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RING SOURCE/FORCE EXCITATION OF AXISYMMETRIC SHELL STRUCTURE

BY

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Summary

Formulae are given for the near field pressures, far field pressures and acoustic particle displacements of a ring source and a ring axial force. These quantities are necessary ingredients for calculations of far field radiation, in the presence of the scattering action of an axisymmetric shell structure. Numerical examples are given.

16 pages
3 figures

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1. INTRODUCTION

Previous work [1] has presented a formulation for calculating numerically the far field scattering by a finite elastic axisymmetric shell structure which is subject to general external acoustical excitation, field quantities being represented by Fourier series expansions in the circumferential coordinate. Specifically, the work shows that if expressions are available for the pressure, particle displacements and far field pressure of the excitation, in the absence of the shell, then the necessary ingredients are present for numerical calculations of the far field pressure in the presence of the shell. Time-harmonic plane wave excitation, the basis of most scattering studies, demonstrated the usefulness of the method.

In many hydroacoustics problems the excitation is located close to the shell's surface: it may be of monopole type due to cavitation sources, dipole type due to fluctuating fluid forces or quadrupole type due to turbulence sources. Herein the excitation cases of an axial ring source and an axial ring force are analysed. They model, in a very simple way, propulsor generated cavitation and fluctuating thrust in the surrounding fluid. The presence of a shell is important because interaction between the source/force and a resonant scatterer may result in considerable amplification or attenuation of the sound field in certain spatial directions.

2. RING SOURCE/FORCE EXCITATION

(a) General

Figure 1 shows an axisymmetric elastic shell which is excited by a time-harmonic ring source or axial force. In order to predict the far field sound radiation using a coupled finite element and Helmholtz integral equation formulation described elsewhere [1] it is first necessary to calculate, in the absence of the shell, the near field pressures, the near field particle displacements and the far field pressures.

(b) Free-Space Pressure

The free-field pressures of an off-axis unit point source and axial force, in a fluid of sound velocity c and density ρ, are well known, viz.

$$\begin{aligned} p_s(r,\phi,z) &= \exp(ikD_0)/D_0 \\ p_f(r,\phi,z) &= -(1/4\pi)(\partial/\partial z)[\exp(ikD_0)/D_0] \end{aligned} \tag{2.1}$$

where

$$D_0^2 = r^2+r_0^2+(z-z_0)^2-2rr_0\cos(\phi-\phi_0) \tag{2.2}$$

is the square of the distance between the source/force located at (r_0,ϕ_0,z_0) and the observation point located at (r,ϕ,z): 'k' is the acoustic wavenumber, ω/c. Time variation $\exp(-i\omega t)$ is omitted throughout.

Expand these pressures as Fourier series in the circumferential coordinate difference $\phi-\phi_0$, viz.

$$p_s(r,\phi,z) = \Sigma_{n=0}^{\infty} p_{sn}(r,z)\cos[n(\phi-\phi_0)]$$
$$p_f(r,\phi,z) = \Sigma_{n=0}^{\infty} p_{fn}(r,z)\cos[n(\phi-\phi_0)] \tag{2.3}$$

where

$$p_{sn}(r,z) = (e_n/2\pi)\int_0^{2\pi} [\exp(ikD_0)/D_0]\cos[n(\phi-\phi_0)]d\phi \tag{2.4}$$

$$p_{fn}(r,z) = -(1/4\pi)(e_n/2\pi)\int_0^{2\pi} (\partial/\partial z)[\exp(ikD_0)/D_0]\cos[n(\phi-\phi_0)]d\phi$$

in which e_n=1 if n=0 and e_n=2 otherwise. Using the periodic property of the cosine function it is not difficult to show that these equations are independent of ϕ_0. Thus

$$p_{sn}(r,z) = (e_n/\pi)\int_0^{\pi} [\exp(ikD)/D]\cos(n\psi)d\psi \tag{2.5}$$

$$p_{fn}(r,z) = -(1/4\pi)(e_n/\pi)\int_0^{\pi} (\partial/\partial z)[\exp(ikD)/D]\cos(n\psi)d\psi$$

where

$$D^2 = r^2+r_0^2+(z-z_0)^2-2rr_0\cos(\psi) \tag{2.6}$$

Consider a ring source/force of radius r_0 and amplitude $B(\phi_0)/2\pi r_0$ per unit length of the circumference: the pressure fields may be synthesized as

$$p_s(r,\phi,z) = \Sigma_{n=0}^{\infty} a_n p_{sn}(r,z)\cos(n\phi)$$
$$+ \Sigma_{n=0}^{\infty} b_n p_{sn}(r,z)\sin(n\phi) \tag{2.7}$$
$$p_f(r,\phi,z) = \Sigma_{n=0}^{\infty} a_n p_{fn}(r,z)\cos(n\phi)$$
$$+ \Sigma_{n=0}^{\infty} b_n p_{fn}(r,z)\sin(n\phi)$$

where

$$a_n = \int_0^{2\pi} [B(\phi_0)/2\pi r_0]\cos(n\phi_0)r_0 d\phi_0$$

$$(2.8)$$

$$b_n = \int_0^{2\pi} [B(\phi_0)/2\pi r_0]\sin(n\phi_0)r_0 d\phi_0$$

Specializing to the case of constant force/source amplitude, per unit length, gives the axisymmetric contribution from the n=0 harmonic alone, viz.

$$p_s(r,z) = B(1/\pi)\int_0^{\pi} [\exp(ikD)/D]d\psi$$

$$(2.9)$$

$$p_f(r,z) = (-B/4\pi)(1/\pi)\int_0^{\pi} (\partial/\partial z)[\exp(ikD)/D]d\psi$$

As a consistancy check let $r_0 \to 0$ in equations (2.1) and (2.9) to give identical results for the pressure of a point force/source on the axis of symmetry.

(c) Derivatives

The derivatives required in the analysis are obtained by application of the formulae

$$\partial/\partial z = (\partial D/\partial z)(d/dD) = [(z-z_0)/D](d/dD)$$

$$\partial/\partial r = (\partial D/\partial r)(d/dD) = [(r-r_0\cos(\psi))/D](d/dD) \qquad (2.10)$$

$$(d/dD)[\exp(ikD)/D] = [ik/D-1/D^2]\exp(ikD)$$

giving

$$(\partial/\partial z)[\exp(ikD)/D] = (z-z_0)(ik/D^2-1/D^3)\exp(ikD)$$

$$(\partial/\partial r)[\exp(ikD)/D] = [(r-r_0\cos(\psi))](ik/D^2-1/D^3)\exp(ikD)$$

$$(2.11)$$

$$(\partial^2/\partial z^2)[\exp(ikD)/D] =$$
$$(ik/D^2-1/D^3)\exp(ikD) + (z-z_0)^2(-3ik/D^4+3/D^5-k^2/D^3)\exp(ikD)$$

$$(\partial^2/\partial r\partial z)[\exp(ikD)/D] =$$
$$(z-z_0)(r-r_0\cos(\psi))(-3ik/D^3+3/D^4-k^2/D^2)\exp(ikD)$$

(d) Acoustic Particle Displacements

The acoustic particle displacement, in the direction normal to the elastic surface, of a pressure field p is

$$W(r,\phi,z) = [\cos(\beta)\partial p/\partial r - \sin(\beta)\partial p/\partial z]/\rho\omega^2 \tag{2.12}$$

where β is the normal angle shown in Figure 1. The displacements for the source and force excitation are found by replacing 'p' by p_s and p_f of equations (2.7).

For the free field ring source the required derivatives of the Fourier amplitudes are

$$\begin{aligned}\partial p_{sn}/\partial r &= (e_n/\pi)\int_0^\pi (\partial/\partial r)[\exp(ikD)/D]\cos(n\psi)d\psi \\ \partial p_{sn}/\partial z &= (e_n/\pi)\int_0^\pi (\partial/\partial z)[\exp(ikD)/D]\cos(n\psi)d\psi\end{aligned} \tag{2.13}$$

and for the free field ring force they are

$$\begin{aligned}\partial p_{fn}/\partial r &= (-1/4\pi)(e_n/\pi)\int_0^\pi (\partial^2/\partial z\partial r)[\exp(ikD)/D]\cos(n\psi)d\psi \\ \partial p_{fn}/\partial z &= (-1/4\pi)(e_n/\pi)\int_0^\pi (\partial^2/\partial z^2)[\exp(ikD)/D]\cos(n\psi)d\psi\end{aligned} \tag{2.14}$$

Equations (2.11) give derivatives of exp(ikD)/D.

(e) <u>Far Field Pressure</u>

The distance factor 'D', given by equation (2.6), is written as

$$\begin{aligned}D &= (r^2+z^2+r_0^2+z_0^2-2zz_0-2rr_0\cos(\psi))^{1/2} \\ &= R[1+(r_0^2+z_0^2)/R^2-2zz_0/R^2-2rr_0\cos(\psi)/R^2]^{1/2}\end{aligned} \tag{2.15}$$

In the acoustic far field, $R\to\infty$, the second term is neglected, giving to a first order of accuracy

$$D = R(1-z_0\cos(\theta)-r_0\sin(\theta)\cos(\psi)) \tag{2.16}$$

Substitute equation (2.16) into equations (2.5) to give the approximations,

$$\begin{aligned}p_{sn}(R,\theta) &= (e_n/\pi)[\exp(ikR)/R]\exp(-ikz_0\cos(\theta)) \times \\ &\quad \int_0^\pi \exp(-ikr_0\sin(\theta)\cos(\psi))\cos(n\psi)d\psi \\ p_{fn}(R,\theta) &= (e_n/\pi)[\exp(ikR)/R]\exp(-ikz_0\cos(\theta)) \times \\ &\quad (-ik/4\pi)\cos(\theta)\int_0^\pi \exp(-ikr_0\sin(\theta)\cos(\psi))\cos(n\psi)d\psi\end{aligned} \tag{2.17}$$

The integral can be evaluated as it is a standard definition of the Bessel function

$$J_n(x) = (i^{-n}/\pi)\int_0^{\pi} \exp(ix.\cos(\psi))\cos(n\psi)d\psi \tag{2.18}$$

Thus

$$p_{sn}(R,\theta) = e_n[\exp(ikR)/R]\exp(-ikz_0\cos(\theta))i^n J_n(kr_0\sin(\theta))$$

$$p_{fn}(R,\theta) = (-ik/4\pi)\cos(\theta)\ X \tag{2.19}$$

$$e_n[\exp(ikR)/R]\exp(-ikz_0\cos(\theta))i^n J_n(kr_0\sin(\theta))$$

The far field pressures for general amplitude variation of the excitation are then

$$p_s(R,\theta,\phi) = \Sigma_{n=0}^{\infty}\ a_n p_{sn}(R,\theta)\cos(n\phi)$$

$$+\ \Sigma_{n=0}^{\infty}\ b_n p_{sn}(R,\theta)\sin(n\phi) \tag{2.20}$$

$$p_f(R,\theta,\phi) = \Sigma_{n=0}^{\infty}\ a_n p_{fn}(R,\theta)\cos(n\phi)$$

$$+\ \Sigma_{n=0}^{\infty}\ b_n p_{fn}(R,\theta)\sin(n\phi)$$

where a_n and b_n are again defined by equations (2.8). Far field pressures for constant amplitude of the excitation are those of the n=0 harmonic alone, viz.

$$p_s(R,\theta) = B[\exp(ikR)/R]\exp(-ikz_0\cos\ \ \ J_0(kr_0\sin(\theta))$$

$$p_f(R,\theta) = (-ikB/4\pi)\cos(\theta)\ X \tag{2.21}$$

$$[\exp(ikR)/R]\exp(-ikz_0\cos(\theta))J_0(kr_0\sin(\theta))$$

In the low frequency regime $kr_0 \ll 1$ the Bessel function may be replaced by its value for a small argument. In this case the far field pressure is proportional to $(kr_0)^n$ which is independent of the ring radius when n=0, and drops rapidly as 'n' increases. However, the near field pressure and hence the excitation are usually strongly dependent on the radius.

3. NUMERICAL TESTS

A Fortran program [1] for calculating far field scattered pressure when the excitation is a plane wave at an arbitrary incidence has been modified to calculate far field pressure when the excitation is a ring source or axial force. As a check on the program consider the case of a spherical steel shell which is excited by a point axial force. The following material and geometric constants in SI units were used for the calculations:

Young's modulus 19.5×10^{10}
Poisson's ratio 0.29
Density 7700
Thickness 0.01
Radius 1.0
Hysteretic loss-factor 0.01
Sound velocity in fluid 1500
Density of fluid 1000
Distance force from shell 0.5

Numerical results of far field pressure, dB ref. 1 micropascal at 1m, are shown in Figure 2. The shell was idealized by 41 conical shell elements. At low frequencies there is excellent agreement with results obtained from a closed-form solution [2]: as the frequency increases the agreement worsens, which is typical of results obtained by the finite element method. Agreement at high frequencies could, no doubt, be improved by doubling the number of elements used in the idealization, but this would be a formidable computational task. Compare the numerical results with those of the point force alone; the presence of the shell has strongly enhanced the radiation in the mid frequency regime. The only differences worthy of note between results at $\theta=0$ (A plot) and $\theta=180$ (B plot) are the latter showing more pronounced radiation at the 340Hz resonance, and reduced radiation above 400Hz because the observation point lies in the shadow zone.

As the source approaches the shell numerical results (not shown) become more and more inaccurate. This is because a linear variation in pressure over an element, assumed in the Helmholtz integral equation discretization [1], is not suitable for representing a surface pressure whose spatial variation becomes increasingly like a delta function. Numerical checks, such as doubling the number of elements, are thus essential.

As an extra demonstration of the capabilities of the Fortran program, Figure 3 shows far field sound radiation when the elastic scatterer is a prolate spheroidal shell of length 6m and width 2m, the ring of radius 1m being located 0.5m aft. Other constants are the same as for the spherical shell. The point of observation is forward of the shell, viz. $\theta=180$. In Figure 3A the excitation is a ring force; resonant enhancement is absent, but there are frequency regions where the combined radiation of the ring and shell cancel. In Figure 3B the excitation is a ring source; again sharp resonant enhancement is absent, but there is a region centred on 250Hz where broad-band enhancement of up to 5dB occurs.

The sharper resonant enhancement of the spherical shell may be due in part to its modes having a smaller radiation efficiency than the modes of the prolate spheroidal shell. Thus the spherical shell response is not limited so much by radiation damping. According to Junger & Feit [3] inefficiently radiating resonances may in certain circumstances radiate more acoustic power than their efficiently radiating counterparts.

4. CONCLUSIONS

The mathematics of the free-space acoustic pressure of a ring source and a ring force have been presented in a form which facilitates computations of far field sound radiation in the presence of an axisymmetric shell structure. Numerical results for specific spherical and prolate spheroidal shells show that the former amplifies the radiation, at resonance, much more than does the latter.

The programs can be used to predict far field sound radiation from branched shells with/without rib stiffening, and this may be the subject of a future investigation.

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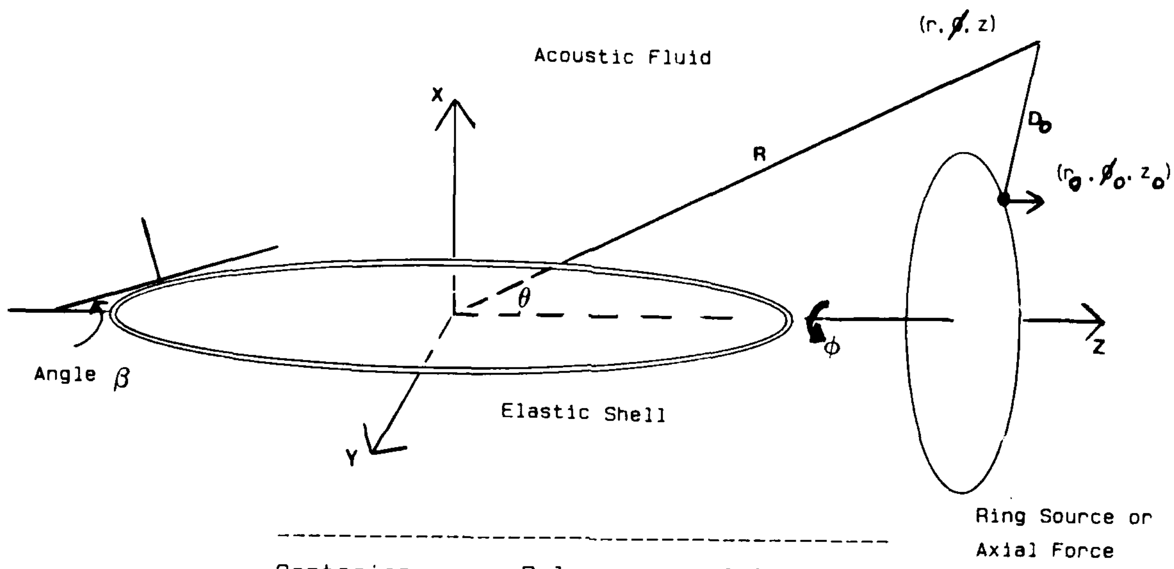

Cartesian	Polar	Cylindrical
x	R.sin (θ) cos (∅)	r.cos (∅)
y	R.sin (θ) sin (∅)	r.sin (∅)
z	R.cos (θ)	z

FIG. 1 RING SOURCE OR AXIAL RING FORCE EXCITATION OF AXISYMMETRIC SHELL STRUCTURE

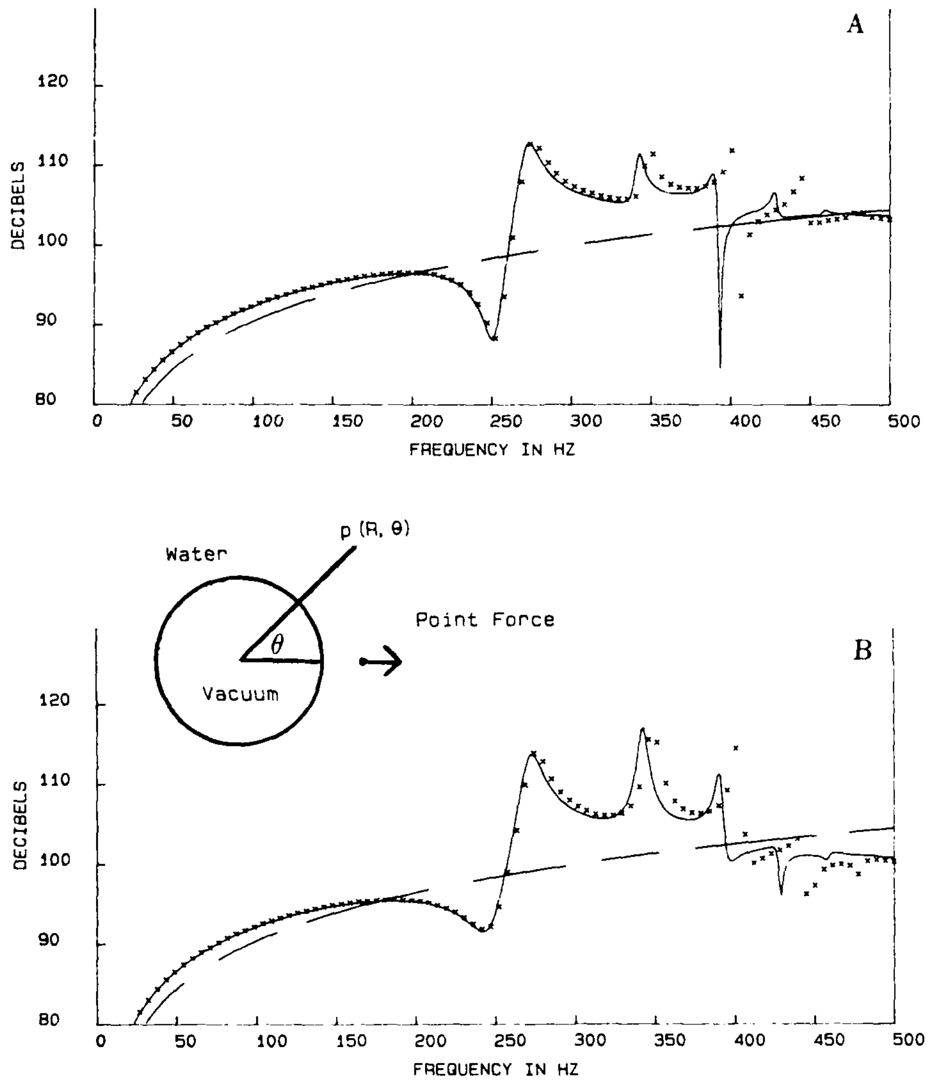

FIG. 2 Far field radiation. Spherical shell excited by an exterior unit point force (a) θ=0.0 (b) θ=180
x x finite element and Helmholtz integral equation;
—— 'exact'; — — no shell.

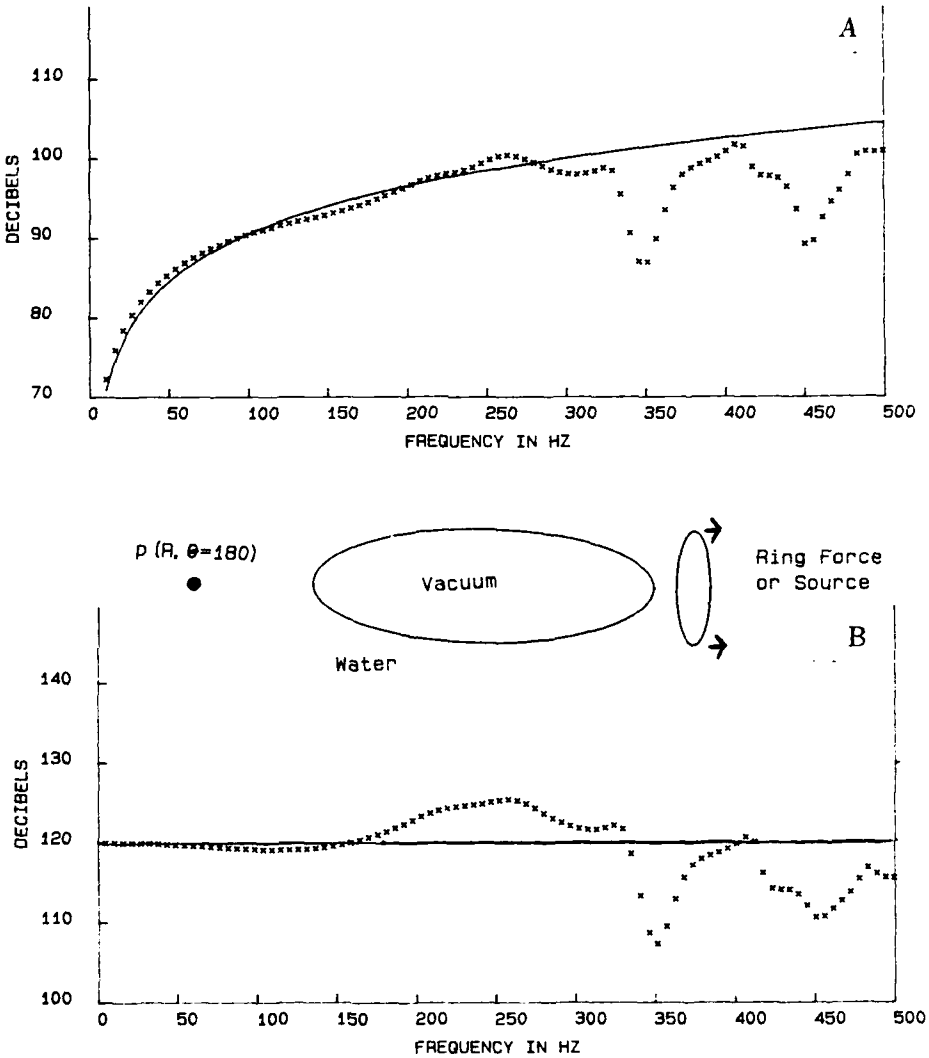

FIG. 3 Far field radiation at θ=180. Prolate 3 : 1 spheroidal shell excited by (a) exterior ring force (b) exterior ring source. x x finite element and Helmholtz integral equation; ——— no shell.

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