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# A Standard Definition for Wind-Generated, Low-Frequency Ambient Noise Source Levels

## Presented at the 116th Meeting of the Acoustical Society of America, 14-18 November 1988, Honolulu, Hawaii

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## Preface

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Low-frequency, wind-generated ambient noise source levels are important input parameters for newly developed ambient noise prediction models such as DUNES. However, there has been a significant variation among recently reported source levels. An analysis is made of these values. Although the actual noise measurements, the assumed source model, and a small angle approximation are similar for all papers; differences arise due to additional geometrical factors or a further approximation to shift the source level to the surface. A standard source level definition and evaluation method are proposed. Standard values of low-frequency, wind-generated source levels are presented based on this strategy.

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# A STANDARD DEFINITION FOR WIND-GENERATED LOW FREQUENCY AMBIENT NOISE SOURCE LEVELS

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Low-frequency, wind-generated ambient noise source levels are important input parameters for newly developed ambient noise prediction models such as DUNES. However, there has been a significant variation among recently reported source levels. An analysis is made of these values. Although the actual noise measurements, the assumed source model, and a small angle approximation are similar for all papers; differences arise due to additional geometrical factors or a further approximation to shift the source level to the surface. A standard source level definition and evaluation method are proposed. Standard values of low-frequency, wind-generated source levels are presented based on this strategy.

# WIND-GENERATED AMBIENT NOISE SOURCE LEVELS

**THIS PAPER**

- PROPOSES A STANDARD DEFINITION AND CONVENTION
- SHOWS A COMPARISON OF MEASUREMENTS
- CONCLUDES THAT NO FUNDAMENTAL DIFFERENCES EXIST

THE PURPOSE IS TO CLARIFY APPARENT CONTRADICTIONS IN REPORTED AMBIENT NOISE SOURCE LEVELS.

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# Wind Generated Ambient Noise Source Levels

For several years we have been examining ambient noise data from omnidirectional hydrophones, horizontal arrays and vertical arrays. Northern hemisphere data for the most part clearly shows the dominance of shipping noise in the low frequency(< 200 Hz) region. However, selected Northern Hemisphere data obtained with both omnidirectional hydrophones below critical depth and high resolution vertical arrays, as well as Southern Hemisphere data, indicates a wind driven noise mechanism in this low to mid frequency range. The numerical estimation of the properties of mid-basin ambient noise fields which result from wind driven noise requires a determination of the source level and the directional characteristic. Source level estimates which have been published are based on experimental data; however, the conversion from the measured omnidirectional or beam level data needs to be carefully examined. We have encountered difficulty in the direct comparison of published results because of this required conversions to source level. Furthermore, the specification of source level in computer codes used to calculate these noise fields also requires care due to the multitude of conventions and the characteristics of the particular propagation model being used. In our opinion, a standard definition of source level and characteristics would have been most helpful. The purpose of this paper is to supply a sound rational and standard specification for wind driven noise source level.

## CURRENT MODEL ASSUMPTIONS AND APPROXIMATIONS FOR WIND-GENERATED AMBIENT NOISE SOURCE LEVELS

**MOST MODELS ASSUME EITHER**

- A MONOPOLE OR DIPOLE LAYER
- A LAYER DEPTH(S)
- A SMALL ANGLE APPROXIMATION (DISTANT SOURCE)
- A SHIFT TO SURFACE (DISTANT SOURCES)
- A SOURCE LEVEL BASED ON MEASURED OMNI-DIRECTIONAL NOISE AND A CONVERSION FACTOR

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## Ambient Noise Models and Calculations

Calculations of mid-basin ambient noise levels require the use of a specific propagation code (PE, RAYTRACE, ASTRAL, NORMAL MODE, etc). The specification of noise intensity per unit area with respect to 1 μPa does not clearly specify the levels one must use with each type of propagation code. Most computations employ one of the approaches shown on this vugraph. For example, calculations performed with PE (Carey 1987) require the specification of a monopole source level and the depth of the source beneath the pressure release surface. A Ray Trace Code, on the other hand, may well use a dipole source with a level based on measured omnidirectional noise level. All methods require a source level specification which can be related to measured data.

## SOURCE LEVEL DEFINITION

$$\text{SOURCE LEVEL (SL)} \equiv 10 \text{ LOG} \left[\frac{\text{POWER RADIATED AT METER}}{\text{REFERENCE POWER}}\right]$$

$$= 10 \text{ LOG} \left[\frac{\text{INTENSITY AT ONE METER}}{\text{REFERENCE INTENSITY}}\right]$$

SL HAS UNITS dB RE 1$\mu$Pa @ 1 METER WHEN THE REFERENCE POWER (WATTS) AND REFERENCE INTENSITY (WATTS/$M^2$) ARE BASED ON A PRESSURE OF 1$\mu$Pa.

## Source Level Specification

This vugraph shows the standard definition of source level. This definition usually applies to a monopole source of sound. The reference power or intensity is commonly based on a plane wave with pressure, $P_R = 1 \mu Pa$ and intensity $I_R = P^2/2\rho c$. This definition has been applied to a dipole by measuring the intensity on the maximum response axis. The purpose of this paper is to provide a method of source level specification for surface distributed sources of ambient noise.

# EXPRESSIONS USED TO CONVERT MEASURED OMNI-DIRECTIONAL NOISE LEVELS ($NL_O$) TO DIPOLE SOURCE LEVEL ($SL_D$)

A AMPLIFICATION FACTOR

P PROFILE FACTOR

| | |
|---|---|
| BURGESS & KEWLEY | $SL_D = NL_o - 8\text{ dB} - A - P$ |
| BANNISTER | $SL_D = NL_o - 4.97\text{ dB}$ |
| WILSON | $SL_D = NL_o - 10\text{ LOG}(\pi) - NL_o - 4.97\text{ dB}$ |
| KUPERMAN | $SL_D = NL_o - 10\text{ LOG}$ (COMPUTER-GENERATED CONSTANT) |

# Source Level Conversion Factors

Shown on this vugraph are equations used by several investigations to convert omnidirectional noise levels $(NL_o)$ to wind driven source levels $(SL_D)$. Burgess and Kewley convert the local omnidirectional noise level to source level by use of an implication factor (A) resulting from multiple reflections at the bottom and surface (depends on bottom loss), 8 = 10Log ($2\pi$) which results from integration of the sound intensity over a hemisphere, and a sound velocity profile factor (P) which is a measure of the deviation away from hemisphere isotropy. The units of source level are at 1 meter. We note that an appropriate value for $A \sim 3$ indicates that when $P \sim 1$ the expression used by Burgess and Kewley agrees with Bannister and Wilson. Kuperman assumes a layer of monopoles a distance $z' \ll \lambda/4\pi$. Integration over the whole plane yields a computer generated constant 6.34dB.

Thus there are some differences in these conversion expressions which can result in factors on the order of 1-5dB.

## THEORETICAL RELATIONSHIPS BETWEEN MONOPOLE AND DIPOLE INTENSITIES USED BY SEVERAL INVESTIGATORS

| | |
|---|---|
| KUPERMAN | $10 \text{ LOG } (I_D/I_M) = 21.9 + 20 \text{ LOG } (h/\lambda) \sim 6.34 \text{ dB}$ |
| TALHAM | $= 10 \text{ LOG } (\pi^2) = 9.94 \text{ dB}$ |
| WILSON | $= 10 \text{ LOG } (\pi^2) = 9.94 \text{ dB}$ |
| BANNISTER | $= 10 \text{ LOG } (4) = 6 \text{ dB}$ |
| BURGESS & KEWLEY | $= 10 \text{ LOG } (\pi^2) = 9.94 \text{ dB}$ |

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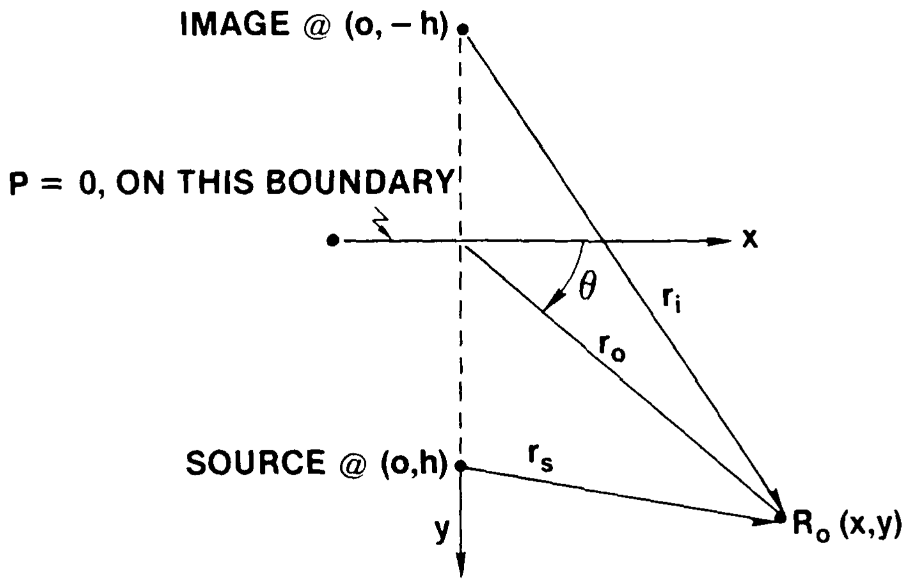
## Theoretical Relationships Between Monopole and Dipole Intensities

The term dipole refers to two point sources of opposite sign separated by one half wavelength. In the context of this discussion we use dipole loosely to mean the field due to a source and its image in the pressure release surface when the distance beneath the pressure release surface is less than a quarter wavelength. Shown on this vugraph are expressions for the maximum response axis intensity ($I_D$) compared to the monopole intensity used by each investigator. We observe differences between 3.6-4dB. The reason for these differences will become apparent in the following vugraphs.

## THE GEOMETRY




$$r_s^2 = x^2 + (y-h)^2$$

$$r_i^2 = x^2 + (y+h)^2$$

$$|r_s| \sim |r_i| \sim |r_o|$$

$$r_o^2 = x^2 + y^2$$

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## The Geometry

The purpose of the vugraph is to illustrate the geometry considered for a point source a distance h beneath the pressure release surface. The convention being adopted is to take $\theta$ as $0$ on the surface and $\pi/2$ in the downward direction. $|r_0|$ is the radial distance to the observation point and in the farfield of the source and its image and is approximately equal to the $|r_s|$ and $|r_i|$. The problem will assume a unity surface reflection coefficient and each source acting as a spherical radiator.

## SOURCE NEAR SEA SURFACE

DIPOLE PRESSURE: $p_D(r_o) = \frac{2\,p_o}{4\pi\, r_o} \text{SIN}\,(k\,h\,\text{SIN}\,\theta)$ (1)

DIPOLE INTENSITY: $I_{D_0} = 4 \bullet I_M \,\text{SIN}^2\,(k\,h\,\text{SIN}\,\theta)$ (2)

$I_{D_1} \simeq 16\pi^2\, I_M\, (h/\lambda)^2\, \text{SIN}^2\,\theta$ (3)

WHEN: $k = 2\pi/\lambda$ AND $h/\lambda = 1/4$ (4)

$I_{D_2} \simeq \pi^2\, I_M\, \text{SIN}^2\theta$

DIPOLE POWER RADIATED INTO A HEMISPHERE, ($r_o = 1$ METER):

$P_{D_0} = 4\pi \bullet I_M \left[1 - \frac{\text{SIN}\,(2k\,h)}{2\,k\,h}\right]$ (5)

$P_{D_1} = 32\,\frac{\pi^3}{3}\,(h/\lambda)^2\, I_M$ (6)

$P_{D_2} = (2/3\;\pi^3\, I_M)$ (7)

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## Source Near the Sea Surface

Shown on this vugraph are the equations governing sound radiation from a point source under a pressure release surface. We are assuming the surface reflection coefficient is equal to unity and that the source is harmonic in time and propagates as an outward going spherical wave. Several approximations can be made concerning expressions for the intensity and power radiated by such a source.

Equation 1 shows the result for the magnitude as a summation of the radiated pressure from the source and its image. The monopole source strength is $P_o$ and the pressure at $r_o$ from a monopole is $P_M = P_o/4\pi r_o$.

Equation 2 shows the intensity at the radial distance $r_o$. The monopole intensity is $I_M = P_M^2/2\rho c = P_o^2/(4\pi)r_o^2(2\rho c)$. Here we have when $\theta = \pi/2$ that $I_{Do}/I_M = 4$. This is the first expression for the relationship between source levels.

Equation 3 shows the result of a small argument of the sine function, ($h/\lambda < 1$). In this instance we find $I_{D1}/I_M = 16\pi^2(h/\lambda)^2$ when $\theta = \pi/2$. The dipole source level depends on $h/\lambda$.

Equation 4 shows the result when we have a true dipole $h/\lambda = 1/4$. Then the intensity $I_{D2}/I_M = \pi^2$ @ $\theta = \pi/2$. At this point we have three possible expressions for the source levels.

Equation 5, 6, and 7 shows the expressions for the radiated power into a hemisphere. We have chosen a hemisphere as only the lower half plane receives radiated energy.

# TABLE OF RELATIVE SOURCE LEVEL VALUES

BASED ON ANALYTICAL EXPRESSIONS FOR THE DIPOLE INTENSITY AND POWER, WE HAVE:

$$S_D - S_M = 10 \text{ LOG } (I_D/I_M) = 10 \text{ LOG } (P_D/P_M)$$

| | '0' | '1' | '2' |
|---|---|---|---|
| $\left(\frac{I_D}{I_M}\right)_{PEAK}$ | 4 | $16\pi^2(h/\lambda)^2$ | $\pi^2$ |
| $P_D/P_M$ | $\left[1 - \frac{\text{SIN}\,(2k\,h)}{2\,k\,h}\right]$ | $\frac{8\pi^2}{3}(h/\lambda)^2$ | $\frac{\pi^2}{6}$ * |

*POWER RADIATED INTO A HEMISPHERE. WHEN THE TOTAL SPHERICAL RADIATION PATTERN IS CONSIDERED: $P_D/P_M = \pi^2/3$.

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## Table of Relative Source Level Values

Based on the expressions on the last vugraph we have summarized the various ratios of intensity and power. Here we are using the monopole intensity or power as a reference. Notice there are six possibilities. Our previous vugraph has shown Bannister employed $10Log(4)\approx 6dB$, where as Wilson, Talham, and Burgess used $10Log(\pi^2)=9.94dB$. All these authors employed the peak or maximum response intensity level.

## SOURCE LEVEL CONSIDERATIONS

IF WE LET $h/\lambda = 1/4$ — THE PROBLEM IS SIMPLIFIED

$$\frac{I_D}{I_M} = \pi^2 \sin^2\theta \qquad \underline{SL_D = SL_M + 10 \log(\pi^2 \sin^2\theta)} \qquad (1)$$

WITH THIS DEFINITION, THE $SL_D$ WOULD BE MEASURED AT $\theta = \pi/2$

$$\boxed{\bullet\ SL_D = SL_M + \underbrace{10 \log(\pi^2)}_{9.94\ \text{dB}}} \quad h/\lambda = 1/4 \qquad \theta = \pi/2 \qquad (2)$$

HOWEVER IF WE USE THE RADIATED POWER <u>INTO THE WATER</u>

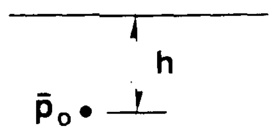
$$\boxed{SL_{DP} = SL_M + \underbrace{10 \log(\pi^2/6)}_{2.16} \qquad h/\lambda = 1/4} \qquad (3)$$

## Source Level Considerations

Carey and Browning have surmised that most physically realizable sources of sound near the surface of the sea have a dipole character. If we are to represent the sources of noise as distribution of dipoles per unit area, then the source depth to wavelength ratio should be equal to 1/4. This implies a frequency dependent source of sound rather than a monopole at fixed depth and that the source of sound is distributed in depth as well as in area. With this implication, we have expressions for the intensity (equation 2) and power (equation 3). The intensity ratio requires the specification of $\theta = \pi/2$ ; the power definition which is based on the total radiated power into the water does not. The specification of the number of independent dipoles per unit area is thus a specification of power radiated per unit area or vice versa. The independence criteria require the spacing between dipole radiated to be greater than the correlation of the hydrodynamic forcing function.

## RECOMMENDED SOURCE LEVEL CONVENTION I

FOR THOSE CALCULATIONS EMPLOYING A POINT SOURCE BELOW THE PRESSURE RELEASE SURFACE




SPECIFY: $p_o$, $h/\lambda$

USE MONOPOLE SOURCE LEVEL $SL_o = 10 \text{ LOG } (p_o^2/p_R^2)$

$p_R$ = REFERENCE PRESSURE, 1 $\mu$Pa

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## Recommended Source Level-Convention I

Based on our previous discussions we now summarize and recommend a standard convention. For those calculations which require a point source below a pressure release surface we recommend the specification of $P_o$ and $h/\lambda$. The monopole source level is specified with respect to $1 \mu Pa$ @ $1 m$. The source level $SL_O = 20 LOG (P_O/P_R)$ has units $dB//((1 \mu Pa)^2/m^2 - Hz)$. The $m^2$ specifies the per unit area dependence of ambient noise.

## SOURCE LEVEL CONVENTION II

FOR THOSE CALCULATIONS AND APPLICATIONS WHICH REQUIRE A DIPOLE IN THE PRESSURE RELEASE SURFACE

DIPOLE

SPECIFY: $h/\lambda = 1/4$

$$I_D \cong \pi^2 I_M \text{ SIN}^2(\theta) \qquad (1)$$

$$P_D = \frac{\pi^2}{6} P_M, \text{ FOR RADIATION INTO THE WATER} \qquad (2)$$

BASED ON THE POWER RADIATED

$$\boxed{SL_D = SL_o + 2.16 \text{ dB}} \qquad (3)$$

## Source Level Convention II

When the calculation requires a surface distribution of dipole sources (in this instance $h/\lambda \approx 1/4$), the power radiated into the water is given equation 2. This results in a dipole level (equation 3) 2.16 dB greater than the monopole source level. The reference units for dipole sources per unit area are $dB//((1\mu Pa)^2/m^2 \text{-} Hz)$ at 1 meter. The specification of the source level in terms of total power radiated into the water provided a basis for comparison with omnidirectional measurements and alleviates the problem of maximum response axis intensity levels.

# CONCLUSIONS
# WIND-GENERATED AMBIENT NOISE SOURCE LEVELS

- A STANDARD DEFINITION AND CONVENTION PROPOSED
- THIS STANDARD AT MINIMUM ALLOWS FOR COMPARISON OF MEASUREMENTS
- IN LIGHT OF THIS STANDARD THERE APPEAR TO BE NO FUNDAMENTAL DIFFERENCES

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## Conclusions

We have concluded that source leads for ambient noise which are distributed over 1 meter should be based on a monopole of strength $P_o$ a distance $h/\lambda = 1/4$ beneath the pressure release surface:

- Monopole

$$SL_o = 20 LOG (P_o/P_R) \qquad dB//((1\mu Pa)^2/m^2 - Hz) .$$

$$\mathcal{P}_M = 4\pi I_M = P_o^2/2\rho c .$$

- Dipole

$$\mathcal{P}_D = (\pi^2/6)\,\mathcal{P}_M \,, \qquad \text{power radiated into the water.}$$

$$SL_D = SL_o + 2.16 dB \qquad dB//((1\mu Pa)^2/m^2 - Hz)$$

We conclude that this allows for comparison with measurements that estimate power radiated per unit area. Furthermore when we apply this standard to measurements previously disclosed, no fundamental differences are observed.

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APPENDIX A: FREQUENCY CONVENTION

| RANGE | FREQUENCY | DESIGNATION | DESCRIPTION |
|---|---|---|---|
| SEISMIC | 0.02 TO 0.2 | | |
| | 0.2 TO 2 | ULF | ULTRA LOW FREQUENCY |
| INFRASONIC | 2 TO 20 | VLF | VERY LOW FREQUENCY |
| | 20 TO 200 | LF (LOW) | LOW FREQUENCY |
| SONIC | 200 TO 2000 | MF (MID) | MID-FREQUENCY |
| | 2000 TO 20,000 | HF (HIGH) | HIGH FREQUENCY |
| | 20,000 TO 200K | VHF | VERY HIGH FREQUENCY |
| ULTRASONIC | 200K TO 2M | UHF | ULTRA HIGH FREQUENCY |

APPENDIX B: Derivation of the Farfield Intensity and Acoustic Power Expressions for the Acoustic Dipole

The Wave Equation with Constants

We assume a source of sound which is "point" like at $\chi_0 = \chi(x_0, y_0, z_s)$ emitting waves with a time dependence $e^{i\omega t}$, $(\omega = 2\pi f)$.

1.) $\nabla^2\Psi - (1/c(\chi)^2)\,\partial^2\Psi/\partial t^2 = K \cdot S \cdot e^{i\omega t}\delta(\chi - \chi_0)$ , where $c(\chi)$ is the sonic speed. $\delta(\chi - \chi_s)$ is the delta function, and K is constant describing the source strength.

2.) $\Psi \equiv$ VELOCITY POTENTIAL $= \Phi(x,y,z)e^{-i\omega t}$; $v = -\nabla\Psi$ ; $P = \rho\,\partial\Psi/\partial t$.

3.) $\nabla^2\Phi + (\omega^2/c^2)\Phi = K \cdot S \cdot \delta(\chi - \chi_0)$. We need to determine the wave function and the constant K for a wave diverging omnidirectronally from the unit source. In the immediate vicinity of the source, the wave diverges spherically.

4.) $\Phi = (1/R)e^{i\omega R/c}$; $\omega R/c \ll 1$ ; $R = |\chi - \chi_0|$.

To find K we intergrate over a small sphere of volume $V_\varepsilon$ with radius $\varepsilon$ about $\chi_0$ . Then we take the limit as $\varepsilon \to 0$ and let $S = 1$.

5.) $\iiint_{V_\varepsilon} dV_\varepsilon \left(\nabla^2\Phi + (\omega^2/c^2)\Phi\right) = \iiint_{V_\varepsilon} dV_\varepsilon \left(K\delta(\chi - \chi_0)\right)$.

6.) It is easily shown that $\iiint dV_\varepsilon (\nabla^2\Phi) = -4\pi$ ; $\iiint dV_\varepsilon (\omega^2/c^2\,\Phi) = 0$.

7.) $\iiint dV_\varepsilon \cdot K \cdot \delta(\chi - \chi_s) = -4\pi$ = K by virture of the delta function.

8.) $\nabla^2\Phi + (\omega^2/c^2)\Phi = -4\pi\delta(\chi - \chi_s)$ ; $\Phi = e^{\pm i\omega R/c}/R$.

9.) Cylindrical coordinates $(1/r)\partial/\partial r\left(r\,\partial\Phi/\partial r\right) + \partial^2\Phi/\partial r^2 + \omega^2/c^2\,\Phi = -4\pi\frac{\delta(r)}{r}\delta(z - z_s)\delta(\phi)$.

10.) After intergration over $\phi$ , $(1/r)\,\partial/\partial r\left(r\,\partial\Phi/\partial r\right) + \partial^2\Phi/\partial r^2 + \omega^2/c^2\,\Phi = -2\frac{\delta(r)}{r}\delta(z - z_s)$.

The Acoustic Dipole The wave equation in spherical coordinates from equation 1.

11.) $\nabla^2\Psi + k^2\Psi = -4\pi g_0 e^{-i\omega t}\delta(r - r_0)$. (a)

12.) The outward propagating wave $\Psi = (q_0/r)\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t)$.[b]

where $q_0$ represents the magnitude of the volume source. The acoustic pressure $P = \rho\, \partial\Psi/\partial t$ and the particle velocity $\underset{\sim}{V} = -\underset{\sim}{\nabla}\Psi$ .

The solution to the wave equation at point $r_0$ from a point source at $r_s$ and its image at $r_i$ is the superposition of individual solutions. In this instance the source is a distance h beneath the pressure release surface.

13.) $\Psi(r_0) = \Psi_s(r_s) + \Psi_i(r_i)$ ; $P(r_0) = P_s(r_s) + P_i(r_i)$ ; $|r_s| \cong |r_i| \simeq |r_0|$ ; $\mu = -1$.

$$r_s^2 = x^2 + (y-h)^2 \; ; \quad r_i^2 = x^2 + (y+h)^2 \; ; \quad r_0^2 = x^2 + y^2.$$

$$\Psi(r_0) = (q_0/r_0)\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r_0} - i\omega t)\,[\exp(-ikyh/r_0) - \exp(+ikyh/r_0)].$$

The resulting velocity potential function is

14.) $\Psi(r_0) = (q_0/r_0)\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r_0} - i\omega t)(-2i)\sin(kh\sin\theta)$.

The pressure is determined by use of $P = \rho\,\partial\Psi/\partial t$. .

15.) $P(r_0) = (-2\rho q_0\omega/r_0)\sin(kh\sin\theta)\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r_0} - i\omega t)$ ; $|P(r_0)| = \dfrac{2\rho q_0\omega}{r_0}\sin(kh\sin\theta)$.

The acoustic intensity is by definition,

16.) $I_D = Re(u^*P) = (4k\omega\rho q_0^2/r_0^2)\sin^2(kh\sin\theta) = (4\omega^2\rho q_0^2/c r_0^2)\sin^2(kh\sin\theta)$.

where D refers to the source and its image.

17.) The point source, monopole, intensity is $I_M = \omega^2\rho q_0^2/r_0^2 c$. The radiated intensity from the source and image.

18.) $I_D = 4I_M\sin^2(kh\sin\theta)$. The radiated power into the lower hemisphere.

19.) $P_D = \int I_D\, dA = \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta\, r_0^2(4\omega^2\rho q_0^2/r_0^2 c)\sin^2(kh\sin\theta)\cos\theta$.

20.) $P_D = (4\pi\omega^2\rho q_0^2/c)[1 - \sin(2kh)/2kh]$. $P_M = 4\pi\omega^2\rho q_0^2/c$ .

The result of the dervation is the following expression for intensity and power.

21.) The general result.

a.) $I_M = \omega^2\rho q_0^2/c$ b.) $I_D = 4I_M\sin^2(kh\sin\theta)$

c.) $P_M = 4\pi I_M$ d.) $P_D = 4\pi I_M[1 - \sin(2kh)/2kh]$

e.) $I_D/I_M = 4\sin^2(kh\sin\theta)$ f.) $P_D/P_M = (1 - \sin(2kh)/2kh)$

22.) The approximation $kh << 1$ $\quad kh = 2\pi h/\lambda$

a.) $I_D = 16\pi^2(h/\lambda)^2 \sin^2\theta \, I_M$ $\quad I_D/I_M = 16\pi^2(h/\lambda)^2 \sin^2(\theta)$

b.) $P_D = (32\pi^3/3)(h/\lambda)^2 I_M$ $\quad P_D/P_M = (8\pi^2/3)(h/\lambda)^2$

23.) The approximation $2\pi h/\lambda = \pi/2$

a. $I_D/I_M = \pi^2 \sin^2\theta$ $\quad$ b.) $P_D/P_M = \pi^2/6$

Note: (a) When the radiation from a point source is taken of the form $1/r \exp(-i\omega t + i\underset{\sim}{k}\cdot\underset{\sim}{r})$, the solution will be Hankel functions of the first kind (i.e., Morse) when the solution is of the form $1/r \exp(i\omega t - i\underset{\sim}{k}\cdot\underset{\sim}{r})$ we must use Hankel function of the second kind.

Note: (b) We have a choice $e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}/r$ or $e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}/4\pi r$, we have choosen the first form.

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