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Wavevector-Frequency Spectral Estimation: A Review of Conventional Signal Processing Techniques

A Paper Presented at the 116th Meeting of the Acoustical Society of America, 16 November 1988, Honolulu, Hawaii

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Preface

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This document consists of the text and slides of an invited paper presented at The 116th Meeting of the Acoustical Society of America held in Honolulu, Hawaii, during 15-18 November 1988. The following abstract was published in the Program for The 116th Meeting (J. Acoust. Soc. Am., Supplement 1, Vol. 84, Fall 1988).

Signal processing techniques for estimating wavevector-frequency spectra of space-time fields from measured data parallel those developed by electrical engineers for estimating frequency spectra from measured samples of temporal fields. This paper reviews a conventional technique for estimating the wavevector-frequency spectrum from samples of a space-time field measured through an array of sensors. The mean value of this spectral estimate can be related to the true spectrum of the space-time field and the wavevector and frequency responses

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19. **ABSTRACT** (Cont'd)

associated with the various components of the signal processing. This relationship permits, under certain conditions, the quality of the spectral estimate to be assessed. The dependence of the quality of the spectral estimate on the various components of the signal processing is discussed.

A-1

WAVEVECTOR-FREQUENCY SPECTRAL ESTIMATION: A REVIEW OF CONVENTIONAL SIGNAL PROCESSING TECHNIQUES

INTRODUCTION

Experimental determination of the statistics of fields that vary randomly in space and time is a problem of practical interest in many branches of acoustics. One fundamental descriptor of such random space-time fields is the wavevector-frequency spectrum.

This paper reviews a technique for estimating the wavevector-frequency spectrum from measured data. This technique parallels one developed by electrical engineers for estimating the frequency spectrum of fields that vary only with time. The purpose of this review is not to promote a particular measurement technique, but to illustrate the variety of signal processing issues that must be addressed in the estimation of wavevector-frequency spectra.

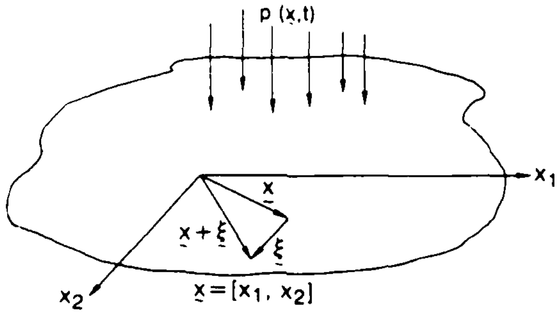
DEFINITIONS

We begin with some definitions. Slide 1 illustrates a pressure field at the surface of a plane defined by the x_1 and x_2 axes. Although a pressure field is used for purposes of illustration, the field could just as well be acceleration, stress, or any other physical quantity of interest.

The correlation of the pressure field over the plane of interest is defined as the average value, over many repetitions of the measurement, of the product of (1) the pressure at the vector location $\underset{\sim}{x}$ and time t and (2) the pressure at the location $\underset{\sim}{x} + \underset{\sim}{\xi}$ and time $t + \tau$. The average over the ensemble of experiments is denoted by E.

In general, the correlation is a function of (1) the absolute spatial position and time of one observation of the pressure and (2) the spatial separation vector and time difference between observations. If the correlation and all other statistical moments are independent of the absolute time of observation, t, the field is said to be stationary.

SLIDE 1



SPACE-TIME CORRELATION OF THE PRESSURE FIELD (Q_{pp})

$$Q_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, t, \tau) = E\left\{ p(\underset{\sim}{x}, t)\, p(\underset{\sim}{x} + \underset{\sim}{\xi}, t + \tau) \right\}$$

STATIONARY FIELD (CORRELATION INDEPENDENT OF ABSOLUTE TIME, t)

$$Q_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, t, \tau) = Q_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, \tau)$$

HOMOGENEOUS FIELD (CORRELATION INDEPENDENT OF ABSOLUTE SPATIAL POSITION, $\underset{\sim}{x}$)

$$Q_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, t, \tau) = Q_{pp}(\underset{\sim}{\xi}, t, \tau)$$

HOMOGENEOUS AND STATIONARY FIELD (CORRELATION INDEPENDENT OF $\underset{\sim}{x}$ AND t)

$$Q_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, t, \tau) = Q_{pp}(\underset{\sim}{\xi}, \tau)$$

Similarly, if the correlation is independent of the absolute spatial location of the observation, $\underset{\sim}{x}$, the field is said to be homogeneous.

We will restrict our attention to stationary fields so that the correlation will have the functional form of either the second or fourth expressions shown on this slide.

The wavevector-frequency spectrum is defined as the Fourier transform of the correlation field in both space and time, as shown in slide 2.

For a homogeneous and stationary pressure field, the wavevector-frequency spectrum is obtained by the Fourier transformation of the correlation field on the spatial separation vector, $\underset{\sim}{\xi}$, and the time difference, τ, as shown at the top of this slide. Here, Φ_p denotes the wavevector-frequency spectrum of the pressure field and the superscript H designates a homogeneous field.

SLIDE 2

DEFINITIONS OF WAVEVECTOR-FREQUENCY SPECTRA

STATIONARY-HOMOGENEOUS FIELD

$$\Phi_p^H(\underset{\sim}{k}, \omega) = \iint_{-\infty}^{\infty} Q_{pp}(\underset{\sim}{\xi}, \tau) \exp[-i(\underset{\sim}{k} \cdot \underset{\sim}{\xi} + \omega\tau)]\, d\underset{\sim}{\xi}\, d\tau$$

STATIONARY-NONHOMOGENEOUS FIELD

SPACE-VARYING SPECTRUM (K_p)

$$K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega) = \iint_{-\infty}^{\infty} Q_{pp}(\underset{\sim}{x}, \underset{\sim}{\xi}, \tau) \exp[-i(\underset{\sim}{k} \cdot \underset{\sim}{\xi} + \omega\tau)]\, d\underset{\sim}{\xi}\, d\tau$$

SPACE-AVERAGED SPECTRUM (Φ_p)

$$\Phi_p(\underset{\sim}{k}, \omega; A) = \frac{1}{A}\int_A K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega)\, d\underset{\sim}{x}$$

TWO-WAVEVECTOR SPECTRUM ($\mathcal{K}_p$)

$$\mathcal{K}_p(\underset{\sim}{\mu}, \underset{\sim}{k}, \omega) = \int_{-\infty}^{\infty} K_p(\underset{\sim}{x}, \underset{\sim}{k}, \omega) \exp[-i\underset{\sim}{\mu} \cdot \underset{\sim}{x}]\, d\underset{\sim}{x}$$

$$\underset{\sim}{\mu} = \{\mu_1, \mu_2\}$$

The wavevector, $\underset{\sim}{k}$, is the Fourier conjugate variable of the spatial separation vector, $\underset{\sim}{\xi}$, and the circular frequency, ω, is the conjugate variable of the time difference, τ.

For a stationary, nonhomogeneous pressure field, three alternative forms of the wavevector-frequency spectrum can be defined. The space-varying spectrum, designated by K_p, is defined by the same multiple Fourier transformation of the correlation field that was used for defining the homogeneous spectrum. However, because the nonhomogeneous correlation field depends on the absolute spatial vector, $\underset{\sim}{x}$, the space-varying spectrum also varies with $\underset{\sim}{x}$.

The space-averaged spectrum, Φ_p, is defined as the average value of the space-varying spectrum over some area, A, in absolute space.

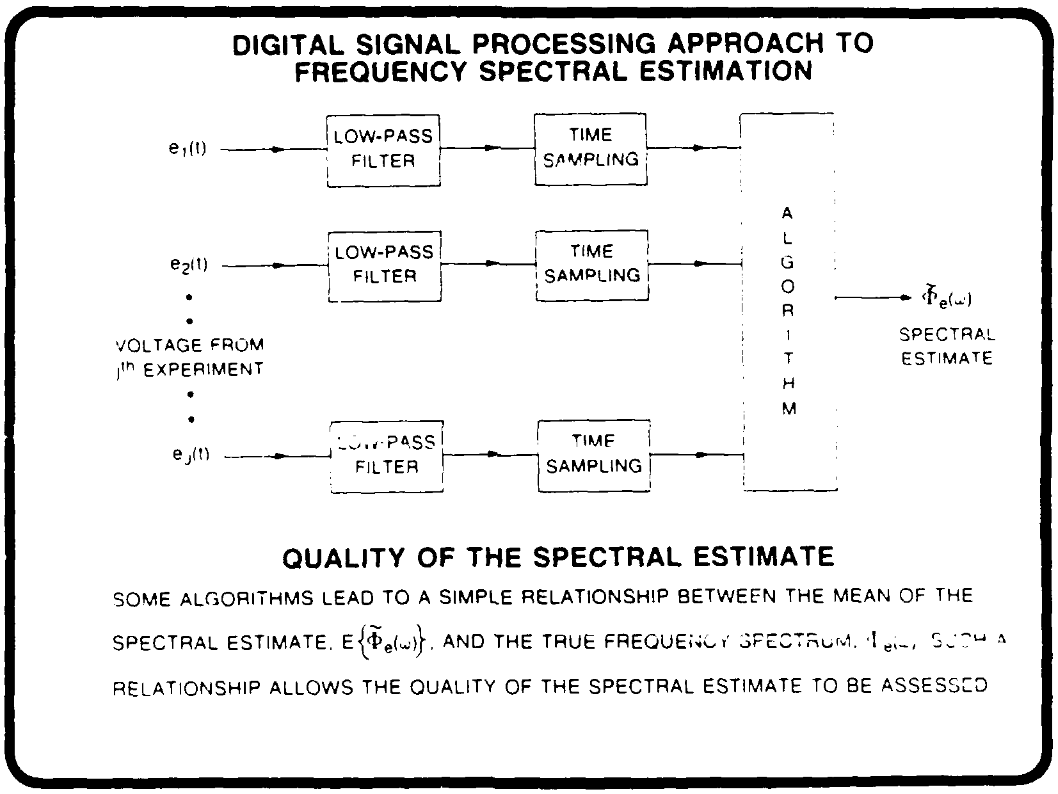
The two-wavevector-frequency spectrum, designated by $\mathcal{K}_p$, introduces a second wavevector variable, $\underset{\sim}{\mu}$, by an additional Fourier transformation of the correlation field. This second wavevector is the Fourier conjugate variable of the absolute spatial vector, $\underset{\sim}{x}$.

Note that all definitions of wavevector-frequency spectra assume knowledge of the correlation field over all space and time. In practice, however, we can measure only samples of a physical field over some finite limits in space and time. Therefore, we cannot measure the exact wavevector-frequency spectrum of a space-time field. Rather, we can only obtain some estimate of it.

CONVENTIONAL SIGNAL PROCESSING APPROACH

The problem of estimating the wavevector-frequency spectrum from a sampling of the space-time field is similar to the problem faced by electrical engineers in estimating the frequency spectrum of a temporal field from time samples of that field. The electrical engineers have developed a variety of signal processing techniques to address this problem. One of these techniques is digital signal processing.

SLIDE 3



A block diagram of the digital signal processing approach to estimating the frequency spectrum of a voltage field, e(t), is shown at the top of slide 3.

Here, in each of the J repetitions of the same experiment, the measured voltage, $e_j(t)$, is first low-pass filtered and then sampled at uniform intervals over some finite duration of time. The resulting ensemble of data is then input to a digital computer, where a computational algorithm is used to obtain a spectral estimate. Here the tilde is used to differentiate the spectral estimate from the true spectrum.

For some spectral estimation algorithms, it is possible to relate the mean of the spectral estimate to the true frequency spectrum and the frequency responses associated with the low-pass filter and the temporal sampling. In such cases, it is often possible to assess the quality of the spectral estimate: that is, how closely the spectral estimate approximates the true spectrum.

The conventional approach to estimating the wavevector-frequency spectrum of a space-time field simply extends this digital signal processing approach to include the spatial variation of the field. That is, the field of interest is sampled at uniform intervals of space as well as time, and the spectral estimation algorithms are modified to accommodate the wavevector dependence of the spectrum.

WAVEVECTOR-FREQUENCY SPECTRAL ESTIMATION

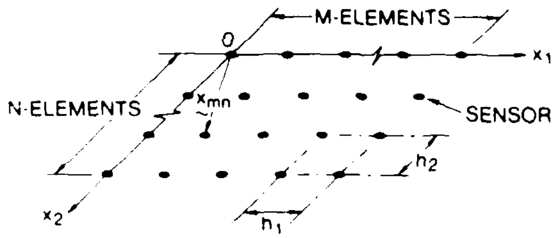
To illustrate this approach, consider the problem of estimating the wavevector-frequency spectrum of a pressure field over a planar surface. It is assumed that this pressure field is known to be statistically stationary.

SPATIAL SAMPLING

The spatial sampling of the surface pressure field is illustrated in slide 4. Here, the pressure is sampled over the plane of interest by a rectangular array of sensors. The sensors are evenly spaced in both the x_1 and x_2 coordinate directions.

SLIDE 4

SPATIAL SAMPLING



OUTPUT VOLTAGE OF SENSOR AT $\underset{\sim}{x}_{mn} = [(m-1)h_1, (n-1)h_2]$

$$e(\underset{\sim}{x}_{mn}, t) = \iint_{-\infty}^{\infty} g(\underset{\sim}{y}, \theta)\, p(\underset{\sim}{x}_{mn} - \underset{\sim}{y}, t - \theta)\, d\underset{\sim}{y}\, d\theta$$

DATA AVAILABLE AFTER SPATIAL SAMPLING

$$e_j(mh_1, nh_2, t) \qquad \begin{array}{l} 0 \leq m \leq M-1 \\ 0 \leq n \leq N-1 \\ 1 \leq j \leq J \\ 0 \leq t \leq t_1 \end{array}$$

WHERE J IS THE NUMBER OF REPETITIONS OF THE EXPERIMENT

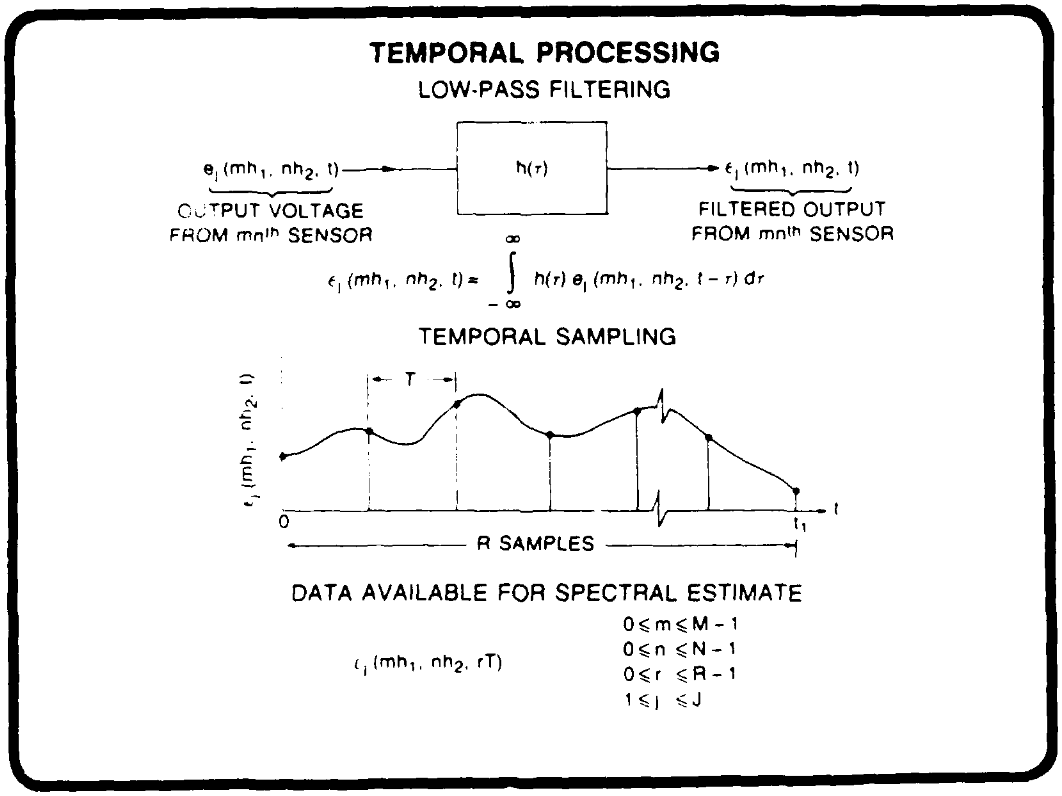
Each sensor converts the pressure acting over its active face to a voltage, denoted by e. The sensors are assumed to be linear, causal devices with identical space-time impulse responses. The relation between the output voltage of a sensor located at $\underset{\sim}{x}_{mn}$, the pressure field, and the impulse response of the sensor, g, is shown in the center of this slide.

If, in each repetition of the measurement, the outputs of all sensors are observed over the time interval 0 to t_1, the data available after the spatial sampling are shown at the bottom of this slide. Here again, the subscript j indicates the j-th repetition of the experiment.

TEMPORAL SAMPLING

The temporal processing of the output from each sensor is identical to that described previously. As shown at the top of slide 5, the output voltage from each

SLIDE 5



sensor is first low-pass filtered. The filtered sensor output, designated by ϵ_j, is related to the unfiltered output voltage, e_j, and the impulse response of the filter, $h(\tau)$, by the expression shown below the block diagram.

The filtered sensor outputs from the array are then sampled simultaneously at periodic intervals in time, as illustrated in the center of the slide.

This sampling process is repeated for each repetition of the experiment, resulting in the ensemble of data shown at the bottom of the slide. These data are input to an algorithm that performs the spectral estimate.

THE ALGORITHM

As indicated in slide 6, the algorithm is the specific computational procedure that produces the spectral estimate from the ensemble of space-time samples of the field. It is at this stage of the signal processing that one must decide (1) whether the field is homogeneous or nonhomogeneous and (2) which of the various forms of the wavevector-frequency spectrum is to be estimated.

SLIDE 6

THE ALGORITHM

DEFINITION:

COMPUTATIONAL PROCEDURE USED TO CONVERT ENSEMBLE OF SPACE-TIME SAMPLES OF THE FIELD TO AN ESTIMATE OF THE WAVEVECTOR-FREQUENCY SPECTRUM.

FACTORS INFLUENCING CHOICE OF ALGORITHM:

1) HOMOGENEITY OF FIELD
   - CHOICE OF SPECTRAL FORM

2) TRADE-OFFS
   - RESOLUTION VERSUS COMPUTATIONAL SPEED AND STABILITY
   - ABILITY TO RELATE THE MEAN SPECTRAL ESTIMATE TO TRUE SPECTRUM.

There are often several alternative algorithms for the estimation of a particular spectral form. These alternative algorithms offer various trade-offs between spectral resolution, stability, computational speed, and complexity. In addition, some algorithms allow the mean of the spectral estimate to be mathematically related to the true spectrum of the field.

The ability to relate the mean of the spectral estimate to the true spectrum of the field of interest is an important factor in the selection of an algorithm. Such a relationship can be used either in hindsight, to assess the quality of a spectral estimate, or in foresight, to design a measurement system that will enhance the probability of a high quality spectral estimate.

THE EXAMPLE

To illustrate the benefits of such a relationship, we consider the estimation of the wavevector-frequency spectrum of a planar pressure field that is both stationary and homogeneous.

SLIDE 7

SPECTRAL ESTIMATE FOR A STATIONARY-HOMOGENEOUS PRESSURE FIELD

THE ALGORITHM:

$$\tilde{\Phi}^{H}_{\epsilon}(\underset{\sim}{k},\omega) = \frac{1}{Mh_1Nh_2RTJ}\sum_{j=1}^{J}\left|\mathcal{E}_j(\underset{\sim}{k},\omega)\right|^2$$

WHERE

$$\mathcal{E}_j(\underset{\sim}{k},\omega) = h_1h_2T\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\sum_{r=0}^{R-1}\epsilon_j(mh_1,nh_2,rT)\exp\left[-i\left(k_1mh_1+k_2nh_2+\omega rT\right)\right]$$

AND

$\tilde{\Phi}^{H}_{\epsilon}$ IS THE ESTIMATE OF THE WAVEVECTOR-FREQUENCY SPECTRUM OF THE LOW-PASS FILTERED OUTPUT VOLTAGE FIELD

ϵ_j IS THE LOW-PASS-FILTERED OUTPUT VOLTAGE FROM A SENSOR

j DENOTES SAMPLES FROM THE j^{th} REPETITION OF THE EXPERIMENT

Recall that the data available for this spectral estimate are the ensemble of time samples of the low-pass filtered output voltages, ϵ_j, from the M-by-N array of sensors used to sample the pressure field. Therefore, the only wavevector-frequency spectrum that we can estimate is that of the filtered output voltage field. The algorithm used to estimate this stationary, homogeneous wavenumber-frequency spectrum is shown at the top of slide 7. Here, we see that the spectral estimate is proportional to the squared magnitude of the discrete Fourier transform, in both wavevector and frequency, of the samples of the filtered output voltage.

SLIDE 8

RELATION BETWEEN THE MEAN OF THE SPECTRAL ESTIMATE AND THE TRUE SPECTRUM OF THE PRESSURE FIELD

$$E\left\{\tilde{\Phi}_e^H(\underset{\sim}{k}, \omega)\right\} = \frac{h_1 h_2 T}{(2\pi)^3 MNR} \iint_{-\infty}^{\infty} \Phi_p^H(\underset{\sim}{\gamma}, \Omega)\,|G(\underset{\sim}{\gamma}, \Omega)|^2\,|H(\Omega)|^2\,|A(\underset{\sim}{\gamma} - \underset{\sim}{k})|^2\,|\mathcal{T}(\Omega - \omega)|^2 d\underset{\sim}{\gamma}\, d\Omega$$

WHERE

$G(\underset{\sim}{\gamma}, \Omega)$ IS THE WAVEVECTOR-FREQUENCY RESPONSE OF THE SENSORS, DEFINED BY

$$G(\underset{\sim}{\gamma}, \Omega) = \int_{-\infty}^{\infty} g(\underset{\sim}{\xi}, \tau)\exp[-i(\underset{\sim}{\gamma}\cdot\underset{\sim}{\xi} + \Omega\tau)]\, d\underset{\sim}{\xi}\, d\tau,$$

$H(\Omega)$ IS THE FREQUENCY RESPONSE OF THE LOW-PASS FILTER, DEFINED BY

$$H(\Omega) = \int_{-\infty}^{\infty} h(\tau)\exp[-i\Omega\tau]\, d\tau,$$

$A(\underset{\sim}{\gamma})$ IS THE WAVEVECTOR RESPONSE OF THE ARRAY, DEFINED BY

$$A(\underset{\sim}{\gamma}) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \exp[-i(\gamma_1 m h_1 + \gamma_2 n h_2)], \quad \text{AND}$$

$\mathcal{T}(\Omega)$ IS THE FREQUENCY RESPONSE ASSOCIATED WITH THE TEMPORAL SAMPLING, DEFINED BY

$$\mathcal{T}(\Omega) = \sum_{r=0}^{R-1} \exp[-i\Omega r].$$

By using the relationships shown in previous slides, the mean of the spectral estimate of the filtered output voltage can be related to the true wavevector-frequency spectrum of the pressure field. As shown at the top of slide 8, the mean of the spectral estimate is proportional to the convolution of the true spectrum of the pressure field, filtered by the wavevector-frequency response of the sensors and the frequency response of the low-pass filter, with the wavevector response of the array and the frequency response associated with the temporal sampling.

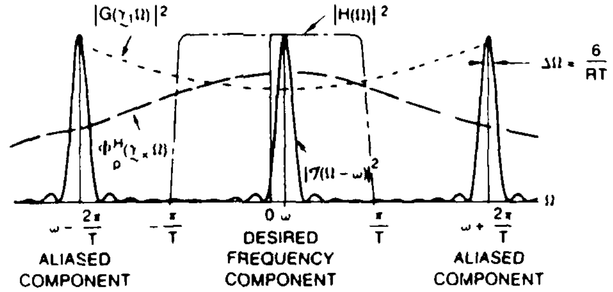
As indicated on the slide, the wavevector-frequency response of the sensor and the frequency response of the low-pass filter are appropriate Fourier transforms of the impulse responses of the sensor and the low-pass filter. The wavevector response of the array and the frequency response associated with the temporal sampling are discrete Fourier transforms of the spatial and temporal sampling patterns.

The equation relating the mean of the spectral estimate and the true pressure spectrum cannot be inverted to produce a unique solution for the wavevector-frequency spectrum of the pressure field. However, under certain conditions, the true spectrum can be inferred from the mean of the spectral estimate. Let us explore one such set of conditions by first examining the integral over frequency.

SLIDE 9

FILTERING IN THE FREQUENCY DOMAIN

$$E\left\{\tilde{\Phi}_{I}^{H}(\underset{\sim}{k},\omega)\right\} = \frac{h_1 h_2 T}{(2\pi)^3 MNR} \int_{-\infty}^{\infty} |A(\underset{\sim}{\gamma}-\underset{\sim}{k})|^2 \left\{ \int_{-\infty}^{\infty} \Phi_p^H(\underset{\sim}{\gamma},\Omega)\, |G(\underset{\sim}{\gamma},\Omega)|^2 |H(\Omega)|^2 |\mathcal{T}(\Omega-\omega)|^2\, d\Omega \right\} d\underset{\sim}{\gamma}$$



IF

(1) THE PRODUCT OF $\Phi_p^H(\underset{\sim}{\gamma},\Omega)\, |G(\underset{\sim}{\gamma},\Omega)|^2 |H(\Omega)|^2$ IS BAND LIMITED IN Ω SUCH THAT IT IS EFFECTIVELY ZERO OUTSIDE THE RANGE $-\pi/T < \Omega < \pi/T$ AND

(2) THE VARIATION OF THIS PRODUCT WITH Ω IS SLOW IN COMPARISON TO THE BANDWIDTH (6/RT) OF THE MAJOR LOBE OF THE FREQUENCY RESPONSE ASSOCIATED WITH THE TEMPORAL SAMPLING.

THEN, BY USE OF THE MEAN VALUE THEOREM, $E\left\{\tilde{\Phi}_{I}^{H}(\underset{\sim}{k},\omega)\right\}$ CAN BE APPROXIMATED BY

$$E\left\{\tilde{\Phi}_{I}^{H}(\underset{\sim}{k},\omega)\right\} \approx \frac{h_1 h_2 |H(\omega)|^2}{(2\pi)^2 MN} \int_{-\infty}^{\infty} \Phi_p^H(\underset{\sim}{\gamma},\Omega)\, |G(\underset{\sim}{\gamma},\Omega)|^2 |A(\underset{\sim}{\gamma}-\underset{\sim}{k})|^2\, d\underset{\sim}{\gamma}$$

For reference purposes, the relationship between the mean of the spectral estimate and the true spectrum is repeated at the top of slide 9, with the integral over the frequency domain isolated by the braces.

As indicated in this integral, the spectral estimate at the frequency ω is obtained by multiplying the product of the true spectrum, the response of the sensors, and the response of the low-pass filter by the frequency response associated with the temporal sampling centered at the frequency of interest. The resultant product is then integrated over all frequency.

The frequency dependence of the various terms in the integrand are illustrated below the equation. The dashed line represents the true spectrum, the dotted line represents the frequency response of the sensor, the broken line represents the response of the low-pass filter, and the solid line represents the filtering associated with the temporal sampling.

The frequency response associated with the temporal sampling is a periodic function of frequency, with major acceptance lobes separated by integer multiples of $2\pi/T$. Thus, when this filter is frequency shifted to sample the product of the true spectrum, the sensor response, and the low-pass filter at the frequency ω, this product is also sampled at the frequencies associated with the periodic replicates of the primary acceptance lobe. In the integration over frequency, the samples from these replicates are added to the sample at ω, thereby contaminating the estimate of the spectrum at ω. This contamination is called aliasing.

The purpose of the low-pass filter is to band limit the output voltage of the sensors such that negligible output occurs at frequencies above π/T and below $-\pi/T$. With such filtering, the aliased contributions to the spectral estimate are virtually eliminated.

If, as noted below the illustration,

(1) The product of the true pressure spectrum with the frequency responses of the sensors and the low-pass filters is band limited to frequencies between $-\pi/T$ and π/T and

(2) The variation of this product with frequency is slow compared with the frequency bandwidth of the primary response lobe of the filter associated with the temporal sampling,

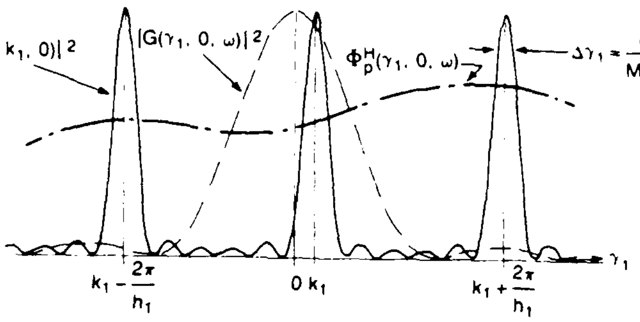
then, by use of mean value theorem, the integral over frequency can be performed, yielding the expression shown at the bottom of the slide.

For reference purposes, this expression is repeated at the top of slide 10. Here, we see that the mean of the spectral estimate is proportional to the integral, over all

SLIDE 10

THE FILTERING IN THE WAVEVECTOR DOMAIN

$$E\left\{\tilde{\Phi}^H_\epsilon(\underset{\sim}{k},\omega)\right\} = \frac{h_1 h_2 |H(\omega)|^2}{(2\pi)^2 MN} \int_{-\infty}^{\infty} \Phi^H_p(\underset{\sim}{\gamma},\Omega)\, |G(\underset{\sim}{\gamma},\Omega)|^2\, |A(\underset{\sim}{\gamma}-\underset{\sim}{k})|^2\, d\underset{\sim}{\gamma}$$



IF:

(1) THE PRODUCT OF $\Phi^H_p(\underset{\sim}{\gamma},\Omega)\,|G(\underset{\sim}{\gamma},\Omega)|^2$ IS BAND LIMITED IN $\underset{\sim}{\gamma}$ SUCH THAT IT IS EFFECTIVELY ZERO OUTSIDE $-\pi/h_1 < \gamma_1 < \pi/h_1$ AND $-\pi/h_2 < \gamma_2 < \pi/h_2$ AND

(2) THE VARIATION OF THIS PRODUCT IN $\underset{\sim}{\gamma}$ IS SLOW IN COMPARISON TO THE WAVEVECTOR BANDWIDTH $(6\pi/h_1, 6\pi/h_2)$ OF THE PRIMARY RESPONSE LOBE OF THE ARRAY.

THEN, BY USE OF THE MEAN VALUE THEOREM, $E\left\{\tilde{\Phi}^H_\epsilon(\underset{\sim}{k},\omega)\right\}$ CAN BE APPROXIMATED BY

$$E\left\{\Phi^H_\epsilon(\underset{\sim}{k},\omega)\right\} \approx \Phi^H_p(\underset{\sim}{k},\omega)\,|G(\underset{\sim}{k},\omega)|^2\,|H(\omega)|^2$$

wavevectors, of the product of the true pressure spectrum and the wavevector-frequency response of the hydrophone filtered by the wavevector response of the array steered to the wavevector $\underset{\sim}{k}$.

The integrand is illustrated below the equation as a function of the wavevector component γ_1. Here, because of the periodic nature of the array response, we see the same potential for aliasing in the wavenumber domain that we saw in the frequency domain of the previous slide.

In the frequency domain, this aliasing problem was averted by use of the low-pass filter. In the wavevector domain, the only low-pass-filtering device is the sensor. Unfortunately, we lack the capability, with current sensor technology, to design and build low-pass filters that match, in the wavevector domain, the performance of filters currently available in the frequency domain.

However, if

(1) We choose the array spacing such that, for all practical purposes, the product of the true pressure spectrum and the wavevector-frequency response of the sensors is band limited within the limits indicated beneath the illustration and

(2) The variation of this product with γ_1 and γ_2 is slow in comparison with the bandwidth components of the primary acceptance lobe of the array,

then we can again use the mean value theorem to show that the mean of the spectral estimate is related to the true wavevector-frequency spectrum of the pressure field by the simple algebraic expression shown at the bottom of the slide.

Here, it is evident that, given knowledge of the wavevector-frequency response of the sensors and the frequency response of the low-pass filter, the true spectrum of the pressure field can be deduced from the mean of the estimated spectrum.

As a consequence of this relationship, we can define what is called an unbiased estimator of the wavevector-frequency spectrum of the pressure field. This estimator, given at the top of slide 11, specifies the normalization of the algorithm required to convert the estimate of the spectrum of the filtered output voltage field to an estimate of the spectrum of the pressure field. This estimator is called unbiased because the mean of this spectral estimate is equal to the true spectrum of the pressure field.

Recall that certain assumptions were required to obtain the relationship on which this unbiased estimate is based. These assumptions define the signal processing required for this normalized algorithm to produce an unbiased estimate of the wavevector-frequency spectrum of the pressure field.

These requirements, which are listed below the definition of the unbiased estimator, are

(1) The temporal sampling rate ($2\pi/T$) must be at least twice the cutoff frequency (ω_c) of the low-pass filter to avoid aliasing in the frequency domain,

SLIDE 11

UNBIASED ESTIMATOR OF THE SPECTRUM OF THE PRESSURE FIELD

$$\tilde{\Phi}_p^H(\underset{\sim}{k}, \omega) = \frac{\tilde{\Phi}_\ell^H(\underset{\sim}{k}, \omega)}{|G(\underset{\sim}{k}, \omega)|^2 \; |H(\omega)|^2}$$

SIGNAL PROCESSING REQUIREMENTS FOR AN UNBIASED ESTIMATE

(1) THE TEMPORAL SAMPLING RATE ($2\pi/T$) MUST BE AT LEAST TWICE THE CUTOFF FREQUENCY (ω_c) OF LOW-PASS FILTER,

(2) THE SPATIAL SAMPLING RATE IN EACH COORDINATE DIRECTION ($2\pi/h_1$ AND $2\pi/h_2$) MUST BE AT LEAST TWICE THE CORRESPONDING WAVENUMBER BANDWIDTHS OF THE PRODUCT $\Phi_p^H(\underset{\sim}{k}, \omega)\,|G(\underset{\sim}{k}, \omega)|^2$,

(3) THE FREQUENCY RESOLUTION ASSOCIATED WITH THE TEMPORAL SAMPLING ($6/RT$) MUST BE SMALLER THAN ANY VARIATION OF $\Phi_p^H(\underset{\sim}{k}, \omega)$ WITH FREQUENCY, AND

(4) THE WAVEVECTOR RESOLUTION OF THE ARRAY ($6/Mh_1$, $6/Nh_2$) MUST BE SMALLER THAN ANY VARIATION OF $\Phi_p^H(\underset{\sim}{k}, \omega)$ WITH k_1 AND k_2, RESPECTIVELY.

(2) The spatial sampling rate in each coordinate direction ($2\pi/h_1$ and $2\pi/h_2$) must be at least twice the corresponding wavenumber bandwidths of the product of the true pressure spectrum and the wavevector-frequency response of the sensors,

(3) The frequency resolution associated with the temporal sampling must be much smaller than any variation of the true spectrum with frequency, and

(4) The wavevector resolution of the array must be much smaller than any variation of the true spectrum in the corresponding wavenumber coordinate.

As indicated in slide 12, these signal processing requirements present us with a dilemma. That is, failure to meet any of these signal processing requirements will result in a biased estimate of the wavevector-frequency spectrum of the pressure field. On the other hand, each of these processing requirements assumes some

SLIDE 12

THE DILEMMA

THE SIGNAL PROCESSING REQUIRED FOR AN UNBIASED SPECTRAL ESTIMATE IS DEFINED FROM SPECTRAL CHARACTERISTICS OF THE TRUE FIELD OF INTEREST.

THE CONSEQUENCE

UNBIASED SPECTRAL ESTIMATES ARE USUALLY OBTAINABLE ONLY THROUGH AN ITERATIVE MEASUREMENT PROCESS.

knowledge of the spectrum that we are trying to estimate. The practical consequence of this dilemma is that an unbiased spectral estimate can only be obtained through an iterative process of measurements.

The procedure for estimating the wavevector-frequency spectrum of a nonhomogeneous, but stationary, field is similar to the one for the homogeneous field. Here again, the key to obtaining a high quality spectral estimate is to select an algorithm for the desired spectral form that can, in the mean, be related to that true spectrum of the field. By this process, it is often possible to define an unbiased estimator of the field and to determine the signal processing required to obtain that estimate.

As a final note, it should be recognized that the variance of a spectral estimate is also an important metric of the quality of the estimate. Unfortunately, time does not permit a discussion of the variance of the spectral estimate.

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