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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE Form Approved  
OMB No. 0704-0188

1a REPORT SECURITY CLASSIFICATION  
UNCLASSIFIED 1b RESTRICTIVE MARKINGS

3 DISTRIBUTION / AVAILABILITY OF REPORT  
Approved for public release;  
Distribution unlimited

AD-A205 598 5 MONITORING ORGANIZATION REPORT NUMBER(S)  
AFOSR-TR-89-0329

6a NAME OF PERFORMING ORGANIZATION  
Yale University 7a NAME OF MONITORING ORGANIZATION  
AFOSR/NM

6c ADDRESS (City, State, and ZIP Code)  
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8a NAME OF FUNDING / SPONSORING ORGANIZATION  
AFOSR 9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER  
AFOSR-88-0080

8c ADDRESS (City, State, and ZIP Code)  
Building 410  
Bolling AFB, DC 20332-6448 10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	A6	

11 TITLE (Include Security Classification)  
Development and Analysis of Arma Parameter Estimation Schemes in the Presence of Noise

12 PERSONAL AUTHOR(S)  
Professor Arye Nehorai

13a TYPE OF REPORT  
Annual Report 13b TIME COVERED  
FROM 11/1/87 TO 10/31/88 14 DATE OF REPORT (Year, Month, Day)  
January 4, 1989 15 PAGE COUNT  
5

16 SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19 ABSTRACT (Continue on reverse if necessary and identify by block number)  
  
See Back  
  
**DTIC**  
**SELECTE**  
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20 DISTRIBUTION / AVAILABILITY OF ABSTRACT  
 UNCLASSIFIED/UNLIMITED  SAME AS RPT  DTIC USERS 21 ABSTRACT SECURITY CLASSIFICATION  
UNCLASSIFIED

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NM

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A testimony to the significance and recognition of our recent research is that we were invited to publish the article [9] as a chapter in a forthcoming book edited by S. Haykin [10].

The research results obtained this year are described in detail in the references [2]-[9]. The work in all of these articles was performed with the support (either whole or partial) of the AFOSR grant, and this financial support is acknowledged in the articles themselves. The work is summarized in the following.

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# Annual Report

## Research Under Grant AFOSR-88-0080

### November 1, 1987 to November 1, 1988

## 1 Introduction

This report describes the work supported by the Air Force Office of Scientific Research under Grant AFOSR-88-0080 during the first year of a two year period. The grant supported the work of the Principal Investigator Professor Arye Nehorai and of his student Mr. David Starer at Yale University.

The amount of work we have done and the number of subjects studied in one year was far beyond what was promised in the original two year proposal [1]. The original proposal focussed on the development and analysis of single-sensor parameter estimation schemes for ARMA signals in noise. In the single sensor problem, we solved several new problems for these signals and also added sine wave signals in noise. Furthermore, we extended the research to multi-sensor (or sensor array) estimation algorithms which are useful for direction-of-arrivals estimation. All of the proposed algorithms have been tested by computer simulation to verify their operation.

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## 2 Single Sensor Algorithms

### 2.1 Algorithm Development

A convenient model for time-series analysis and system identification is the autoregressive moving-average (ARMA) or rational transfer function model: In time-series analysis, a measured signal can be modeled as the output of a rational transfer function driven by white noise. The estimated model can then be used to determine the signal spectrum. In system identification, an unknown system can be modeled as a rational transfer function whose parameters can be estimated from measurements of the input and output signals using ARMA identification algorithms. Knowledge of these parameters can be used, for example, by an adaptive regulator to control the system.

Most existing ARMA estimators employ "unconstrained" rational models. However, in many applications the measured signal is known to have special properties, and it is preferable to constrain the model to conform to these properties. Furthermore, most existing ARMA estimators ignore the contamination of the signal by noise, and this leads to incorrect estimates. To solve these problems, we developed in [2] a general family of ARMA

adaptive algorithms which utilize *a priori* known information concerning the signal's properties and solve the problems associated with non-linearities which arise from the presence of noise. These algorithms can track time-varying parameters, are more accurate and are computationally more efficient than the unconstrained ARMA algorithms. Applications of these algorithms include signal analysis in multipath scenarios, image deblurring, structured signal (e.g. band-pass) spectrum analysis.

In many ARMA spectral estimation problems, the parameters of interest are the poles (or singularities) of the rational transfer function. These are useful, for example, for spectral peak estimation, robust speech communication, and directions of arrivals (DOA's) estimation. Thus there exists a need for an algorithm which can satisfy the simultaneous requirements of a) directly providing estimates of a system's poles and of b) providing new pole estimates as each data sample is received. Our research in [3] has provided an algorithm which satisfies the above requirements. In this method, the system is parametrized directly in terms of its poles. Convergence analysis is provided and it is shown that the algorithm converges to the true parameters. The research has already provided some spinoff results in the form of new polynomial factorization algorithms which will be published at a forthcoming conference [4].

One of the common problems in system identification is that the noise characteristics may not be known or may vary from application to application. In [5] we have developed a new system identification algorithm that is robust against unknown noise properties. The method is an optimal min-max instrumental variable (IV) method that gives the smallest estimation error variance in the worst noise case from a prespecified class of noises. Implementation of the new method does not require knowledge of system or noise parameters and therefore can be done *without iteration*. It is useful for example for dead-beat controllers where there is no need for the noise parameters to be estimated. This method was applied successfully to both artificial and real data from gas furnace measurements.

## 2.2 Performance Analysis of Algorithms

Performance analysis includes derivation of the error covariance matrices of estimation algorithms and derivation of generic bounds on estimation accuracy. A common bound is the Cramér-Rao bound (CRB) which is a lower bound on the estimation error covariance matrix of any unbiased estimator. The CRB is useful for evaluating the quality of algorithms and for gaining insight into the problem of interest.

The main difficulty in deriving the covariance matrix of estimation algorithms stems from their estimation non-linearities. In [6] we have provided a statistical analysis of two non-linear least-squares estimators (NLSE's) of sine wave parameters in the colored noise case. These estimators are the basic NLSE, which ignores the possible correlation of the noise, and the optimal NLSE, which estimates the noise correlation (appropriately parametrized) as well as the sine-wave parameters. It is shown that these two NLS estimators have the *same accuracy* in large samples. This result provides complete justification for preferring the computationally less expensive basic NLSE over the "optimal" NLSE. Both estimators are shown to achieve the Cramér-Rao bound as the sample size increases. A simple explicit expression for the asymptotic CRB matrix is provided, which should be useful in studying the performance of sine-wave parameter estimators designed to work in the colored noise

case.

One of the well known methods for sine wave frequency estimation is the Pisarenko method (see corresponding reference in [7]). The method consists of determining the minimum eigenvalue of the data covariance matrix and its associated eigenfilter zeros. In [7] we provide a self-contained statistical analysis of this method. An explicit formula is provided for the asymptotic covariance matrix of the method. This covariance is then compared with the Cramér-Rao bound and the Yule-Walker method (see [7]). Our results show that both the Pisarenko and the Yule-Walker methods are quite inefficient in the statistical sense. Their error variances are much larger than the CRB. It is also shown that the variances of these two methods increase faster than the CRB when the signal-to-noise ratio decreases. Since both the Pisarenko and the Yule-Walker methods are quite inefficient statistically, they are attractive only when computational simplicity is a must.

### 3 Sensor Array Processing

Sensor array processing is a common name for several problems associated with array of sensors. The sensors may be electromagnetic antennas, microphones, geophones, underwater sonars, etc. These sensors are arranged in an arbitrary geometry to form a *sensor array*. The sensor array records signal waveforms of distant emitters. These in turn are used to estimate unknown emitter parameters, such as directions of arrivals, location, signal waveforms, etc. The great interest in sensor array processing stems from their wide applicability in radar, sonar, radio and microwave communication, seismology and hydroacoustics. An interesting note is that damped/undamped sinewave parameter estimation in noise turns out to be a special case of the sensor array processing problem.

The sensor output data is modeled as a superposition of individual wave forms. In the narrow-band case (i.e. when the power of all the emitter signals is in the same narrow frequency band) the sensor data model is:

$$y(t) = A(\theta)x(t) + e(t) \quad t = 1, 2, \dots, N \quad (1)$$

where  $\{y(t)\}$  are the observed sensor data vectors,  $\{x(t)\}$  are the unknown signal wave form vectors, and  $e(t)$  is an additive noise. The matrix  $A(\theta)$  models the characteristics of the array (e.g. geometry and sensor directivity). Based on the sensor observations  $\{y(t)\}$ , the parameters of interest ( $\theta$  and  $\{x(t)\}$ ) are estimated.

Two classes of methods for estimating the unknown parameters of (1) have received significant attention in recent years. The first is the multiple signal characterization (MUSIC) method which is based on the eigendecomposition of the data sample covariance matrix and is in fact an ad-hoc method. The second is the maximum likelihood (ML) which is based on statistical foundations.

In the following we summarize our research results in the area of sensor array processing on these algorithms and the corresponding CRB.

#### 3.1 Algorithm Development

The problem of estimating the direction of arrivals of far-field source wave forms by uniform linear arrays and the related one of estimating the parameters of multiple exponential signals

in noise is important in many applications of signal processing. As yet, no optimal estimation methods tailored to this problem with guaranteed convergence have been found since the estimation involves the difficult problem of maximizing the highly nonlinear likelihood function. Our research in [8] presents an ML solution to the problem. Using the properties of shift matrices we derive simple, exact, closed-form expressions for the gradient vector and the Hessian matrix. Based on these results, a Newton algorithm is presented for the estimation of source DOA's and multiple exponential signals in noise. The main advantages of this algorithm include ease of implementation with matrix software packages (such as MATLAB), as well as fast and guaranteed convergence to a local maximum of the likelihood function. A globally convergent ML algorithm is currently being developed.

### 3.2 Performance Analysis of Algorithms

Since the MUSIC is computationally simpler than the ML method, and since these methods are widely used, it has been of great interest and importance to evaluate the performance of these two methods, comparing them with one another and with the CRB.

Our paper [9] provides extensive studies of the performance of the MUSIC and ML methods, and analyzes their statistical efficiency. Closed-form expressions are derived for the MUSIC covariance matrix and the CRB for the estimation problem (1). It is shown that unless the emitter signals are uncorrelated (i.e. absence of multipaths) and the number of sensors is large, the MUSIC estimator cannot achieve the CRB. The relationship between the MUSIC and the ML is also investigated. It is shown that the MUSIC is a large sample (i.e. large  $N$ ) realization of the ML estimate if and only if the emitter signals are uncorrelated. The paper also contains results on the resolvability of the MUSIC and ML algorithms for the problem of finding the DOA's of plane waves using a uniform linear array.

One of the well known results in estimation theory is that the ML method achieves the CRB asymptotically. Our results prove that, for the problem (1), the ML method cannot achieve the CRB unless the number of sensors is very large, even if the number of data ( $N$ ) is large. This "shocking" result is explained in [9]. It raises the question: Do better estimators than the ML method exist for a finite number of sensors? This question is currently being investigated.

## 4 Future Outlook

Our research on algorithm development and analysis supported by this grant is rapidly growing. In the first year of this grant we have solved in [2]-[9] many important and difficult problems which currently exist in the signal processing and system identification areas. Most of these problems were not raised in the original proposal. However, there are many problems that we still have to address.

In the future we will continue the development and analysis of algorithms for signal processing and system identification. We will derive the ML covariance matrix for (1) and analyse methods other than MUSIC and the MLE, such as weighted MUSIC, ESPRIT and subspace rotation methods (SRM's). We will consider sensor array models that are more general than (1). For instance, models of near-field sources, correlated sensor noises, signals

modeled through their covariances rather than their amplitudes, and finite data models. We will develop new algorithms and analyse their performance for these problems. A globally convergent ML algorithm for (1) will be developed. Optimal methods that could outperform the ML method will be developed for a finite number of sensors and data samples. Bounds that are more appropriate than the CRB will be considered for the problems of closely spaced emitters and damped (transient) signals.

The support of the Air Force Office of Scientific Research has been essential for the research in [2]-[9] and is gratefully acknowledged. We hope that this support will continue beyond the next year and will include more students to solve more problems.

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