ defined by Eq. (89) may be expressed in terms of the components ϕ_t and ϕ_s of $\nabla\phi$ along two unit vectors $\vec{t}$ and $\vec{s}$ tangent to the hull.

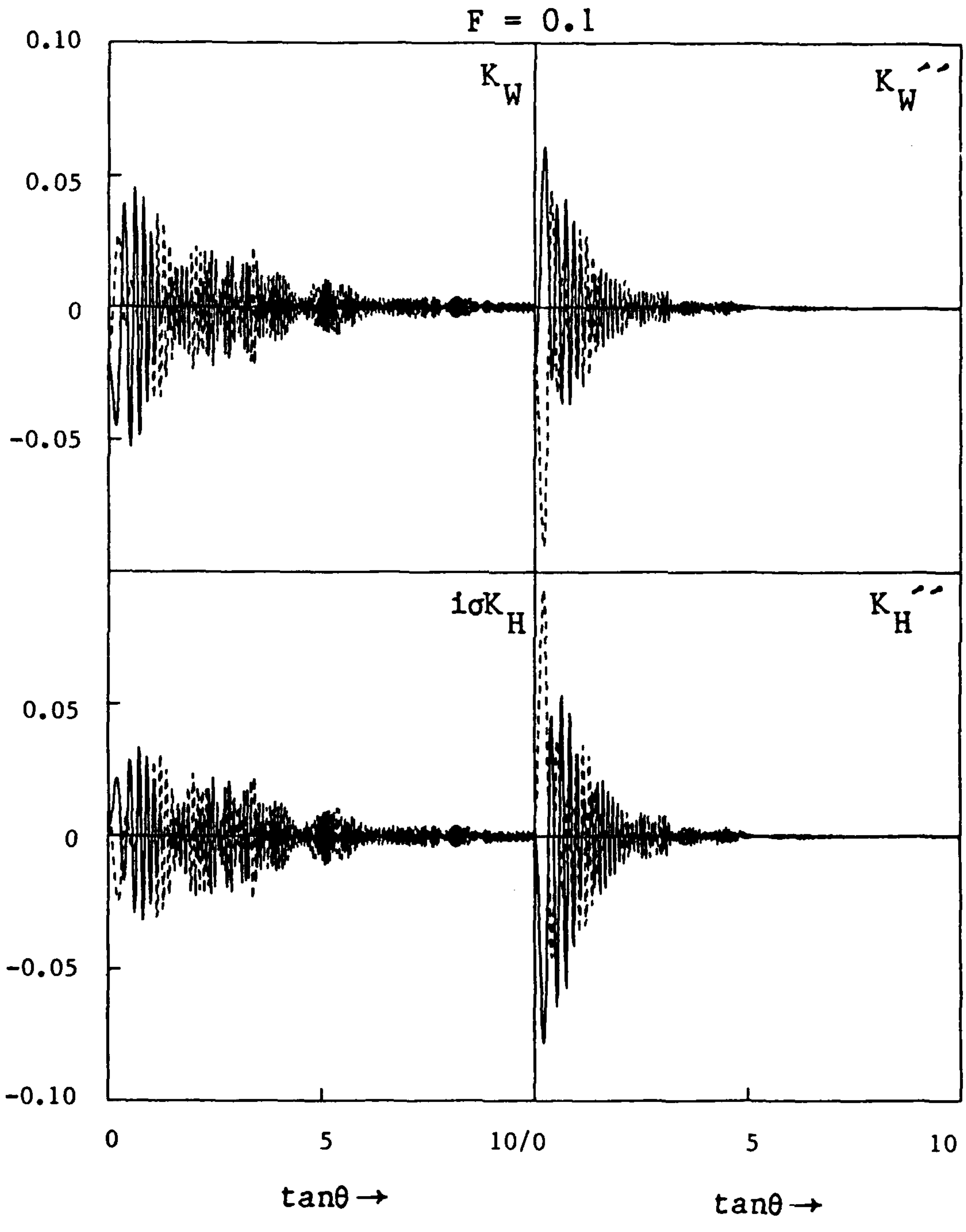

Fig. 6a The functions K_W , $i\sigma K_H$, K_W'' and K_H'' for $F = 0.1$.

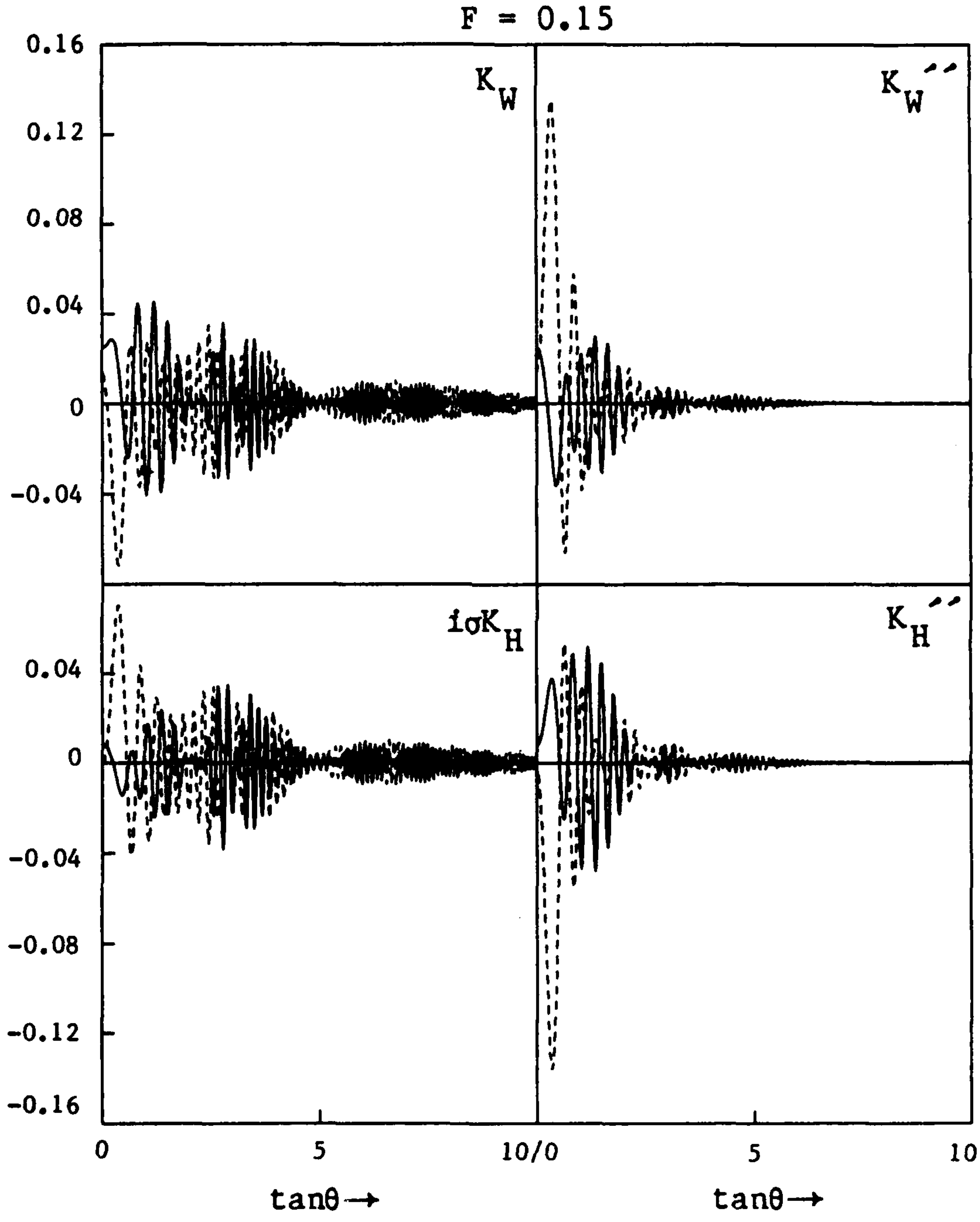

Fig. 6b The functions K_W , $i\sigma K_H$, K_W'' and K_H'' for $F = 0.15$.

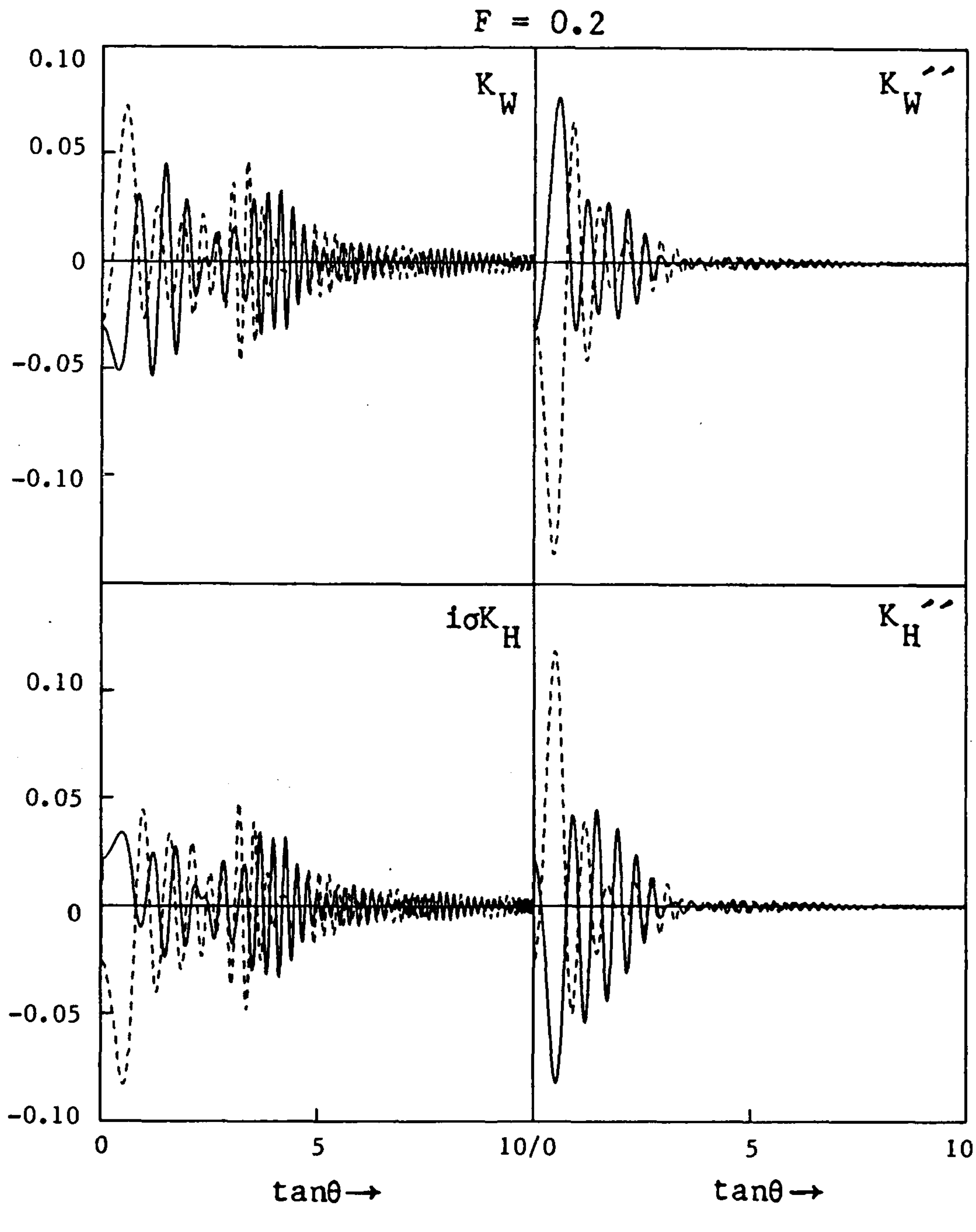

Fig. 6c The functions K_W , $i\sigma K_H$, K_W'' and K_H'' for F = 0.2 .

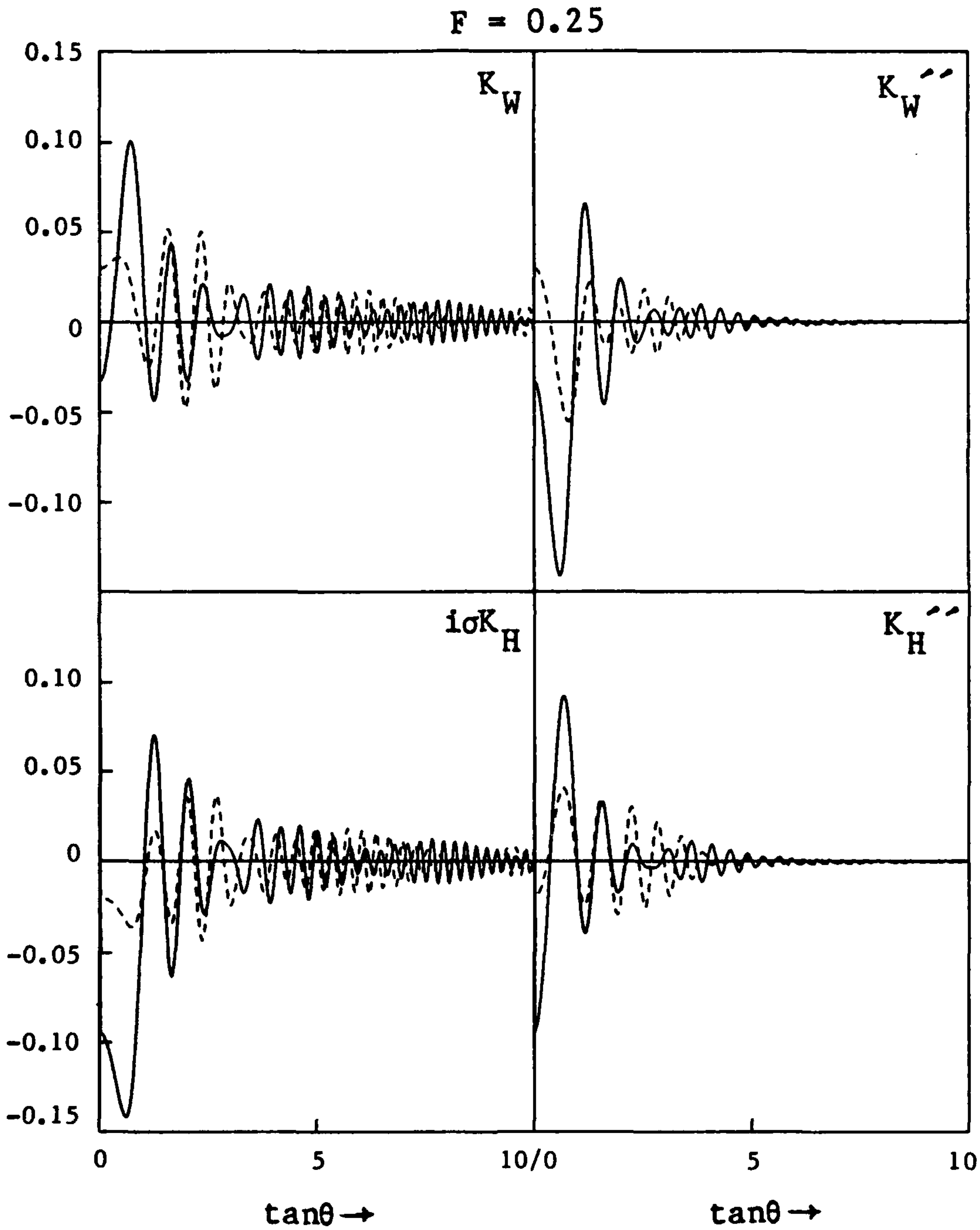

Fig. 6d The functions K_W , $i\sigma K_H$, K_W'' and K_H'' for $F = 0.25$.

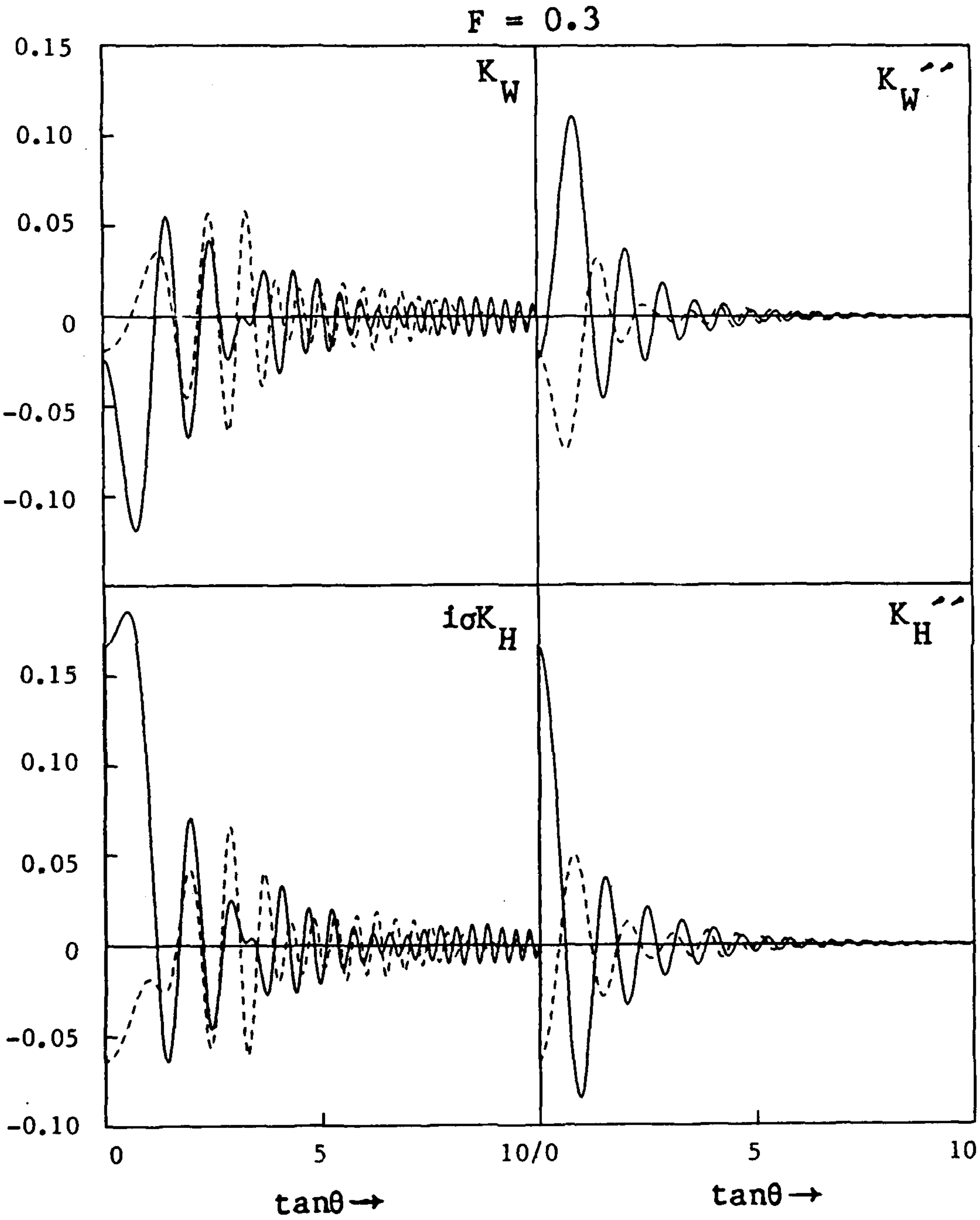

Fig. 6e The functions K_W , $i\sigma K_H$, K_W'' and K_H'' for F = 0.3 .

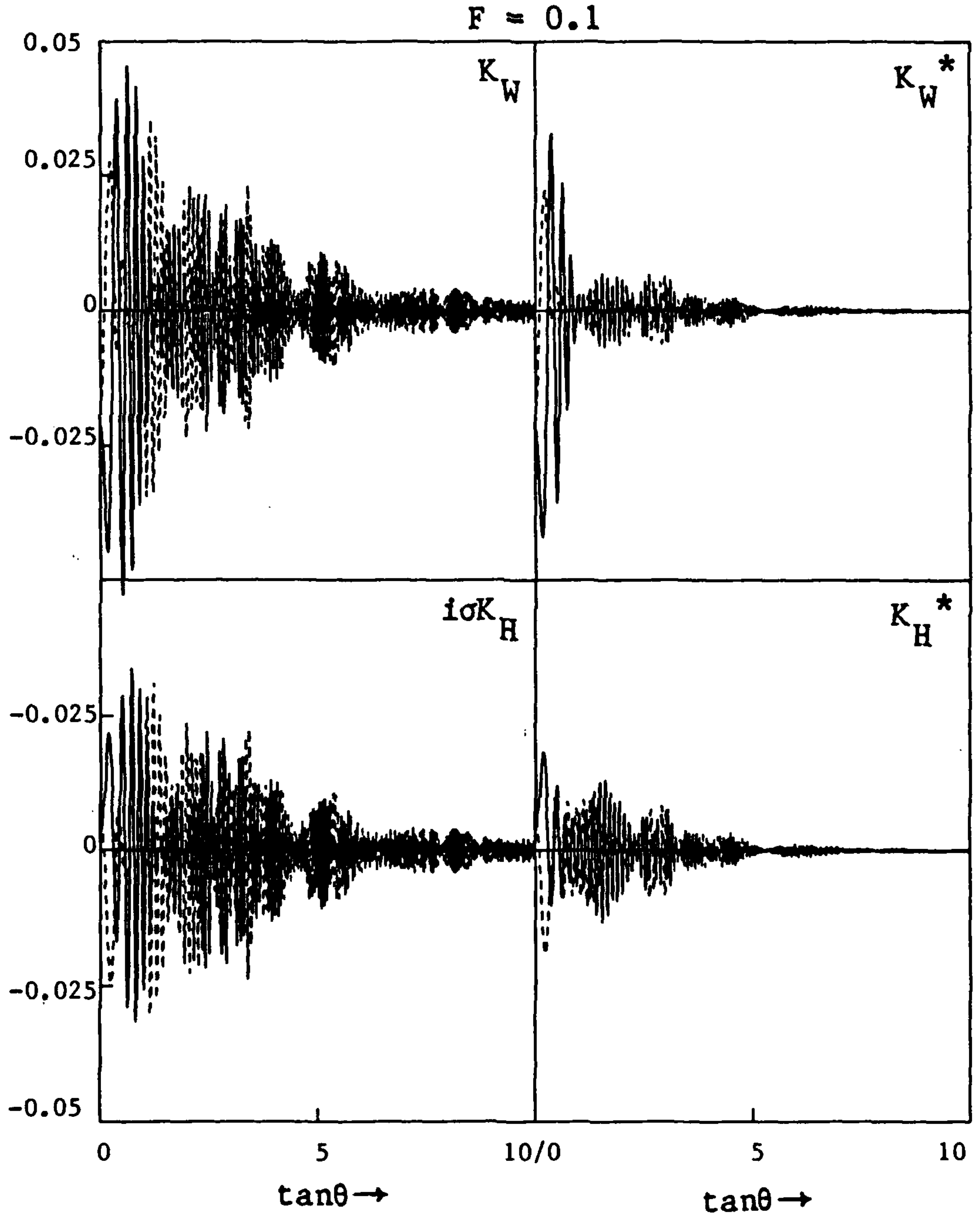

Fig. 7a The functions K_W , $i\sigma K_H$, $K_W{}^*$ and $K_H{}^*$ for $F = 0.1$.

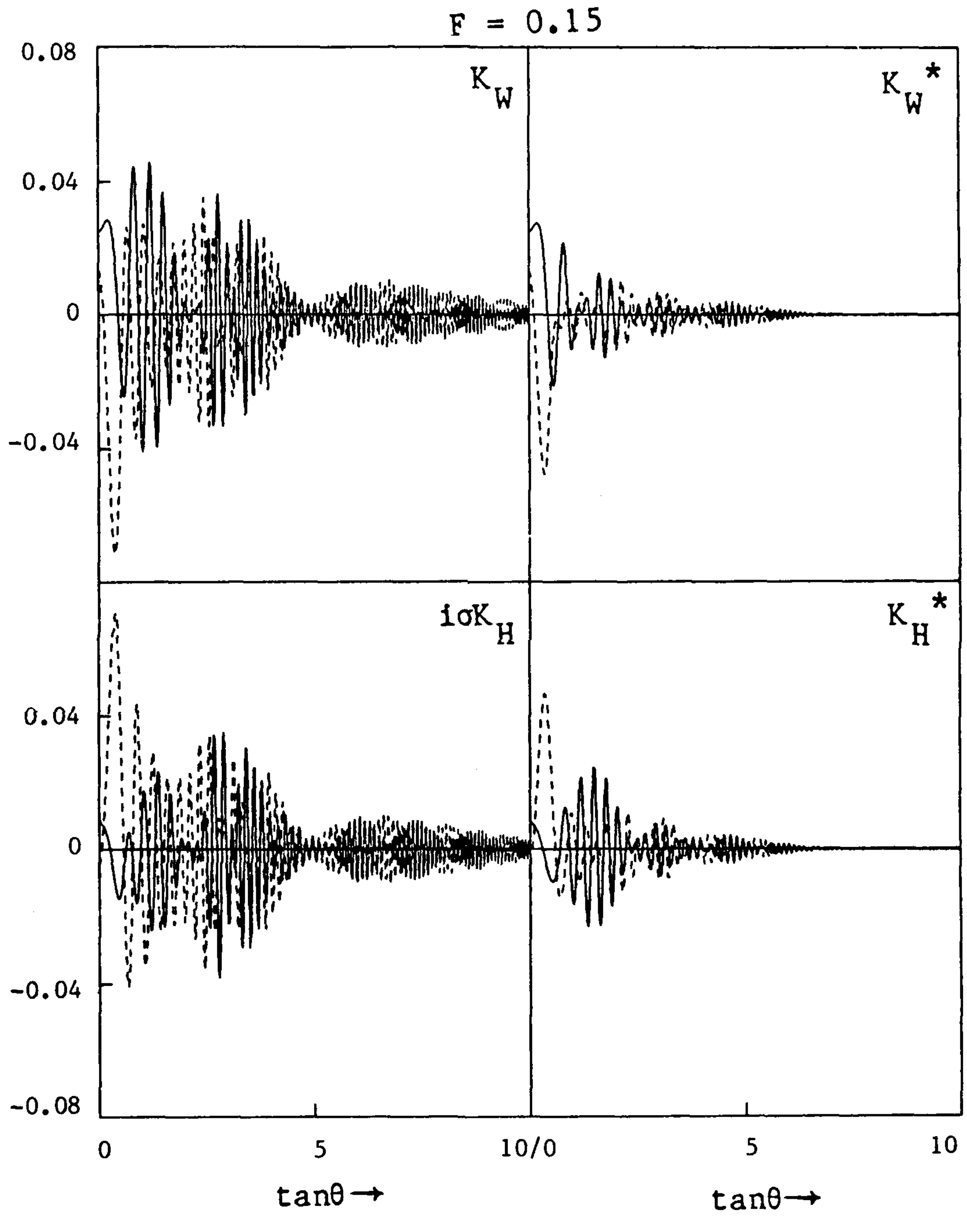

Fig. 7b The functions K_W , $i\sigma K_H$, K_W^* and K_H^* for $F = 0.15$.

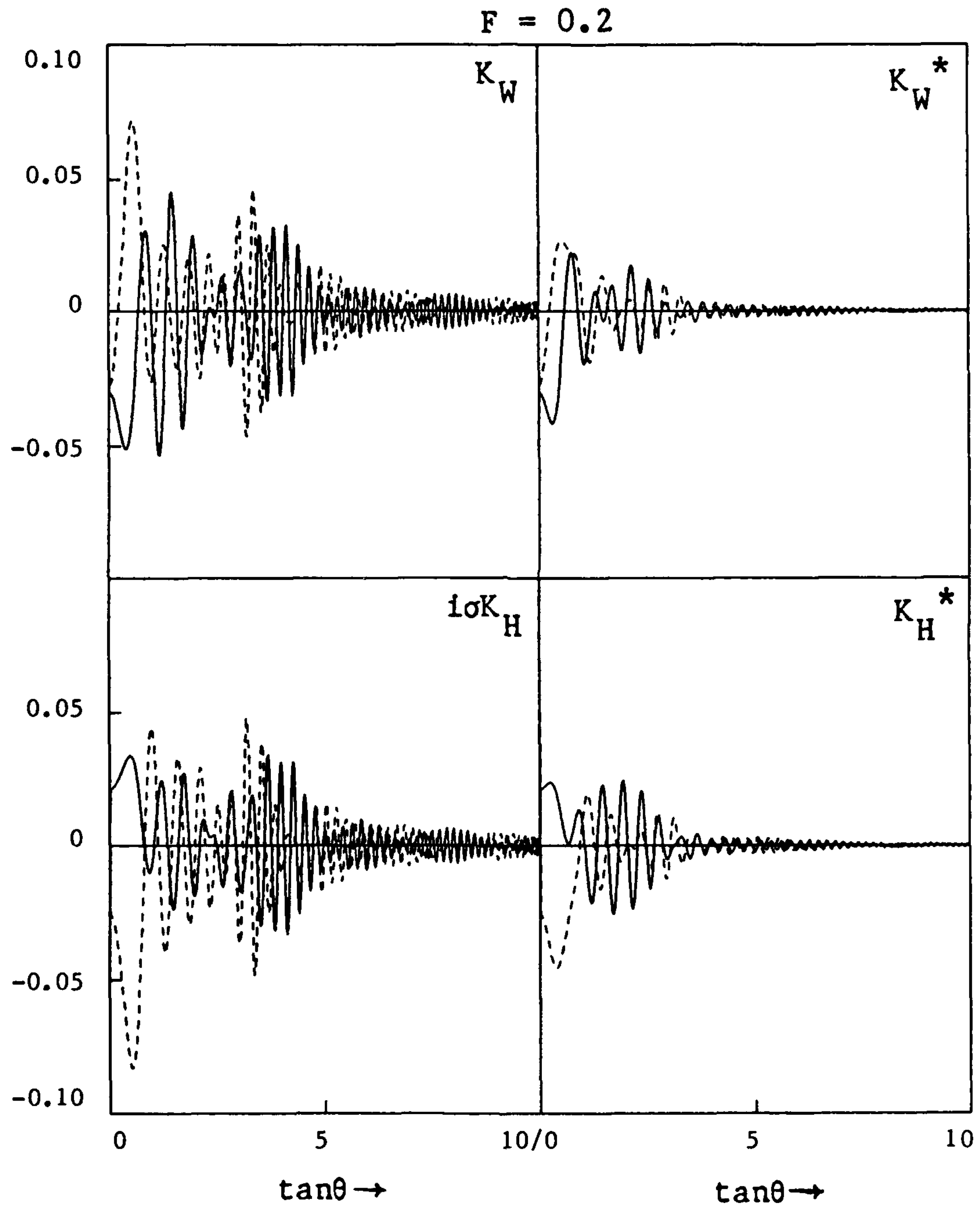

Fig. 7c The functions K_W , $i\sigma K_H$, K_W^* and K_H^* for $F = 0.2$.

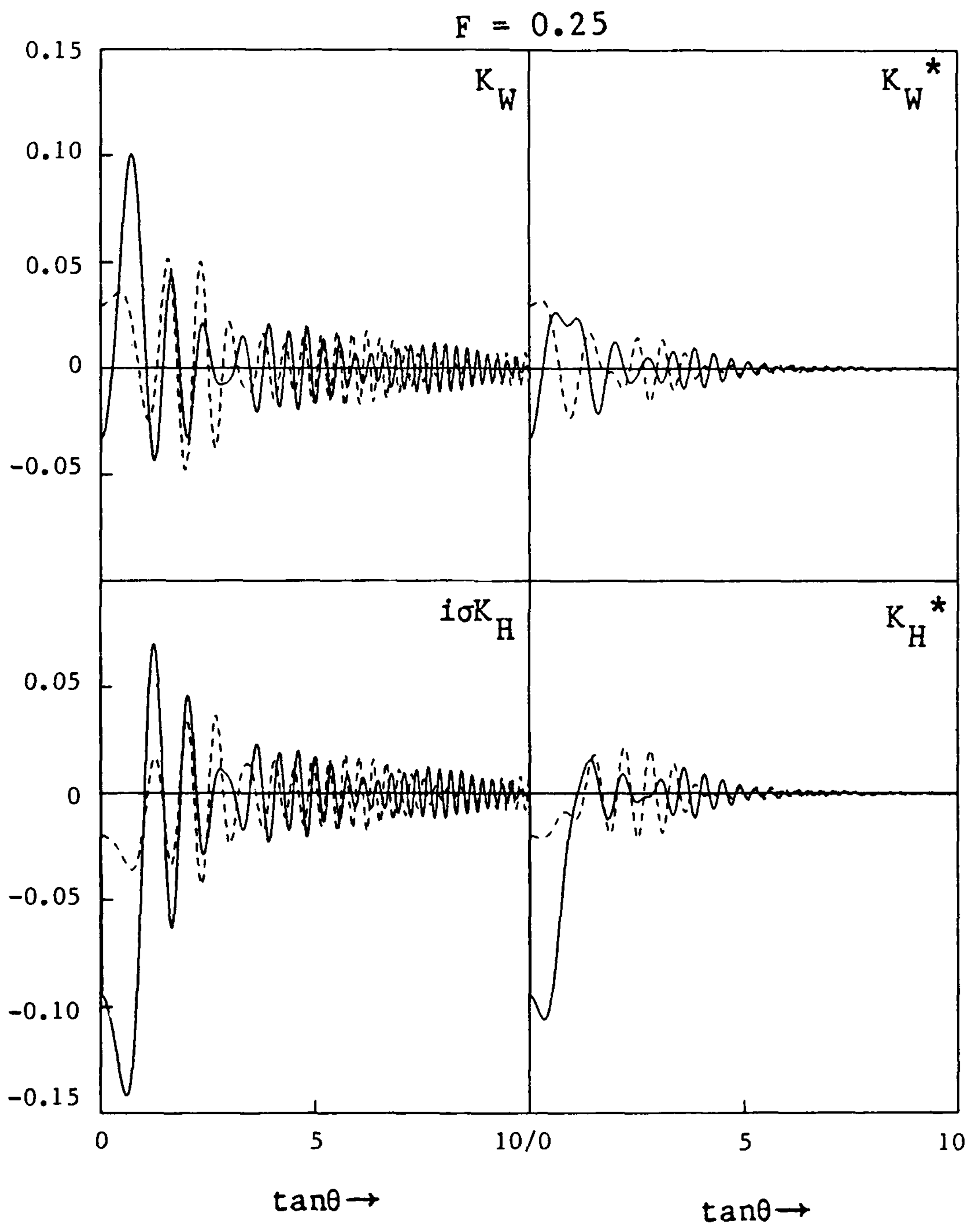

Fig. 7d The functions K_W , $i\sigma K_H$, K_W^* and K_H^* for $F = 0.25$.

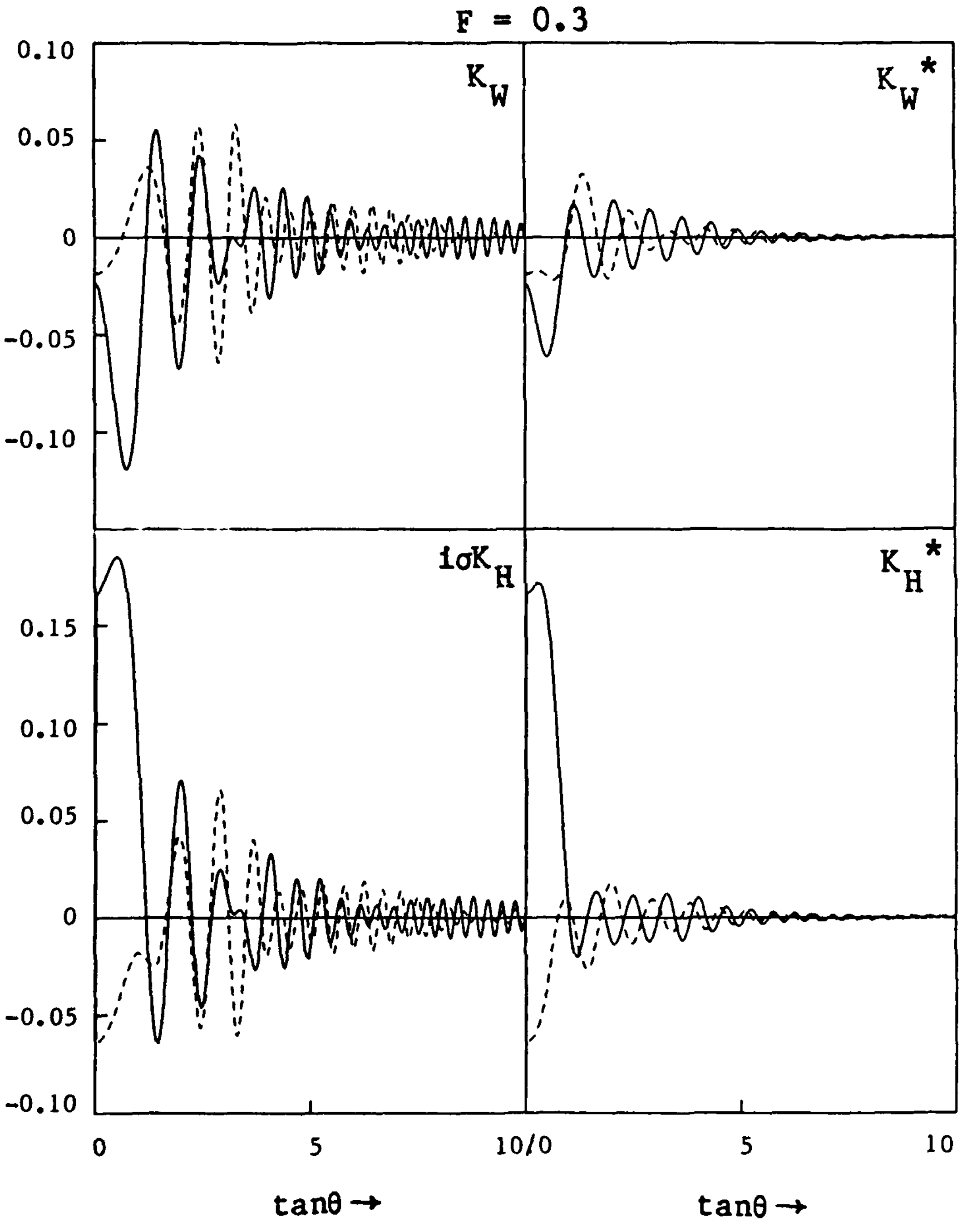

Fig. 7e The functions K_W , $i\sigma K_H$, K_W^* and K_H^* for $F = 0.3$.

CONCLUSION

In summary, the wave potential $\phi_W(\vec{\xi})$ at a point $\vec{\xi} = (\xi, \eta, \zeta \leq 0)$ behind the stern of a ship is defined by Eq. (20), that is we have

$$\phi_W(\vec{\xi}) = (2/\pi) \int_0^\infty \exp(\nu^2\zeta p^2) \cos(\nu^2\eta pt) \text{ Im } \exp(i\nu^2\xi p) \left[K_+(t) + K_-(t)\right] dt \ , \qquad (104)$$

where ν and p are defined by Eqs. (1) and (14b), respectively, and the wave-spectrum functions $K_\pm(t)$ are expressed in the form of Eq. (21), as follows:

$$K_\pm(t) = K_0^{\,\pm}(t) + K_\phi^{\,\pm}(t) \ . \qquad (105)$$

In this expression, $K_0^{\,\pm}$ represent the zeroth-order slender-ship approximation and $K_\phi^{\,\pm}$ the Neumann-Kelvin correction to the slender-ship approximation.

The slender-ship approximation $K_0^{\,\pm}$ is given by Eq. (22) or by the recommended modified Eq. (34). We then have

$$K_0^{\,\pm}(t) = \int_w E_\pm(n_x^{\,2}-u^2)t_y \, d\ell + u^2 \int_{w'} \exp(P^2z)E_\pm \, t_y \, d\ell$$
$$- i\nu^2 u \int_s \exp(P^2z)E_\pm \, n_z \, da + \nu^2 \int_b \exp(P^2z)E_\pm \, n_x \, da \ , \qquad (106)$$

where w' represents a waterline-like curve separating the hull side s and the hull bottom b. In Eq. (105), $E_\pm$ are the trigonometric functions defined by Eq. (19), that is

$$E_\pm(x,y;t) = \exp\left[-iP^2(ux \pm vy)\right] \ , \qquad (107)$$

where P, u and v are given by Eqs. (14a,b) and (15a,b).

The Neumann-Kelvin correction terms $K_\phi^{\,\pm}$ are defined by Eqs. (23) and (24). This well-known expression was modified into the form given by Eqs. (40) and (41) via a first mathematical transformation. A second mathematical transformation led to the alternative expression given by Eqs. (74) and (75), which involve the arbitrary function C(t). Mathematical and numerical considerations led to the selection $C(t) = -uv$ and to the recommended expression for the Neumann-Kelvin correction $K_\phi^{\,\pm}$ given by Eqs. (89), (90a,b) and (91). We then have

$$K_\phi^{\,\pm} = \int_w E_\pm \, a_\pm \, d\ell \pm i\nu^2 \int_h \exp(P^2z)E_\pm \, A_\pm \, da \ , \qquad (108)$$

$$a_{\pm} = (t_x\phi_t + s_x\phi_s)t_y \pm uv^3\partial\phi/\partial t \ , \tag{109a}$$

$$|\vec{t}\times\vec{s}|\,A_{\pm} = \left[v^3 t_x \mp u(1+v^2)t_y - iuvt_z\right]\partial\phi/\partial s - \left[v^3 s_x \mp u(1+v^2)s_y - iuvs_z\right]\partial\phi/\partial t \ , \tag{109b}$$

where Eqs. (17) and (102) were used. In Eq. (109b), $\vec{t}$ and $\vec{s}$ are unit vectors tangent to the hull surface along curves which approximately correspond to waterlines and framelines, respectively. The vectors $\vec{t}$ and $\vec{s}$ point towards the bow and the keel line, respectively. They are roughly (but not necessarily exactly) orthogonal. At the mean free surface, the vector $\vec{t}$ is tangent to the top waterline (and we thus have $t_z=0$). The components ϕ_s and ϕ_t of $\nabla\phi$ along the unit tangent vectors $\vec{s}$ and $\vec{t}$ and the velocities $\partial\phi/\partial s = \nabla\phi\cdot\vec{s}$ and $\partial\phi/\partial t = \nabla\phi\cdot\vec{t}$ are related as follows

$$\partial\phi/\partial s = \phi_s + \varepsilon\phi_t \quad \text{and} \quad \partial\phi/\partial t = \phi_t + \varepsilon\phi_s \ , \tag{110a,b}$$

$$\phi_s = (\partial\phi/\partial s - \varepsilon\partial\phi/\partial t)/(1-\varepsilon^2) \quad \text{and} \quad \phi_t = (\partial\phi/\partial t - \varepsilon\partial\phi/\partial s)/(1-\varepsilon^2) \tag{111a,b}$$

where ε is defined as

$$\varepsilon = \vec{t}\cdot\vec{s} \ . \tag{112}$$

The free-surface integral in Eq. (23) associated with the nonlinear terms in the free-surface boundary condition has been ignored in Eq. (108), which thus corresponds to the usual linearized Neumann-Kelvin approximation. The generalized Neumann-Kelvin expression incorporating the free-surface nonlinear term $\pi(\phi)$ defined by Eq. (5) is then given by

$$K_{\phi}^{\pm}(t) + \int_{f_\xi} E_{\pm}\,\pi(\phi)\,dxdy \ . \tag{113}$$

The usual expression for the functions $K_{\phi}^{\pm}$ defined by Eqs. (23) and (24) involves both the velocity potential ϕ and the velocity vector $\nabla\phi$. The alternative mathematically-equivalent expressions for the functions $K_{\phi}^{\pm}$ given by Eqs. (40) and (41), Eqs. (74) and (75), and Eqs. (89) and (90a,b) only involve the velocity vector $\nabla\phi$, not the potential ϕ. More precisely, these alternative modified expressions are defined in terms of the velocity components ϕ_t and ϕ_s along the vectors $\vec{t}$ and $\vec{s}$

tangent to the hull, as may be seen from Eq. (102) and is indicated explicitly in Eqs. (109a,b). The alternative modified expressions obtained in this study therefore define the wave potential behind the stern of a ship in terms of the speed and the size of the ship, the form of its hull and the tangential velocity at the mean hull surface. These expressions are directly suitable for use in conjunction with a boundary-integral equation method based on a source distribution, or any other numerical method in which the velocity vector (but not the potential) is determined on the mean hull surface.

However, the main recommendation of the alternative modified expressions for the functions $K_\phi^{\pm}$ obtained in this study resides in the fact that the cancellations occurring between the waterline and hull integrals in the usual expression (23) are considerably reduced in the modified expressions, especially the recommended expression defined by Eqs. (108) and (109a,b). The sum of the port and starboard contributions to the function $K_\phi = K_\phi^{+} + K_\phi^{-}$ may be expressed in the alternative forms

$$K_\phi = K_W + i\sigma K_W' + \sigma^2 K_H', \tag{114a}$$

$$K_\phi = K_W^* + K_H^*, \tag{114b}$$

corresponding to the usual expression (23) and the recommended modified expression (108), respectively. The waterline integrals K_W and $i\sigma K_W'$ and the hull integral $\sigma^2 K_H'$ in Eq. (114a) are defined by Eqs. (28a-c) and (29), and the terms K_W^* and K_H^* correspond to the waterline and hull integrals in the modified expression (108). The real and imaginary parts of the functions K_W, $i\sigma K_W'$, $\sigma^2 K_H'$, K_W^*, K_H^* and K_ϕ are depicted in Figs. 8a-e for the simple cases considered previously in Figs. 2, 4a-e, 5a-e, 6a-e and 7a-e. The function K_ϕ is appreciably smaller and vanishes much more rapidly with increasing values of $\tan\theta$ than its components $\sigma^2 K_H'$, $i\sigma K_W'$ and K_W. Large cancellations therefore occur among these components and the usual expression (23) is quite ill suited for accurate numerical calculations, notably for evaluating the short divergent waves in the wave spectrum corresponding to large values of $\tan\theta$. It may be seen from Figs. 8a-e that the modified waterline and hull integrals K_W^* and K_H^* in expression (108) are appreciably smaller and vanish much faster than the functions K_W , $i\sigma K_W'$ and $\sigma^2 K_H'$ and are comparable to the function K_ϕ. The modified expression (108) thus is considerably better suited than the well-known usual expression (23) for accurate numerical calculations of the steady wave spectrum of a ship.

For large values of $t = \tan\theta$, the major contribution to the hull integral K_H^* stems from the upper part of the hull surface in the vicinity of the mean waterline due to the exponential function $\exp(P^2 z)$. The hull integral K_H^* , and consequently the function K_ϕ , may then be approximated by a waterline integral for large values of $\tan\theta$, as has indeed been shown previously in this study for the special case of a wall-sided hull. This asymptotic approximation can be extended to arbitrary ship forms, i.e. ships having flare, and refined by retaining the first few terms in the asymptotic approximation. A detailed short-wave asymptotic analysis will be reported elsewhere as it is important for evaluating the short divergent waves of interest for applications to remote-sensing of ship wakes.

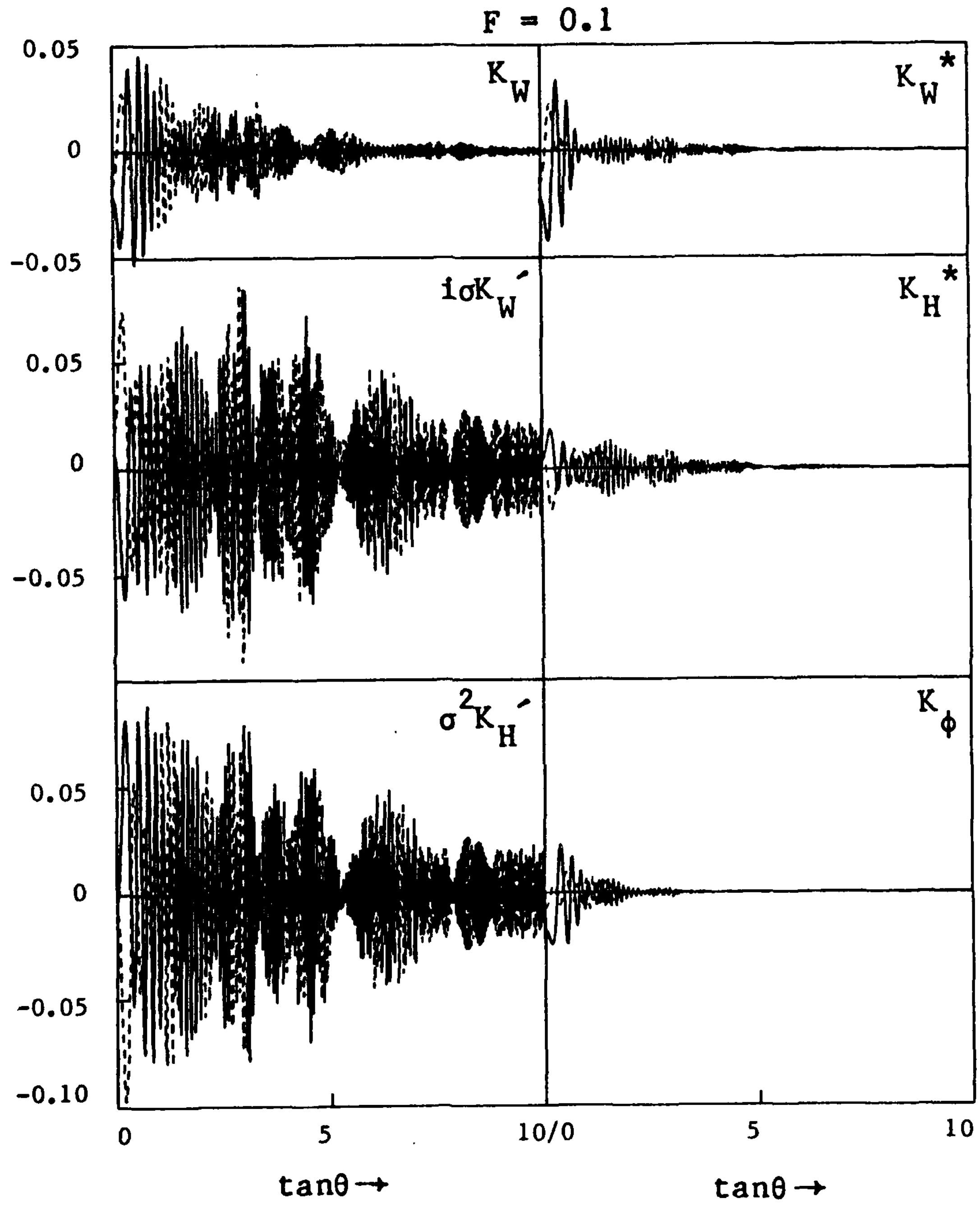

Fig. 8a The functions K_W , $i\sigma K_W'$, $\sigma^2 K_H'$, K_W^* , K_H^* and K_ϕ for $F = 0.1$.

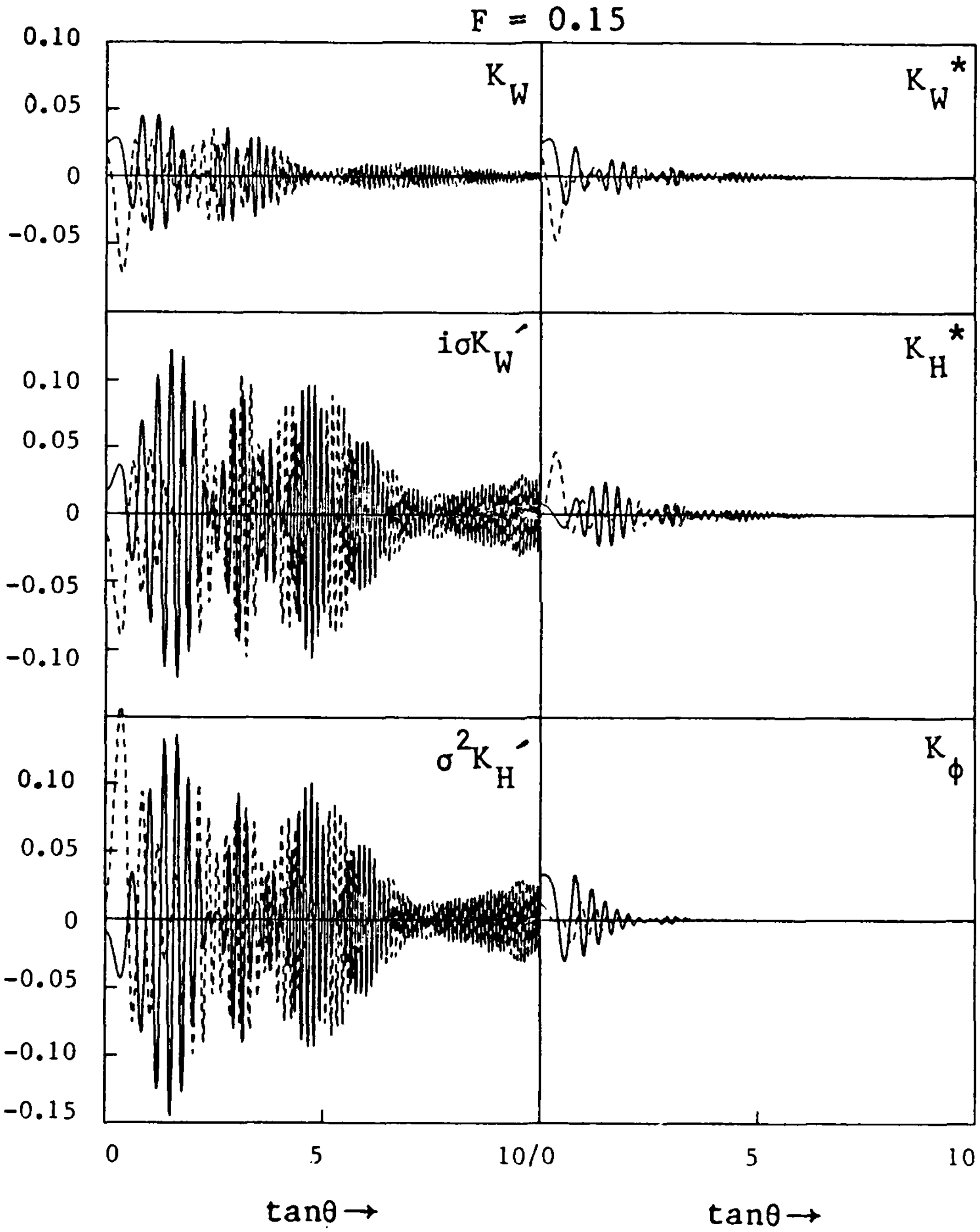

Fig. 8b The functions K_W , $i\sigma K_W'$, $\sigma^2 K_H'$, K_W^* , K_H^* and K_ϕ for F = 0.15 .

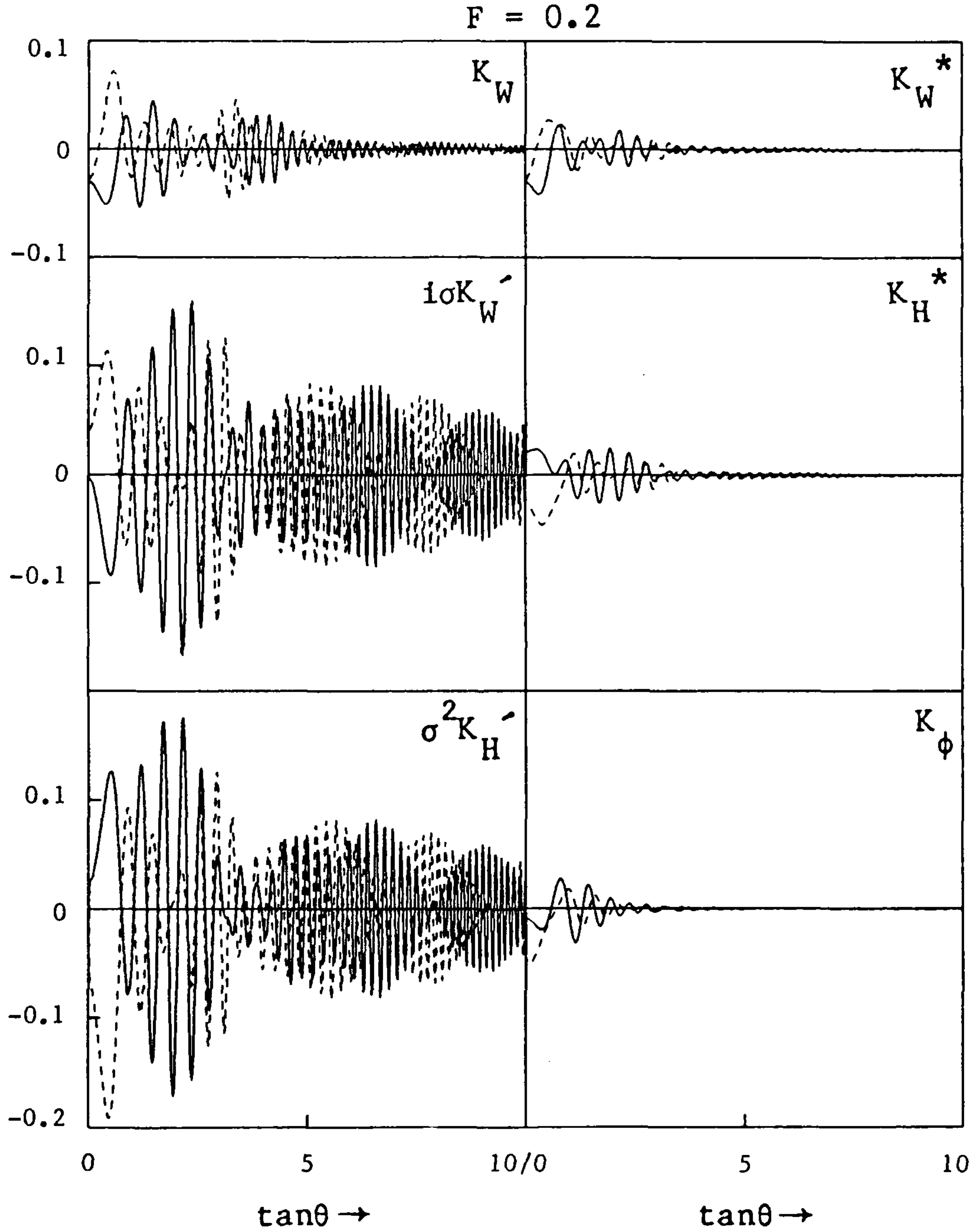

Fig. 8c The functions K_W , $i\sigma K_W'$, $\sigma^2 K_H'$, K_W^* , K_H^* and K_ϕ for $F = 0.2$.

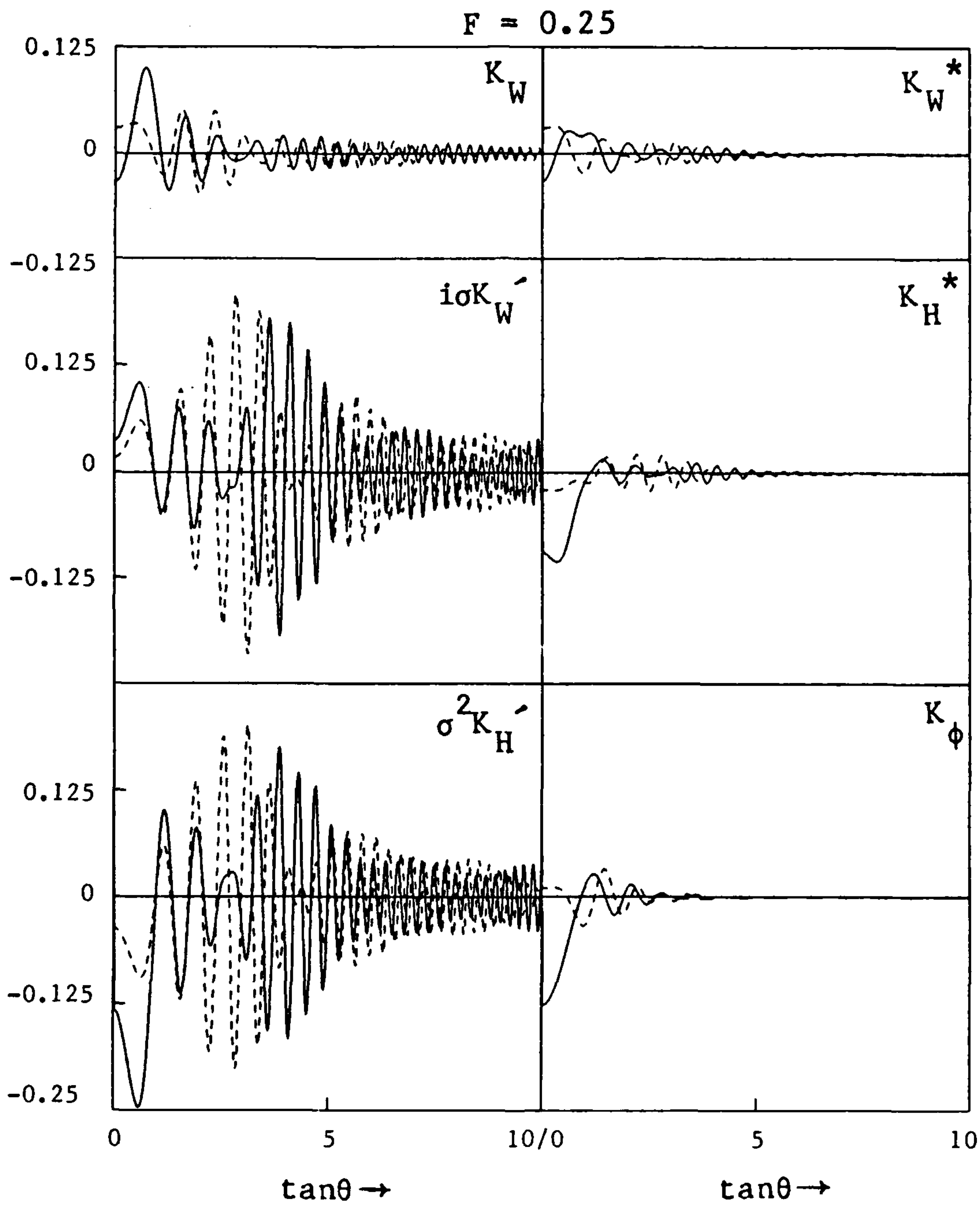

Fig. 8d The functions K_W , $i\sigma K_W'$, $\sigma^2 K_H'$, K_W^* , K_H^* and K_ϕ for F = 0.25 .

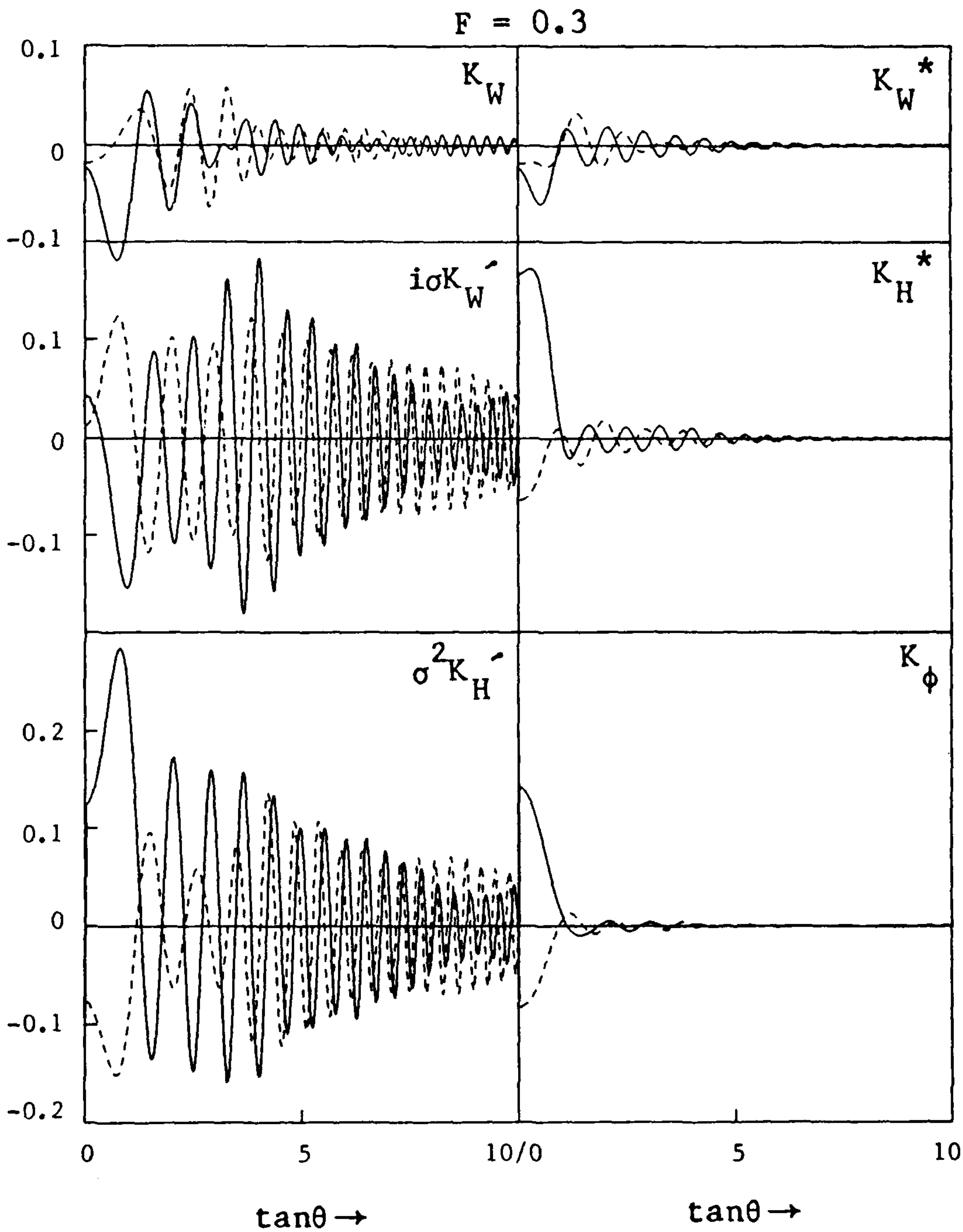

Fig. 8e The functions K_W , $i\sigma K_W{}'$, $\sigma^2 K_H{}'$, $K_W{}^*$, $K_H{}^*$ and K_ϕ for $F = 0.3$.

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