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A New Perspective on Rutting in Flexible Pavements

Final Technical Report

PREPARED FOR AIR FORCE OFFICE OF SCIENTIFIC RESEARCH, BOLLING AIR FORCE BASE, WASHINGTON, D.C. 20332-6448

FEBRUARY 24, 1989

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This report presents a new approach to the prediction of rutting in flexible pavements. It recognizes the prominent role played by the granular layer in the stress distribution and strain accumulation in flexible pavements and diverts from the conventional continuum approach to modeling stresses in the layer. Fundamentally, the conventional requirement for the existence of the second derivatives of strains in the granular layer is replaced by a stress continuity equation that provides for stress transfer between particles through these contacts. This is developed assuming that deformation is due primarily to particle movements and is shown to be based on a nonlinear stress-strain relationship unique for granular material. In order to evaluate the rutting in a flexible pavement, the particulate theory of stress transfer is extended to predict stresses in multilayered systems consisting of granular and elastic layers. Knowledge of the nature of stress transfer and stress-strain response is next incorporated into a theory for strain accumulation with repetitive loading.

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PREFACE

This report was prepared by The BDM Corporation, 7915 Jones Branch Drive, McLean, Virginia 22102 under Contract Number F4962088-C-0019 for the Air Force Office of Scientific Research, Bolling Air Force Base, Washington, D. C. 20332-6448.

This report summarizes work done between December 15, 1987, and February 15, 1989. AFOSR Program Manager was Major Steven Boyce, USAF.

This report has been reviewed by the Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication.

Steven Boyce, Major (USAF)
Program Manager
Directorate of Aerospace Sciences

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SECTION I
INTRODUCTION

A. OBJECTIVES

The objective of this research is to develop basic theories that can be used to predict rutting in flexible pavements with granular layers. The approach taken in this research is to treat the granular material as particulate in nature and in this way provide a more rational model than conventional theory that treats the particulate material as a continuum. It takes a premise that the major reason for the inability of conventional elastic or elasto-plastic theories to model granular soil behavior is the fact that these soils consist of randomly-arranged, irregularly-shaped discrete particles that are free to displace relative to each other. It assumes that the mechanism responsible for the deformation of the material subject to induced loading is not caused so much by deformation of individual particles as by relative movements of the more mobile particles forming a denser state.

In linear and nonlinear continuum theory an equation ensuring the existence of the second derivatives of strains throughout the media is introduced. This equation, through constitutive relations, is combined with the equilibrium equations for stresses to provide problem solution. One inaccuracy in using this theory to model the behavior of granular material is the inappropriateness of this strain requirement. This requirement inevitably results in the prediction of horizontal tensile stresses when the granular media is subjected to inclined loads. It will also predict stresses if the granular media is subjected to purely horizontal surface loads, an unrealistic condition since cohesionless material will provide no resistance to these loads. This is the reason why the continuing development of sophisticated nonlinear empirical equations to model the experimentally-observed stress-strain relations of the soil does not greatly improve the continuum mechanics prediction. Khedr (1985) observed that even finite element methods using elasto-plastic theory predicts unrealistic radial pressure because they characterize the granular layer as a continuum.

In the particulate theory developed here the requirement for the existence of the second derivatives of strains in the granular media is replaced by a stress continuity equation that guarantees stress transfer between particles through their contacts. This continuity equation is derived from two points of view, namely: (1) from the assumption that deformation is caused by particle movements rather than particle compression; and (2) that in the definition of stress and strain, any representative element of soil must be composed of particles and voids. In this way, the elemental volume of soil cannot be made to approach zero but must, instead, approach a finite minimal volume with enough particles such that the particle movements produce strain. The combination of this continuity equation with the equilibrium equation allows the determination of all the components of stress. In addition, the particulate theory results in a stress-strain response model unique for granular soils.

The specific objectives of this research are to:

(1) Develop a particulate theory to predict stress transfer through granular material subjected to inclined loads,

(2) Extend the particulate theory to predict the stress-strain response under static loading;

(3) Use the particulate theory to predict strain accumulation under repeated loading; and

(4) Combine the results of objectives (1), (2), and (3) to predict rutting in multilayered, flexible pavements with a granular layer.

B. BACKGROUND

Flexible pavement design requires the ability to predict pavement performance. One of the major indicators of pavement performance is the distress caused by rutting. This rutting is the accumulation of permanent settlement with traffic application, and for flexible pavements it occurs in all inelastic components of the pavement. Rutting will eventually result in a reduction of pavement serviceability due to loss in riding comfort and may also lead to hydroplaning and icing due to the collection of the water in the deformations. As a result, any rational method of predicting rutting in flexible pavement must be able to model both the stress transfer mechanism and the stress-strain characteristics of each layer. It must also be able to combine the behavior of each layer in the way that it contributes to the overall performance of the pavement.

It is common practice in pavement design to assume stress distribution using the multiple elastic layers theory because of the relative ease at which solutions may be obtained (Monismith and Finn, 1977; Monismith, Finn, and Epps, 1986). The derived stresses which are based on linear stress-strain relationships that acknowledge no strain accumulation are next used with empirical or elasto-plastic nonlinear stress-strain relationships to predict strain and strain accumulation with repeated loading. However, experimental data show that the response of soil, and especially granular materials, depends strongly on the state of stress. Therefore, it is essential to know the correct stress conditions in the flexible pavements before strain accumulation can be rationally predicted. Most of the research in the last decade has concentrated on developing stress prediction that includes the nonlinearity of the stress-strain relationship of granular material. This usually takes the form of empirically relating the resilient modulus, defined as the ratio of the repeated deviator stress to the recoverable strain, to the sum of the principal stresses. Besides requiring complex finite element solutions, this model has serious limitations. More complex and sophisticated models giving better descriptions of resilient response do not greatly improve the prediction (Uzan, 1985; Brown and Pappin, 1981).

It is very difficult to accurately predict rutting in flexible pavement because the granular layer is the most important load-carrying component of the pavement. To adequately design flexible pavements, an accurate understanding of the behavior of granular material is necessary. This is more essential now than ever since existing pavements and future pavements are expected to accommodate heavier and heavier loads. This material consists of discrete particles, and it deforms as the result of particle movements rather than particle deformation. Experimental evidence abounds (Haggarty, 1963; Morgan and Gerrard, 1971) for the inability of classical elastic or inelastic continuum mechanics to predict granular materials behavior under load. The main reason for this is that even the most sophisticated of these solutions requires the second derivatives of strains to exist at all points in the granular media. This requirement is unrealistic as voids always exist in the material, and the deformations are the result of discrete particle displacements.

An alternative approach to stress distribution in granular materials can also be found in the literature (Golden, 1984, 1986; Harr, 1977; Hill and Harr, 1982; Endley and Peyrot, 1977; Chikwendu and Alimba, 1979; Sergeev, 1969). This approach considers the discrete nature of the soil and assumes that when a normal point force is applied on the surface of an infinite half-space, the influence of this force travels from particle to particle in a fashion analogous to a random walk or continuous Markov process. The resulting equation is the diffusion equation for vertical stresses. This approach ignores the requirement for the existence of the second derivatives of strains and requires knowledge of the diffusion coefficient, a material property. The results of this approach show that it provides a better qualitative fit to observed stress distributions in granular media and can be used to predict the stresses in layered media if the diffusion coefficients of the layers are known (Harr, 1977; Golden, 1984). The limitations of this theory at this point are the description of the diffusion coefficient and the role the constitutive nature of the material plays in the stress diffusion.

Even from a conceptual viewpoint, any particulate theory is an improvement over continuum models. As granular soils are particulate in nature, stresses are transmitted at particle contacts, and strains are the result of particle movements rather than particle compression. The vertical stress distribution predicted by the stochastic theory for a point load on the surface agrees with experimental results which show a more bell-shaped stress distribution than the elastic prediction (Harr, 1977). Any particulate theory based on stress diffusion from a source should give a bell-shaped distribution as this is the solution of the diffusion equation for a point source. Further, with the requirement for the specification of a material property in the form of the diffusion coefficient, the stochastic theory shows that stress transfer is dependent on particle sizes, shapes, packing, load history, etc. This is in opposition to the elastic theory which predicts the same vertical stress at a given point in sand as it does in steel.

C. SCOPE

This report presents a new approach to the prediction of rutting in flexible pavements. It recognizes the prominent role played by the granular layer in the stress distribution and strain accumulation in flexible pavements and diverts from the conventional continuum approach to modeling stresses in the layer. The methodology of the new approach is as follows:

(1) The requirement for the existence of the second derivatives of strains in the granular layer conventionally used in the determination of stress is relaxed. It is replaced by a stress continuity equation that provides for stress transfer between particles through these contacts. This is developed assuming that deformation is due primarily to particle movements;

(2) It is shown that the development of the stress continuity equation is based on a nonlinear stress-strain relationship unique for granular material. This relationship is derived by the particulate approach and reduces to the hyperbolic model for soils that do not exhibit a distinct peak stress;

(3) Knowledge of the nature of stress transfer and stress-strain response in granular media is next incorporated into a theory for strain accumulation with repetitive loading; and

(4) In order to evaluate the rutting in a flexible pavement, the particulate theory of stress transfer is extended to predict stresses in multi-layered systems consisting of granular and elastic layers. In this way, more accurate predictions of the stresses in the layers will lead to more accurate predictions of rutting.

D. ORGANIZATION OF TECHNICAL REPORT

This technical report is divided into eight sections and an appendix. Section I is the introduction which outlines the objectives, background and scope of the research. Section II investigates the stress transfer mechanism in granular media. After a look at the traditional methods, a new particulate approach to stress transfer is introduced. Section III presents a particulate theory for stress-strain response in granular materials. It addresses both the stress-strain response due to static loading and permanent strain accumulation under repeated loading. Section IV presents the validation of the theories developed in Sections II and III using information found in the literature. Methods of determining the experimental constants identified in the theories of Sections II and III are presented in Section V. Section VI presents a method for predicting rutting in flexible pavements with granular layers. Section VII presents the Conclusion and Recommendations, and Section VIII is the List of References. Finally, a paper published from this work is presented in an appendix.

SECTION II

STRESS TRANSFER IN GRANULAR MATERIALS UNDER INCLINED LOADS

A. INTRODUCTION

Granular soils generally consist of randomly arranged, irregularly-shaped, discrete particles that are free to displace relative to each other. The deformation of this material under load is not caused so much by deformation of individual particles as by relative movement of the more mobile particles forming a denser state. As a result, experiments have shown that deformation predicted by the theory of elasticity is incorrect in magnitude and distribution (Turnbull, Maxwell, and Ahlvin, 1961; Morgan and Gerrard, 1981). Really obvious discrepancies between experiments and elastic predictions are that horizontal tensile stresses are predicted under inclined loads and stresses are predicted under purely horizontal surface loads. In actuality, tensile stresses cannot exist in cohesionless materials and cohesionless materials will provide no resistance to purely horizontal surface loads.

The main reason for the elastic theory's prediction of tensile stresses in granular materials under inclined loads and the prediction of stress transmission in granular material under purely horizontal surface loading is the requirement for the second derivatives of strains to exist at all points in the material. In elastic predictions, using linear stress-strain relationships, this condition is combined with the equilibrium equations to give the stresses in the media. Traditional methods of improving the predictions of stresses in granular methods introduce more realistic nonlinear stress-strain behavior. However, the requirement for the existence of the second derivatives of strains at all points in the media is still imposed. It seems unlikely that these methods of improving the prediction of stresses in granular material can avoid the condition of predicting tensile horizontal stresses under inclined load or the transmission of stresses under purely horizontal loads.

An alternative approach to stress distribution in granular media that does not require the existence of the second derivatives of strains at all points but on stress transfer through particle contacts can be found in References 10 and 13. Their approach assumes that the influence of a surface force travels from particle to particle in a fashion analogous to a continuous Markov process. The resulting equation is the diffusion equation for vertical stresses. However, there is no constitutive relationship identified in the stochastic approach (Reference 10).

In this section, the stress transfer mechanism for an inclined load on a granular material is developed by recognizing that particles are of finite size and that stress and strain cannot be defined at a point in granular material but only with respect to some finite volume. It is shown that this approach, although entirely deterministic, leads to the diffusion equation of the stochastic approach. More importantly, it shows that there is a built-in nonlinear stress-strain relationship. The approach avoids the requirement for the existence of the second derivatives of strains and avoids the prediction of tensile stresses in granular soils and the prediction of stresses *under purely horizontal* surface loads.

B. TRADITIONAL APPROACH TO STRESS DISTRIBUTION IN GRANULAR MEDIA

The solution of stresses in any body is obtained by solving the equations of equilibrium. In two dimensions, neglecting the weight of the soil, these equilibrium equations are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0 \tag{1}$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = 0 \tag{2}$$

and

$$\tau_{xz} = \tau_{zx} \tag{3}$$

where σ_z and σ_x are the normal stresses on the element in the vertical and horizontal directions, respectively; and τ_{xz} and τ_{zx} are the vertical and horizontal shear stresses. Since these are only three equations for the four unknown stresses, a fourth equation is necessary for the solution of the stresses. Traditionally, this fourth equation is supplied by assuming the existence of the second derivatives of strains at all points in the granular media. This requirement results in the compatibility equation (Timoshenko, 1951)

$$\frac{\partial^2 \varepsilon_x}{\partial x^2} + \frac{\partial^2 \varepsilon_z}{\partial z^2} = \frac{\partial^2 \gamma}{\partial x \partial z} \tag{4}$$

where ε_x and ε_z are axial strains and γ is the shear strain. The stresses and strains are then related by an appropriate constitutive relationship. For the special case of linear elastic assumption, closed-form solutions for the stresses are available. These, however, fail to appropriately model the observed conditions in granular media (Morgan and Gerrard, 1981). With the use of empirical stress-strain models, such as the hyperbolic model, intense numerical modeling is necessary. The complexity of solution and the assumption of the existence of the second derivatives of strains at all points in the media are the limitations of the approach.

C. NEW APPROACH TO STRESS DISTRIBUTION IN GRANULAR MEDIA

The new approach considers any representative element of soil of volume dxdydz to be composed of particles and must satisfy the equilibrium equations. However, rather than assuming the existence of the second derivatives of strains to exist at all points in the granular media, the particulate nature of soil deformation is examined to develop the additional equations necessary for solution of the stresses.

Since any representative element of soil must be composed of particles and voids, its volume dxdydz cannot be made to approach zero but must approach a finite limiting volume, say ijh, with enough particles so that the relative movements of particles in ijh caused by forces on these particles produce strain. If w is the average change in vertical displacement of particles in the element, then the vertical strain in the element is

$$\varepsilon_z = \lim_{dz \to h} w/dz = w/h \tag{5}$$

Also, if F is the vertical component of force on a horizontal plane through the element, then the vertical stress is

$$\sigma_z = \lim_{\substack{dx \to i \\ dy \to j}} F/(dxdy) = F/(ij) \tag{6}$$

This is analogous to the continuum definition where ijh approaches a point of zero volume on a macroscopic scale but is composed of discrete atoms at a microscopic level.

For simplicity, a microscopic stiffness coefficient k is introduced to represent the average resistance of particles to movement in the z direction such that the vertical component of force on a particle in ijh is kw. The magnitude of k depends on the packing, the roughness of the particles, and the confining pressure. The vertical force in ijh is F = Nkw, where N is the number of particles in ijh. The vertical stress in ijh is

$$\sigma_z = \frac{Nkw}{(ij)} = \frac{Nkh\ \varepsilon_z}{(ij)} \tag{7}$$

Since ijh is the smallest possible volume of soil that can be used for the definition of stress and strain, it serves as a control volume analogous to a point in a continuum and particles enter and leave ijh as deformation takes place. This means that N varies with deformation, and Equation (7) is nonlinear. This can easily be seen as $N = ijh/[V_p(1+e)]$, where V_p is the average volume of a particle and e is the void ratio;

and by definition, the volumetric strain is related to the void ratio as $\varepsilon_v = (e_o - e)/(1+e_o)$, where e_o is the initial void ratio. Therefore

$$N = \frac{ijh}{[V_p\ (1 + e_o)\ (1 - \varepsilon_v)]} \tag{8}$$

Consider two elements of soil adjacent to each other but separated by a surface dydz in the yz plane as shown in Figure 1. Element 1 with center at location x has particles with average change in vertical displacement w, and the element with center at x+dx has average change in vertical displacement $w+(\partial\omega/\partial x)dx$. The vertical force at the right face of element 1 is $F_1 = N_1kw$, and the vertical force at the left face of element 2 is $F_2 = N_2\ k\ [w+(\partial w/\partial x)dx]$ where N_1 is the number of particles at the right face of element 1 and N_2 is the number of particles at the left face of element 2. The vertical shear stress at the interface between the elements is $(F_1 - F_2)/(dydz)$, or

$$\tau_{xz} = \frac{\left[(N_1 - N_2)\ kw - N_2 ik\frac{\partial w}{\partial x} \right]}{(jh)} \tag{9}$$

From Equation (7), the derivative is

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x} \left(\frac{ij}{Nk} \sigma_z \right) \tag{10}$$

The substitution of Equations (7) and (10) into Equation (9) gives

$$\tau_{xz} = b\sigma_z - D\frac{\partial \sigma_z}{\partial x} \tag{11}$$

where

$$b = \frac{N_1 - N_2}{N} \frac{i}{h} - N_2 \frac{ik}{jh} \frac{\partial}{\partial x} \left(\frac{ij}{Nk}\right) \tag{12}$$

and

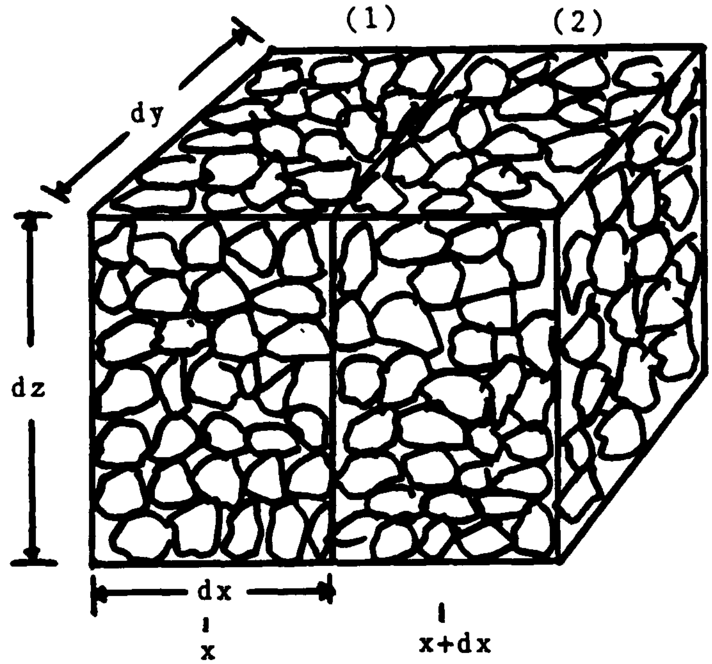

Figure 1. Adjacent Soil Elements.

$$D = \frac{N_2 i^2}{N h} \tag{13}$$

It can be noted at once that in the case of a normal load changes of i, j, and N with x can be assumed small; hence $N_1 = N_2$ and Equation (12) gives $b = 0$. However, if there is an x component of the load, lateral changes of N with x may not be neglected.

1. Two Dimensions

The substitution of Equation (11) into Equation (2) gives the diffusion equation

$$\frac{\partial \sigma_z}{\partial z} = \frac{\partial}{\partial x}\left(D\frac{\partial \sigma_z}{\partial x}\right) - \frac{\partial}{\partial x}(b\sigma_z) \tag{14}$$

Equation (14) is identical to that of References 9 and 11, however, here the diffusion coefficient is derived from the relationship of stress to particle displacement. Equation (14) is nonlinear as D is a variable that depends on the size ijh of the representative element of soil; and this, in turn, depends on the magnitude of the particle displacements or stress intensity. For small particle displacements, a small volume of soil ijh may be used to adequately define stress and strain while for large particle movements, a larger volume of soil is needed to make these definitions. If the horizontal displacements are small compared to the vertical, then ij can be treated as a constant and h is seen to be decreasing away from the loaded area. In this case, D increases with z and the absolute value of x. This is apparently the same type of diffusion that occurs in elastic material, for it can easily be shown by back substitution that with $D = (x^2 + z^2)/(2z)$ and $b = q_1D/(2 + q_1x)$ where q_1 is the ratio of horizontal to vertical component of the load, Equation (14) gives the elastic solution for a line load at the origin of coordinates.

If D and b are known, then with known boundary conditions Equation (14) can be solved for the vertical stresses. The substitution of Equation (11) into Equation (1) gives the normal horizontal stress as

$$\sigma_x = \frac{\partial}{\partial z}(D\sigma_z) + \int_x^{\infty} \frac{\partial}{\partial z}(b\sigma_z)dx \tag{15}$$

The boundary conditions $\sigma_x = \sigma_z = 0$ at $x = -\infty$ gives $\frac{\partial}{\partial z}\int_{-\infty}^{\infty} b\sigma_z dx = 0$, which is satisfied only if b is a constant. Also from Equation (11) at x = 0 and z = 0 it is known that $\partial\sigma_z/\partial x = 0$, σ_z = the vertical load, and τ_{xz} = the horizontal load; hence $b = q_1$. Therefore, Equations (11) and (15) become

$$\tau_{xz} = q_1\sigma_z - D\frac{\partial\sigma_z}{\partial x} \tag{16}$$

and

$$\sigma_x = \frac{\partial}{\partial z}(D\sigma_z) + q_1\,\tau_{xz} \tag{17}$$

2. Three Dimensions

In three dimensions, the equilibrium equations are

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z} = 0 \tag{18}$$

$$\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{zy}}{\partial z} = 0 \tag{19}$$

$$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} = 0 \tag{20}$$

$$\tau_{xy} = \tau_{yx},\ \tau_{yz} = \tau_{zy},\ \tau_{xz} = \tau_{zx} \tag{21}$$

Development similar to that in the derivation of Equation (11) says that changes in vertical particle displacement across the xz plane produce

$$\tau_{xz} = b_{xz}\sigma_z - D_{xz}\frac{\partial\sigma_z}{\partial x} \qquad (22)$$

and changes in vertical particle displacement across the yz plane produce

$$\tau_{yz} = b_{yz}\sigma_z - D_{yz}\frac{\partial\sigma_z}{\partial y} \qquad (23)$$

In like manner, the shear stress τ_{xy} is created by changes in particle displacement in the y direction across the xy plane which leads to

$$\tau_{xy} = b_{xy}\sigma_z - D_{xy}\frac{\sigma_z}{\partial y} \qquad (24)$$

The subscripts on b and D refer to b and D in the respective planes.

If the inclined load has X and Z components only, then $b_{xz} = q_1$, $b_{yz} = 0$, and $b_{xy} = 0$. Substituting this into Equation (20) gives the three-dimensional diffusion equation

$$\frac{\partial\sigma_z}{\partial z} = \frac{\partial}{\partial x}\left(D_{xz}\frac{\partial\sigma_z}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{yz}\frac{\partial\sigma_z}{\partial y}\right) - q_1\frac{\partial\sigma_z}{\partial x} \qquad (25)$$

Hence, in theory, Equations (18) through (25) can be solved for all of the stresses.

D. SOLUTION FOR STRESSES UNDER INCLINED LOADS

1. Two Dimensional

The nature of D in Equation (13) is unknown; however, as stated above, for the two dimensional case D is a function of i and h where i and h are the horizontal and vertical components of the representative element of soil. The magnitudes of i and h depend on the magnitudes of the particle displacements in the x and z directions, respectively. Directly under the center of the load i and h vary with depth only. This means that in the region under the load D can be approximated by D(z), a function of z only. In this case, one can make the transformation

$$W(z) = \int_0^z D(z)dz \tag{26}$$

and the solution of Equation (14) for a line load with vertical component P and horizontal component q_1P is

$$\sigma_z = \frac{P}{[4\pi W(z)]^{1/2}} \exp\left(-\frac{(x-q_1 z)^2}{4W(z)}\right) \tag{27}$$

The shear stress can then be found from Equation (16) as

$$\tau_{zx} = \left(q_1 + \frac{(x-q_1 z)D(z)}{2W(z)}\right)\sigma_z \tag{28}$$

The horizontal stress can also be solved from Equation (17) as

$$\sigma_x = \left\{D'(z) - \frac{D^2(z)}{2W(z)} + \frac{D^2(z)}{4W^2(z)}\left[x - q_1 z + \frac{2q_1 W(z)}{D(z)}\right]^2\right\}\sigma_z \tag{29}$$

where

$$D'(z) = \frac{\partial D(z)}{\partial z} \tag{30}$$

One remarkable property noted by Golden (1984) is that, although nonrealistic horizontal tensile stresses are predicted by elastic theory if horizontal forces are present, they can be avoided by this theory if D(z) is such that $D'(z) \geq D^2(z)/2W(z)$. Another observation is that for a purely horizontal force the elastic theory predicts stresses in granular material, while in these equations predict zero stresses.

2. Three Dimensions

In three dimensions with $D_{xz} = D_{yz} = 0$, the solution of Equation (25) for a point load with vertical component Q and horizontal component Qq_1 in the x direction is

$$\sigma_z = \frac{Q}{4\pi W(z)} \exp\left\{-\left[\frac{(x-q_1 z)^2 + y^2}{4W(z)}\right]\right\} \tag{31}$$

It can also be shown (Golden, 1986) that without knowledge of D_{xy}, the other stresses can be described in terms of σ_z by the following equations

$$\tau_{xz} = q_1\sigma_z - D\frac{\partial\sigma_z}{\partial x} \tag{32}$$

$$\tau_{yz} = -D\frac{\partial\sigma_z}{\partial y} \tag{33}$$

$$\tau_{xy} = \left[-q_1 D\frac{\partial}{\partial y} + D^2\frac{\partial^2}{\partial x\partial y}\right]\sigma_z \tag{34}$$

$$\sigma_x = \left[q_1^2 - 2q_1 D\frac{\partial}{\partial x} + D' + D^2\frac{\partial^2}{\partial x\partial y}\right]\sigma_z \tag{35}$$

and

$$\sigma_y = \left[D' + D^2\frac{\partial^2}{\partial y^2}\right]\sigma_z \tag{36}$$

where D' is described in Equation (30).

SECTION III
STRAIN ACCUMULATION IN GRANULAR MATERIALS UNDER STATIC AND REPEATED LOADS

A. INTRODUCTION

The stress-strain behavior of granular soils is created primarily by individual particle movements to form a denser matrix rather than elastic compression of the particles. As a result, the experimental curve is always nonlinear and depends on the loading conditions. In general, the shape of the stress-strain curve is concave towards the strain axis in triaxial compression, and concave towards the stress axis in one dimensional (uniaxial strain) and isotropic compression. Further, it is noted that in triaxial conditions some sands show a distinct peak stress while others do not. As a result, it is difficult to model the behavior of this material by conventional elasto-plastic theories, and recourse is usually taken to empirical methods like the hyperbolic model for matching triaxial test results with no distinct peak stress (Desai and Siriwardane, 1984) or numerical curve fitting techniques (Desai, 1971).

In this section, traditional approaches to stress-strain modeling in granular media is identified and a new, more rational, approach is introduced. This new approach is particulate and is in agreement with the stress transmission theory of the previous section. That is, the deformation is considered to be the result of particle movement rather than deformation of particles. The result is a general model that derives the stress-strain response for one dimensional, isotropic, and triaxial loading conditions as special cases.

B. TRADITIONAL APPROACH TO STRESS-STRAIN MODELING OF GRANULAR MATERIAL

The experimentally-observed stress-strain curves of granular soils are always nonlinear. The shape of the curve also differs based on the boundary conditions imposed in the radial directions. As a result, traditional elastic and elasto-plastic theories have failed to adequately model the behavior of this material. So far, no general model exists that accounts for all imposed boundary conditions and traditional methods consist of applying curve-fitting techniques to curves obtained under specific conditions.

Of primary importance in settlement calculations is the need to model the triaxial compression condition. One simple empirical method proposed by Konder and Zelasko (1963) is the widely-used hyperbolic model. This model appears to be a natural fit to soils that do not exhibit a distinct peak stress. For soils that exhibit a distinct peak stress, a modification of the Ramberg-Osgood empirical model used for dynamic loading is proposed for static conditions (Desai and Siriwardane, 1984). This is based on fitting a curve to the initial tangent modulus, the modulus of the plastic zone, the yield stress, and a parameter defining the order of the curve. For order one, this curve becomes a hyperbola. One other approach suggested is to treat the tangent shear modulus and tangent bulk modulus as variables. These are usually taken as linear functions of the octahedral normal and shear stresses (Nelson and Barron, 1971).

The difficulty in modeling the nonlinear stress-strain behavior of sand under load has led many investigators to propose numerical curve-fitting techniques. One of the most popular of these is the piecewise linear method. Here, the nonlinear experimental curve is divided into pieces of linear elastic sections for numerical analysis. Very often these are the incremental Hooke's law or the hypoelastic law (Desai and Wu, 1976). Another numerical method is the use of spline functions to fit experimentally-observed curves. These are functions that use the data to provide an analytic curve similar to the graphical process of using a French curve (Desai, 1971). These also require intense numerical procedures, and the data must be presented in a smooth form and not scattered as observed experimentally.

C. NEW APPROACH TO STRESS-STRAIN MODELING OF GRANULAR MATERIAL

1. General Theory

Granular media is composed of voids and particles. Deformation is caused primarily by particle movements. As the particle movements are discrete, their derivative at a point does not exist, and the strain in this media cannot be defined at a point. Strain can only be defined with respect to an elemental volume of soil with enough particles so that the relative movements of the particles in the elemental volume can produce deformation. Like the strain, the stress in granular soils does not exist in a void and should not be described at a point. The stress corresponding to the strain in the elemental volume is the result of forces on the particles in the elemental volume and, like the strain, can only be defined with respect to the elemental volume. These definitions of strain and stress are given analytically as Equations (5) and (6) of Section II.

The substitution of Equation (5) into Equation (6) gave the stress-strain relationship of Equation (7). Referring to the z direction as the axial direction and letting subscript "a" refer to the axial direction, the stress-strain relationship in the axial direction as obtained from Equation (7) is

$$\sigma_a = \frac{Nkh\ \varepsilon_a}{(ij)} \tag{37}$$

where N is a function of the volumetric strain as shown in Equation (8). The substitution of Equation (8) into Equation (37) gives the stress-strain relationship in the axial direction as

$$\sigma_a = \frac{E_o\ \varepsilon_a}{(1-\beta\varepsilon_v)} \tag{38}$$

where

$$E_o = \frac{h^2 \ k}{\left[V_p \ (1+e_o) \right]} \tag{39}$$

Here, $\beta = 1/\varepsilon_{v1}$ and ε_{v1} is the maximum value obtainable by ε_v. This latter term was added since $\varepsilon_v = 1$ is unattainable under conventional loads.

The derivative of Equation (38) gives the slope of the stress-strain curve as

$$\frac{\partial \sigma_a}{\partial \varepsilon_a} = \frac{(1-\beta\varepsilon_v) \ E_o \ + \ E_o\beta\varepsilon_a \frac{d\varepsilon_v}{d\varepsilon_a}}{(1-\beta\varepsilon_v)^2} \tag{40}$$

It is apparent from this that at $\varepsilon_a = \varepsilon_v = 0$, the slope is E_o. Therefore, E_o is the initial tangent modulus of the soil, and knowledge of h and k is not necessary if E_o can be measured.

2. Application To One Dimensional And Isotropic Loading Conditions

In one dimensional compression of soils no lateral strains are allowed, and the volumetric strain is equal to the axial strain. That is $\varepsilon_v = \varepsilon_a$ and $\varepsilon_{v1} = \varepsilon_{aL}$, where ε_{aL} is the asymptotic axial strain. Therefore, in this case, Equation (38) becomes

$$\sigma_a = \frac{E_o \ \varepsilon_a}{1 - \dfrac{\varepsilon_a}{\varepsilon_{aL}}} \tag{41}$$

In isotropic loading conditions, the strains are equal in all directions. Hence, $\varepsilon_v = 3\varepsilon_a$ and $\varepsilon_{v1} = 3\varepsilon_{aL}$, and Equation (41) also applies for this case.

It should be noted that Equation (41) can be written as

$$\sigma_a = E_o \left(\varepsilon_a + \frac{\varepsilon_a^2}{\varepsilon_{aL}} + \ldots\right) \tag{42}$$

and

$$\frac{\varepsilon_{aL}\, E_o}{2} \left[\exp(2\, \varepsilon_a/\varepsilon_{aL} - 1) - 1\right] = E_o \left(\varepsilon_a + \varepsilon_a^2/\varepsilon_{aL} + \ldots\right) \tag{43}$$

Therefore letting $\alpha_1 = 2/\varepsilon_{aL}$ and $C = E_o/\alpha_1$ gives the alternate expression

$$\sigma_a = C \exp(\alpha_1 \varepsilon_a) - C \tag{44}$$

Equation (44) is the same as that derived by the hypoelastic analysis for these loading conditions (Desai and Siriwardane, 1984).

3. Application To Triaxial Loading Conditions

In elastic material under triaxial compression the radial strain, ε_r, is proportional to the axial strain, and the proportional constant, Poisson's ratio, is determined empirically. In effect, if $v_t = -d\varepsilon_r/d\varepsilon_a$ and $v_s = -\varepsilon_r/\varepsilon_a$, then in elastic material $v_t = v_s$. However, in triaxial compression of sands, the relationship of radial strain to axial strain is nonlinear. To represent this nonlinear condition one can let $v_s - v_t = D_2$, where D_2 is an empirical constant representing the average difference between v_s and v_t. If $D_2 = 0$ the relationship is linear, and the nonlinearity increases as the magnitude of D_2 increases. The definition $v_s = -\varepsilon_r/\varepsilon_a$ gives

$$\frac{dv_s}{d\varepsilon_a} = (\varepsilon_r + \varepsilon_a\, v_t/\varepsilon_a^2) = -\frac{D_2}{\varepsilon_a} \tag{45}$$

Also since $\varepsilon_v = \varepsilon_a + 2\varepsilon_r$ then $\nu_s = (\varepsilon_a - \varepsilon_v)/(2\varepsilon_a)$, which shows that

$$\frac{d\nu_s}{d\varepsilon_a} = -\ 0.5\ \frac{d(\varepsilon_v/\varepsilon_a)}{d\varepsilon_a} \tag{46}$$

Equating Equations (45) and (46) gives $d(\varepsilon_v/\varepsilon_a)/d\varepsilon_a = 2D_2/\varepsilon_a$, which has solution

$$\varepsilon_v = B_2\varepsilon_a + 2D_2\varepsilon_a \ln\varepsilon_a \tag{47}$$

where B_2 is an integration constant. It should be noted that $x\ln x$ approaches zero as x approaches zero. For example, $(0.001)\ln 0.001 = -0.007$ and $(0.0001)\ln 0.0001 = -0.0009$.

The substitution of Equation (47) into Equation (38) yields

$$\sigma_a = \frac{E_o\ \varepsilon_a}{(1 - \beta B_2\varepsilon_a - 2\beta D_2\varepsilon_a \ln\varepsilon_a)} \tag{48}$$

Letting $\sigma_a = \sigma_u$ and $\varepsilon_a = \varepsilon_u$ at maximum stress, Equation (48) gives $\beta B_2 = 1/\varepsilon_u - E_o/\sigma_u - 2\beta D_2 \ln\varepsilon_u$. Also setting $d\sigma_a/d\varepsilon_a = 0$ at maximum stress given $2\beta D_2 = -1/\varepsilon_u$. The substitution of these into Equation (48) yields the general relationship

$$\sigma_a = \frac{\varepsilon_a}{\left[\ a + a(\varepsilon_a/\varepsilon_u)\ln\varepsilon_a + b\varepsilon_a\right]} \tag{49}$$

where $a = 1/E_o$ and $b = 1/\sigma_u - (1+\ln\varepsilon_u)/(E_o\varepsilon_u)$. In soils with no distinct peak stress such as loose sands and sands under high pressure, ε_u approaches infinity, therefore, Equation (49) reduces to the hyperbolic model

$$\sigma_a = \frac{\varepsilon_a}{\left(a + b\varepsilon_a\right)} \tag{50}$$

where $a = 1/E_o$ and $b = 1/\sigma_u$.

The constants B_2 and D_2 in Equation (47) can also be evaluated in terms of the values at maximum volumetric strain. If $\varepsilon_a = \varepsilon_o$ at $d\varepsilon_v/d\varepsilon_a = 0$, then $B_2 = -2D_2(1+\ln\varepsilon_o)$. Also, since at $\varepsilon_a = \varepsilon_o$ the volumetric strain $\varepsilon_v = \varepsilon_{vm}$, the maximum volumetric strain, Equation (47) becomes

$$\varepsilon_v = \varepsilon_a(\varepsilon_{vm}/\varepsilon_o)\ [1 + \ln\ (\varepsilon_o/\varepsilon_a)] \tag{51}$$

This equation shows that $\varepsilon_v = 0$ at the two points $\varepsilon_a = 0$ and $\varepsilon_a = \varepsilon_1$. The value of ε_1 is obtained from Equation (51) as $\varepsilon_1 = 2.718\varepsilon_o$. Further, comparison of these values of B_2 and D_2 with those found in Equation (48) gives $\beta = \varepsilon_o/(\varepsilon_u\varepsilon_{vm})$ and ε_u satisfying $E_o\varepsilon_u - \sigma_u\ln(\varepsilon_u/\varepsilon_o)$.

D. PERMANENT AXIAL STRAIN ACCUMULATION IN GRANULAR MATERIALS

1. Static Loading

Equation (49) gives the axial stress-strain relationship under triaxial conditions. In this equation, ε_a is the total strain accumulated under the static stress increment σ_a. Upon removal of the load, the soil rebounds. The amount of rebound is the elastic portion of the total strain and can be written as

$$\varepsilon_a^e = \frac{1}{E_r}\sigma_a \tag{52}$$

where E_r is the resilient modulus of the soil. The permanent strain accumulated due to one application of σ_a is

$$\varepsilon_a^p = \varepsilon_a - \varepsilon_a^e = \frac{1}{E_o}h(\sigma_a) - \frac{1}{E_r}\sigma_a \tag{53}$$

where $h(\sigma_a)$ is obtained from Equation (49) by solving for ε_a in terms of σ_a. For the hyperbolic case it is obtained from Equation (50) as $h(\sigma_a) = a\sigma_a/(1-b\sigma_a)$. Equation (33) may be written in the more compact form:

$$\varepsilon_a^p = \frac{1}{E_o} f(\sigma_a) \tag{54}$$

where

$$f(\sigma_a) = h(\sigma_a) - k_1\sigma_a \tag{55}$$

and $k_1 = E_o/E_r$.

2. Repeated Loading

At every load application there is some readjustment of the grains in a soil sample. This rearrangement becomes less pronounced as the number of load applications increases because the soil becomes more packed. Therefore, the rate of change of initial stiffness at any load application depends on the particle arrangement (or on the stiffness) at that load application and decreases with increasing load applications. The simplest way to represent this behavior is by a power function

$$\frac{dE_{oi}}{di} = K_1 E_{oi}^{-n} \tag{56}$$

where Eoi is the initial target modulus at the ith cycle of loading, K_1 is a proportional constant, and n is a parameter reflecting the dependence of the rate of change of Eoi on Eoi. The solution of Equation (56) gives

$$K_1 i = \int E_{oi}^{n} dE_{oi} = \frac{E_{oi}^{n+1}}{n+1} + A_1 \tag{57}$$

where A_1 is an integration constant. Solving for E_{oi} gives

$$E_{oi} = [Ai + B]^{m} \tag{58}$$

where

$$A = (n+1)K_1, \quad B = -A_1(n+1), \quad m = \frac{1}{n+1} \tag{59}$$

The permanent strain due cycle i is then obtained from Equation (54) as

$$\varepsilon_{ai}^{p} = \frac{1}{E_{oi}} f_i(\sigma_a) \tag{60}$$

where $f_i(\sigma_a)$ is the relation defined by Equation (55) for the ith load application.

The permanent strain accumulated in N cycles is then

$$\varepsilon_{aN}^{p} = \int_0^N f_i(\sigma_a) \, [Ai + B]^{-m} \, di \tag{61}$$

For soils where the hyperbolic model presents a good fit, it can be shown that $f_i(\sigma_a) = f(\sigma_a)$ if σ_a is constant, that is, it is independent of load cycle. This is because for these soils, the assymptotic stress σ_u and the ratio $k_1 = E_o/E_r$ are fairly constant during load applications. For this case, $f_i(\sigma_a)$ can be taken out of the integral, and the result of the integration of Equation (61) depends on whether m is equal to one or not.

a. Case Where m = 1

In this case, n = 0 or the change of E_{oi} where load application is constant. In this case, Equation (61) becomes

$$\varepsilon_{aN}^{p} = f(\sigma_a)\int_0^N \frac{di}{Ai+B} = \frac{f(\sigma_a)}{A}\left[\ln\left(N + \frac{B}{A}\right) - \ln\frac{B}{A}\right] \tag{62}$$

From Equation (59) for n = 0, it is seen that $B/A = -A_1/K_1$, and from Equation (57) for i = 1 and i = 2, one finds $K_1 = E_{02} - E_{01}$ and $A_1 = E_{02} - 2E_{01}$. This means that

$$\frac{B}{A} = \frac{2E_{o1} - E_{o2}}{E_{o2} - E_{o1}} \tag{63}$$

In general, E_{02} is greater than E_{01}; hence B/A is small compared to N. This means that as an approximation, the permanent axial strain accumulated is obtained from Equation (62) as

$$\varepsilon_{aN}^{p} = a_1 + b_1 \ln N \tag{64}$$

Where $b_1 = f(\sigma_a)/A$ and $a_1 = (1/A)f(\sigma_a)\ln(B/A)$ are constants. It is also seen from Equation (64) that $a_1 = \varepsilon_{a1}^{p}$ as defined in Equation (60). Equation (64) is that proposed by Lentz and Baladi (1981) based on its goodness of fit to their experimental data.

b. Case Where $m \neq 1$

In this case, Equation (61) becomes

$$\varepsilon_{aN}^{p} = f(\sigma_a)\int_0^N [Ai + B]^{-m}\, di = \frac{A^{1-m} f(\sigma_a)}{1-m}\left\{ \left[N+\frac{B}{A}\right]^{1-m} - \left(\frac{B}{A}\right)^{1-m}\right\} \tag{65}$$

Again from Equation (59) it is seen that $B/A = -A_1/K_1$, and from Equation (57), that $(n+1)K1 = E_{02}^{n+1} - E_{01}^{n+1}$ and $(n+1)A1 = E_{02}^{n+1} - 2\,E_{01}^{n+1}$. From this, B/A is obtained as

$$\frac{B}{A} = \frac{2E_{o1}^{n+1} - E_{o2}^{n+1}}{E_{o1}^{n+1} - E_{o2}^{n+1}} \tag{66}$$

Since E_{02} is greater than E_{01}, it is seen that B/A is small compared to N, and Equation (65) can be approximated by

$$\frac{\varepsilon_{aN}^{p}}{N} = B_1\ N^{-m} \tag{67}$$

where $B_1 = A^{1-m}f(\sigma_a)/(1-m)$. This equation is the same as that suggested by Khedr (1986) and Diyaljee and Raymond (1982) based on their experimental work. It should also be noted that from Equation (67) $B_1 = \varepsilon_{a1}^p$ as defined in Equation (60).

E. PERMANENT RADIAL STRAIN ACCUMULATION IN GRANULAR MATERIALS

The total radial strain during static loading is $\varepsilon_r = (\varepsilon_v - \varepsilon_a)/2$ where ε_v is given by Equation (51). The elastic radial strain is

$$\varepsilon_r^e = -\nu\varepsilon_a^e = -\frac{\nu}{E_r}\sigma_a \tag{68}$$

where ν is the elastic Poisson ratio during unloading. This means that the permanent radial strain is

$$\varepsilon_r^p = \varepsilon_r - \varepsilon_r^e \tag{69}$$

The cumulative radial strain for N cycles of loading is then

$$\varepsilon_{rN}^p = \int_0^N \left\{\frac{\varepsilon_{vm}}{2\varepsilon_o}\varepsilon_{ai}\left[1 - \ln(\varepsilon_o/\varepsilon_{ai})\right] - \frac{\varepsilon_{ai}}{2} + \frac{\nu}{E_{ri}}\sigma_a\right\} di \tag{70}$$

where

$$\varepsilon_{ai} = \frac{1}{E_{oi}} h(\sigma_a) \tag{71}$$

and E_{oi} is given by Equation (58). The integration presented by Equation (70) is clumsy. An approximate more straightforward estimate can be evaluated using the formula presented by Chang and Whitman (1988). In their evaluation, they found that

$$\frac{\varepsilon_v^p}{\varepsilon_\gamma^p} = \frac{M^2 - \eta^2}{2\eta} \tag{72}$$

where $\varepsilon_v^p = \varepsilon_a^p + 2\,\varepsilon_r^p$ is the permanent volumetric strain, $\varepsilon_\gamma^p = 2/3\left(\varepsilon_a^p - \varepsilon_r^p\right)$ is the permanent shear strain. Also, M is the ratio of mean and deviatoric stress at $\varepsilon_a = \varepsilon_0$, that is at minimum volumetric strain, and η is the ratio of mean and deviatoric stress at $\varepsilon_a = 0$, or at the start of loading. The mean stress is $(\sigma_a + 2\sigma_r)/3$, and the deviatoric stress is $\sigma_a - \sigma_r$. From Equation (72), the permanent radial strain can be solved in terms of the permanent axial strain as

$$\varepsilon_r^p = \frac{M^2 - \eta - 3\eta}{M^2 - \eta^2 + 6\eta}\,\varepsilon_a^p \tag{73}$$

The permanent radial strain accumulated after N cycles is

$$\varepsilon_{rN}^p = \frac{M^2 - \eta - 3\eta}{M^2 - \eta^2 + 6\eta}\,\varepsilon_{aN}^p \tag{74}$$

where ε_{aN}^p is the permanent axial strain accumulated after N cycles as given in Equation (64) or (67).

SECTION IV

VALIDATION OF PARTIULATE THEORY FOR GRANULAR MATERIAL

A. INTRODUCTION

In this chapter, the assumptions made in the theory leading up to the stress continuity equation of particulate media (Equation 11), the constitutive equation of particulate media (Equation 38), and the strain growth equation of particulate media (Equation 61) is verified. The method of validation is by the comparison of these analytical expressions with experimental observations found in the literature.

To validate the stress continuity concept for granular media, the stresses predicted by the particulate theory developed in Section II is evaluated against measured stresses in particulate media. The results are further compared with the predictions of the linear elastic theory. The stress-strain theory for granular media developed in Section III is also compared with measured stress-strain data on granular media. First, the stress-strain responses under static isotropic and static triaxial compression are examined. Next, the theory on permanent strain accumulation in granular material under repeated loading is compared with experimental observations using different load magnitudes.

B. VALIDATION OF PARTICULATE THEORY OF STRESS TRANSFER

It was already shown in Section II that the particulate theory of stress transfer in granular media does not predict horizontal tensile stresses for inclined loads and predicts no stress transfer for purely horizontal loads. These two phenomena are unique to granular material and cannot be avoided by any theory that imposes the requirement for the existence of the second derivatives of strains in the media. In this section, the comparison of the theory for the prediction of vertical stresses under a loaded area with observed experimental results will be made.

Morgan and Gerrard (1981) presented experimental results of several investigators on the stress distribution in sands under load. Fairly complete information was obtainable from their report on the results of the Waterways Experiment Station (WES) tests and the tests at Melbourne University (M1 and M2). These tests were performed using uniform vertical loads (q) over circular areas of radius "a" and the results included the distribution of vertical stress with depth directly under the center of the areas as shown in Figure 2.

Directly under the center of a circular area of radius "a", the elastic solution predicts from a vertical uniform load q

$$\frac{\sigma_z}{q} = 1 - \left[(a/z)^2 + 1 \right]^{-3/2} \tag{75}$$

This is shown as the dashed line of Figure 2.

For a vertical uniform distributed load of intensity q over a circular area of radius "a", with coordinate origin taken at the center of the loaded area, Equation (31) gives

$$\sigma_z = \frac{q}{4\pi W(z)} \int_0^{2\pi} \int_0^{a} \exp \left[- \frac{1}{4W(z)} (r^2 + \rho^2 - 2r\rho\cos\theta) \right] r dr d\theta \tag{76}$$

where $\rho^2 = x^2 + y^2$. That is, in Equation (31) let $q_1 = 0$, replace Q by $q r dr d\theta$, and employ the law of cosines (Figure 3). Directly under the center of the area, $\rho = 0$, and the integration of Equation (76) gives

$$\frac{\sigma_z}{q} = 1 - \exp\left\{ -a^2/[4W(z)] \right\} \tag{77}$$

Reference 11 showed that the assumption that D(z) is a linear function of z satisfies all granular soil requirements. From Equation (26), this means that W(z) can be expressed as

$$\frac{W(z)}{z^2} = d_1 + d_2 \left(\frac{a}{z} \right) \tag{78}$$

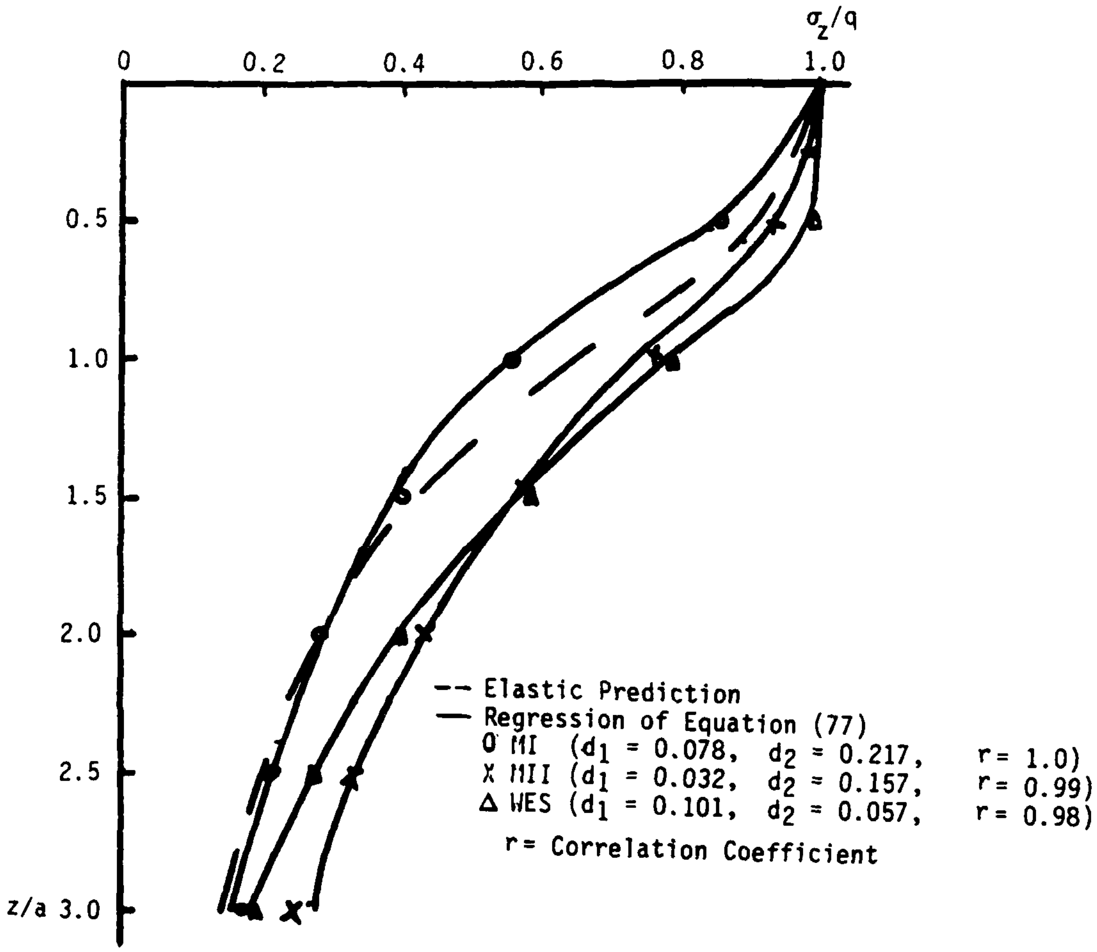

Figure 2. Vertical Stress Below The Center Of A Uniformly-Loaded Circular Area (Data From Reference 22).

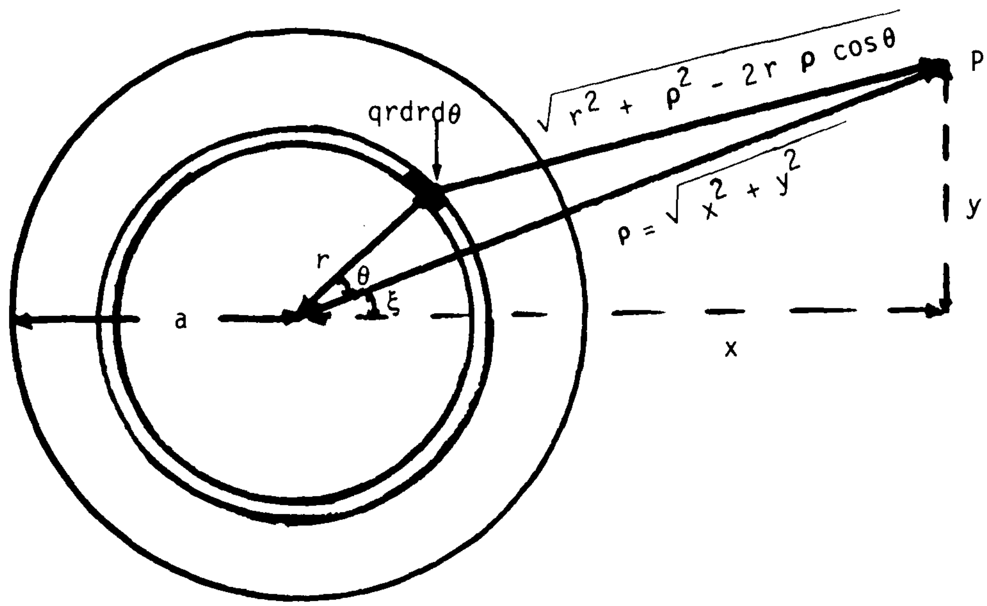

Figure 3. Location Of A Point P From Circular Loaded Area Of Radius A.

The substitution of Equation (78) into Equation (77) gives the transformed equation

$$\frac{-(a/z)^2}{4\ln(1-\sigma_z/q)} = d_1 + d_2\left(\frac{a}{z}\right) \tag{79}$$

This equation is suitable for linear regression on the data of Figure 2. This linear regression produced the solid lines in Figure 2. For each soil the fit is remarkable, indicating that the form of Equation (78) is appropriate for this type of loading. The regression revealed that $d_1 = 0.078$, $d_2 = 0.217$ for the M1 data, $d_1 = 0.032$, $d_2 = 0.157$ for the M_2 data, and $d_1 = 0.101$, $d_2 = 0.057$ for the WES data.

Figure 2 can also give an indication of the relative lateral spread of the distribution. To do this, the approximate method of analysis for vertical stress directly below the loaded area is employed. This says for $\rho = 0$

$$\sigma_z \approx \frac{q(\pi a^2)}{\pi\,(a + z\tan\alpha)^2} \tag{80}$$

Where tan α is a measure of the magnitude lateral spread with depth. A value of $\alpha = 30$ degrees estimated from the theory of elasticity, is usually assumed (Dunn, Anderson, and Kiefer, 1980). From Equation (80), it is found that

$$\tan\alpha \approx \frac{\left(\sqrt{\frac{q}{\sigma_z}} - 1\right)}{\left(\frac{z}{a}\right)} \tag{81}$$

The best fit value for the angle α from the data of Figure 2 is $\alpha = 26.01$ degrees for MI, $\alpha = 19.33$ degrees for MII, $\alpha = 23.81$ degrees for WES and $\alpha = 29.31$ degrees for the elastic prediction. The larger angle of α for the elastic prediction indicates that the elastic solution predicts a wider lateral spread of the vertical pressure distribution. The method of compaction is seen to be a determining factor in the spread, as the MI sand was identical to the MII sand except that the former was compacted by a vibrating plate and the latter by pluvial compaction.

C. VALIDATION OF PARTICULATE THEORY OF STRESS-STRAIN RESPONSE

It was shown in Section III that Equation (38) is a general expression for the relationship of axial stress to axial strain. For the special conditions of one dimensional and isotropic compression, this equation reduced to Equation (41) and, for the case of isotropic compression, to Equation (49).

The validation of Equation (41) was made using data for isotropic loading conditions on two samples of McCormic Ranch sand presented by Desai and Siriwardane (1984, pg. 193) and shown in Figure 4. In both cases, the regression of Equation (41) on the data revealed 99.9 percent correlation. The regression curve is shown as the dashed lines in Figure 4.

The validation of Equation (49) was also made on data for a medium dense sand presented by Desai and Siriwardane (1984, pg 177). The measured stress-strain response of this sand at two different confining pressures (σ_3) are shown in Figure 5. Again, the fit of the equation was excellent and a regression of Equation (49) on the data revealed correlation coefficients of 99.8 percent. The predicted curves are shown as the dashed line in Figure 5.

Equation (49) is based on the nonlinear relationship of the volumetric strain to the axial strain derived in Equation (47). This equation was also evaluated using the volumetric versus axial strain measurements of the sand in Figure 5 taken at the two confining pressures. This data is shown in Figure 6. The regression of Equation (47) on this data is shown as the dashed line in the figure. The correlation coefficient was 99 percent for each case.

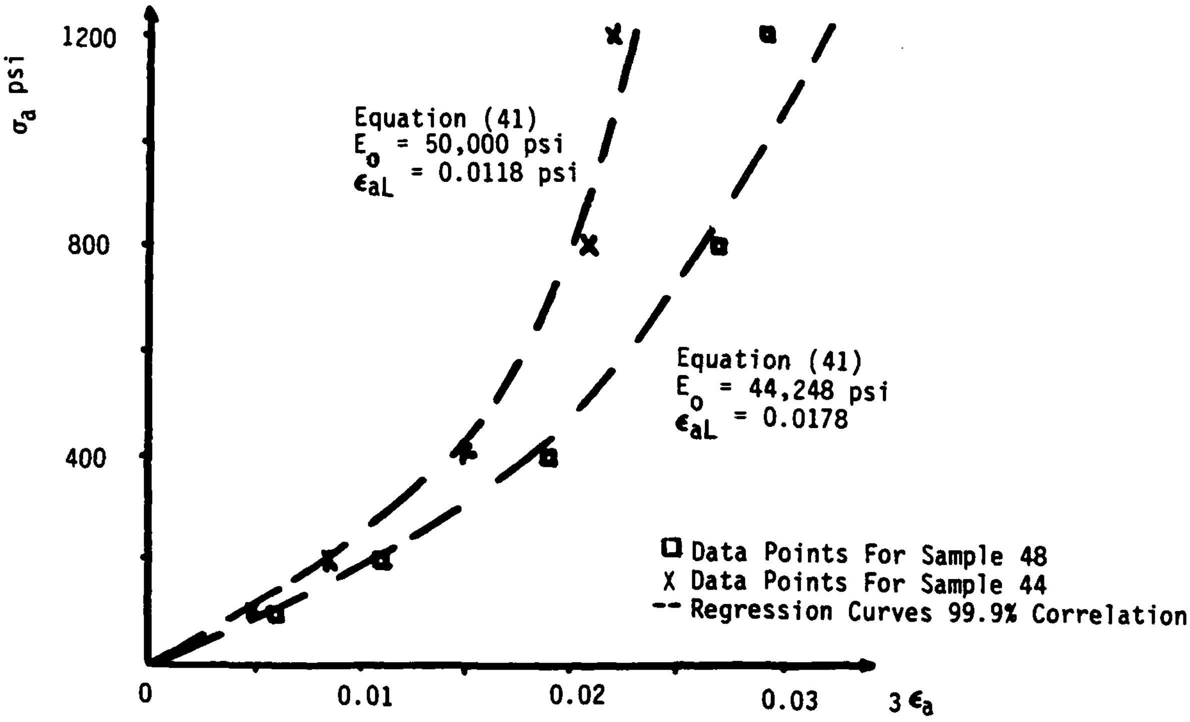

Figure 4. Stress-Strain Behavior For Isotropic Loading (Data From Reference 6, p193, 1 psi = 6.895 kPa).

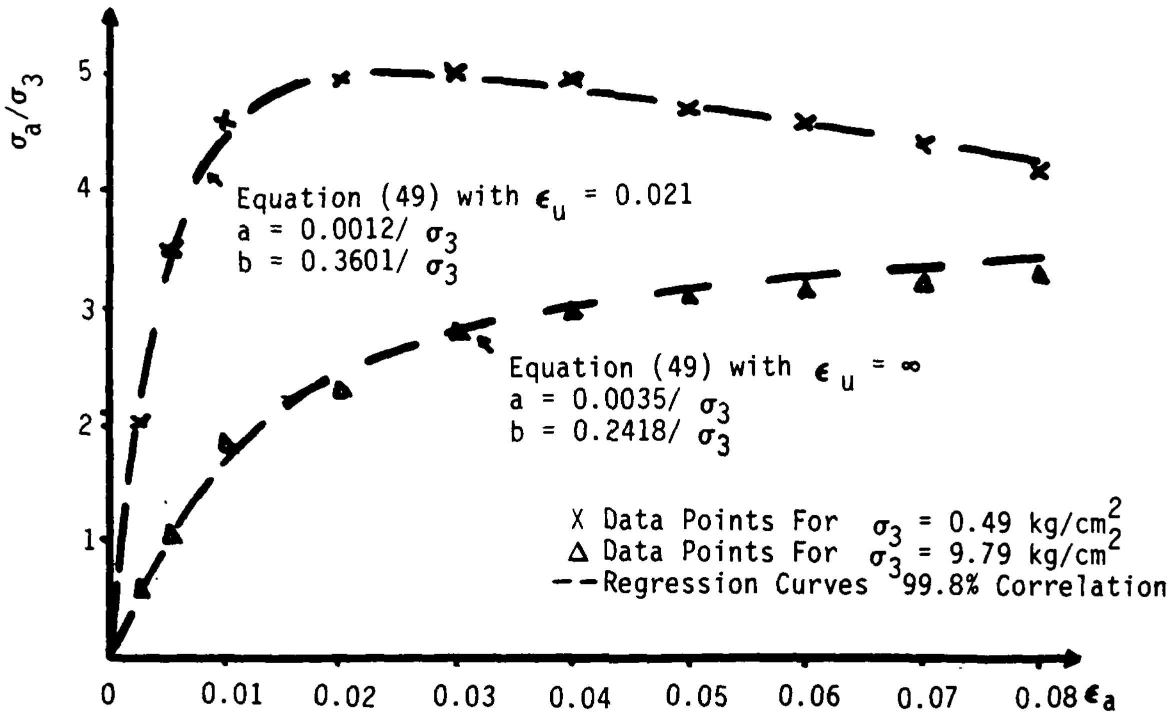

Figure 5. Stress-Strain Behavior For Triaxial Compression (Data From Reference 6, p177).

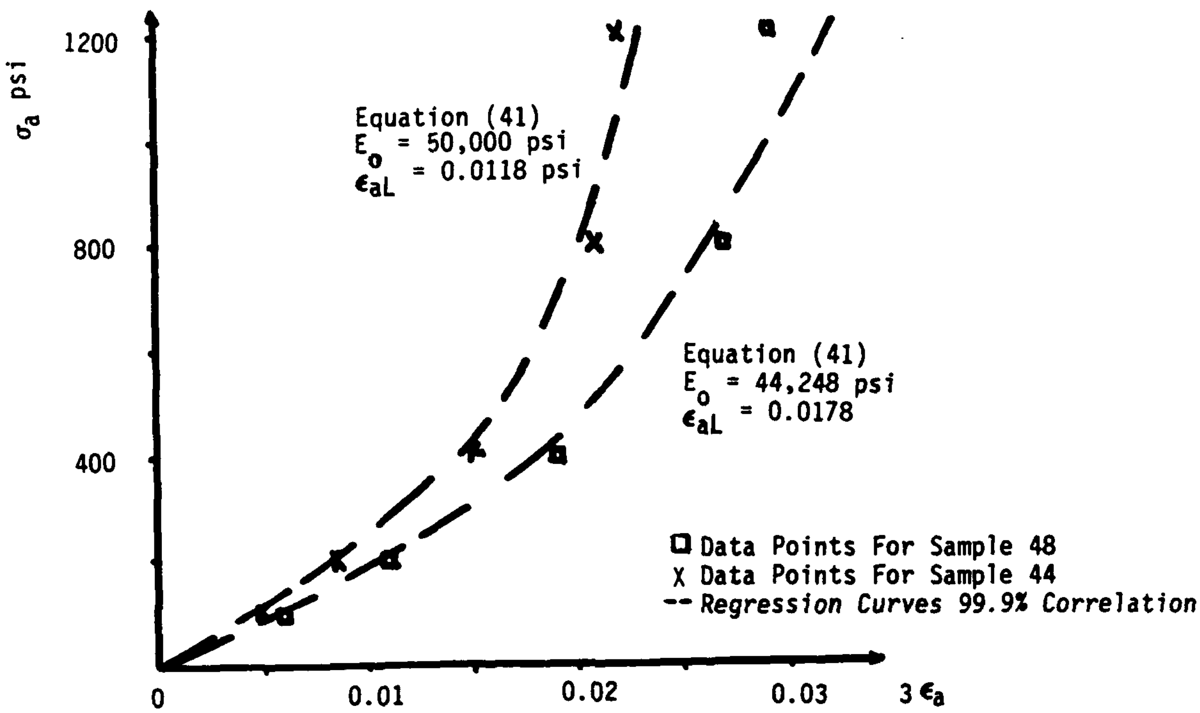

Figure 6. Relationship Of Volumetric And Axial Strains (Data From Reference 6, p177).

D. VALIDATION OF THE PARTICULATE THEORY OF STRAIN ACCUMULATION UNDER REPEATED LOADING

The nature of strain accumulation in granular material is presented in the general form of Equation (61). This equation is a function of a parameter m that reflects not only the nature of the material but also the magnitude of the applied load. To see this, Equation (39) can be written as

$$E_o = \frac{h^2 k(1-n_o)}{V_p} \tag{82}$$

where $n_o = e_o/(1+e_o)$ is the initial porosity of the soil. For small loads, the soil compacts in fairly equal increments, and changes in the porosity are of equal increments. From Equation (82), this means that for small loads, the initial tangent modulus increases linearly with the number of load applications. In this case, Equation (58) says that m is close to unity and Equation (61) gives the semi-log relationship of Equation (64). For large loads, the soil compacts in more uneven increments with larger increments occurring at the early cycles of loading. This reveals the m in Equation (58) should be less than unity, and Equation (61) becomes the log-log expression of Equation (67).

To validate Equations (64) and (67), data on permanent axial strain accumulation with number of cycles of loading for five load magnitudes on a Dolomite Ballast presented by Diyaljee and Raymond (1982) were used. These experimental results are plotted on a log-log plot in Figure 7. In each case, Equation (67) represented a perfect fit, with m = 0.87 for the four smaller loads and m = 0.80 for the largest load.

The logic behind Equation (64) can also be tested using the data of Figure 7. To do this, the data are also plotted on a semi-log scale as shown in Figure 8. The data with m closest to unity should plot more linear than those with values of m furthest from unity. This is seen in Figure 8 where the data with m = 0.87 also plotted a straight line on

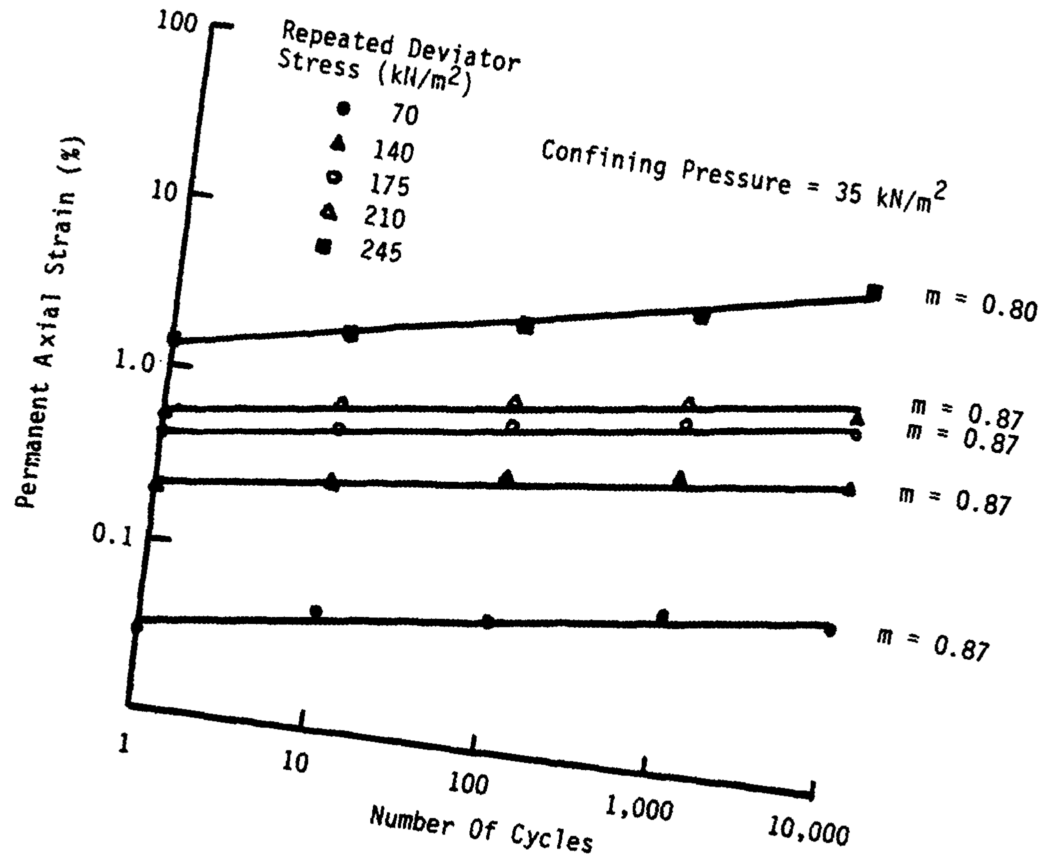

Figure 7. Log-Log Plot Of Permanent Axial Strain Versus Number of Load Cycles (Data From Reference 7).

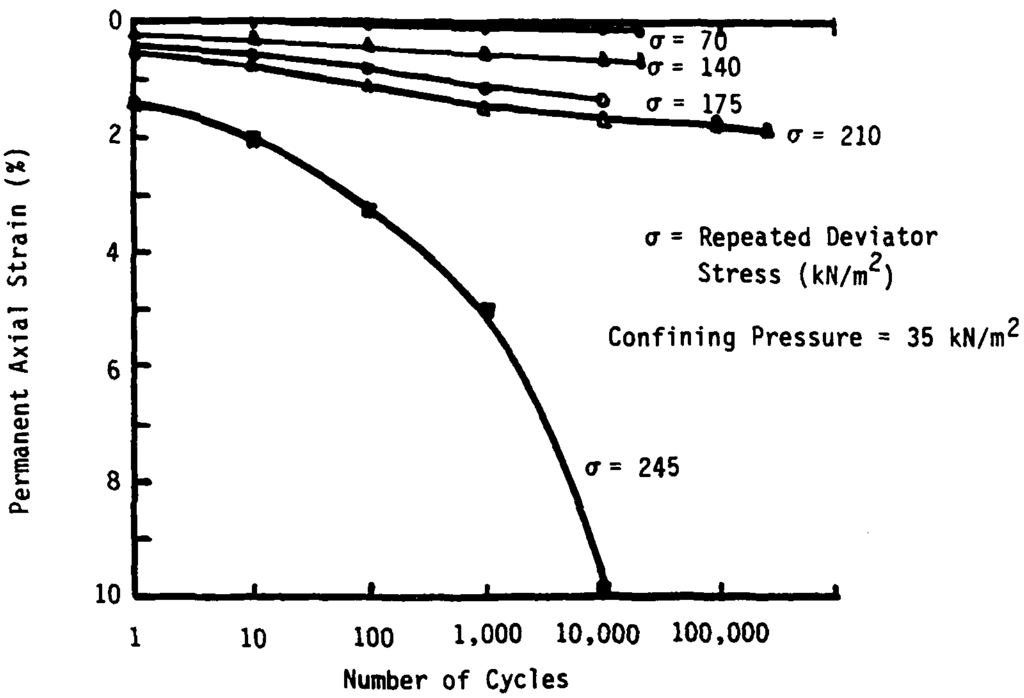

Figure 8. Semi-Log Plot Of Permanent Axial Strain Versus Number Of Load Cycles (Data From Reference 7).

this semi-log scale. However, the case of the heaviest load, with $m = 0.80$, shows a distinct non-linear plot on the semi-log scale.

SECTION V
EVALUATION OF MATERIAL PARAMETERS IN PARTICULATE THEORY

A. INTRODUCTION

In the development of the stress distribution theory of Section II and the stress-strain and strain accumulation theories of Section III, several material parameters were identified. These parameters are material specific and, like Young's modulus and Poisson's ratio of the theory of elasticity, are best evaluated from experimental observations. However, it must first be determined if these parameters are also functions of geometry and load. If they are, their dependence on these quantities should be identified such that experimental procedures can be scaled to observe only the material characteristics of the parameters.

In this section, the material parameters arising in the stress distribution theory, the stress-strain theory, and the strain accumulation theory are examined and methods of determining them are discussed. In particular, their relationship to other parameters such as depth in the soil, confining pressure and load intensity are identified.

B. MATERIAL CONSTANTS IN PARTICULATE THEORY OF STRESS TRANSFER

The theory of stress transfer in granular material presented in Equation (14) depends on two constants D and b. It is shown in Section II that b = q_1, the ratio of the horizontal component to the vertical component of the applied load. The parameter D is the diffusion coefficient of the soil. In Equation (13) N_2 is a fraction of N; therefore D is dependent only on the size of the elemental volume ijh needed to adequately define stress and strain in the material. In particular, D is proportional to the square of the horizontal component, i^2, and inversely proportional to the vertical component h. For small particle displacement, a small volume ijk of soil may be used to adequately define stress and strain, while for large particle displacements, a larger volume of soil is needed to make these definitions. As a result, the magnitudes of i, j, and h at a given location are dependent on the stress level at that location. It is shown in Section IV that for a vertical load on a circular foundation of radius "a", the coefficient D can be approximated by a linear function of depth z as

$$D = 2d_1z + ad_2 \tag{83}$$

where d_1 and d_2 are material parameters to be determined from experiments. The linear relationship of D to z is also observed by Golden (1986) and Hill and Harr (1971). Equation (83) indicates that ad_2 is a term reflecting the boundary condition at z = 0, and d_1 is a parameter reflecting the lateral spread of the vertical stress distribution.

As shown in Section IV, both d_1 and d_2 are dependent on the method of compaction of the granular soil. However, an accurate determination of the stresses below the loaded area can be made if d_1 and d_2 are determined for that soil. Since both d_1 and d_2 are material parameters, this can be determined by laboratory tests for small-scale loads. The linear nature of Equation (76) in terms of loads indicates that the values of d_1 and d_2 obtained in the laboratory can be used for field predictions. The determination of d_1 and d_2 in the laboratory requires the measurement of vertical pressure with depth directly below the center of the loaded area as illustrated by Morgan and Gerrard (1981). The values of d_1 and d_2 are then evaluated by regression of Equation (79) on the data.

C. MATERIAL CONSTANTS IN PARTICULATE THEORY OF STRESS-STRAIN RESPONSE

The general stress-strain response of particular media is given by Equation (38). For one dimensional and isotropic loading, this reduces to Equation (41) and for triaxial loading to Equation (49). Since the triaxial condition is applicable directly to the rutting problem, the nature of the constants in Equation (49) is examined here. Equation (49) can be rewritten as

$$\sigma_a = \frac{E_o \varepsilon_a}{1 + (\varepsilon_a/\varepsilon_e) + (\varepsilon_a/\varepsilon_u)\left[\ln(\varepsilon_a/\varepsilon_u) - 1\right]} \tag{84}$$

where $\varepsilon_e = \sigma_u/E_o$ is the strain predicted at maximum stress if the material was elastic and ε_u is the strain observed at maximum stress. This equation is hyperbolic if ε_u is infinite and is linear if $\varepsilon_e = \varepsilon_u$ and finite as in the elastic case. The three material parameters E_o, ε_e, and ε_u will be examined separately in this section.

The initial soil modulus is developed in Equation (39) as a function of the particle size, initial void ratio, the height h of the minimal volume of granular soil necessary to determine stress and strain, and a parameter k representing the average resistance of a particle to movement in the z direction. The parameters h and k are not easily determined. Fortunately, the initial modulus can be obtained from the regression of Equations (49) or (84) on measured stress-strain data. As the magnitude of k depends on the roughness of the particles, the packing, and the confining pressure, the dependence of the initial soil modulus on the confining pressure is apparent from Equation (39). This dependence as observed experimentally is usually expressed as (Seed, et al, 1986; Richart, Hall and Woods, 1970)

$$E_o = C_1 \sigma_3^{1/2} \tag{85}$$

where C_1 is a constant and σ_3 is the confining pressure. Figure 9 shows that the relationship holds for the medium dense sand presented by Desai and Siriwardane (1984).

Figure 9 shows that ε_e and ε_u are also proportional to the square root of the confining pressure. Since the confining pressure insitu is the product of the coefficient of earth pressure at rest, the effective unit weight and the depth, the proportional relationships of E_o, σ_u, and ε_u to the square root of the confining pressure allow extrapolation and

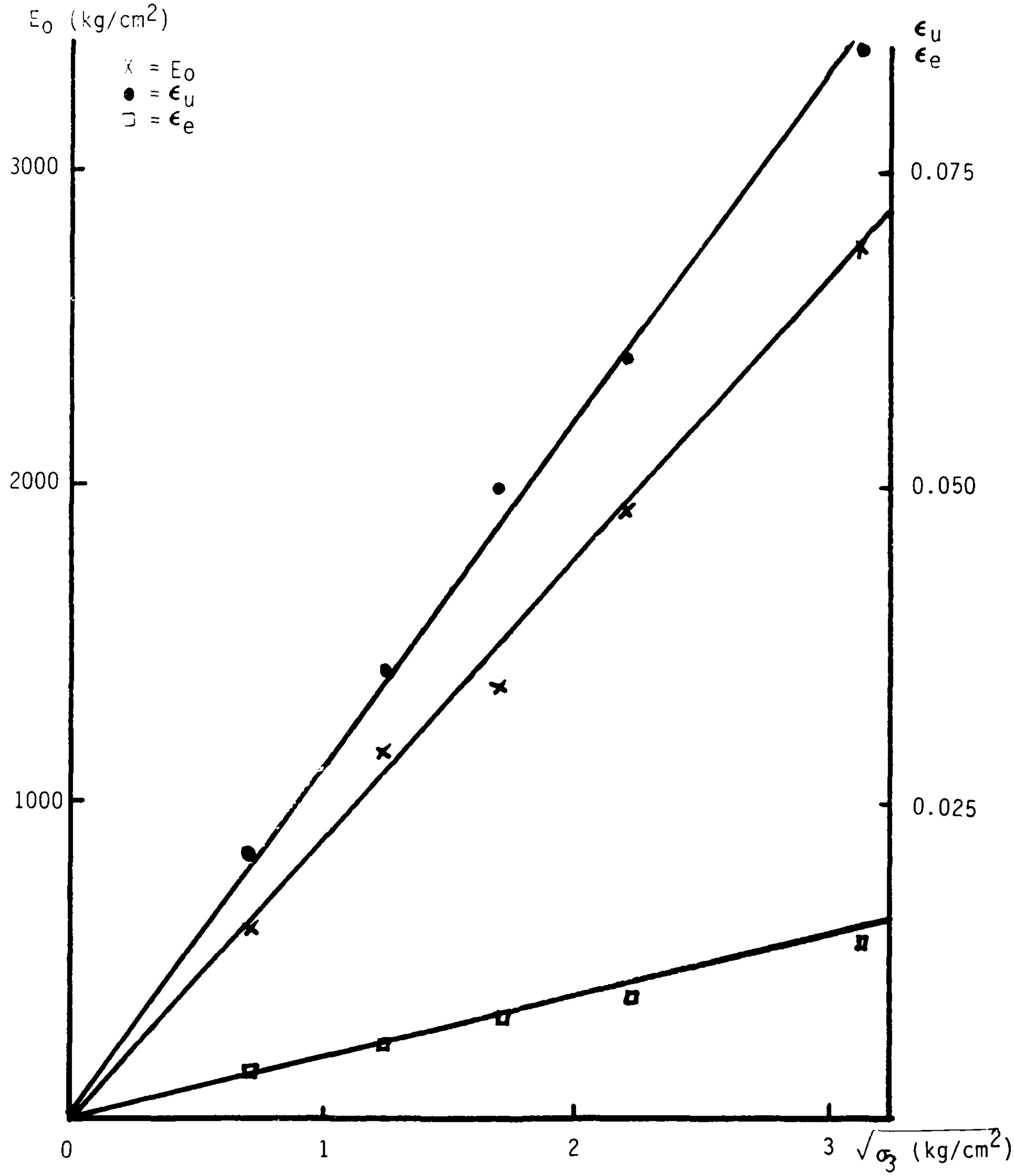

Figure 9. Relation of E_o, ϵ_u, and ϵ_e To Confining Pressure (Data From Reference 6).

interpolation using the experimental results of only one confining pressure.

D. MATERIAL CONSTANTS IN PARTICULATE THEORY OF STRAIN ACCUMULATION

The accumulation of permanent stress after N cycles of load applications is given by Equation (67), which for small values of n can be approximated by Equation (64). In both equations, the constant can be obtained by regression of the equations on experimental data. Unfortunately, these constants are functions of the magnitude of the applied load. In Equation (67) the constant B_1 is the permanent strain accumulated after the first application of the load and 1-m is the slope of the log-log transformation of the equation. In Equation (64), the constant a_1 is the permanent strain accumulated after the first application of the load, and b_1 is the slope of the semi-log relationship.

The permanent strain accumulated after the first application of the load is given in Equation (53). However, in the deviation of Equation (67), it was seen that $B_1 = A^{1-m}f(\sigma_a)/(1-m)$ where from Equation (59) $m = 1/(n+1)$ and $A = (n+1)K_1$, and from Equation (57) it is seen that $(n+1)K_1 = E_{02}^{n+1} - E_{01}^{n+1}$. These developments show that m satisfies the relationship

$$\left(E_{02}^{1/m} - E_{01}^{1/m}\right)^{1/m} = 1 - m \tag{86}$$

The value of a_1 in Equation (64) is given by Equation (53). Also in the deviation of Equation (64) it was seen that $a_1 = (1/A)f(\sigma_a)\ln(B/A)$ where B/A is defined in Equation (63). This means that $E_{01}(1/A)\ln(B/A) = 1$ or $b_1 = f(\sigma_a)/A$ becomes

$$b_1 = \frac{f(\sigma_a)}{E_{01}\ln\left(\frac{2E_{01} - E_{02}}{E_{02} - E_{01}}\right)} \tag{87}$$

In Equations (86) and (87) the values of ϵ_{02} depends on the magnitude of the applied load as it must be evaluated after the removal of the first load application.

Although this section defines the dependence of the parameters of Euqations (67) and (64) on the load, it may be more expedient to determine B_1 and m for Equation (67) or a_1 and b_1 for Equation (64) from experimental observations on repeated loadings. In this case, the experiment must be made using the vertical and confining stress expected in the field. The vertical stress is predictable from stress transfer theory as shown in Equation (76), and the confining pressure is the product of the coefficient of earth pressure at rest, the effective unit weight, and the depth.

SECTION VI

RUTTING MODEL FOR MULTILAYERED PAVEMENTS WITH PARTICULATE MATERIAL

A. INTRODUCTION

The designs against rutting are currently either empirical or quasi-elastic. The empirical method selects pavement thicknesses based on correlations of excessive deformations to subgrade strength or to satisfy an allowable value of the vertical subgrade strain. The subgrade strength is usually taken as its California Bearing Ratio (CBR) value or more recently recommended is its resilient modulus (Thompson, 1984). These methods cannot be used to predict the amount of deformation present after load applications. The quasi-elastic method is more direct and has the capability of obtaining cumulative deformations. This approach uses elastic theory, either linear or nonlinear, to predict the expected stress state within the pavement. The approach is termed "quasi-elastic" since it uses this predicted elastic stress state to design laboratory tests to measure the relationship of permanent strains to number of repeated load applications.

In linear elastic analysis the pavement is assumed to be composed of homogeneous, isotropic linear elastic layers that are infinite in the horizontal extent. The loading is vertical and uniformly distributed on a circular base with no surface shear. It is also assumed that there is full continuity between the layers and within the layers. The main limitation of this approach is that the aggregate base of the pavement is not linear elastic. In nonlinear elastic analysis, the same assumptions are made except that the elastic modulus of the granular layer is described as a function of the stress state based on an empirical curve that fits experimental results. This approach requires complex numerical modeling, and, through its requirement for full continuity between the layers and among the aggregate and voids, violates the no tensile stress requirement of the aggregate.

Designs based primarily on the stress transfer of layered linear or nonlinear elastic theory fail to evaluate the benefits of properly graded and compacted granular material. In general, the method of compaction and gradation of the granular material affects the nature of the stress distribution in the material and if properly controlled can be used to increase the carrying capacity of the pavement by spreading the load, thereby reducing stresses in the lower layers and decreasing the stress concentration that causes rutting. It was already shown in Section II that the particulate theory of stress transfer includes a diffusion coefficient that is a function of the method of compaction and gradation of the granular material. It is shown in this section how this method of stress transfer can be modified to predict stresses in layered pavements with particulate and elastic layers. The strain accumulation theory of Section III is also extended to multiple layers in order to predict rutting in the pavement.

B. STRESS TRANSFER IN MULTILAYERED MEDIA

Flexible pavements generally consist of a 2- to 8-inch asphalt surface over a 12- to 36-inch granular base or subgrade. Therefore, it is a three-layer system, with each layer transmitting stress in a different manner due to the difference in the type of material. In this section, the particulate stress transfer theory of Section II will be modified to represent the condition of stress transfer through pavement layers of varying properties.

The general expression for the vertical stress induced by an inclined point load of vertical component Q and horizontal component q_1Q in the X direction is given by Equation (31) and can be rewritten as

$$\sigma_z = \frac{Q}{4\pi W(z)} \exp\left\{\frac{-\ \rho^2 - 2q_1x + q_1^2 -}{4W(z)}\right\} \tag{88}$$

where $\rho^2 = x^2 + y^2$.

To determine the vertical stress under an inclined uniform load q on a circular area of radius "a", Equation (88) can be modified by replacing Q by $qrdrd\theta$, ρ by $r^2 + \rho^2 - 2r\rho\cos\theta$ and x by $x - r\cos(\xi+\theta)$ where $\xi = \tan^{-1}(y/x)$. The final equation is

$$\sigma_z = \frac{Q}{4\pi W(z)} \int_0^{2\pi}\int_0^{a} \exp\left\{\frac{r^2+\rho^2-2r\rho\cos\theta-2q_1x+2q_1r\cos(\theta+\xi)+q_1{}^2z^2}{-4W(z)}\right\} rdrd\theta \quad (89)$$

For the special case of a vertical load (q_1=0), this equation can be integrated over r to give

$$\sigma_z = \frac{q}{2\pi} \exp\left(\frac{-\rho^2}{4W(z)}\right) \int_0^{2\pi} \left\{\frac{1 - \exp\left[\frac{-a^2}{4W(z)} + \frac{\rho a\cos\theta}{2W(z)}\right]}{1 - \rho^2\cos^2\theta/W(z)}\right\} d\theta \qquad (90)$$

For known values of "a" and W(z), this latter equation can be easily integrated numerically at any value of ρ. Directly under the center of the vertically loaded area (ρ=0), Equation (90) gives

$$\sigma_z = q\left\{1 - \exp\left[\frac{-a^2}{4W(z)}\right]\right\} \qquad (91)$$

Equations (89), (90), and (91) present progressively simpler cases. However, in each of these equations, all of the geometric and material properties are contained in the function W(z). Therefore, the determination of the stress distribution in layered media requires only a modification of W(z) in that layer. Directly below the center of a vertically-loaded circular area of radius "a" on granular material it was shown in Section IV that $W(z) = W_g(z)$, where $W_g(z)$ refers to granular layer and is given by

$$W_g(z) = d_1 z^2 + d_2az \qquad (92)$$

here a_1 and a_2 are material-specific constants reflecting the particle sizes, packing, and previous loading. An equivalent function, W(z), can also be found for materials that transmit stress close to that predicted by elastic theory. In this case, setting Equation (77) equal to Equation (75) gives $W(z) = W_e(z)$, where

$$W_e(z) = \frac{a^2}{\left\{6\ln\left[\left(\frac{a}{z}\right)^2 + 1\right]\right\}} \tag{93}$$

In order to determine the modification of W(z) in a multilayered pavement, the method presented by Golden (1984) is used. In this method, $W_i(z)$ represents the function W(z) for stress distribution in a media composed only of the material in the ith layer, and the function W(z) in Equation (89), (90), or (91) is taken as

$$W(z) = \begin{cases} W_1(z) \text{ for } z \leq h_1 \\ W_2(z) - W_2(h_1) + W_1(h_1) \text{ for } h_1 \leq z \leq h_1 + h_2 \\ W_3(z) - W_3(h_1+h_2) + W_2(h_1+h_2) - W_2(h_1) + W_1(h_1) \text{ for } z \geq h_1+h_2 \end{cases} \tag{94}$$

where h_i is the thickness of the ith layer.

The stress distribution in the asphalt layer is elastic in nature. Therefore, the value of $W_1(z)$ for flexible pavements is given by Equation (93). The second layer of the pavement is the granular layer, and the value of $W_2(z)$ is given by Equation (90). The third layer is the subgrade. For cohesive subgrade it was observed that due to its cohesive nature the elastic prediction of stresses is acceptable (Morgan and Gerrard, 1981). In this case, Equation (93) gives $W_3(z)$. For sand subgrade, the elastic solution is not acceptable (Morgan and Gerrard, 1981), and $W_3(z)$ is of the form of Equation (92). The vertical stress on the subgrade ($z = h_1 + h_2$) directly below the center of the loaded area is found from Equation (91), where from Equation (94)

$$W(z) = d_1 \left(h_2^2 + 2h_1h_2 \right) + d_2\, a h_2 + \frac{a^2}{6 \ln\left[\left(\frac{a}{h_1}\right)^2 + 1 \right]} \tag{95}$$

The role played by the layer thicknesses and material constants on distributing the stress to the weaker subgrade layer is apparent from this equation. From Equation (89) a larger $W(z)$ translates into smaller vertical stresses. Therefore, increases in all quantities h_1, h_2, d_1, and d_2 decrease the stress transferred to the weaker subgrade. However, since parameters d_1 and d_2 are functions of the gradation and compaction of the granular layer, Equation (95) can be used to evaluate the benefits of increased layer thicknesses versus increased compaction and gradation.

C. RUTTING PREDICTION MODEL

The majority of research on subgrade and granular materials has shown a log-log relationship of the permanent strain to the number of load applications (Yoder and Witczak, 1974). This log-log relationship was also derived theoretically for asphalt materials (Khedr, 1988) and observed experimentally (Majidzadeh, Khedr, and El-Mojarrish, 1979). This relationship is identical to Equation (67) derived for granular media with m not equal to one.

For granular material, it was also observed that for values of m approaching one, Equation (64) becomes the more appropriate model. The parameter, B_1 in Equation (67) (and a_1 in Equation (64)) is the permanent deformation after the first cycle of loading and for granular material is given by Equation (54). In this sense, B_1 is an explicit function of the vertical stress at the point in question in the media. The parameter m is also a function of the stress level, as shown in Section V. Therefore, it is imperative that in the evaluation of B_1 and m that the correct stress be used.

The theoretical result of Section III and those of Khedr (1986) indicate that at any depth z under the loaded area the accumulated permanent strain after N cycles is given by

$$\varepsilon^{p}_{aN}(z) = B(\sigma_z) N^{m(\sigma_z)} \tag{96}$$

The parameters $B(\sigma_z)$ and $m(\sigma_z)$ are functions of the vertical stress and material characteristics. If subscripts a, g, and s refer to the asphalt layer, granular layer, and subgrade, then the permanent depression in the flexible pavement after N cycles is

$$\delta_N = \int_0^{h_1} B_a(\sigma_z)\, N^{m_a(\sigma_z)} dz + \int_{h_1}^{h_1+h_2} B_g(\sigma_z)\, N^{m_g(\sigma_z)} dz + \int_{h_1+h_2}^{\infty} B_s(\sigma_z)\, N^{m_s(\sigma_z)} dz \tag{97}$$

Equation (97) is the general prediction model. Although it is possible to express the $B(\sigma_z)$ parameters as continuous known functions of z, this is not possible for the $m(\sigma_z)$ parameters. This is illustrated in Section V for the granular layer and in Khedr (1986) for the asphalt layer. As a result, the integrals of Equation (97) must be replaced by summations, and the pavement divided into discrete sections of magnitude Δz. The values of $B(\sigma_z)$ and $m(\sigma_z)$ are then evaluated using the stress expected in the center of each section. Further, although it is shown in Section V that it is possible to express $B(\sigma_z)$ and $m(\sigma_z)$ in terms of other material parameters, it is more efficient to determine these parameters directly from repeated load tests using linear regression on the log-log transformation of the measured values of ε^{p}_{an} and N. This method concentrates on the evaluation of only two parameters B and m per laboratory test. However, it is imperative that the stress state expected in the field be used to perform the laboratory test. As an improvement over the use of stresses determined from elastic theory, the values of $B(\sigma_z)$ and $m(\sigma_z)$ should be evaluated using the stress predicted by the multilayered theory described above. A simple closed-form description of the vertical stress to be applied to the laboratory sample is provided by the combination of Equations (91) and (94) where $W_1(z)$ and $W_3(z)$ are described by Equation (91) and $W_2(z)$ by Equation (92).

SECTION VII
CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The report's objective is to present the results of research investigating rutting in flexible pavements from a perspective whereby the granular layer is treated as particulate rather than a continuum. It shows that the stress transfer in the granular layer can be derived without the assumption of the existence of the second derivation of strains. This assumption is generally used for both linear and nonlinear elastic stress analysis. However, in the granular layer, the particles are discrete and deformation is the result of discrete particle motion rather than continuous particle compression.

From the results of the report, the following conclusions can be drawn:

- A particulate definition of stress and strain in granular media is necessary in order to adequately model the behavior of the granular layer in flexible pavements. This definition is necessary since stresses are transferred only at particle contacts and not through the voids in the material, and strains are the result of particle movements rather than particle compression.
- The particulate definition of stress and strain allows the development of a stress continuity relationship which when combined with the equilibrium equation gives the stresses in the material. This development has three major advantages over conventional linear and nonlinear continuum analyses. First, the assumption that the second derivatives of strains exist at all points is not unrealistically imposed on the discrete medium; secondly, the observed stress-strain behavior of the material is built into the stress continuity relationship; and thirdly, the stresses are expressed in terms of a material term W(z) that is a function of the gradation and method of compaction of the medium.

- The particulate theory prediction of stress and strain conforms to experimental observations and all material constants can be evaluated with the use of conventional experimental methods.
- The particulate theory of stress distribution reduces to the elastic case with the appropriate choice of W(z), and the evaluation of stress transfer through multilayered media is easily determined with a systematic representation of W(z) in terms of the thicknesses and materials composing the layers.
- The particulate approach to modeling granular behavior confirms the log-log relationship between accumulated permanent strain and number of cycles of loading. However, it magnifies the fact that parameters in this relationship are highly dependent on the stress state at the point where the strain is evaluated.
- The particulate theory of stress distribution through multilayered media can be used to predict the stress state to be used for evaluation of the material constants in the permanent strain accumulation versus number of cycles of loading. This is an improvement over conventional linear and nonlinear elastic analysis due to its simplicity and the rationality of the particulate approach to stress distribution in granular media.

B. RECOMMENDATIONS

The evaluation of rutting in flexible pavements presented in this report is based on a new, more rational definition of stress and strain in particulate media. This definition assumes that deformation is the result of particle movement rather than particle compression and that stress is transferred only at particle contacts. It is gratifying that this definition very simply leads to models that adequately represent observed stress-strain behavior for granular soils under one dimensional, isotropic, triaxial, and shear loading conditions (see Appendix A). It also easily illustrates the diffusive nature of stress transfer in granular material under inclined loading. Therefore, it is recommended that other problems concerned with the transmission of stress and strain in granular media be approached with this new definition. Included among these are the following:

- Modeling the performance of sand grids under loads. Sand grids are used increasingly for expeditionary airfields and their design will be greatly improved if the mechanism of their behavior is rationally modeled. This approach is ideal for such investigation.

- Developing improved methods of interpreting the information obtained from nondestructive testing of flexible pavement using devices such as the falling weight deflectometer. This new approach will allow for the evaluation of more characteristic pavement properties rather than the resilient modulus currently measured.

- Investigating the mechanics of the nonlinear nature of stress wave transmission in granular soils, thereby improving the understanding of blast and earthquake effects on structures and potentially liquefiable soil.

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APPENDIX A

A GENERAL STRESS-STRAIN MODEL FOR GRANULAR SOILS

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A GENERAL STRESS-STRAIN MODEL FOR GRANULAR SOILS

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ABSTRACT

A general stress-strain model is derived for granular soils based on the premise that the deformation in these soils is not caused so much by the deformation of individual particles as by the relative movements of the more mobile particles. The model is general and specific relationships are obtained for different loading conditions. For one dimensional and isotropic loading conditions a stress-strain curve concave towards the stress axis is derived, and for triaxial and shear loading conditions the derived stress-strain curve is concave towards the strain axis. In particular soils that show a distinct peak stress are well modeled, and the model reduces to the hyperbolic model for soils exhibiting no distinct peak stress. All cases show excellent fit to experimental data.

INTRODUCTION

The stress-strain behavior of granular soils is created primarily by individual particle movements to form a denser matrix rather than elastic compression of the particles. As a result the experimental curve is always nonlinear.In general it is concave towards the strain axis under triaxial and shear loading, and concave towards the stress axis in one dimensional and isotropic loading conditions. To adequately predict the settlement under a load one needs to be able to model this nonlinear behavior.

Of primary importance in settlement calculations is the need to model the triaxial compression condition. One simple empirical method proposed by Konder and Zelasko (4) is the widely used hyperbolic model. This model appears to be a natural fit for soils that do not exhibit a distinct peak stress. Another empirical model proposed is a modification of the Ramberg-Osgood model used for dynamic loading for static conditions (2). This is based on fitting a curve to the initial tangent modulus, the modulus of the plastic zone, the yield stress and a parameter defining the order of the curve. For order one this curve becomes a hyperbola. One other approach suggested is to treat the tangent shear modulus and tangent bulk modulus as variables (5). These are usually taken as linear functions of the octahedral normal and shear stresses.

The difficulty in modelling the nonlinear stress-strain behavior of sand under load has led many investigators to propose numerical curve fitting techniques. One of the most popular of these is the piecewise linear method. Here the nonlinear experimental curve is divided into pieces of linear elastic sections for numerical analysis. Very often these are the incremental Hooke's law or the hypoelastic law (2). Another numerical method is the use of spline functions to fit experimentally observed curves. These are functions that uses the data to provide an analytic curve similar to the graphical process of using a French curve (1). These also require intense numerical procedures, and the data must be presented in a smooth form and not scattered as observed experimentally.

In this paper a general stress-strain model is derived from basic principles. This model is shown to apply to one dimensional, isotropic, triaxial, and shear loading conditions as special cases based on the boundary conditions they impose.

STRESS AND STRAIN IN GRANULAR SOILS

In the determination of stresses and strains in granular soils any representative element of the soil with volume dxdydz must be composed of particles and voids. The element must also consist of enough particles such that the relative movements of the particles as a result of forces on these particles produce strains. This means that in the limit the volume dxdydz cannot be made to approach zero but must instead approach some minimal finite volume ijh. This minimal volume of fixed dimensions i, j and h in the x, y and z directions, respectively, is the smallest volume of granular soil that can be used to define stress and strain. In effect, this volume is analogous to a point in continuous material. The latter having a fixed volume of zero magnitude on a macroscopic scale but at a microscopic level is composed of discrete atoms that enter and leave the point.

Since particles are allowed to enter and leave the minimal element, if w is the average relative displacement of a particle in the element in the z direction, then the normal strain in the element in the z direction is

$$\varepsilon_z = w/h \tag{1}$$

For simplicity we introduce a microscopic stiffness coefficient k_z to represent the average resistance of a particle to movement in the z direction such that the force on a particle in the minimal element in the z direction is $k_z w$. The magnitude of k_z depends on the roughness of the particles and the confining pressure. The force in the element in the z direction is $F_z = Nk_z w$, where N is the number of particles in the element. The normal stress in the element in the z direction is then $F_z/(ij)$

$$\sigma_z = Nk_z w/(ij) \tag{2}$$

In this equation $N = ijh/[V_p(1+e)]$, where V_p is the average volume of a particle and e is the void ratio. By definition the volumetric strain is related to the void ratio as $\varepsilon_v = (e_0-e)/(1+e_0)$, where e_0 is the initial void ratio. Therefore,

$$N = ijh/[V_p(1+e_0)(1-\varepsilon_v)] \tag{3}$$

The substitution of equations (1) and (3) into equation (2) and taking the z direction as the axial direction give the general relationship of axial stress to axial strain as

$$\sigma_a = E_0\varepsilon_a/((1-\beta\varepsilon_v) \tag{4}$$

where

$$E_0 = h^2 k_z/[V_p(1+e_0)] \tag{5}$$

and the parameter β was added since the condition $\varepsilon_v = 1$ is unattainable under conventional loads. From equation (4), the derivative of σ_a with respect to ε_a at $\varepsilon_v = \varepsilon_a = 0$ is E_0. Therefore, E_0 is the initial tangent modulus of the soil.

To look at shear stresses in granular soils consider two minimal elements of soil adjacent to each other but separated by a surface dxdy in the xy plane. Let element 1 be at location z with average particle displacement u in the x direction, and element 2 be at location z+dz with average relative particle displacement $u+(\partial u/\partial z)dz$ in the x direction. Also let k_x represent the average resistance of a particle to movement in the x direction. Therefore, the force in the x direction in the lower half of element 1 is $F_{x1} = N_1 k_x u$, and the force in the x direction in the upper half of element 2 is $F_{x2} = N_2 k_x[u+(\partial u/\partial z)dz]$. Here N_1 is the number of particles in the lower half of element 1 and N_2 is the number of particles in the upper half of element 2. We assume that N does not change much with z as compared with changes in u, therefore, $N_1 = N_2 = N/2$. The shear stress in the x direction at the interface between the two elements is $(F_{x1}-F_{x2})/(dxdy)$, or since dxdydz approaches ijh in the limit the shear stress is

$$\tau = [Nk_x h/(2ij)](\partial u/\partial z) \tag{6}$$

ONE DIMENSIONAL AND ISOTROPIC COMPRESSION

In one dimensional compression of soils $\varepsilon_v = \varepsilon_a$, and in isotropic compression we have $\varepsilon_v = 3\varepsilon_a$. From equation (4) these two conditions can be represented by the single equation

$$\sigma_a = E_0\varepsilon_a/(1-\eta\varepsilon_a) \tag{7}$$

where η is the reciprocal of the assymptotic strain observed at large stress. Figure (1) shows the excellent fit of equation (7) to data for isotropic loading conditions on two samples of McCormic Ranch sand obtained from reference (2, pg. 193).

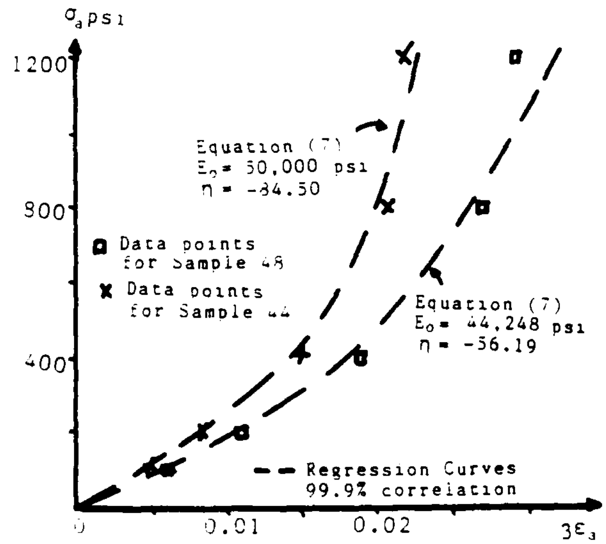


Fig. 1. Stress-Strain in Isotropic Loading (1 psi = 6.895 KPa)

One should also note that equation (7) can be written as $\sigma_a = E_0(\varepsilon_a + \eta\varepsilon_a^2 + \ldots)$ and by series expansion we have that $[\exp(2\eta\varepsilon_a)-1]/(2\eta) = \varepsilon_a + \eta\varepsilon_a^2 + \ldots$

Therefore, letting $\alpha = 2^n$ and $A = E_o/\alpha$, gives the alternate expression

$$\sigma_a = A\exp(\alpha \varepsilon_a) - A \tag{8}$$

Equation (8) is the same as that derived by the hypoelastic analysis for these loading conditions (2, pg 139).

TRIAXIAL COMPRESSION

In triaxial compression of elastic material the radial strain, ε_r, is proportional to the axial strain, and the proportional constant is the empirical Poisson's ratio. In effect, if $\nu_t = - d\varepsilon_r/d\varepsilon_a$ and $\nu_s = - \varepsilon_r/\varepsilon_a$, then in elastic material $\nu_t = \nu_s$. However, in triaxial compression of sands the relationship of radial strain to axial strain is nonlinear (2). To represent this nonlinear condition one can let $\nu_s - \nu_t = D$, where D is an empirical constant representing the average difference between ν_s and ν_t. If D = 0 the relationship is linear, and the nonlinearity increases with the magnitude of D. The definition $\nu_s = - \varepsilon_r/\varepsilon_a$ gives $d\nu_s/d\varepsilon_a = (\varepsilon_r+\varepsilon_a\nu_t)/\varepsilon_a^2 = - D/\varepsilon_a$, also since the volumetric strain $\varepsilon_v = \varepsilon_a + 2\varepsilon_r$ we have $\nu_s = (\varepsilon_a-\varepsilon_v)/(2\varepsilon_a)$, which shows that $d\nu_s/d\varepsilon_a = -0.5d(\varepsilon_v/\varepsilon_a)/d\varepsilon_a$. Equating the expressions for $d\nu_s/d\varepsilon_a$ gives the expression $d(\varepsilon_v/\varepsilon_a)/d\varepsilon_a = 2D/\varepsilon_a$, which has solution

$$\varepsilon_v = B\varepsilon_a + 2D\varepsilon_a \ln\varepsilon_a \tag{9}$$

where B is an integration constant. It should be noted that xlnx approaches zero as x approaches zero.

The relationship of volumetric strain to axial strain under triaxial compression for a medium dense sand at two different confining pressures as obtained from reference (2, pg. 177) is shown in Figure (2a). The regression of equation (9) on the data in this figure is shown as the dashed lines. An excellent fit is observed for each case.

The substitution of equation (9) into equation (4) yields

$$\sigma_a = E_o\varepsilon_a/(1-\beta B\varepsilon_a - 2\beta D\varepsilon_a\ln\varepsilon_a) \tag{10}$$

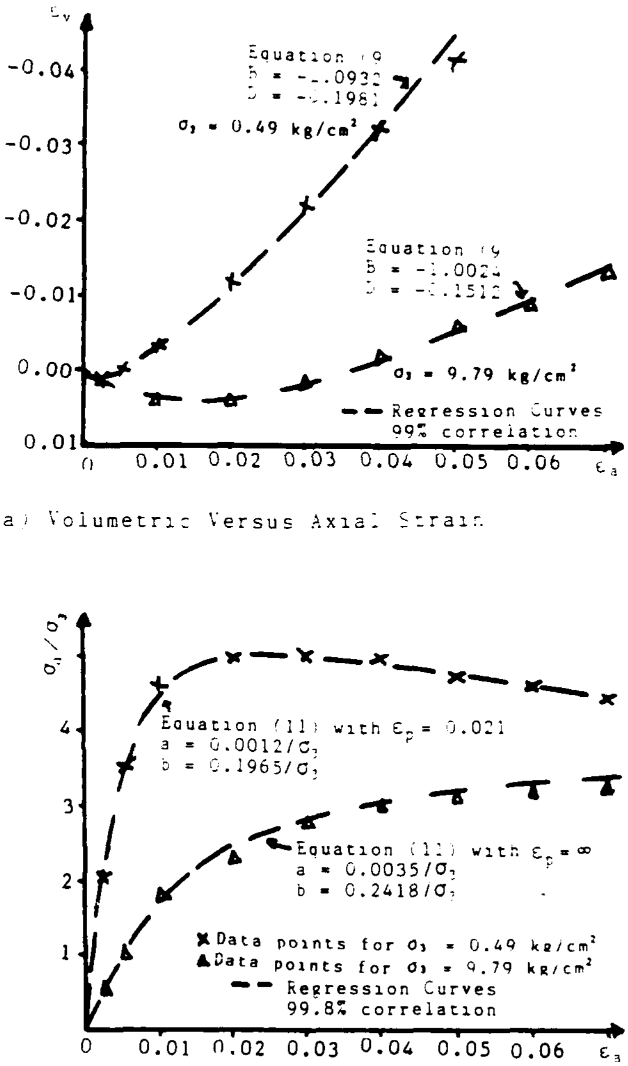
Letting $\sigma_a = \sigma_p$ and $\varepsilon_a = \varepsilon_p$ at maximum stress, we get $\beta B = 1/\varepsilon_p - E_o/\sigma_p - 2\beta D\ln\varepsilon_p$. Also, setting $d\sigma_a/d\varepsilon_a = 0$ at maximum stress gives $2\beta D = -1/\varepsilon_p$. The substitution of these into equation (10) gives the relationship

$$\sigma_a = \varepsilon_a/[af(\varepsilon_a) + b\varepsilon_a] \tag{11}$$

where $a = 1/E_o$, $b = 1/\sigma_p$, and

$$f(\varepsilon_a) = 1 + (\varepsilon_a/\varepsilon_p)[\ln(\varepsilon_a/\varepsilon_p) - 1] \tag{12}$$

In soils with no distinct peak stress such as loose sands and sands under high



(a) Volumetric Versus Axial Strain

(b) Axial Stress-Strain Behavior

Fig. 2. Triaxial Compression

pressure ε_p approaches infinity. In this case equation (12) gives $f(\varepsilon_a) = 1$, and equation (11) reduces to the hyperbolic model

$$\sigma_a = \varepsilon_a/(a + b\varepsilon_a) \tag{13}$$

The excellent fit of equation (11) to the stress-strain responses of a medium dense sand at two different confining pressures is shown in Figure (2b).

SHEAR LOADING

The shear strain in the element of soil during shear loading is $\gamma = \partial u/\partial z$. Therefore, the substitution of equation (3) into equation (6) gives

$$\tau = G_o\gamma/(1-\beta\varepsilon_v) \tag{14}$$

where

$$G_0 = h^2 k_x / [2V_p(1+e_0)] \tag{15}$$

and the parameter β was added since the condition $\epsilon_v = 1$ is unattainable under conventional loads. Evaluating $d\tau/d\gamma$ at $\gamma = \epsilon_v = 0$ shows that G_0 is the initial tangent shear modulus.

In elastic material under shear loading the volumetric strain is proportional to the shear strain. This means that in elastic material $d\epsilon_v/d\gamma = \epsilon_v/\gamma$. In granular soils, however, this linear relation does not hold (6). To represent the nonlinear relationship between ϵ_v and γ we let $d\epsilon_v/d\gamma - \epsilon_v/\gamma = C$, where C is an empirical constant representing the average difference between the values $d\epsilon_v/d\gamma$ and ϵ_v/γ. This expression says that $\epsilon_v = \gamma[(d\epsilon_v/d\gamma) - C]$. Differentiating both sides of this with respect to γ gives $d^2\epsilon_v/d\gamma^2 = C/\gamma$, which has solution

$$\epsilon_v = B_2\gamma + C\gamma\ln\gamma \tag{16}$$

where B_2 is an integration constant.

Substituting equation (16) into equation (14) and letting $\tau = \tau_p$ and $\gamma = \gamma_p$ at peak stress gives the first constant as $\beta B_2 = 1/\gamma_p - G_0/\tau_p - \beta C\ln\gamma_p$. Further, setting $d\tau/d\gamma = 0$ at peak stress gives the constant $\beta C = -1/\gamma_p$. The substitution of these into equation (14) gives the expression

$$\tau = \gamma/[af(\gamma) + b\gamma] \tag{17}$$

where $a = 1/G_0$, $b = 1/\tau_p$, and

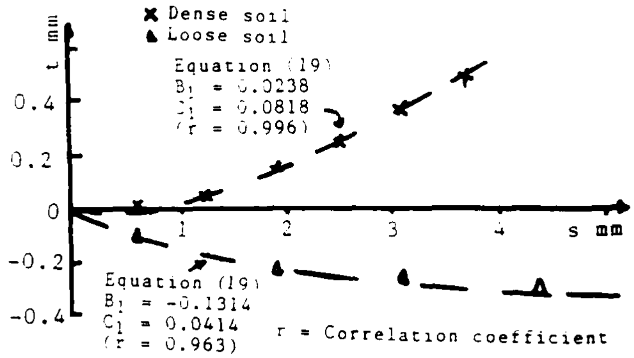
$$f(\gamma) = 1 + (\gamma/\gamma_p)[\ln(\gamma/\gamma_p) - 1] \tag{18}$$

For soils with no distinct peak stress we have the condition $\gamma_p = \infty$ and equation (18) gives $f(\gamma) = 1$. In this case equation (17) reduces to the hyperbolic model.

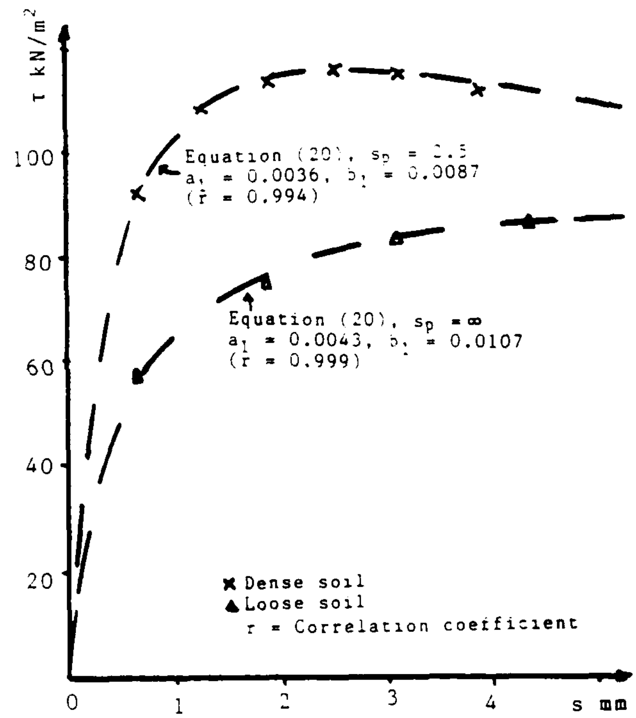
The direct shear experimental data for the change in sample thickness, t, with respect to shearing displacement, s, for a sand in a loose and a dense state as presented by Sutton (6, pg. 99), are shown in Figure (3a). The corresponding shear stress versus displacement data for this sand is shown in Figure (3b). The sand was tested under drained conditions and a constant normal stress of 150 kN/m² was applied by the piston in both tests. In the direct shear test the change in sample thickness t is proportional to the volumetric strain ϵ_v and the shearing displacement s is proportional to the shear strain γ. Therefore, the relationship of t to s is of the same form as equation (16). That is

$$t = B_1 s + C_1 s\ln s \tag{19}$$

where B_1 and C_1 are constants. Also, the relationship of τ to s is of the same form



(a) Change in Sample thickness



(b) Shear Stress Versus Displacement

Fig. 3. Direct Shear loading

as equations (17) and (18), or

$$\tau = s/[a_1 f(s) + b_1 s] \tag{20}$$

and

$$f(s) = 1 + (s/s_p)[\ln(s/s_p) - 1] \tag{21}$$

where s_p is the displacement at peak stress, and a_1 and b_1 are constants.

The regression of equation (19) on the data of Figure (3a) are shown as the dashed lines of the figure. Also the regression of equation (20) on the data of Figure (3b) are shown as the dashed lines on that figure. In each case the fit is excellent.

SUMMARY

Based on the assumption that the stress-strain behavior of granular media is controlled by the displacements of individual particles rather than particle compression a general stress-strain model for granular material is derived. This model says that for both axial and shear loading the stress is proportional to [strain/($1-\beta\epsilon_v$)], where the proportional constant is the initial modulus of the soil and β is a constant. For one dimensional and isotropic loading conditions the volumetric strain ϵ_v is proportional to the axial strain. For triaxial and shear loading conditions nonlinear relationships of ϵ_v to the axial strain and ϵ_v to the shear strain are developed. It is shown that the model can be used for soils that show distinct peak stresses when under triaxial and shear loading, and that it reduces to the hyperbolic model for soils that shows no distinct peak stress.

ACKNOWLEDGEMENT

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