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# Improved Sea Surface Scattering Model

by

Gary S. Brown

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Scientific Officer: Dr. Ronald J. Repka

1400 North Wilson Boulevard

Arlington, Virginia 22209-2308

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This report develops an integral equation for the fluctuating zero mean part of the field scattered by a randomly rough perfectly conducting surface. The development starts with Magnetic Field Integral Equation for the current and then Fourier transforms this equation to obtain an integral equation for the scattered field in transform space. The method of smoothing is then used to develop an integral equation for the fluctuating part of the scattered field. The Born term in this integral equation of the second kind contains much more information than the classical Kirchhoff approximation. It is shown to reduce to the Kirchhoff approximation for gently sloping surface with large rms roughness and the Rice perturbation result for small surface roughness. This new Born term contains factors which appear to have the potential capability for going beyond existing approximations but further analytical and numerical work is required to quantify this hypothesis.

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## EXECUTIVE SUMMARY

The purpose of this work is to develop new approximate theories for scattering from randomly rough surfaces which are less restrictive than conventional approximations. The approach used was to apply the method of smoothing to the Fourier transformed Magnetic Field Integral Equation to obtain a new integral equation for the fluctuating part of the scattered field. This new integral equation contained a new Born term which was the sum of the transformed Kirchhoff approximation and a new term proportional to the transform of the fluctuating part of the surface propagator. This new Born term reduced to known asymptotic results in both the high and low frequency limits. Further study is required to fully understand how this new Born term increases the range of validity of our present approximations. However, it appears that it seems to remove the small slope approximation contained in the Rice approximation. If so, this result will greatly help in the understanding of scattering from small scale waves on the ocean surface.

Abstract

This report develops an integral equation for the fluctuating zero mean part of the field scattered by a randomly rough perfectly conducting surface. The development starts with Magnetic Field Integral Equation for the current and then Fourier transforms this equation to obtain an integral equation for the scattered field in transform space. The method of smoothing is then used to develop an integral equation for the fluctuating part of the scattered field. The Born term in this integral equation of the second kind contains much more information than the classical Kirchhoff approximation. It is shown to reduce to the Kirchhoff approximation for gently sloping surfaces with large rms roughness and the Rice perturbation result for small surface roughness. This new Born term contains factors which appear to have the potential capability for going beyond existing approximations but further analytical and numerical work is required to quantify this hypothesis.

Table of Contents

## 1.0 INTRODUCTION

Scattering of electromagnetic energy by randomly rough surfaces such as the ocean surface has many important applications. All of these are based on the link between the scattered field behavior and the character of the rough surface. In its most basic form, this link is established through the physics represented in Figure 1.1. The surface is illuminated by an incident electromagnetic field $\vec{E}_i$. This field induces a surface current $\vec{J}_s$ on the surface which reradiates the scattered field $\vec{E}_s$. The total field above the surface is $\vec{E}_s + \vec{E}_i$. The knowns in this problem are the incident field $\vec{E}_i$ and the <u>statistics</u> of the surface. It is important to note that it is the statistical characteristics of the surface which are known <u>and</u> not any or all realizations of the surface. The purpose of reviewing these basics is to point out that the best one can hope to solve for are the <u>statistical characteristics</u> of the scattered field.

### 1.1 Statement of the Problem

In order to calculate these characteristics of the scattered field, it is first necessary to treat the problem as deterministic. This is usually done by setting up an integral equation for the surface current induced by the incident field in terms of the incident field and the shape of the surface. Assuming that this integral equation can be solved, the resulting current weighted by a phase factor related to the surface height $\zeta(x,y)$ is then Fourier transformed with respect to the x and y surface coordinates to yield the far field portion of the scattered field. This part of the scattered field is adequate when the field is measured sufficiently far from the surface; a condition that is usually always met in practice. With this knowledge of the scattered field and its dependence

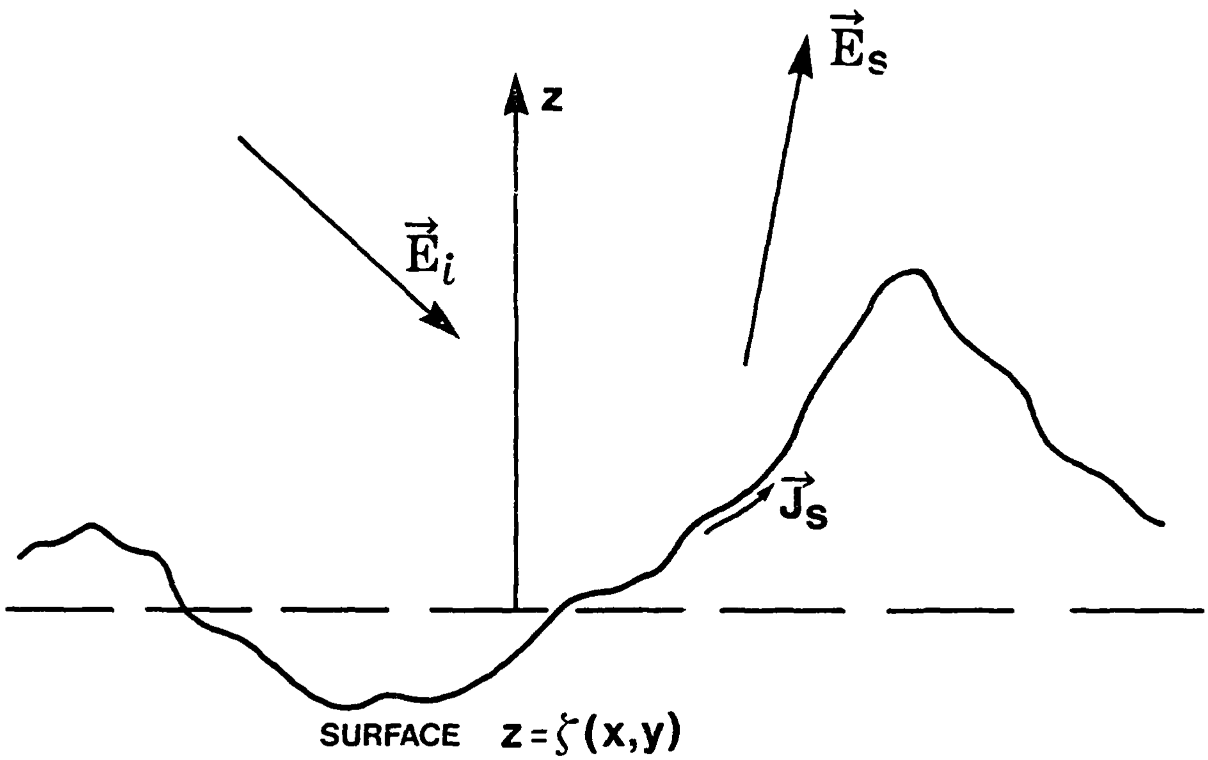


Figure 1.1. Representation of an incident field $\vec{E}_i$ impinging on a surface defined by $z=\zeta(x,y)$ and inducing a current $\vec{J}_s$ on the surface which reradiates the scattered field $\vec{E}_s$.

on the surface shape, fundamental theorems of probability theory can be used, in principle, to compute any statistical moment of the scattered field. Of course, what is really desired is the two-point probability density function (pdf) of the scattered field or even the less complicated single point pdf. The former provides estimates of how far one must move in space before the scattered field decorrelates. Unfortunately, neither of these pdfs are very easy to calculate because of the complicated manner in which a pdf is related to the functional dependence of $\vec{E}_s$ on the surface characteristics. Thus, even with detailed knowledge of how $\vec{E}_s$ depends on the shape of the surface, it may be practical to calculate only the first few statistical moments of the scattered field including its temporal and spatial decorrelation properties.

The description given above concerning calculating the statistical properties of the scattered field is predicated on the very important caveat that the integral equation for the induced surface current <u>can be solved</u>. This, of course, is not an easy task and, in fact, is the <u>major difficulty</u> in this entire problem. If the surface current density can be determined then all other quantities of interest can be obtained via straightforward numerical integrations. It is essential however that the integral equation for the current be solved to a known level of accuracy.

## 1.2 Possible Solutions

The number of options available to solving the integral equation for the current are rather limited. One can employ approximate techniques which are usually related to high or low frequency asymptotic methods. For example, Brown [1.1] has shown how the current induced on a perfectly conducting rough surface behaves in the high frequency limit while Holliday [1.2] obtained the behavior of the current in the low frequency limit from the same integral equation.

Although it has not been carried through to completion, it is not difficult to see how these two asymptotic analyses could be appropriately combined to obtained a first order estimate of the current induced on a surface having two scales of roughness. However, since such a combination will only produce a first order approximation, it is doubtful that it will generate results which are significantly different from other two scale theories such as in Brown [1.3]. There has been some recent work appear on modeling waves as wedges and estimating the resulting scattering via wedge diffraction [1.4]. However, this particular scattering mechanism appears to be important only near grazing incidence and it depends upon the statistics of wedge like waves which are still largely unknown. In general, it appears that approximate solutions for the induced surface current based on asymptotic methods have yielded about as much information and insight as they are capable of.

The other approach to solving the integral equation for the current is a pure numerical procedure wherein one uses the methods of moments to convert the integral equation to a matrix equation comprising discrete values of the current along the surface. Taking the Fourier transform of the current yields the field scattered by this particular realization of the surface. Using Monte Carlo methods to generate other realizations of the surface from a known density and spectrum, one can repeat this process until enough scattered field values are available to construct the desired statistical moments. While this is a very powerful method, it is computationally intensive and even with super computers it appears to be practical for two-dimensional (2D) surfaces only. Two dimensionally rough surfaces are ones where the surface height depends on one variable only; this is like a corduroy surface in which the period and height are random functions of position.

While the 2D surface computations do produce some useful results and insight, it is not clear how much of this translates to the more general 3D roughness case. Clearly, there is no accounting for some very important polarization effects such as depolarization or the conversion, by scattering, from one polarization state to another. This particular phenomenon is very important because it forms the basis for remotely sensing certain types of surface features. There are, in general, no guidelines for translating 2D scattering results into 3D interpretations and so it is necessary to consider solving the 3D problem via numerical means. Unfortunately, this dimensional extension is not trivial. This is because one must overcome the difficulty associated with the inversion of very large three dimensional matrices. Researchers working with scattering by finite size bodies have found that a purely numerical approach is not practical when the size of the body exceeds a few electromagnetic wavelengths [1.5]. This result does not bode well for the extended rough surface problem where the surface is necessarily very large compared to the electromagnetic wavelength.

Given this very practical limitation, what options are available for solving the general 3D roughness problem? One option is the so-called "Full Wave" approach of Bahar [1.6]. This method is one in which the mathematical approach is not at all obvious, the basic physics of the solution are obscure, and the first order approximate results are claimed to contain significantly more than similar results obtained by more obvious means. In short, the "Full Wave" method appears to raise more questions (as to its capabilities and limitations) than it answers. For this reason, the method, in its present state of development, does not appear to be an attractive alternative. Certainly this situation could change if the method were more carefully explained.

Another possible approach might entail a numerical solution of Maxwell's partial differential equations and associated boundary conditions by some form of mesh grid scheme. This technique is attracting attention in wave propagation problems where it makes maximum use of the computational capability of so called super computers. However, it is not completely clear how this approach will perform when there is a single arbitrarily shaped interface. It is not at all obvious that it is superior to the integral equation formulation and so this approach requires much more study.

The only alternative that appears to be practical is the development of a new integral equation which has a much more robust approximate solution. That is, while an exact solution of this alternate integral equation may be no more feasible than with the original integral equation, its approximate solution may be much more accurate over a wider range of parameters than that for the original equation.

## 1.3 Thrust of This Report

The purpose of this report is to develop a new integral equation such as described above. This new integral equation will be obtained by a sequence of very straightforward steps. The starting point will be the Magnetic Field Integral Equation (MFIE) for the surface current induced on a perfectly conducting surface by an incident field. The next step in the process is to Fourier transform the terms in this integral equation to obtain an integral equation for the scattered field; a reasonably well known $\vec{k}$–space integral equation formulation for the scattered field.

The next step entails the application of the method of smoothing to this integral equation. This amounts to splitting the scattered field into an average value and a zero mean fluctuating part. This apparently trivial decomposition has a number of very positive benefits. First, it permits the derivation of an integral equation for the fluctuating scattered field which is exactly what is needed. That is, there is no need for the average scattered field because it propagates away from the surface in the specular direction only. The second benefit is that all the effects of the mean scattered field and the mean propagator can be summed and this leads to a much more robust or informative Born term in this integral equation than in the original equation for the total (mean plus fluctuating parts) scattered field. It is exactly this Born term which is the desired result.

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## 2.0 DEVELOPMENT OF AN ALTERNATE INTEGRAL EQUATION

For this initial work, it is desirable to keep the scattering problem as simple as possible without obscuring the basic elements which are attributable to surface roughness. To accomplish this goal, the case of a perfectly conducting rough interface will be considered. Techniques developed to deal with this problem can also be applied to the imperfectly conducting interface [2.1] but the complication is that both electric and magnetic currents must be considered. In view of the very lossy nature of sea water, it may be possible to use approximate boundary conditions to simplify the lossy interface problem even further (with some additional work).

### 2.1 The Basic Integral Equation for the Current

For an interface which is perfectly conducting, there are two distinctly different kinds of integral equations for the surface current [2.2]. The first is derived from the continuity of the tangential electric field across the interface and it is called the Electric Field Integral Equation (EFIE). It is an integral equation of the first kind which does not lend itself to an obvious approximate solution. The second integral equation for the current is based on the discontinuity of the tangential magnetic field across the interface and so it is called the Magnetic Field Integral Equation (MFIE). It is an integral equation of the second kind whose first order approximate solution is outside of the integral term. Its specific form is as follows:

$$\vec{J}_s(\vec{r}) = \vec{J}_s^{\,i}(\vec{r}) + 2\hat{n}(\vec{r})\times\int_{S_o} \hat{J}_s(\vec{r}_o)\times\nabla_o g(|\vec{r}-\vec{r}_o|)dS_o \qquad (2.1)$$

where $\vec{J}_s$ is the unknown surface current induced on the surface $S_o$, and

$$\vec{J}_s^{i}(\vec{r}) = 2\hat{n}(\vec{r})\times\vec{H}^{i}(\vec{r}) \tag{2.2}$$

$$\hat{n} = (-\zeta_x\hat{x} - \zeta_y\hat{y} + \hat{z})/\sqrt{1+\zeta_x{}^2+\zeta_y{}^2} \tag{2.3}$$

and

$$g(|\vec{r}-\vec{r}_o|) = \exp(-jk_o|\vec{r}-\vec{r}|)/4\pi|\vec{r}-\vec{r}_o|.$$

In the above $\hat{n}(\vec{r})$ is the unit normal to the surface at the point $\vec{r} = x\hat{x} + y\hat{y} + \zeta(x,y)\hat{z}$ on the surface while $\vec{H}^{i}(\vec{r})$ is the incident magnetic field at the point $\vec{r}$. The symbols $\zeta_x$ and $\zeta_y$ signify the x and y–slopes of the surface, $k_o$ is the wavenumber for free space ($=2\pi/\lambda_o$), and g is the free space Green's function. Although the term $\vec{J}_s^{i}$ is frequently called the Born term, when one sets

$$\vec{J}_s \approx \vec{J}_s^{i} \tag{2.4}$$

this is known in electromagnetics as the Kirchhoff approximation.

The MFIE is an interesting integral equation because all indications point to the fact that the Kirchhoff approximation is not, in general, as accurate as one might expect [2.3]. That is, for surfaces containing many scales of roughness, the Kirchhoff approximation does a good job of predicting the scattered power about the specular direction but not in other directions. This is due largely to the very strong coherent effect of the phase of $\vec{H}^{i}$ in the specular direction and the concomitant lack of sensitivity to the surface structure comparable to $\lambda_o$. The reason this is mentioned here is that it provides a first clue as to what an alternate integral equation is going to have to do to improve on (2.5). That is, such an improvement is going to have to deemphasize the strong coherent effects of $\vec{J}_s^i$ in the specular direction and amplify the effects of the small scale surface structure in the off-specular directions. As will be shown, this is exactly what the alternate integral equation does.

## 2.2 The Integral Equation For The Scattered Field

An integral equation for the scattered field can be developed directly from (2.1) in the so called far field approximation. That is, if the point of observation of the scattered field is sufficiently far from the surface, it is easy to convert (2.1) into an equation for the scattered field. One first multiplies (2.1) by the factor

$$\sqrt{1 + (\nabla_t \zeta)^2} \exp(jk_{s_z} \zeta)$$

where

$$\nabla_t \zeta = \frac{\partial \zeta}{\partial x} \hat{x} + \frac{\partial \zeta}{\partial y} \hat{y}$$

and

$$k_{s_z} = k_o \cos \theta_s$$

where $\theta_s$ is the angle of the scattering direction relative to the normal to the mean surface (see Fig. 2.1). Eqn. (2.1) can then be written as follows;

$$\vec{J}(\vec{r}) = \vec{J}^i(\vec{r}) + 2\vec{N}(\vec{r}) \times \int \vec{J}(\vec{r}_o) \times \vec{G}(\vec{r}-\vec{r}_o)\, d\vec{r}_{t_o} \qquad (2.5)$$

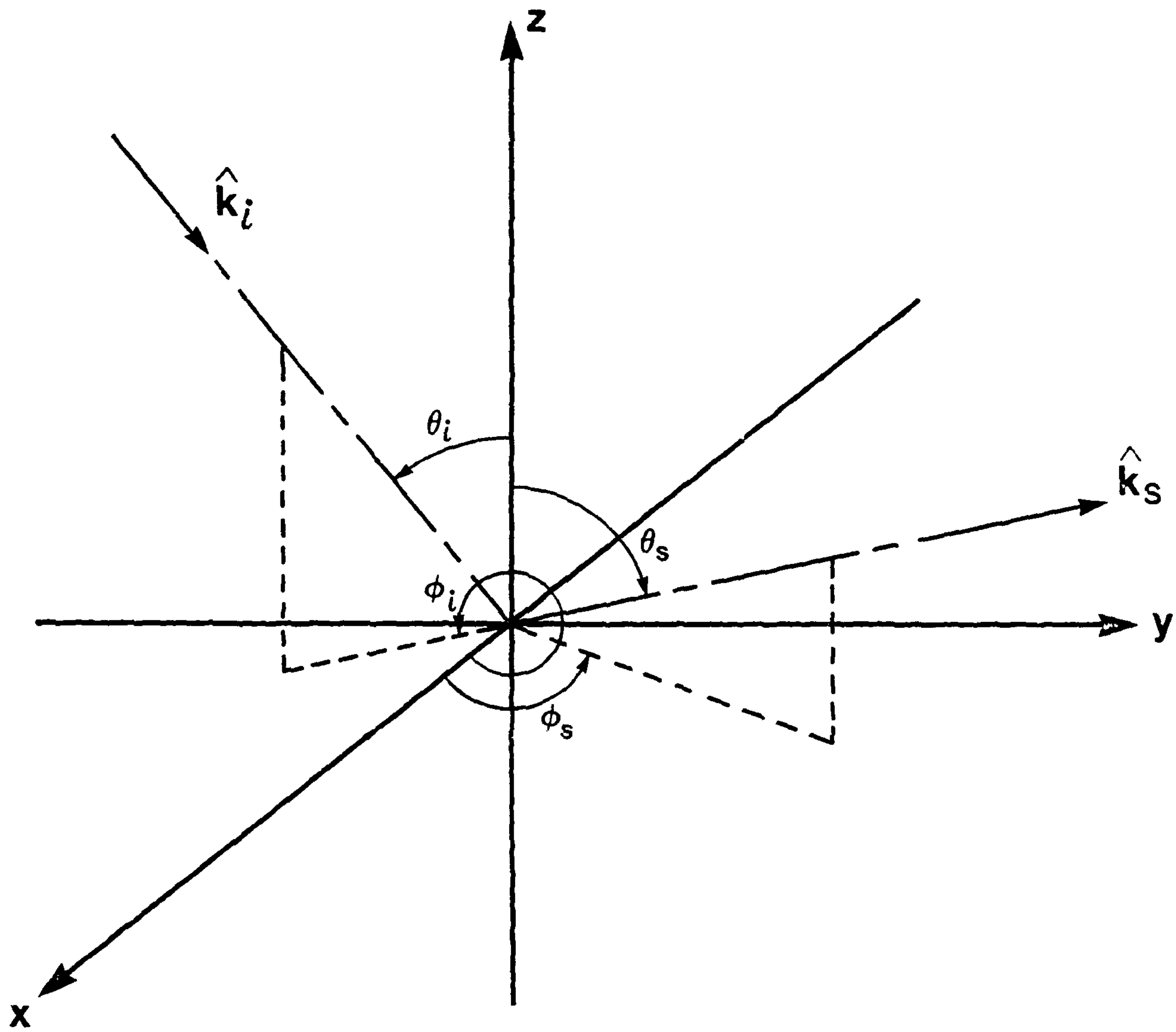


Figure 2.1. Incidence and scattering geometry

where

$$\vec{J}(\vec{r}) = \sqrt{1 + (\nabla_t \zeta)^2}\, \exp(jk_{s_z} \zeta)\, \vec{J}_s(\vec{r}) \tag{2.6}$$

$$\vec{J}^i(\vec{r}) = 2\vec{N}(\vec{r}) \times \vec{H}^i(\vec{r}) \exp(jk_{s_z} \zeta) \tag{2.7}$$

$$\vec{N}(\vec{r}) = -\nabla_t \zeta + \hat{z} \tag{2.8}$$

and

$$\vec{G}(\vec{r} - \vec{r}_o) = \left[\nabla_o g(|\vec{r} - \vec{r}_o|)\right] \exp\left[jk_{s_z}(\zeta - \zeta_o)\right] \tag{2.9}$$

The integration in (2.5) is now over the z=0 plane rather than the surface; this results from the conversion

$$dS_o = \sqrt{1 + (\nabla_t \zeta_o)^2}\, d\vec{r}_{t_o}$$

where $d\vec{r}_{t_o} = dx_o dy_o$.

The second step in the conversion process is to note that the current must be normal to the surface, i.e. $\vec{N} \cdot \vec{J} = 0$, or

$$J_z = \frac{\partial \zeta}{\partial x} J_x + \frac{\partial \zeta}{\partial y} J_y \tag{2.10}$$

Substituting this relationship in the right hand side of (2.5) leads to the following matrix equation for $J_x$ and $J_y$;

$$\bar{J}(\vec{r}) = \bar{J}^{i}(\vec{r}) + \int \bar{\bar{K}}(\vec{r}_t - \vec{r}_{t_o}, \zeta - \zeta_o, \nabla_t \zeta_o, \nabla_t \zeta) \cdot \bar{J}(\vec{r}_o)\, d\vec{r}_{t_o} \tag{2.11}$$

where

$$\bar{J} = \begin{bmatrix} J_x \\ J_y \end{bmatrix} \qquad \bar{\bar{K}} = \begin{bmatrix} \gamma_{xx} & \gamma_{xy} \\ \gamma_{yx} & \gamma_{yy} \end{bmatrix} \tag{2.12}$$

and

$$\gamma_{xx} = 2\left[\frac{\partial \zeta_o}{\partial x_o}\frac{\partial g}{\partial x} + \frac{\partial \zeta}{\partial y}\frac{\partial g}{\partial y} - \frac{\partial g}{\partial \zeta}\right] \cdot \exp\left[jk_{s_z}(\zeta - \zeta_o)\right] \tag{2.12a}$$

$$\gamma_{xy} = -2\left[\frac{\partial \zeta}{\partial y} - \frac{\partial \zeta_o}{\partial y_o}\right]\frac{\partial g}{\partial x} \cdot \exp\left[jk_{s_z}(\zeta - \zeta_o)\right] \tag{2.12b}$$

$$\gamma_{yy} = 2\left[\frac{\partial \zeta}{\partial x}\frac{\partial g}{\partial x} + \frac{\partial \zeta_o}{\partial y_o}\frac{\partial g}{\partial y} - \frac{\partial g}{\partial \zeta}\right] \cdot \exp\left[jk_{s_z}(\zeta - \zeta_o)\right] \tag{2.12c}$$

$$\gamma_{yx} = -2\left[\frac{\partial \zeta}{\partial x} - \frac{\partial \zeta_o}{\partial x_o}\right]\frac{\partial g}{\partial y} \cdot \exp\left[jk_{s_z}(\zeta - \zeta_o)\right] \tag{2.12d}$$

The argument of the propagator $\bar{\bar{K}}$ shows explicitly which variables it depends upon. The notation $\vec{r}_t - \vec{r}_{t_o}$ implies a dependence on $x-x_o$ and $y-y_o$, i.e. a dependence on the difference in the transverse (to z) variables. If (2.11) can be solved for $J_x$ and $J_y$ then $J_z$ can be found from (2.10).

The last step in the process of converting (2.1) into an integral equation comprises multiplying (2.11) by $\exp(j\vec{k}_t \cdot \vec{r}_t)$ and Fourier transforming the result with respect to x and y. The result is shown below

$$\bar{e}(\vec{k}_t) = \bar{e}^i(\vec{k}_t) + (2\pi)^{-2} \int \tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_o}) \cdot \bar{e}(\vec{k}_{t_o})\, d\vec{k}_{t_o} \tag{2.13}$$

where

$$\bar{e}(\vec{k}_t) = \int \bar{J}(\vec{r}) \exp(j\vec{k}_t \cdot \vec{r}_t) d\vec{r}_t \tag{2.14a}$$

$$\bar{e}^i(\vec{k}_t) = \int \bar{J}^i(\vec{r}) \exp(j\vec{k}_t \cdot \vec{r}_t) d\vec{r}_t \tag{2.14b}$$

and

$$\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_o}) = \int\int \tilde{\bar{\bar{K}}}(\vec{r}_t - \vec{r}_{t_o}, \zeta - \zeta_o, \nabla_t \zeta_o, \nabla_t \zeta)$$

$$\cdot \exp\left[ j(\vec{k}_t \cdot \vec{r}_t - \vec{k}_{t_o} \cdot \vec{r}_{t_o}) \right] d\vec{r}_t d\vec{r}_{t_o} \tag{2.14c}$$

The term in (2.14c) is obtained in the Appendix. If (2.13) can be solved for $\bar{e}(\vec{k}_t)$ then $e_z(\vec{k}_t)$ can be found from transforming (2.10), i.e.

$$e_z(k_t) = \tilde{\nabla}_t \zeta \otimes \bar{e}(\vec{k}_t) \tag{2.15}$$

where the tilde over $\nabla_t \zeta$ denotes its Fourier transform and $\otimes$ is the symbol for convolution.

A solution of (2.13) and (2.15) is sufficient to determine the far zone scattered field $\vec{E}_s$ because it is related to $\vec{e} = e_x \hat{x} + e_y \hat{y} + e_z \hat{z}$ via

$$\vec{E}_s = jk_o \eta_o g(R) \hat{k}_s \times \hat{k}_s \times \vec{e}(\vec{k}_{t_s}) \tag{2.16}$$

or, using a vector identity for the double cross product,

$$\vec{E}_s = -jk_o\eta_o g(R)\left\{\vec{e}(\vec{k}_{t_s}) - [\hat{k}_s\cdot\vec{e}(\vec{k}_{t_s})]\hat{k}_s\right\} \tag{2.17}$$

In the above relationships, $\eta_o$ is the characteristic impedance of free space, R is the distance from the origin on the surface to the point of observation, g is the free space Green's function, and $\vec{k}_s$ is

$$\vec{k}_s = k_o\hat{k}_s = k_o\left[\sin\theta_s\cos\phi_s\hat{x} + \sin\theta_s\sin\phi_s\hat{y} + \cos\theta_s\hat{z}\right] = \vec{k}_{t_s} + k_{s_z}\hat{z}$$

which is shown in Figure 2.1. Note that in order to determine $\vec{e}$ from (2.13) and 2.15), one must find it for <u>all values of $\vec{k}_t$</u>; however, only its value at $\vec{k}_t = \vec{k}_{t_s}$ is required to determine the scattered field.

## 2.4 Using Smoothing For The Fluctuating Scattered Field

Although (2.13) is not a well known result, it fails to provide the kind of new information in the Born term, i.e. $\bar{e}^i(\vec{k}_t)$, that we seek. This is because $\bar{e}^i$ is just the transform of the Kirchhoff approximation. Something additional must be done to convert (2.13) into an integral equation which contains a more informative Born term. To accomplish this task, the method of smoothing (MoS) will be used. This has previously been used to determine the average scattered field [2.4], but it will be applied here to find the fluctuating scattered field.

To simplify matters, (2.13) is written in a more compact form, i.e.

$$\bar{e}(\vec{k}_t) = \bar{e}^i(\vec{k}_t) + L\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_o}) \cdot \bar{e}(\vec{k}_{t_o}) \qquad (2.18)$$

where L is the integral operator

$$L = (2\pi)^{-2} \int (\cdot) d\vec{k}_{t_o}$$

The key element in the Method of Smoothing is to split $\bar{e}$ and all of the other quantities in (2.18) into the sum of an average part and a zero mean fluctuating part, e.g.

$$\bar{e} = \langle\bar{e}\rangle + \delta\bar{e} \qquad (2.19)$$

where $\langle\bar{e}\rangle$ is the average part and $\delta\bar{e}$ is the fluctuating part and such that

$$\langle\delta\bar{e}\rangle = 0$$

Decomposing all of the other functions in (2.18) in a similar manner leads to the following result:

$$\begin{aligned}\langle\bar{e}\rangle + \delta\bar{e} = \langle\bar{e}^i\rangle + \delta\bar{e}^i + L\langle\tilde{\bar{\bar{K}}}\rangle\langle\bar{e}\rangle + L\langle\tilde{\bar{\bar{K}}}\rangle\delta\bar{e} \\ + L\delta\tilde{\bar{\bar{K}}}\langle\bar{e}\rangle + L\,\delta\tilde{\bar{\bar{K}}}\,\delta\bar{e}\end{aligned} \qquad (2.20)$$

Applying the fluctuating operator, $1-\langle\ \rangle$, to both sides of (2.20) yields

$$\delta\bar{e} = \delta\bar{e}^i + L\langle\tilde{\bar{\bar{K}}}\rangle\delta\bar{e} + L\,\delta\tilde{\bar{\bar{K}}}\langle\bar{e}\rangle + (1-P)L\,\delta\tilde{\bar{\bar{K}}}\,\delta\bar{e} \qquad (2.20a)$$

where P = < > is the averaging operator. There are some important simplifications that occur in (2.20a). From (2.14c), the average of $\tilde{\bar{\bar{K}}}$ can be written as follows:

$$<\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_o})> = \int\int <\bar{\bar{K}}(\vec{r}_t - \vec{r}_{t_o}, \zeta - \zeta_o, \nabla_t\zeta, \nabla_t\zeta_o)>$$

$$\cdot \exp\left[j(\vec{k}_t \cdot \vec{r}_t - \vec{k}_{t_o} \cdot \vec{r}_{t_o})\right] d\vec{r}_t \, d\vec{r}_{t_o} \qquad (2.21)$$

When $\bar{\bar{K}}$ is averaged, all of the dependence upon $\zeta$, $\zeta_o$, $\nabla_t\zeta$, and $\nabla_t\zeta_o$ disappears. What is left in $<\bar{\bar{K}}>$ is a dependence on $\vec{r}_t - \vec{r}_{t_o}$ only; this is a consequence of the form of $\bar{\bar{K}}$ and the assumed statistical homogeneity of the surface. Thus, (2.21) can be written as follows;

$$<\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_o})> = \int\int <\tilde{\bar{\bar{K}}}(\vec{r}_t, -\vec{r}_{t_o})>$$

$$\exp\left[j(\vec{k}_t \cdot \vec{r}_t - k_{t_o} \cdot \vec{r}_{t_o})\right] d\vec{r}_t \, d\vec{r}_{t_o} \qquad (2.22)$$

With

$$\Delta\vec{r}_t = \vec{r}_t - \vec{r}_{t_o},$$

(2.21) can be rewritten as

$$<\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_o})> = \int\int <\bar{\bar{K}}(\Delta\vec{r}_t)> \exp(j\vec{k}_t \cdot \Delta\vec{r}_t)$$

$$\cdot \exp\left[j(\vec{k}_t - \vec{k}_{t_o}) \cdot \vec{r}_{t_o}\right] d\Delta\vec{r}_t \, d\vec{r}_{t_o} \qquad (2.23)$$

which becomes

$$<\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_o})> = (2\pi)^2 \, \delta\,(\vec{k}_t - \vec{k}_{t_c}) \, <\tilde{\bar{\bar{K}}}(\vec{k}_t)> \qquad (2.24)$$

where

$$(2\pi)^2 \delta(\vec{k}_t - \vec{k}_{t_o}) = \int \exp\left[j(\vec{k}_t - \vec{k}_{t_o}) \cdot \vec{r}_{t_o}\right] d\vec{r}_{t_o}$$

Using this result gives

$$L<\tilde{\bar{\bar{K}}}> \delta\,\bar{e} = <\tilde{\bar{\bar{K}}}(\vec{k}_t)> \cdot \int \delta\,(\vec{k}_t - \vec{k}_{t_o}) \, \delta\,\bar{e}\,(\vec{k}_{t_o}) d\vec{k}_{t_o}$$

or

$$L<\tilde{\bar{\bar{K}}}> \delta\,\bar{e} = <\tilde{\bar{\bar{K}}}(\vec{k}_t)> \cdot \delta\,\bar{e}\,(\vec{k}_t) \qquad (2.25)$$

From earlier work with the smoothing method [2.4], it is known that $<\bar{e}>$ can be written as

$$<\bar{e}(\vec{k}_{t_o})> = (2\pi)^2 \, \vec{a}(k_{s_z}) \, \delta\,(\vec{k}_{t_o} - \vec{k}_{t_i}) \qquad (2.26)$$

where $\vec{a}(k_{s_z})$ is the complex vector amplitude of the average scattered field and

$$\vec{k}_{t_i} = k_o\left[-\sin\theta_i \cos\phi_i \hat{x} - \sin\theta_i \sin\phi_i \hat{y}\right]$$

Using (2.26) leads to

$$L\,\delta\tilde{\bar{\bar{K}}} \cdot \langle\bar{e}\rangle = \vec{a}(k_{s_z}) \cdot \delta\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_i}) \tag{2.27}$$

Combining (2.26) and (2.27) in (2.20) yields

$$\delta\,\bar{e}(\vec{k}_t) = \left\{1-\langle\tilde{\bar{\bar{K}}}(\vec{k}_t)\rangle\right\}^{-1}\left[\delta\,\bar{e}^{i}(\vec{k}_t) + \vec{a}(k_{s_z}) \cdot \delta\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_i})\right]$$

$$+ \left\{1-\langle\tilde{\bar{\bar{K}}}(\vec{k}_t)\rangle\right\}^{-1}(1-P)\,L\,\delta\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_o}) \cdot \delta\,\bar{e}(\vec{k}_{t_o}) \tag{2.28}$$

which is the desired result.

## 2.5 Discussion of The Resulting Integral Equation

(2.28) is an integral equation of the second kind in transform space for the fluctuating part of the scattered field. In order to solve it, one must know the average scattered field, i.e. the $\vec{a}(k_{s_z})$ in (2.26). However, as shown in [2.4], $\vec{a}(k_{s_z})$ is itself the solution of an integral equation. The generation of separate integral equations for both the average and the fluctuating parts of the scattered field is the real power of the method of smoothing. The reason this is important is that the separate integral equations usually lead to more robust approximations than first making an approximation in (2.13) and then taking the average and fluctuating parts of the approximation. With the latter approach, it is seldom clear how good the resulting average and fluctuating approximations are. With the smoothing approach and the resulting separate integral equations, it is much easier to estimate the validity of a given approximation.

There is nothing in the form of (2.28) to suggest that it is, any general, an easier to solve than (2.13). However, it does contain a more informative Born term than (2.13).

### 2.5.1 Significance of The Born Term

The fluctuating part of the Born term in (2.13) is the fluctuating part of the Kirchhoff approximation. As discussed earlier, this is a very restrictive approximation. The Born term in (2.28) contains more information than just the fluctuating part of the Kirchhoff approximation. The factor $\delta\,\bar{e}^{i}(\vec{k}_t)$ with $\vec{k}_t = \vec{k}_{t_s}$ is the Kirchhoff term. The factor $\vec{a}(k_{s_z}) \cdot \delta\,\tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_i})$ with $\vec{k}_t = \vec{k}_{t_s}$ appears to bring about some accounting for the small scale structure on the surface. In fact, when it is combined with $\delta\,\bar{e}^{i}(\vec{k}_{t_s})$, the sum leads to the Rice approximation to order $k_o^{\,2}\langle\zeta^2\rangle$. This follows from the form of the sum, the fact that

$$\delta\,\tilde{\bar{\bar{K}}}(\vec{k}_{t_s}, -\vec{k}_{i_t}) \rightarrow \tilde{\bar{\bar{K}}}(\vec{k}_{t_s}, -\vec{k}_{i_t})$$

for small surface roughness, and the work of Holliday [2.5]. Thus, the Born term in (2.28) contains the appropriate behavior for small surface roughness.

As the roughness becomes large, the second term will become small due to the decrease of the mean scattered field, i.e. $\vec{a}(k_{s_z})$ decays almost exponentially with increasing roughness. Furthermore, if the slopes are sufficiently small then $\langle \tilde{\bar{\bar{K}}}(\vec{k}_{t_s})\rangle$ should also be small. Thus, for a surface having large roughness height, $\langle \zeta^2 \rangle$, but small slopes, $\langle (\nabla_t \zeta)^2 \rangle$, the Born term in (2.28) reproduces the proper Kirchhoff limit.

For surface conditions other than the above, it is necessary to study the behavior of both $\langle \tilde{\bar{\bar{K}}} \rangle$ and $\delta \tilde{\bar{\bar{K}}}$ because they will obviously bear on the behavior of the Born term in (2.28). The factor involving $\langle \tilde{\bar{\bar{K}}} \rangle$ appears to be particularly important because it involves a degree of apparent multiple scattering on the surface. If one traces this term back to its source, it comes from the interaction of the current on one part of the surface with the current at another part through the effects of the average propagator $\langle \bar{\bar{K}} \rangle$. While this appears to be multiple scattering relative to the current, (2.28) shows that it has a single scattering effect on the scattered field. This is the first time that a term of this form has been identified in rough surface scattering.

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Appendix

The purpose of this Appendix is to obtain the Fourier transform of the integral term in (2.11). The Fourier transform of the integral term can be written as

$$I = \int\int \bar{\bar{K}}(\vec{r}_t, -\vec{r}_{t_o}) \cdot \vec{J}(\vec{r}_o) \exp(j\vec{k}_t \cdot \vec{r}_t) d\vec{r}_{t_o}\, d\vec{r}_t \qquad \text{(A-1)}$$

where we have been less specific in showing the argument of $\bar{\bar{K}}$ than in (2.11). Since only $\bar{\bar{K}}$ depends on $\vec{r}_t$, (A-1) can be written as

$$I = \int \underset{\sim}{\bar{\bar{K}}}(\vec{k}_t, \vec{r}_{t_o}) \cdot \vec{J}(\vec{r}_o)\, d\vec{r}_{t_o} \qquad \text{(A-2)}$$

Using Parseval's theorem in two dimensions, this integral can be written as follows;

$$\int \underset{\sim}{\bar{\bar{K}}}(\vec{k}_t, \vec{r}_{t_o}) \cdot \vec{J}(\vec{r}_o)\, d\vec{r}_{t_o} = (2\pi)^{-2} \int \tilde{\bar{\bar{K}}}(\vec{k}_t, -\vec{k}_{t_o}) \cdot \tilde{\vec{J}}(\vec{k}_{t_o}) d\vec{k}_{t_o} \qquad \text{(A-3)}$$

which is the desired result because $\tilde{\vec{J}} = \vec{e}$.

## 3.0 CONCLUSIONS AND RECOMMENDED FUTURE WORK

The purpose of this work was to derive a new integral equation for the average scattered field which would contain a more robust Born term. This goal has been achieved in the form of (2.28). While the new integral equation does not, in general, appear to be any more soluable than previous integral equations, it does contain a much more informative Born term. This term reproduces both the Kirchhoff and the Rice perturbation limits and appears to have the capability to extend beyond these asymptotic limits. In fact, this new Born term has the potential to expand our existing approximate results to the point where further numerical work on the full 3–D integral equation may not be necessary.

In order to fully exploit this new result, further analytical and numerical work is required. The analytical work is needed in order to understand the physics behind each of the factors in the Born term in (2.28); that is, their source and meaning. Such insight is essential to estimating just how far the Born term can be pushed before it becomes an inadequate approximation.

Numerical work is needed to determine the influence of the average and fluctuating parts of the propagator $\tilde{\bar{\bar{K}}}$. Since their influence represents the new parts of the Born term in (2.28), this numerical work is essential. As a first cut at the numerics, the 2–D form of the Born term in (2.28) should be compared to some of the numerical solutions of the MFIE which have been obtained in 2–D. This would provide some guidelines as to when the new Born term is accurate. The evaluation of this term in 3–D is more complicated and will require more time.