AFIT/GST/ENS/89M-11

A MULTIVARIATE MAPPING FUNCTION FOR COMBAT POWER APPRAISAL IN THE AIRLAND RESEARCH MODEL

THESIS

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Preface

The purpose of this study was to develop an analytical model based on expert opinion. Currently, the Air Land Research Model deterministically assigns combat power to units. My purpose was to survey former combat arms battalion commanders and officers with staff experience and to develop a function that models combat power appraisal.

In developing and administering the survey, I am deeply indebted to the following people : Dr. Morris Peterson, Army Personnel Survey Division; Mrs. Ellen Godfrey, Combined Arms and Services Staff School; COL Davis, Army War College; and the students at CAS3 and the Army War College who participated in the survey. Without their assistance, this thesis would not have been completed.

A word of thanks goes to my thesis advisor, MAJ Dan Reyen and my reader MAJ Joe Litko for their support. Final acknowledgement to the Engineering Sciences faculty of AFIT, who put up with my puzzled looks and endless questions in the classroom.

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Abstract

This thesis attempts to develop an equation, based on expert opinion, that models the appraisal of combat power. The motivation came from identifying a shortcoming in the assignment of combat power in the Air Land Research Model (ALARM). A link is needed to join the Basic Inherent Power (BIP) of a unit and its Adjusted Basic Inherent Power (ABIP). The ABIP is a discounted portion of the BIP based on the current mission and status of the unit.

The unit and mission explored in this thesis was a mechanized infantry task force in the attack. The survey required combat arms officers to give categorical judgments on unit effectiveness, based on the state of the unit. Four unit state variables were used. 144 different unit profiles were generated and divided into four versions of the survey. Surveys were completed by students at the Army Combined Arms and Services Staff School and the Army War College. Response rate was approximately 80%.

Survey responses were transformed to numerical values using an interval scaling technique. These values and the variable settings were used in regression analysis. The best fit model was used to develop the Unit Appraisal Function (UAF). The UAF can now link the BIP to the ABIP, based on the mission and status of the unit.

A MULTIVARIATE MAPPING FUNCTION FOR COMBAT POWER APPRAISAL IN THE AIRLAND RESEARCH MODEL

I. Introduction

The United States Army intends to fight its battles according to the AirLand Battle doctrine. Several recent research efforts have focused on developing methodologies to model combat conducted under this doctrine. The AirLand Research Model (ALARM) provides the framework within which these methodologies are explored. The specific goals of ALARM are :

A. A two-sided, force-on-force model at the Blue Corps, Red Front level.

B. A primarily systematic (no human intervention) decision architecture, but with the provision to selectively insert human decision-makers if required for the development of rule-based systems.

C. A generalized network methodology and multidimensional coordinate system to represent transportation systems, terrain, communication links, and both fixed and mobile combat assets.

D. A resolution determined by the function and situation being modeled.

E. An ability to represent planning based on future-time extrapolation of the possible results of plan execution.

F. Detailed audit trails of cause and effect/effect relationships between decisions made and the results of the decision execution (9:1).

The latest attempts to move toward this goal of a systematic decision architecture, have involved initiatives to quantify the decision making process. In 1986, Schoenstadt and Parry developed the Axiomatic General Value System (GVS). GVS is a structure for assigning value to the numerous targets that are expected to inhabit the AirLand battlefield (9:3). Values are assigned to units on the battlefield using an analytical equation (9:6). GVS then formed the basis for the next attempt to model a portion of the decision making process.

In 1986, Kilmer expanded the GVS value assignment idea to include the evaluation of combat power and a methodology for making decisions in the present time based on what the situation is expected to be in the future (6:61-64). At this point, decision making was being modeled by present time value assignment (subjective in nature) and exponential discounting (analytical in nature). The next logical step in the evolution of modeling a decision making process was to combine the subjective features of a man in the loop with the analytical equation.

In 1988, Crawford explored a group of factors that impacted on combat power. Through a survey of Army officers, Crawford was able to develop a multivariate mapping function that accounted for the decision makers's appraisal of his unit at a future time (1:7). Crawford's degraded power function fit directly into Kilmer's future state decision making process (1:24-25). This last step of combining the subjective nature of human decision making with the analytical equation partially fulfills the goal of a systematic decision architecture.

Thesis Motivation

In order to properly portray decision making, a family of multivariate mapping functions is required for the power calculations for different types of operations and at the various echelons modeled by ALARM. FM 100-5 lists five types of offensive operations (2:9-1), seven types of defensive operations (2:10-4), and three types of retrograde operations (2:12-1). ALARM models the different echelons between battalion and corps. At this time, there is no link between a unit's Basic Inherent Power and the situationally specific Adjusted Basic Inherent Power. This thesis attempts to develop a multivariate mapping function which models a unit commander's appraisal of his combat power.

Problem Statement

> Given a set of unit state variables, develop a multivariate mapping function which models a decision maker's appraisal of the unit Basic Inherent Power based on the mission and state of the unit.

Scope

The decision maker's appraisal of unit capability is situationally dependent. As a result, the multivariate mapping function models only one scenario. The scenario for this thesis was restricted to the European theater and mechanized infantry task force in the offense.

Assumptions

This thesis is not meant to be a definitive study of what factors impinge on the decision maker's appraisal of the unit's combat capability. Instead, this thesis used four factors determined from a pilot study. These factors were then used to explore their relationships in the decision making process.

Summary

In this chapter, the AirLand Research Model was introduced and its goals explained. The evolutionary steps in creating the systematic decision architecture have been listed. The requirement to link a unit's Basic Inherent Power to its Adjusted Basic Inherent Power provided the motivation for this thesis. Chapter II contains a review of the literature that pertains to the thesis problem statement. Chapter III is composed of the methodology for collecting the needed data from the sample population and the techniques for developing the multivariate mapping function. Chapter IV contains the analysis and Chapter V lists the results and the recommendations for further study.

II. Literature Review

Combat Power

According to FM 100-5, combat power is the combined effects of maneuver, firepower, protection, and leadership on the enemy force; however, it is not absolute. Combat power is relative with respect to the opposing enemy force and situation (2:2-4). This situational aspect of combat power is especially important when related to one of the four AirLand battle tenets, agility.

Agility requires leaders "... who can act faster than the enemy" (2:2-2). This means that commanders must be mentally flexible. As the enemy counters one plan, a new action is developed which upsets this enemy initiative (2:2-2). In practice, agility means that commanders must make decisions affecting future operations using present time information. Simulating this type of decision making, "... planning based on future-time extrapolation of the possible results of plan execution" is one of the goals of ALARM (9:1).

Generalized Value System

The first step in modeling the combat decision process in ALARM took the form of an algorithm that assigned a value to each potential target on the battlefield (9:2). The generalized value system was developed by Schoenstadt and

Parry at the Naval Post Graduate School. The generalized value system is based on three assumptions :

> The purpose of an army is to wage war,and therefore the only elements/units that have inherent value are the fighting elements,i.e. maneuver and fire support.
>
> The value of CS/CSS units derives totally from the increase or decrease in value they provide to the combat (inherent value) units they support.
>
> Uncommitted units and usable, but unused, support are analogous to financial assets which mature at some time in the future- that is their current value is at a discounted version of their nominal (inherent or derived) value (9:3-4).

The authors note that value must ultimately be related to combat ability and that the value of a unit is based on the "state of the unit" (9:5). The state of the unit is then based upon a multidimensional measurement depending on the following factors :

> Number of operational weapon systems,
> Effective personnel strength,
> Available ammunition, and
> Available POL (9:5).

Using these factors as a state variable, the authors define a unit's situationally dependent value.

> The Situationally Dependent Value of a unit is its basic value,either inherent or derived, decremented by an exponential factor based on the time interval before that unit is available for commitment or can provide support (9:6).

The analytical expression of a unit's situationally dependent value is then :

$$V = V[s(t_p)]\exp[-A(t-t_p)] \tag{1}$$

where

$V[s(t_p)]$ = value based on the state of the unit at present time t_p

$\exp[-A(t-t_p)]$= the exponential discounting factor due to the difference in time between time of arrival t and the present time t_p (9:6).

This equation provides the connection between the situational aspect of combat power mentioned in FM 100-5 and the ALARM goal of representing planning based on future-time extrapolation.

Future State Decision Making

In March 1986, Kilmer wrote his thesis on the problem of modeling the decision process in ALARM. He used the generalized value system as the basis for a method termed future state decision making. Future state decision making models decisions made in the present time based on what the situation is expected to be in the future (6:11). Kilmer defined numerous terms and relationships in creating his model. Those that are pertinent to this thesis are listed below.

State. The state SX1(t) of an entity X1 at time, t,is the condition of X1 at time, t, expressed as a vector of the entity's attributes.

Power. The power of an entity determined by a particular hierarchical level is its ability to change or influence either directly or indirectly the states of entities that the level will face that belong to the enemy or that the enemy is planning to use.

Inherent Power. The inherent power of an entity is its ability to directly affect the states of enemy entities or of entities that enemy is using or planning to use.

Basic Inherent Power. The basic inherent power (BIP(X1)) is the inherent power possessed by entity X1 at full strength, when it is in position to engage its most likely adversary as a direct result of X1's ability to conduct combat operations.

Adjusted Basic Inherent Power. The adjusted basic inherent power ABIP($\underline{SX1}$(t)) of the entity X1 at time, t, is the BIP of X1 adjusted for the specific mission and condition of the entity at time,t.

Predicted Adjusted Basic Inherent Power. The predicted adjusted basic inherent power PABIP(X1(t))$|\underline{SX1}(t_p)$) of entity X1 at time, t_p, is the ABIP that X1 is predicted to have at time, t ($t>t_p$) (6:28-34).

This last term is expressed by the equation :

$$PABIP(X1(t)|\underline{SX1}(t_p)) = ABIP(\underline{SX1}(t_p))\exp[-L(t-t_p)] \quad (2)$$

where

$\exp[-L(t-t_p)]$ = the monotonical decrease in power that the combat unit experiences because of logistical needs, over the time period ($t-t_p$).

The decay constant, L, is due to non-combat related attrition of supplies, equipment, and personnel (6:34). This expression for the decrease in combat power is a concept related to the Lanchester attrition equations.

Situational Inherent Power. The situational inherent power, SIP(X1(t)$|\underline{SX1}(t_p)$), of an entity X1 is the inherent power that X1 is predicted at time,t_p, to have at time t (6:38).

At this point, Kilmer introduces the exponential discount factor used in the general value system, based on the time interval ($t-t_a$). This is the time interval between the time a unit arrives in position and the present time. Putting this exponential function into equation form gives :

$$SIP(X1(t)|\underline{SX1}(t_p)) = PABIP(X1(t_a)|\underline{SX1}(t_p))\exp[D(t_a-t)] \quad \text{for } 0 \le t \le t_a$$

$$SIP(X1(t)|\underline{SX1}(t_p)) = PABIP(x1(t)|\underline{SX1}(t_p)) \quad \text{for } t \ge t_a \text{ (6:39)} \qquad (3)$$

To summarize these equations, the power that an entity can exercise in combat is exponentially discounted depending on how far away, in time, it is from being in position. Simultaneously, while the entity is moving to its position, the entity is being attrited of personnel, ammunition, equipment, fuel, and other supplies. The above listed equations can be optimized, thereby modeling a decision process. The next step in modifying these equations is to input the "human factor".

Degraded Power Function.

In March 1988, Crawford proposed a method for determining the predicted adjusted basic inherent power (PABIP). Rather than modeling a decision-maker's prediction of his unit's power for a specific situation with an analytical expression such as Kilmer proposed, Crawford collected data from a pool of experts and developed a multivariate mapping function that replaced the logistical exponential decay function, $\exp[-L(t-t_p)]$ (1:5).

Crawford's version of the PABIP equation is :

$$\mathrm{PABIP}(X1(t)\,|\,\underline{SX1}(t)) = \mathrm{ABIP}(\underline{SX1}(t)) \times \mathrm{DPF}(\underline{SX1}(t)) \quad \text{for } t_p \le t \le t_e \tag{4}$$

where

$\mathrm{DPF}(\underline{SX1}(t))$ = the degraded power function based on the state of the entity between the present time, t_p, and the end of the time horizon, t_e (1:25).

Crawford used the state variables defined in the general value system : personnel (PER), ammunition (AMMO), vehicles and equipment (VEH), and fuel (POL). His survey required the respondents to judge whether a mechanized infantry battalion in the defense was capable of performing the mission at different levels of the state variables (1:32). By transforming the respondents' categorical judgments to an interval scale, Crawford was able to determine the equation of the curve which best fit the data. The degraded power function (DPF) for this particular unit and mission was found to be :

$$\mathrm{DPF} = \frac{88.978 - .0056 \times X1 - .0055 \times X2 - .0054 \times X3 - .0005 \times X4}{88.978} \tag{5}$$

where

$X1 = (\mathrm{PER}-100)^2$
$X2 = (\mathrm{AMMO}-100)^2$
$X3 = (\mathrm{VEH}-100)^2$
$X4 = (\mathrm{POL}-100)^2$ (1:22)

This degraded power function models the decision-maker's appraisal of the unit's power based on the projected state of the unit.

Summary

The generalized value system provided the initial algorithm for modeling a combat decision making process in ALARM. Kilmer expanded GVS into a series of equations which he termed future state decision making. Future state decision making utilized exponential functions to model the decrease of combat power based on the expected state of the unit at present and predicted times. Calculation starts with the BIP and culminate with the SIP. The SIP becomes planning value based on a prediction of future status. Kilmer's future state decision making uses the SIP to make decisions in the present time to affect future outcomes. The missing link is between the BIP and the ABIP. How does a commander judge his mission effectiveness based on his current status and mission ?

An analytical equation modelling combat power appraisal is needed. This equation should take into account those factors that influence this appraisal process. In Chapter 3, the factors and their levels used for an offensive scenario at the battalion level of command are addressed.

III. Methodology

Determination of Factors

The initial phase of this experiment was to determine what state variables impacted on the commander's appraisal of unit combat power. A pilot study was conducted to determine these state variables. The pilot study consisted of one open ended question in which the respondents were required to rank six factors which impacted on their appraisal of unit combat power. Respondents could choose from a list of suggested factors or add their own. The factors that respondents could choose from were Crawford's four state variables (PER, VEH, AMMO, and POL), chain of command, combat service support, combat support, training level, unit cohesion, and morale. With each ranked factor, the respondent was required to explain how each factor could be measured and if the factor could be influenced by the commander. The survey insturment for the pilot study is contained in Appendix A.

The pilot study confirmed that Crawford's state variables were important in the decision making process; however, due to the offensive scenario, POL,VEH, and AMMO were grouped together.

State Variables

The four state variables used for this experiment were a combination of grouped and non-grouped factors. The grouped factors were equipment and supply status (ESS), combat support (CS), and time (TIME). The non-grouped factor was the personnel status (PER). These state variables are explained below.

PER : Measured as the percentage of the Table of Organization and Equipment (TOE) authorizations currently available. This measurement also includes the leadership structure from battalion through platoon.

ESS : Measured as the percentage of operational weapon systems and vehicles as authorized by TOE, currently available. Also included are the supplies needed to operate the systems, fuel and ammunition. These are measured as the percentage of the unit basic load (UBL) remaining.

CS : Measured by the number of systems supporting the battalion. This state variable is a mixture of three of the seven operating systems stressed at the National Training Center (NTC), engineer, fire support, and combat support.

TIME : Measured by the equation -

$$t_A/t_C \tag{6}$$

where

t_A = time in hours until the attack is initiated
t_C = time in hours the commander feels he needs to prepare his unit.

This state variable incorporates the NTC operating system of intelligence and the intangible factors of leadership and training. Time is needed for the organic and non-organic intelligence assets to identify enemy locations and dispositions, for troop leading procedures and for rehearsals.

The method for grouping the equipment and supply factors was developed by Etheridge and Anderson (4:3-2). The different levels of the state variables that were used are :

PER - 100%, 75%, 60%, 40%
ESS - 100%, 80%, 60%, 40%
CS - 100%, 60%, 20%
TIME- 100%, 60%, 20%

The low settings for PER and ESS were taken from Etheridge and Anderson. In their study of unit reconstitution, they determined that units were considered mission ineffective at these points (4:7-3). Appendix B contains the explanations for each level of the state variables. Where appropriate, the explanations matched those presented by Etheridge and Anderson (4:3-3).

Experimental Design

The population targeted for this experiment was composed of combat arms officers who have had command and staff experience. Combat arms officers enrolled at the Army War College and the Combined Arms and Services Staff School (CAS3) were chosen as the sample frames.

The mass administered questionnaire was chosen as the method of data collection. This method was the most economical and allowed for the completion of the experiment within time constraints. The survey method also had the advantages of providing respondent anonymity, reducing interviewer bias, and facilitating the transformation of respondent answers to code.

This experiment was a full factorial design with $3^2 x 4^2$ or 144 different unit profiles (state variables set at the different levels). The questionnaire portion of the survey consisted of unit profiles followed by the respondent's categorical judgments on mission effectiveness. The fixed alternative question format was used. It was determined that since an answer to any unit profile question was bounded by the answers "totally ineffective" and "totally effective" that the fixed alternative format was an appropriate method (5:157-158). This format had the additional advantages that it is easier for the respondent to answer and simplifies tabulating and interpreting the data (11:285).

The full factorial matrix was randomly sorted and then divided into four groups of 36 questions. It was considered that a survey length of 36 questions would elicit responses without loss of respondent attention. The construction of the full factorial matrix and the survey was completed using the VP-PLANNER spread sheet (8) and WORD PERFECT 5.0 (10).

A pre-test using the survey questionnaire was conducted and comments were elicited from the respondents using open ended questions. The finalized survey is contained in Appendix B.

Analysis

After all the surveys were completed, the responses were recorded in a rectangular data structure. The method for constructing interval scales from categorical judgments developed by Lindsay was then used to transform the data

(7:2-33). Once this transformation was completed, methods of linear and multiple regression were used to determine a predictive model that fit the data. The results of the analysis are contained in Chapter IV.

Unit Appraisal Function

Using the predictive model determined by the regression, a multivariate mapping function was developed. This function, termed the Unit Appraisal Function (UAF) is the link between the Basic Inherent Power (BIP) and the Adjusted Basic Inherent Power (ABIP). The UAF accounts for mission and unit status and degrades the BIP in a fashion which models the unit commander's appraisal process. The UAF will decrement the BIP, so that the ABIP is some fraction of the BIP based on current mission and status. To do this, the best fit regression equation will be evaluated where all state variables are at 100 %. This value is the unit maximum. To create a percent function, the UAF becomes:

$$\text{UAF} = \frac{\text{Best Fit Model}(\text{SX}(1))}{\text{Unit Maximum}}$$

In equation form, the UAF is used as :

$$\text{ABIP}(\underline{\text{SX1}}(t)) = \text{UAF}(\underline{\text{SX1}}(t)) \times \text{BIP}(\text{X1}) \quad \text{for } 0 \le t \le t_m \tag{7}$$

where

> UAF(SX1(t)) is the unit commander's appraisal of his combat power based on his current mission and status, during the time period $t - t_m$, where t_m is the time of mission initiation.

In this thesis, a mechanized infantry task force with an attack mission and different unit status' were used to develop a particular UAF.

IV. Analysis

Survey Data

A total of 315 surveys were sent to the Combined Arms and Services Staff School and the Army War College. Of this number, 257 were returned. The table below gives data on the four personal history questions that the respondents were asked to answer. Years of command and years of active duty are averaged. Branch information is contained in Appendix C.

Table 1.

	CPT	MAJ	LTC	COL
Number Surveyed	196	1	52	8
Years Active Duty	8	10	19	21
Years Command	2	2	5	6

Constructing the Interval Scale

In order to take advantage of the expert opinion of the respondents, a method of constructing an interval scale from categorical judgments was needed. Lindsay's Method of Successive Intervals was determined to be the best method.

This method is fairly simple to implement and has the advantage of assigning a specific numerical value to the unit profiles (7:15). This feature facilitated the regression analysis.

Lindsay's technique requires four assumptions :

1. The respondents' feeling about the scale value assigned to a particular profile is a normally distributed random variable with mean S_i and variance $\sigma^2{}_i$.

2. The categories (Totally Ineffective through Totally Effective) are assumed to be a mutually exclusive set of successive intervals which collectively exhaust the judgement continuum.

3. The respondents' feelings about a category's upper bound is a normally distributed random variable. For each category j, the upper bound would be normally distributed with mean b_j and variance $v^2{}_j$

4. All category bounds have the same variance, so that for all j, $v^2{}_j = c$. (7:1-7)

Lindsay's technique requires ten steps. These steps are listed below. Each step is illustrated with calculations using data from the questionnaires. Appendix D contains the calculations for the survey itself.

Step 1. Arrange the raw frequency data in a table where the rows are the unit profiles and the columns are the categories. Place the categories in rank order, with the left most column representing the least effective category.

Table 2. Raw Frequencies

No.	A	B	C	D	E
24	2	5	46	14	0
33	9	34	24	2	0
38	1	1	4	49	0
70	3	27	23	7	0

Step 2. Compute the relative frequencies for each row by dividing the frequencies by the number of respondents who answered that question. Calculate the cumulative relative frequencies for the array by summing each column with the values of the columns to the left. This is the P array. Remove all values of $p_{ij} > 0.98$ and $p_{ij} < 0.02$.

Table 3. Relative Frequencies

No.	A	B	C	D	E
24	.03	.07	.69	.21	0
33	.13	.49	.35	.03	0
38	.02	.02	.07	.89	0
70	.05	.45	.38	.12	0

Table 4. Cumulative Relative Frequencies

No.	A	B	C	D	E
24	.03	.1	.79	1	1
33	.13	.62	.97	1	1
38	.02	.04	.11	1	1
70	.05	.5	.88	1	1

Table 5. Remove 1s and 0s

No.	A	B	C	D	E
24	.03	.1	.79		
33	.13	.62	.97		
38	.02	.04	.11		
70	.05	.5	.88		

Step 3. Consolidate groups with same remaining categories. Treat all remaining p_{ij} as leftward areas under a Standard Normal (0,1) curve and find the z values for these areas. Record these new z_{ij} in a new table.

Table 6. Combine Categories

No.	A	B	C
24	.03	.1	.79
33	.13	.62	.97
38	.02	.04	.11
70	.05	.5	.88

Table 7. Normalized Values

No.	A	B	C
24	-1.881	-1.282	.807
33	-1.126	.306	1.881
38	-2.052	-1.751	-1.226
70	-1.645	0	1.175

Step 4. For each row i in the z_{ij} matrix, compute the row average, z_i.

Step 5. For each column j in the z_{ij} matrix, compute the column average, b_j. These values are the upper bounds of category j.

Step 6. Compute the grand average of all z_{ij} in the matrix. This is easily done by averaging the column averages. Call the grand average, b.

Table 8. Row, Column, and Grand Average

No.	A	B	C	Row Avg
24	-1.881	-1.282	.807	-.78533
33	-1.126	.306	1.881	.353667
38	-2.052	-1.751	-1.226	-1.67633
70	-1.645	0	1.175	-.156667
Col Avg	-1.676	-.68175	.65925	
b	-.566167			

Step 7. Compute $B = \Sigma (b_j - b)^2$, the sum squares of the difference between column averages and grand average.

$$B = (-1.676 - (.566167))^2 + (-.68175 - (-.566167))^2 + (.65925 - (-.566167))^2$$

$$B = 2.746736$$

Step 8. For each row, compute $A_i = \Sigma (z_{ij} - z_i)^2$.

Table 9. Sum of Square Differences

$A_{ij} = (z_{ij} - z_i)^2$				
No.	A	B	C	$A_i = \Sigma A_{ij}$
24	1.2006	.2467	2.5355	3.9827
33	2 1894	.0023	2.3327	4.5244
38	.1411	.0056	.2028	.3493
70	2.2151	.0245	1.7733	4.0130

<u>Step 9</u>. For each row, compute $(B/A_i)^{.5}$.

<u>Step 10</u>. For each row, compute $S_i = b - z_i x(B/A_i)^{.5}$

Table 10. Scale Value of Instances

No.	$S_i = b - z_i x(B/A_i)^{.5}$
24	$.086024 = -.56617 - (-.785)(2.75/3.98)^{.5}$
33	$-.841729 = -.56617 - (.3537)(2.75/4.52)^{.5}$
38	$4.133259 = -.56617 - (-1.68)(2.75/.349)^{.5}$
70	$-.436553 = -.56617 - (-.157)(2.75/4.01)^{.5}$

This last table contains the scale value of instances. These scale values need to be transformed to an interval. For the purpose of this thesis an interval of 1 - 100 was chosen for the lower bound of A (Totally Ineffective) to the upper bound of D (Effective). Using the linear transformation, $Y = \alpha + \beta x$, $\beta > 0$ and the computed upper bounds of the categories (Step 5, Table 8), a system of simultaneous equations is formed.

Let 75 equal the upper bound of C and 50 the upper bound of B. These values anchor the interval at two points and allow solving the following equations :

$$50 = \alpha + \beta(-.68175)$$
$$75 = \alpha + \beta(.65925)$$
$$A = \alpha + \beta(-1.676)$$

Solving the first two equations, yields $\alpha = 62.7097$ and $\beta = 18.6428$. Using the values for α and β, the upper bound for the remaining category A is 31.46. Complete the transformation using the equation :

$$T_i = \alpha + \beta(S_i) \tag{8}$$

Table 11. Transformed Values

No.	T_i
24	64.313 = 62.7097 + 18.6428(.8060235)
33	47.017 = 62.7097 + 18.6428(-.841729)
38	139.765 = 62.7097 + 18.6428(4.133259)
70	54.571 = 62.7097 + 18.6428(-.436553)

This completes the transformation of categorical responses to an interval scale for group ABC. Steps 3 through 10 are repeated for the different groups of like categories.

Transformed Category Bounds

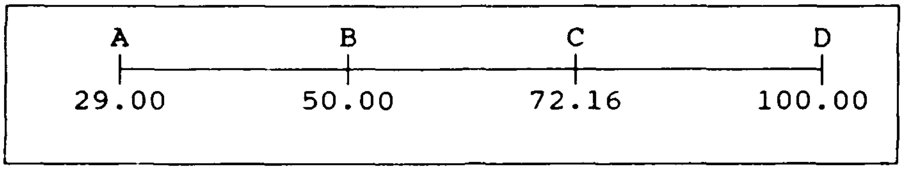
After Lindsay's method had been applied to the data, an interval scale was developed. 100 was chosen as the upper bound for category D and 50 as the upper bound for category B. Using these two values, the upper bound for all categories were determined.

Table 12. Category Upper Bounds

Category	Upper Bound
A (Totally Ineffective)	29.00
B (Ineffective)	50.00
C (Marginal)	72.16
D (Effective)	100.00

The upper bounds were then placed on a scale.

Figure 1. Transformed Category Bounds on Interval Scale



The respondents' answers determined a numerical value for each unit profile. For example, unit profile 48 (No. 48 in Appendix D) has a transformed value of 69.28. This places that unit profile in the upper end of the Ineffective portion of the scale. All unit profiles with their respective transformed values are contained in Appendix E. Appendix E is also the data matrix used for regression.

Regression Analysis

SAS, a statistics program available on the main frame at the Air Force Institute of Technology was used to analyze the data. Various types of curve equations were tried, including linear, logarithmic, and square root. The results are contained in Appendix F. A fourteen term polynomial equation with main effects, quadratics, and two-way interaction terms was then used (three-way interaction was assumed to be negligible). According to Draper and Smith, a minimum of 3 criteria should be addressed when choosing the best model. These criteria are :

1. maximize Coefficient of Determination (R^2)
2. minimize Sample variance (s^2)
3. Mallow's Cp = number of regression terms (including the intercept)(3:296)

A fourth criteria used in this thesis was to minimize the complexity of the model. Once the final model was chosen, the residuals were examined for any type of recognizable pattern. No pattern indicates that the residuals are normally distributed. Additionally, the F ratio should exceed the selected percentage point of the F-distribution by at least a factor of four (3:93).

The fourteen term polynomial regression results are contained in Appendix G. This regression yielded 3 reasonable models to chose from. These models were the 6,7,and 8 variable models. Table 13 lists the criteria mentioned above.

Table 13. Model Criteria

No. Variables	R^2	s^2	Cp	F Ratio	$F(v_1,v_2,.95)$
6	.8267	65.94	13.7	109.0	2.10
7	.8314	64.19	11.9	95.9	2.01
8	.8398	61.44	6.9	88.5	1.94

Draper and Smith's criteria indicated that the 8 variable model was the best fit; however, the 6 variable model was almost as good at predicting results and easier to explain. For the purpose of this thesis, the 6 variable model was used for further analysis. The 6 variable model was of the form :

$$Y = \beta + PER + X1 + X2 + X3 + PER^2 + ESS^2 \qquad (9)$$

where : β = intercept
X1 = PER*ESS
X2 = PER*CS
X3 = PER*TIME

The model with regression coefficients was found to be :

$$Y = -2.826 + .690PER + .0077X1 + .0027X2 + .0024X3 - .0071PER^2 - .0011ESS^2 \qquad (10)$$

The residuals for this equation were plotted and are contained in Appendix H. From this plot, it was determined that the normality of error assumptions held.

Unit Appraisal Function

Substituting the maximum value of 100 for the unit status variables PER,ESS, CS, and TIME into Equation 10, yields the unit maximum = 111.17. By utilizing this number in the denominator of Equation 11, the mission specific Unit Appraisal Function (UAF) was formed.

$$UAF = (-2.826 + .690PER + .0077X1 + .0027X2 + .0024X3 - .0071PER^2 - .0011ESS^2)/111.17 \tag{11}$$

This particular UAF degrades the BIP of a mechanized infantry task force, due to mission and unit status.

It is appropriate at this point to examine the terms in the UAF. PER appears in 5 terms in the model. Once as a main effect, 3 times as part of two-way interaction and once as a square term. The main effect PER term coefficient indicates a substantial contribution to the overall function. Heuristically this makes sense in an equation that models an attack mission. The two-way interaction are best explained graphically. Figure 2 illustrates the effect of PER, when all other factors are set at high, medium ,and low values. The overall degradation is low when the PER value is high and high when PER is set low.

The PER^2 and ESS^2 terms can be explained algebraically. In models of few variables, linear terms dominate the equations (see the 1,2,and 3 variable models in Appendix G).

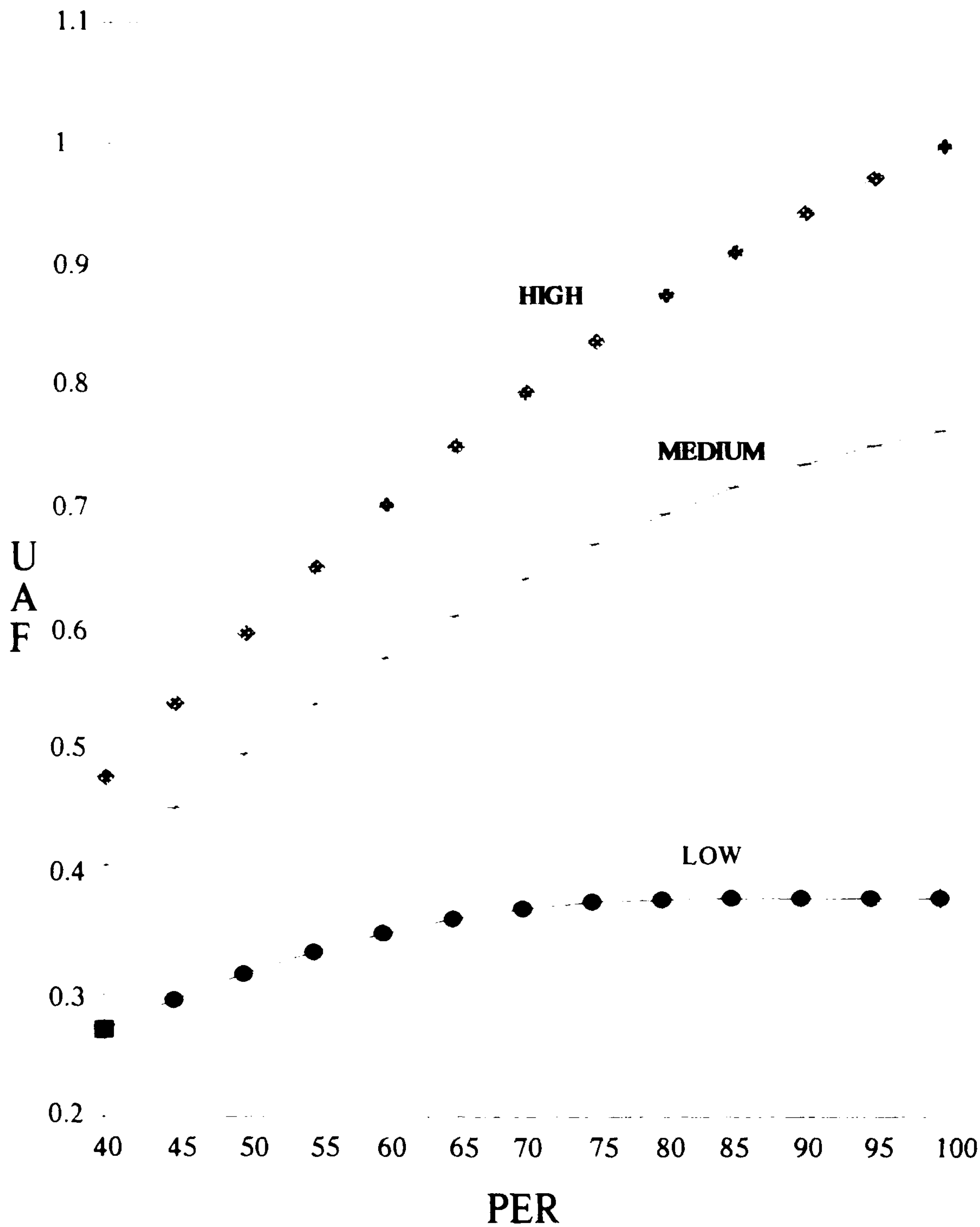


Figure 2. Two-Way Interaction

As the number of variables is increased, the square terms take into account the curve of the equation. This curve is analogous to a decreasing rate of return for each additional asset. In this case, both PER^2 and ESS^2 have negative coefficients indicating this decreasing rate. This also makes sense heuristically. Given that PER and ESS are past the linear level, each additional gain contributes less to the UAF, than previous additions.

V. Results

UAF and Future State Decision Making

The Basic Inherent Power of a unit (BIP(X1)), is the inherent power that a unit possess at full strength when it is in position to engage its most likely adversary. It is a measure of its ability to conduct combat operations (6:28). The Adjusted Basic Inherent Power is the BIP of X1 adjusted for the mission and the specific conditions of the unit. Prior to the UAF, there was no link between the BIP and ABIP for a unit. Utilizing the UAF to join the BIP and ABIP yields Equation 7 in Chapter III.

Example

The following example illustrates the use of the UAF.

A Blue mechanized infantry task force is in an assembly area. At time t=0, the unit receives the mission to attack as part of a larger coordinated attack. The expected Red force is a motorized rifle company. The current state of the Blue unit is :

PER 75% - 75% of TOE authorized strength is available, 1 field grade officer is lost, 75% of company chain of command is intact.

ESS 80% - 80% of TOE authorized systems are operational, ammunition and POL UBLs (unit basic loads) are at 80%.

CS 60% - The task force has priority of fires from the brigades DS (direct support) artillery battalion. 1 combat engineer platoon and 1 air defense platoon are attached to the task force. There are 3 CAS (close air support) sorties available.

TIME 60% - Organic intelligence assets have located 50% of the enemy positions, companies have not completed troop leading procedures, some rehearsals have been completed.

Assigning a BIP of 1000 STAPOWs (standard power units) to the task force and using the UAF, the ABIP for this unit is :

$$UAF = (-2.826 + .690x75 + .0077x75x80 + .0027x75x60 + .0024x75x60 - .0071x75^2 - .0011x80^2)/111.17$$

$$UAF = .734$$

$$ABIP(\underline{SX1}(t)) = .734x1000 \text{ STAPOWs}$$

$$ABIP(\underline{SX1}(t)) = 734 \text{ STAPOWs}$$

734 STAPOWs is the combat power that this unit starts with in the future state decision making process. This value would be carried forward into the PABIP and SIP calculations.

Conclusions

The UAF can be incorporated into ALARM to model a portion of the decision making process. The UAF is a model based on a limited distribution fixed alternative questionnaire involving categorical judgments. The survey was limited to one specific command hierarchy and to one specific mission. Lindsay's Method of Successive Intervals was a relatively uncomplicated method to employ in this research effort.

Recommendations for Further Study

The 257 surveys are contained in a data base. Additional study should focus on rank and branch differences among respondents; if any. In their study, Etheridge and Anderson found no significant differences between ranks and branches (4:4-3). Resampling techniques should be applied to the existing data for the purpose of reproducing the results found in this thesis. Further research and additional work are needed to develop similar models for different unit and mission combinations. The ultimate objective being a family of equations which accurately model human decision making.

Appendix A

Combat Factor Pilot Study

PURPOSE : The purpose of this pilot study is to determine what elements should be included in a survey of factors that impact on a mechanized infantry battalion in the attack.

BACKGROUND : Currently, the AirLand Research Model (ALARM) uses analytical equations to model the unit commander's decision-making process. If a multivariate mapping function, based on expert opinion, can be developed; certain portions of the command and control module of ALARM will be more realistic.

SCENARIO : You are the commander of 1st Battalion 36th Infantry (M). Your current mission is to attack a company from a motorized rifle regiment. You are attacking as part of a brigade and division effort. Follow on missions are possible after the objective is seized. Expect no resupply after you cross the LD\LC.

Factors Already in Use : Personnel - % of TOE strength available
Equipment - % of TOE weapon systems on hand and operational
Ammunition - % of UBL on hand
POL - % of UBL on hand

Other Factors : Chain of Command - are key leaders leaders on hand and effective ?
CSS - is CSS able to support unit needs?
CS - status of FA and CAS
Unit Cohesion - present ?
Training Level - effective, marginal ?
Morale - good, bad ?

QUESTION : List 6 factors in order of importance in view of the scenario given. Use the ones listed above or any others you think are important. Include how this factor is measured and if, in your opinion, a commander can actually influence this factor. Task organization is not a factor. One study has shown that there is no statistical difference between LTC and branch qualified captains in the answers given to questions such as these.

Example : Personnel - measured as a % of TOE authorizations. A commander can divert or take soldiers from one unit to another.

POL - % UBL on hand. Commanders request POL resupply.

APPENDIX B

Survey Approval Authority : U.S. Army Soldier Support Center
Survey Control Number : ATNC-AO-89-9
RCS : MILPC-9

BACKGROUND : One purpose of modelling combat is to gain insight and aid decision making. The AirLand Research Model (ALARM) is one particular model currently in use. One process in ALARM is the commander's perception of his unit's capabilities. It is felt that this process can be done better.

PURPOSE : In order to improve the unit commander's appraisal of his combat capability in the model, it is felt that expert military opinion can help. The purpose of this questionnaire is to collect your opinions about a task force in the attack. The results will then aid in improving the current method of modelling a commander's appraisal of his unit.

PROCEDURE : The questionnaire will ask you to assume the role of a battalion commander and to assess the capabilities of your battalion task force. The questionnaire package includes the following sections :

a. Unit Variables : This section defines a set of variables that encompass the key attributes of the unit. These variables are to be considered in your evaluation and decision process.

b. Variable Levels : This section gives the different levels of each variable and the corresponding descriptions as it applies to your task force within the given scenario.

c. Scenario : This is a short scenario describing the general and special situation of a battalion performing an offensive mission. The scenario is oriented specifically to your battalion task force. The scenario provides only the information considered necessary for you to develop your judgments.

d. Questions : Each question starts with a unit profile. The unit profile consists of the four variables set at different levels. You, as the commander, are then asked to indicate your current effectiveness in accomplishing the mission.

e. Personal History : The final section of this package contains a few questions about your military experience and your opinion about this survey. Your name is not required.

CONCLUSION : This questionnaire is being used to gather information on a particular decision making process. The objective is to model military decision making in some fashion that approaches reality. Your cooperation in this effort is most appreciated. Thank you for your time and consideration.

UNIT VARIABLES

There are four variables used in this study. Some of the variables are composed of more than one factor. The decision of which factors to use and how to group them stems from research conducted prior to this survey. The four variables are : personnel (PER), equipment and supplies (ESS), combat support (CS), and time (TIME). The variables are described below.

PER : Measured as the percentage of the Table of Organization and Equipment (TOE) personnel authorizations currently available. This measurement includes the leadership structure from battalion through platoon.

ESS : Measured as the percentage of operational weapon systems as authorized by TOE currently available. Also included are the supplies needed to operate the systems, fuel and ammunition. These are measured as the percentage of the unit basic load (UBL) remaining.

CS : Measured by the number of systems supporting the battalion. This state variable is a mixture of three of the seven operating systems stressed at the National Training Center (NTC), engineer, fire support, and combat support.

TIME : Measured by the equation -

$$t_A/t_C$$

where

t_A = time in hours until the attack is initiated
t_C = time in hours the commander feels he needs to prepare his unit.

This state variable incorporates the NTC operating system of intelligence and the intangible factors of leadership and training. Time is needed for the organic and non-organic intelligence assets to identify enemy locations and dispositions. Time is needed for troop leading procedures and rehearsals.

Example : If an attack is in 8 hours and the battalion commander feels 12 hours preparation is needed, TIME is at 9/12 or 75 %.

VARIABLE LEVELS

The levels of the unit variables are listed along with a physical description.

PER :

100% - 100% of TOE authorized strength is available and the chain of command is intact.

75% - 75% of TOE authorized strength is available, 1 field grade officer is lost, 75% of company chain of command is intact.

60% - 60% of TOE authorized strength is available, 1 field grade officer is lost, 60% of company chain of command is intact.

40% - less than 40% of TOE authorized strength is available, 2 field grade officers are lost, less than 40% of company chain of command is intact.

ESS :

100% - 100% of TOE authorized systems are operational, ammunition and POL UBLs are at 100%.

80% - 80% of TOE authorized systems are operational, ammunition and POL UBLs are 80%.

60% - 60% of TOE authorized systems are operational, ammunition and POL UBLs are at 60%.

40% - less than 40% of TOE authorized systems are operational, ammunition and POL UBLs are below 40%.

CS :

100% - The battalion has a dedicated battery from the brigade's DS artillery battalion, 1 combat engineer platoon, 1 NBC smoke platoon, and 1 air defense platoon are attached to the battalion, 6 close air support (CAS) sorties are available.

60% - The battalion has priority of fires from the brigadesDS artillery battalion. 1 combat engineer platoon and 1 air defense platoon are attached to the battalion.There are 3 CAS sorties available.

20% - Artillery support is available. There is no combat engineer support available. 1 air defense platoon is attached to the battalion. There are no CAS sorties available.

TIME :

100% - Organic and non-organic intelligence assets have located 90% of enemy positions, troop leading procedures completed as well as rehearsals.

60% - Organic intelligence assets have located 50% of enemy positions, companies have not completed troop leading procedures, some rehearsals have been completed.

20% - Organic intelligence assets have located less than 20% of enemy positions, companies have not completed troop leading procedures, no rehearsals have been completed.

SCENARIO

Introduction : This scenario presents the context in which you will be asked to make judgments concerning the 131st Mechanized Infantry Battalion. The scenario describes the friendly and enemy forces of interest, the general and special situations, and the mission and activities of the 131st Mech.

General Situation : On 12 October 198_ , USSR and Warsaw Pact forces initiated a non-nuclear attack against NATO. The initial phase of the attack was aimed at destroying Allied defensive systems and capturing territory before adequate reinforcements arrive from CONUS. NATO forces have offered much stronger resistance than had been anticipated by the Warsaw Pact. Your battalion, the 131st Mech, is assigned to the 3rd Brigade of the 58th Infantry Division (Mechanized). The division has been in combat since the onset of hostilities and has fought elements of two motorized rifle divisions (BMP) as they breached the inter-German border in the division's sector. All units within the division have suffered varying degrees of combat loss. After 10 days of defending and staging local counter attacks, the corps commander feels that the situation is at a stalemate and he is ready to conduct offensive operations.

The 131st Mech is a J series mechanized infantry battalion equipped with M113 APCs. One infantry company has been detached from the battalion, and one M1A1 tank company has been attached.

The 3rd Brigade is supported by a DS 155mm SP battalion.

Special Situation : It is now 23 October 198_, 131st Mech is occupying defensive positions along the FEBA. You have received a warning order to prepare for an attack against a motorized rifle company (BMP) as part of brigade and division offensive operations. The battalion objective is known. The time of attack has not been specified yet. You, as the task force commander, are appraising your combat power with respect to this particular mission.

QUESTIONS

This questionnaire consists of 36 questions. Each question consists of a unit profile and the answer range. The unit profiles are composed of the different variables set at various levels. Below are two sample questions.

The current status of your unit is : * Your ability to accomplish the mission is :

PER	ESS	CS	TIME	*	TOTALLY INEFF	INEFF	MARG	EFF	TOTALLY EFF
60 %	80 %	60 %	100 %	*	( )	( )	(x)	( )	( )
100 %	100 %	60 %	100 %	*	( )	( )	( )	(x)	( )

This ends the instructions and sample questions. The survey questions follow.

****

VARIABLE LEVELS

The levels of the unit variables are listed along with a physical description.

PER :

- 100% - 100% of TOE authorized strength is available and the chain of command is intact.
- 75% - 75% of TOE authorized strength is available, 1 field grade officer is lost, 75% of company chain of command is intact.
- 60% - 60% of TOE authorized strength is available, 1 field grade officer is lost, 60% of company chain of command is intact.
- 40% - less than 40% of TOE authorized strength is available, 2 field grade officers are lost, less than 40% of company chain of command is intact.

ESS :

- 100% - 100% of TOE authorized systems are operational, ammunition and POL UBLs are at 100%.
- 80% - 80% of TOE authorized systems are operational, ammunition and POL UBLs are at 80%.
- 60% - 60% of TOE authorized systems are operational, ammunition and POL UBLs are at 60%.
- 40% - less than 40% of TOE authorized systems are operational, ammunition and POL UBLs are below 40%.

CS :

- 100% - The battalion has a dedicated battery from the brigade's DS artillery battalion, 1 combat engineer platoon, 1 NBC smoke platoon, and 1 air defense platoon are attached to the battalion, 6 close air support (CAS) sorties are available.
- 60% - The battalion has priority of fires from the brigade's DS artillery battalion. 1 combat engineer platoon and 1 air defense platoon are attached to the battalion.There are 3 CAS sorties available.
- 20% - Artillery support is available. There is no combat engineer support available. 1 air defense platoon is attached to the battalion. There are no CAS sorties available.

TIME :

- 100% - Organic and nonorganic intelligence assets have located 90% of enemy positions, troop leading procedures completed as well as rehearsals.
- 60% - Organic intelligence assets have located 50% of enemy positions, companies have not completed troop leading procedures, some rehearsals have been completed.
- 20% - Organic intelligence assets have located less than 20% of enemy positions, companies have not completed troop leading procedures, no rehearsals have been completed.

OPINION SCALE

The opinion scale contains 5 parts. These parts are totally ineffective, ineffective, marginal, effective, and totally effective. For this survey, the following operational definitions apply :

Totally Ineffective Unit fails mission and is combat inoperable.

Totally Effective Unit completes mission and is ready for any number of subsequent missions.

* You may detach this sheet and use it for reference.

COMBAT POWER APPRAISAL SURVEY

The current status of your unit is :				*	Your ability to accomplish the mission is :				
PER	ESS	CS	TIME	*	TOTALLY INEFF	INEFF	MARG	EFF	TOTALLY EFF
75 %	80 %	20 %	100 %	*	( )	( )	( )	( )	( )
100 %	40 %	20 %	60 %	*	( )	( )	( )	( )	( )
60 %	60 %	20 %	60 %	*	( )	( )	( )	( )	( )
60 %	100 %	100 %	20 %	*	( )	( )	( )	( )	( )
40 %	80 %	60 %	100 %	*	( )	( )	( )	( )	( )
100 %	60 %	100 %	60 %	*	( )	( )	( )	( )	( )
40 %	40 %	60 %	60 %	*	( )	( )	( )	( )	( )
40 %	100 %	60 %	100 %	*	( )	( )	( )	( )	( )
75 %	80 %	60 %	100 %	*	( )	( )	( )	( )	( )
100 %	60 %	60 %	60 %	*	( )	( )	( )	( )	( )
40 %	100 %	20 %	60 %	*	( )	( )	( )	( )	( )
40 %	80 %	100 %	60 %	*	( )	( )	( )	( )	( )
40 %	60 %	60 %	60 %	*	( )	( )	( )	( )	( )
40 %	100 %	60 %	60 %	*	( )	( )	( )	( )	( )
75 %	60 %	20 %	60 %	*	( )	( )	( )	( )	( )
60 %	60 %	20 %	20 %	*	( )	( )	( )	( )	( )
75 %	80 %	60 %	60 %	*	( )	( )	( )	( )	( )
100 %	60 %	60 %	20 %	*	( )	( )	( )	( )	( )
100 %	80 %	20 %	20 %	*	( )	( )	( )	( )	( )
40 %	60 %	100 %	20 %	*	( )	( )	( )	( )	( )
60 %	80 %	100 %	20 %	*	( )	( )	( )	( )	( )
60 %	40 %	100 %	20 %	*	( )	( )	( )	( )	( )
40 %	80 %	100 %	100 %	*	( )	( )	( )	( )	( )
75 %	60 %	100 %	20 %	*	( )	( )	( )	( )	( )
60 %	80 %	20 %	60 %	*	( )	( )	( )	( )	( )
75 %	80 %	20 %	20 %	*	( )	( )	( )	( )	( )
100 %	60 %	100 %	100 %	*	( )	( )	( )	( )	( )
60 %	80 %	20 %	100 %	*	( )	( )	( )	( )	( )
100 %	60 %	20 %	100 %	*	( )	( )	( )	( )	( )
100 %	40 %	100 %	60 %	*	( )	( )	( )	( )	( )

75 %	100 %	100 %	60 %	*	( )	( )	( )	( )	( )
100 %	100 %	100 %	20 %	*	( )	( )	( )	( )	( )
75 %	60 %	100 %	60 %	*	( )	( )	( )	( )	( )
40 %	60 %	20 %	20 %	*	( )	( )	( )	( )	( )
40 %	40 %	20 %	100 %	*	( )	( )	( )	( )	( )
60 %	80 %	60 %	100 %	*	( )	( )	( )	( )	( )

****

This completes the question portion of the survey. Please fill out the following personal history form.

Branch :

IN	AR	FA	AD	EN
( )	( )	( )	( )	( )

Time spent on active duty (in years):

less than 6	6-10	11-13	14-16	17-19	20-22	23-25	over 25
( )	( )	( )	( )	( )	( )	( )	( )

Time spent as a maneuver unit commander (in years):

less than 2	3-5	6-8	9-10	11-12	over 12
( )	( )	( )	( )	( )	( )

Present rank :

CPT	MAJ	LTC	COL
( )	( )	( )	( )

Are there any other factors that should be included in this survey ?

No	Yes	If Yes, please list.
( )	( )	

Appendix C: Officer Branch Information

	Rank			
Branch	CPT	MAJ	LTC	COL
IN	53	0	21	4
AR	24	0	8	1
FA	46	1	11	2
EN	23	0	4	0
AV	35	0	5	1
AD	15	0	2	0

Appendix D : Construction of Transformed Values

Group 1 Raw Frequecies

No.	A	B	C	D	E
1	1	0	3	0	56
2	1	3	5	19	32
3	1	2	6	37	21
4	1	1	3	34	21
5	1	0	10	38	15
6	0	3	23	27	4
7	2	2	2	39	10
8	1	9	20	33	5
9	1	12	25	21	1
10	0	0	1	19	44
11	1	2	3	45	10
12	1	7	15	39	7
13	0	3	10	37	17
14	0	2	12	37	7
15	1	4	24	27	1
16	4	5	15	28	8
17	1	5	37	21	3
18	4	15	32	15	1
19	1	0	13	34	18
20	0	4	26	30	5
21	0	5	33	19	2
22	1	3	28	29	8
23	0	0	28	31	3
24	2	5	46	14	0
25	0	10	33	22	2
26	1	15	36	16	0
27	4	25	29	2	1
28	2	12	26	17	0
29	2	16	33	15	1
30	2	27	35	6	0
31	0	16	35	10	0
32	1	21	29	8	0
33	9	34	24	2	0
34	1	29	29	1	0
35	10	32	22	1	1
36	9	38	13	1	0
37	1	1	1	38	13
38	1	1	4	49	0
39	1	2	30	32	3
40	0	0	12	35	6
41	1	1	25	39	1
42	2	9	32	25	1
43	2	6	32	25	4
44	1	1	38	22	0
45	4	18	28	9	1
46	0	2	12	25	20
47	0	3	9	43	5
48	2	8	30	29	0

Group 2 Raw Frequencies

No.	A	B	C	D	E
49	0	3	17	36	8
50	0	3	31	29	3
51	1	10	35	19	0
52	1	10	24	30	2
53	2	10	28	17	1
54	6	19	36	4	0
55	0	2	27	35	5
56	1	1	20	42	1
57	2	2	38	14	1
58	2	8	23	21	5
59	1	9	43	15	0
60	4	18	30	7	0
61	2	2	37	13	1
62	3	24	28	11	0
63	5	25	25	5	1
64	0	17	29	13	1
65	2	15	28	9	2
66	6	21	27	5	0
67	4	18	29	7	0
68	4	26	27	11	0
69	5	33	20	3	0
70	3	27	23	7	0
71	10	39	19	1	0
72	16	40	10	2	0
73	3	7	14	35	9
74	2	10	27	17	1
75	4	15	30	18	0
76	0	8	21	26	2
77	2	8	39	20	0
78	3	19	30	7	1
79	5	10	27	18	0
80	3	19	37	9	1
81	10	10	19	6	1
82	4	6	19	27	3
83	1	10	34	15	1
84	3	14	37	9	0
85	1	5	35	19	0
86	3	13	28	13	0
87	1	10	35	19	2
88	1	17	31	17	1
89	3	24	34	5	0
90	4	34	20	3	0
91	1	8	24	26	1
92	2	13	28	13	0
93	6	15	37	9	0
94	1	10	32	16	0
95	5	10	37	16	0
96	7	26	32	3	0

Group 3 Raw Frequencies

No.	A	B	C	D	E
97	3	14	27	16	0
98	7	21	33	4	0
99	12	37	17	0	0
100	2	14	34	6	1
101	3	30	25	2	0
102	5	33	25	3	0
103	6	22	20	12	0
104	8	36	17	4	0
105	13	35	9	3	0
106	3	33	17	2	0
107	12	25	19	3	0
108	31	31	6	1	0
109	8	15	26	10	0
110	8	22	22	8	0
111	8	30	18	3	0
112	8	31	23	4	0
113	14	30	18	4	0
114	13	32	20	3	0
115	10	32	15	2	1
116	15	31	14	2	0
117	25	20	14	1	0
118	9	24	18	12	1
119	11	26	19	9	0
120	13	35	17	3	0
121	10	26	18	10	0
122	9	24	19	6	0
123	10	35	11	3	0
124	14	19	21	1	0
125	14	30	13	2	0
126	34	28	5	1	0
127	7	30	14	9	0
128	11	27	20	2	0
129	17	31	19	0	0
130	12	31	21	4	1
131	20	20	20	1	0
132	13	31	12	3	0
133	9	41	9	2	0
134	26	25	10	0	0
135	23	37	7	0	0
136	15	26	16	3	0
137	18	32	10	1	0
138	32	24	8	1	0
139	14	27	13	4	0
140	28	28	9	2	0
141	21	31	3	4	0
142	22	32	10	0	0
143	27	22	6	4	0
144	37	14	2	4	2

Group 1 Relative Frequencies

No.	A	B	C	D	E
1	.02	0	.05	0	.93
2	.02	.05	.08	.32	.53
3	.01	.03	.09	.55	.31
4	.02	.02	.05	.57	.35
5	.02	0	.16	.59	.23
6	0	.05	.4	.47	.07
7	.04	.04	.04	.71	.18
8	.01	.13	.29	.49	.07
9	.02	.2	.42	.35	.02
10	0	0	.02	.3	.69
11	.02	.03	.05	.74	.16
12	.01	.1	.22	.57	.1
13	0	.04	.15	.55	.25
14	0	.03	.21	.64	.12
15	.02	.07	.42	.47	.02
16	.07	.08	.25	.47	.13
17	.01	.07	.55	.31	.04
18	.06	.22	.48	.22	.01
19	.02	0	.2	.52	.27
20	0	.06	.4	.46	.08
21	0	.08	.56	.32	.03
22	.01	.04	.41	.42	.12
23	0	0	.45	.5	.05
24	.03	.07	.69	.21	0
25	0	.15	.49	.33	.03
26	.01	.22	.53	.24	0
27	.07	.41	.48	.03	.02
28	.04	.21	.46	.3	0
29	.03	.24	.49	.22	.01
30	.03	.39	.5	.09	0
31	0	.26	.57	.16	0
32	.02	.36	.49	.14	0
33	.13	.49	.35	.03	0
34	.02	.48	.48	.02	0
35	.15	.48	.33	.02	.02
36	.15	.62	.21	.02	0
37	.02	.02	.02	.7	.24
38	.02	.02	.07	.89	0
39	.01	.03	.44	.47	.04
40	0	0	.23	.66	.11
41	.01	.01	.37	.58	.01
42	.03	.13	.46	.36	.01
43	.03	.09	.46	.36	.06
44	.02	.02	.61	.35	0
45	.07	.3	.47	.15	.02
46	0	.03	.2	.42	.34
47	0	.05	.15	.72	.08
48	.03	.12	.43	.42	0

Group 2 Relative Frequencies

No.	A	B	C	D	E
49	0	.05	.27	.56	.13
50	0	.05	.47	.44	.05
51	.02	.15	.54	.29	0
52	.01	.15	.36	.45	.03
53	.03	.17	.48	.29	.02
54	.09	.29	.55	.06	0
55	0	.03	.39	.51	.07
56	.02	.02	.31	.65	.02
57	.04	.04	.67	.25	.02
58	.03	.14	.39	.36	.08
59	.01	.13	.63	.22	0
60	.07	.31	.51	.12	0
61	.04	.04	.67	.24	.02
62	.05	.36	.42	.17	0
63	.08	.41	.41	.08	.02
64	0	.28	.48	.22	.02
65	.04	.27	.5	.16	.04
66	.1	.36	.46	.08	0
67	.07	.31	.5	.12	0
68	.06	.38	.4	.16	0
69	.08	.54	.33	.05	0
70	.05	.45	.38	.12	0
71	.14	.57	.28	.01	0
72	.24	.59	.15	.03	0
73	.04	.1	.21	.51	.13
74	.04	.18	.47	.3	.02
75	.06	.22	.45	.27	0
76	0	.14	.37	.46	.04
77	.03	.12	.57	.29	0
73	.05	.32	.5	.12	.02
79	.08	.17	.45	.3	0
80	.04	.28	.54	.13	.01
81	.22	.22	.41	.13	.02
82	.07	.1	.32	.46	.05
83	.02	.16	.56	.25	.02
84	.05	.22	.59	.14	0
85	.02	.08	.58	.32	0
86	.05	.23	.49	.23	0
87	.01	.15	.52	.28	.03
88	.01	.25	.46	.25	.01
89	.05	.36	.52	.08	0
90	.07	.56	.33	.05	0
91	.02	.13	.4	.43	.02
92	.04	.23	.5	.23	0
93	.09	.22	.55	.13	0
94	.02	.17	.54	.27	0
95	.07	.15	.54	.24	0
96	.1	.38	.47	.04	0

Group 3 Relative Frequencies

No.	A	B	C	D	E
97	.05	.23	.45	.27	0
98	.11	.32	.51	.06	0
99	.18	.56	.26	0	0
100	.04	.25	.6	.11	.02
101	.05	.5	.42	.03	0
102	.08	.5	.38	.05	0
103	.1	.37	.33	.2	0
104	.12	.55	.26	.06	0
105	.22	.58	.15	.05	0
106	.05	.6	.31	.04	0
107	.2	.42	.32	.05	0
108	.45	.45	.09	.01	0
109	.14	.25	.44	.17	0
110	.13	.37	.37	.13	0
111	.14	.51	.31	.05	0
112	.12	.47	.35	.06	0
113	.21	.45	.27	.06	0
114	.19	.47	.29	.04	0
115	.17	.53	.25	.03	.02
116	.24	.5	.23	.03	0
117	.42	.33	.23	.02	0
118	.14	.38	.28	.19	.02
119	.17	.4	.29	.14	0
120	.19	.51	.25	.04	0
121	.16	.41	.28	.16	0
122	.16	.41	.33	.1	0
123	.17	.59	.19	.05	0
124	.25	.35	.38	.02	0
125	.24	.51	.22	.03	0
126	.5	.41	.07	.01	0
127	.12	.5	.23	.15	0
128	.18	.45	.33	.03	0
129	.25	.46	.28	0	0
130	.17	.45	.3	.06	.01
131	.33	.33	.33	.02	0
132	.22	.53	.2	.05	0
133	.15	.67	.15	.03	0
134	.43	.41	.16	0	0
135	.34	.55	.1	0	0
136	.25	.43	.27	.05	0
137	.3	.52	.16	.02	0
138	.49	.37	.12	.02	0
139	.24	.47	.22	.07	0
140	.42	.42	.13	.03	0
141	.36	.53	.05	.07	0
142	.34	.5	.16	0	0
143	.46	.37	.1	.07	0
144	.63	.24	.03	.07	.03

Group 1 Cumulative Relative Frequencies

No.	A	B	C	D	E
1	.02	.02	.07	.07	1
2	.02	.07	.15	.47	1
3	.01	.04	.13	.69	1
4	.02	.03	.08	.65	1
5	.02	.02	.17	.77	1
6	0	.05	.46	.93	1
7	.04	.07	.11	.82	1
8	.01	.15	.44	.93	1
9	.02	.22	.63	.98	1
10	0	0	.02	.31	1
11	.02	.05	.1	.84	1
12	.01	.12	.33	.9	1
13	0	.04	.19	.75	1
14	0	.03	.24	.88	1
15	.02	.09	.51	.98	1
16	.07	.15	.4	.87	1
17	.01	.09	.64	.96	1
18	.06	.28	.76	.99	1
19	.02	.02	.21	.73	1
20	0	.06	.46	.92	1
21	0	.08	.64	.97	1
22	.01	.06	.46	.88	1
23	0	0	.45	.95	1
24	.03	.1	.79	1	1
25	0	.15	.64	.97	1
26	.01	.24	.76	1	1
27	.07	.48	.95	.98	1
28	.04	.25	.7	1	1
29	.03	.27	.76	.99	1
30	.03	.41	.91	1	1
31	0	.26	.84	1	1
32	.02	.37	.86	1	1
33	.13	.62	.97	1	1
34	.02	.5	.98	1	1
35	.15	.64	.97	.98	1
36	.15	.77	.98	1	1
37	.02	.04	.06	.76	1
38	.02	.04	.11	1	1
39	.01	.04	.49	.96	1
40	0	0	.23	.89	1
41	.01	.03	.4	.99	1
42	.03	.16	.62	.99	1
43	.03	.12	.58	.94	1
44	.02	.03	.65	1	1
45	.07	.37	.83	.98	1
46	0	.03	.24	.66	1
47	0	.05	.2	.92	1
48	.03	.14	.58	1	1

Group 2 Cumulative Relative Frequencies

No.	A	B	C	D	E
49	0	.05	.31	.88	1
50	0	.05	.52	.95	1
51	.02	.17	.71	1	1
52	.01	.16	.52	.97	1
53	.03	.21	.69	.98	1
54	.09	.38	.94	1	1
55	0	.03	.42	.93	1
56	.02	.03	.34	.98	1
57	.04	.07	.74	.98	1
58	.03	.17	.56	.92	1
59	.01	.15	.78	1	1
60	.07	.37	.88	1	1
61	.04	.07	.75	.98	1
62	.05	.41	.83	1	1
63	.08	.49	.9	.98	1
64	0	.28	.77	.98	1
65	.04	.3	.8	.96	1
66	.1	.46	.92	1	1
67	.07	.38	.88	1	1
68	.06	.44	.84	1	1
69	.08	.62	.95	1	1
70	.05	.5	.88	1	1
71	.14	.71	.99	1	1
72	.24	.82	.97	1	1
73	.04	.15	.35	.87	1
74	.04	.21	.68	.98	1
75	.06	.28	.73	1	1
76	0	.14	.51	.96	1
77	.03	.14	.71	1	1
78	.05	.37	.87	.98	1
79	.08	.25	.7	1	1
80	.04	.32	.86	.99	1
81	.22	.43	.85	.98	1
82	.07	.17	.49	.95	1
83	.02	.18	.74	.98	1
84	.05	.27	.86	1	1
85	.02	.1	.68	1	1
86	.05	.28	.77	1	1
87	.01	.16	.69	.97	1
88	.01	.27	.73	.99	1
89	.05	.41	.92	1	1
90	.07	.62	.95	1	1
91	.02	.15	.55	.98	1
92	.04	.27	.77	1	1
93	.09	.31	.87	1	1
94	.02	.19	.73	1	1
95	.07	.22	.76	1	1
96	.1	.49	.96	1	1

Group 3 Cumulative Relative Frequencies

No.	A	B	C	D	E
97	.05	.28	.73	1	1
98	.11	.43	.94	1	1
99	.18	.74	1	1	1
100	.04	.28	.88	.98	1
101	.05	.55	.97	1	1
102	.08	.58	.95	1	1
103	.1	.47	.8	1	1
104	.12	.68	.94	1	1
105	.22	.8	.95	1	1
106	.05	.65	.96	1	1
107	.2	.63	.95	1	1
108	.45	.9	.99	1	1
109	.14	.39	.83	1	1
110	.13	.5	.87	1	1
111	.14	.64	.95	1	1
112	.12	.59	.94	1	1
113	.21	.67	.94	1	1
114	.19	.66	.96	1	1
115	.17	.7	.95	.98	1
116	.24	.74	.97	1	1
117	.42	.75	.98	1	1
118	.14	.52	.8	.98	1
119	.17	.57	.86	1	1
120	.19	.71	.96	1	1
121	.16	.56	.84	1	1
122	.16	.57	.9	1	1
123	.17	.76	.95	1	1
124	.25	.6	.98	1	1
125	.24	.75	.97	1	1
126	.5	.91	.99	1	1
127	.12	.62	.85	1	1
128	.18	.63	.97	1	1
129	.25	.72	1	1	1
130	.17	.62	.93	.99	1
131	.33	.66	.98	1	1
132	.22	.75	.95	1	1
133	.15	.82	.97	1	1
134	.43	.84	1	1	1
135	.34	.9	1	1	1
136	.25	.68	.95	1	1
137	.3	.82	.98	1	1
138	.49	.86	.98	1	1
139	.24	.71	.93	1	1
140	.42	.84	.97	1	1
141	.36	.88	.93	1	1
142	.34	.84	1	1	1
143	.46	.83	.93	1	1
144	.63	.86	.9	.97	1

Group 1 Remove 1s and 0s

No.	A	B	C	D	E
1	.02	.02	.07	.07	
2	.02	.07	.15	.47	
3		.04	.13	.69	
4	.02	.03	.08	.65	
5	.02	.02	.17	.77	
6		.05	.46	.93	
7	.04	.07	.11	.82	
8		.15	.44	.93	
9	.02	.22	.63	.98	
10			.02	.31	
11	.02	.05	.1	.84	
12		.12	.33	.9	
13		.04	.19	.75	
14		.03	.24	.88	
15	.02	.09	.51	.98	
16	.07	.15	.4	.87	
17		.09	.64	.96	
18	.06	.28	.76		
19	.02	.02	.21	.73	
20		.06	.46	.92	
21		.08	.64	.97	
22		.06	.46	.88	
23			.45	.95	
24	.03	.1	.79		
25		.15	.64	.97	
26		.24	.76		
27	.07	.48	.95	.98	
28	.04	.25	.7		
29	.03	.27	.76		
30	.03	.41	.91		
31		.26	.84		
32	.02	.37	.86		
33	.13	.62	.97		
34	.02	.5	.98		
35	.15	.64	.97	.98	
36	.15	.77	.98		
37	.02	.04	.06	.76	
38	.02	.04	.11		
39		.04	.49	.96	
40			.23	.89	
41		.03	.4		
42	.03	.16	.62		
43	.03	.12	.58	.94	
44	.02	.03	.65		
45	.07	.37	.83	.98	
46		.03	.24	.66	
47	0	.05	.2	.92	
48	.03	.14	.58		

Group 2 Remove 1s and 0s

No.	A	B	C	D	E
49		.05	.31	.88	
50		.05	.52	.95	
51	.02	.17	.71		
52		.16	.52	.97	
53	.03	.21	.69	.98	
54	.09	.38	.94		
55		.03	.42	.93	
56	.02	.03	.34	.98	
57	.04	.07	.74	.98	
58	.03	.17	.56	.92	
59		.15	.78		
60	.07	.37	.88		
61	.04	.07	.75	.98	
62	.05	.41	.83		
63	.08	.49	.9	.98	
64		.28	.77	.98	
65	.04	.3	.8	.96	
66	.1	.46	.92		
67	.07	.38	.88		
68	.06	.44	.84		
69	.08	.62	.95		
70	.05	.5	.88		
71	.14	.71			
72	.24	.82	.97		
73	.04	.15	.35	.87	
74	.04	.21	.68	.98	
75	.06	.28	.73		
76		.14	.51	.96	
77	.03	.14	.71		
78	.05	.37	.87	.98	
79	.08	.25	.7		
80	.04	.32	.86		
81	.22	.43	.85	.98	
82	.07	.17	.49	.95	
83	.02	.18	.74	.98	
84	.05	.27	.86		
85	.02	.1	.68		
86	.05	.28	.77		
87		.16	.69	.97	
88		.27	.73		
89	.05	.41	.92		
90	.07	.62	.95		
91	.02	.15	.55	.98	
92	.04	.27	.77		
93	.09	.31	.87		
94	.02	.19	.73		
95	.07	.22	.76		
96	.1	.49	.96		

Group 3 Remove 1s and 0s

No.	A	B	C	D	E
97	.05	.28	.73		
98	.11	.43	.94		
99	.18	.74	1		
100	.04	.28	.88	.98	
101	.05	.55	.97		
102	.08	.58	.95		
103	.1	.47	.8		
104	.12	.68	.94		
105	.22	.8	.95		
106	.05	.65	.96		
107	.2	.63	.95		
108	.45	.9			
109	.14	.39	.83		
110	.13	.5	.87		
111	.14	.64	.95		
112	.12	.59	.94		
113	.21	.67	.94		
114	.19	.66	.96		
115	.17	.7	.95	.98	
116	.24	.74	.97		
117	.42	.75	.98		
118	.14	.52	.8	.98	
119	.17	.57	.86		
120	.19	.71	.96		
121	.16	.56	.84		
122	.16	.57	.9		
123	.17	.76	.95		
124	.25	.6	.98		
125	.24	.75	.97		
126	.5	.91			
127	.12	.62	.85		
128	.18	.63	.97		
129	.25	.72			
130	.17	.62	.93		
131	.33	.66	.98		
132	.22	.75	.95		
133	.15	.82	.97		
134	.43	.84			
135	.34	.9			
136	.25	.68	.95		
137	.3	.82	.98		
138	.49	.86	.98		
139	.24	.71	.93		
140	.42	.84	.97		
141	.36	.88	.93		
142	.34	.84			
143	.46	.83	.93		
144	.63	.86	.9	.97	

Combine Categories : Group ABCD

No.	A	B	C	D
1	.02	.02	.07	.07
91	.02	.15	.55	.98
53	.03	.21	.69	.98
100	.04	.28	.88	.98
43	.03	.12	.58	.94
37	.02	.04	.06	.76
73	.04	.15	.35	.87
45	.07	.37	.83	.98
19	.02	.02	.21	.73
57	.04	.07	.74	.98
63	.08	.49	.9	.98
118	.14	.52	.8	.98
61	.04	.07	.75	.98
65	.04	.3	.8	.96
58	.03	.17	.56	.92
56	.02	.03	.34	.98
27	.07	.48	.95	.98
82	.07	.17	.49	.95
5	.02	.02	.17	.77
78	.05	.37	.87	.98
7	.04	.07	.11	.82
2	.02	.07	.15	.47
144	.63	.86	.9	.97
4	.02	.03	.08	.65
81	.22	.43	.85	.98
74	.04	.21	.68	.98
35	.15	.64	.97	.98
16	.07	.15	.4	.87
15	.02	.09	.51	.98
83	.02	.18	.74	.98
9	.02	.22	.63	.98
115	.17	.7	.95	.98
11	.02	.05	.1	.84

Combine Categories : Group ABC

No.	A	B	C
102	.08	.58	.95
103	.1	.47	.8
99	.18	.74	1
101	.05	.55	.97
89	.05	.41	.92
90	.07	.62	.95
92	.04	.27	.77
84	.05	.27	.86
85	.02	.1	.68
86	.05	.28	.77
96	.1	.49	.96
97	.05	.28	.73
98	.11	.43	.94
93	.09	.31	.87
94	.02	.19	.73
95	.07	.22	.76
104	.12	.68	.94
130	.17	.62	.93
131	.33	.66	.98
132	.22	.75	.95
128	.18	.63	.97
124	.25	.6	.98
125	.24	.75	.97
127	.12	.62	.85
133	.15	.82	.97
140	.42	.84	.97
141	.36	.88	.93
143	.46	.83	.93
139	.24	.71	.93
136	.25	.68	.95
137	.3	.82	.98
138	.49	.86	.98
123	.17	.76	.95
110	.13	.5	.87
111	.14	.64	.95
112	.12	.59	.94
109	.14	.39	.83
105	.22	.8	.95
106	.05	.65	.96
107	.2	.63	.95
113	.21	.67	.94
120	.19	.71	.96
121	.16	.56	.84
122	.16	.57	.9
119	.17	.57	.86
114	.19	.66	.96
116	.24	.74	.97
117	.42	.75	.98
29	.03	.27	.76
51	.02	.17	.71

No.	A	B	C
30	.03	.41	.91
54	.09	.38	.94
60	.07	.37	.88
24	.03	.1	.79
28	.04	.25	.7
32	.02	.37	.86
44	.02	.03	.65
42	.03	.16	.62
38	.02	.04	.11
36	.15	.77	.98
48	.03	.14	.58
33	.13	.62	.97
34	.02	.5	.98
62	.05	.41	.83
75	.06	.28	.73
72	.24	.82	.97
18	.06	.28	.76
80	.04	.32	.86
79	.08	.25	.7
77	.03	.14	.71
68	.06	.44	.84
67	.07	.38	.88
66	.1	.46	.92
69	.08	.62	.95
70	.05	.5	.88

Combine Categories : Group BCD

No.	B	C	D
22	.06	.46	.88
8	.15	.44	.93
64	.28	.77	.98
3	.04	.13	.69
21	.08	.64	.97
87	.16	.69	.97
76	.14	.51	.96
17	.09	.64	.96
20	.06	.46	.92
6	.05	.46	.93
47	.05	.2	.92
49	.05	.31	.88
46	.03	.24	.66
12	.12	.33	.9
39	.04	.49	.96
13	.04	.19	.75
14	.03	.24	.88
25	.15	.64	.97
55	.03	.42	.93
50	.05	.52	.95
52	.16	.52	.97

Combine Categories : Group AB

No.	A	B
129	.25	.72
126	.5	.91
135	.34	.9
134	.43	.84
108	.45	.9
71	.14	.71
142	.34	.84

Combine Categories : Group BC

No.	B	C
41	.03	.4
59	.15	.78
88	.27	.73
26	.24	.76
31	.26	.84

Combine Categories : Group CD

No.	C	D
40	.23	.89
23	.45	.95
10	.02	.31

Normalize Group ABCD

No.	A	B	C	D	Row Avg
1	-2.052	-2.052	-1.475	-1.475	-1.7635
91	-2.052	-1.037	.126	2.052	-.22775
53	-1.881	-.807	.496	2.052	-.035
100	-1.751	-.583	1.175	2.052	.22325
43	-1.881	-1.175	.202	1.556	-.3245
37	-2.052	-1.751	-1.556	.706	-1.16325
73	-1.751	-1.037	-.385	1.126	-.51175
45	-1.475	-.332	.954	2.052	.29975
19	-2.052	-2.052	-.807	.613	-1.0745
57	-1.751	-1.475	.643	2.052	-.13275
63	-1.405	-.025	1.282	2.052	.476
118	-1.08	.05	.842	2.052	.466
61	-1.751	-1.475	.675	2.052	-.12475
65	-1.751	-.524	.842	1.751	.0795
58	-1.881	-.954	.151	1.465	-.30475
56	-2.052	-1.881	-.412	2.052	-.57325
27	-1.475	-.05	1.645	2.052	.543
82	-1.475	-.954	-.025	1.645	-.20225
5	-2.052	-2.052	-.954	.739	-1.07975
78	-1.645	-.332	1.126	2.052	.30025
7	-1.751	-1.475	-1.226	.915	-.88425
2	-2.052	-1.475	-1.037	-.075	-1.15975
144	.332	1.08	1.282	1.881	1.14375
4	-2.052	-1.881	-1.405	.385	-1.23825
81	-.772	-.176	1.037	2.052	.53525
74	-1.751	-.807	.468	2.052	-.0095
35	-1.037	.358	1.881	2.052	.8135
16	-1.475	-1.037	-.253	1.126	-.40975
15	-2.052	-1.34	.025	2.052	-.32875
83	-2.052	-.915	.643	2.052	-.068
9	-2.052	-.772	.332	2.052	-.11
115	-.954	.524	1.645	2.052	.81675
11	-2.052	-1.645	-1.282	.995	-.996

Col Avg	-1.66621	-.910879	.2016667	1.523909
Grand Avg		-.212879	B =	2.77123

Normalize Group ABC

No.	A	B	C	Row Avg
102	-1.405	.202	1.645	.1473333
103	-1.282	-.075	.842	-.171667
99	-.915	.643	2.052	.5933333
101	-1.645	.126	1.881	.1206667
89	-1.645	-.227	1.465	-.135667
90	-1.475	.306	1.645	.1586667
92	-1.751	-.613	.739	-.541667
84	-1.645	-.613	1.08	-.392667
85	-2.052	-1.282	.468	-.955333
86	-1.645	-.583	.739	-.496333
96	-1.282	-.025	1.751	.148
97	-1.645	-.583	.613	-.538333
98	-1.226	-.176	1.556	.0513333
93	-1.34	-.496	1.126	-.236667
94	-2.052	-.878	.613	-.772333
95	-1.475	-.772	.706	-.513667
104	-1.175	.468	1.556	.283
130	-.954	.306	1.475	.2756667
131	-.44	.412	2.052	.6746667
132	-.772	.675	1.645	.516
128	-.915	.332	1.881	.4326667
124	-.675	.253	2.052	.5433333
125	-.706	.675	1.881	.6166667
127	-1.175	.306	1.037	.056
133	-1.037	.915	1.881	.5863333
140	-.202	.995	1.881	.8913333
141	-.358	1.175	1.475	.764
143	-.1	.954	1.475	.7763333
139	-.706	.553	1.475	.4406667
136	-.675	.468	1.645	.4793333
137	-.524	.915	2.052	.8143333
138	-.025	1.08	2.052	1.035667
123	-.954	.706	1.645	.4656667
110	-1.126	0	1.126	0
111	-1.08	.358	1.645	.3076667
112	-1.175	.227	1.556	.2026667
109	-1.08	-.279	.954	-.135
105	-.772	.842	1.645	.5716667
106	-1.645	.385	1.751	.1636667
107	-.842	.332	1.645	.3783333
113	-.807	.44	1.556	.3963333
120	-.878	.553	1.751	.4753333
121	-.995	.151	.995	.0503333
122	-.995	.176	1.282	.1543333
119	-.954	.176	1.08	.1006667
114	-.878	.412	1.751	.4283333
116	-.706	.643	1.881	.606
117	-.202	.675	2.052	.8416667
29	-1.881	-.613	.706	-.596
51	-2.052	-.954	.553	817667

No.	A	B	C	Row Avg
30	-1.881	-.227	1.34	-.256
54	-1.34	-.306	1.556	-.03
60	-1.475	-.332	1.175	-.210667
24	-1.881	-1.282	.807	-.785333
28	-1.751	-.675	.524	-.634
32	-2.052	-.332	1.08	-.434667
44	-2.052	-1.881	.385	-1.18267
42	-1.881	-.995	.306	-.856667
38	-2.052	-1.751	-1.226	-1.67633
36	-1.037	.739	2.052	.5846667
48	-1.881	-1.08	.202	-.919667
33	-1.126	.306	1.881	.3536667
34	-2.052	0	2.052	0
62	-1.645	-.227	.954	-.306
75	-1.556	-.583	.613	-.508667
72	-.706	.915	1.881	.6966667
18	-1.556	-.583	.706	-.477667
80	-1.751	-.468	1.08	-.379667
79	-1.405	-.675	.524	-.518667
77	-1.881	-1.08	.553	-.802667
68	-1.556	-.151	.995	-.237333
67	-1.475	-.306	1.175	-.202
66	-1.282	-.1	1.465	.0276667
69	-1.405	.306	1.645	.182
70	-1.645	0	1.175	-.156667
Col Avg	-1.25687	-.014693	1.292133	
Grand Avg		.0068578	B =	3.249397

Normalize Group BCD

No.	B	C	D	Row Avg
22	-1.556	-.1	1.175	-.160333
8	-1.037	-.151	1.475	.0956667
64	-.583	.739	2.052	.736
3	-1.751	-1.126	.496	-.793667
21	-1.405	.358	1.881	.278
87	-.995	.496	1.881	.4606667
76	-1.08	.025	1.751	.232
17	-1.34	.358	1.751	.2563333
20	-1.556	-.1	1.465	-.063667
6	-1.645	-.1	1.475	-.09
47	-1.645	-.842	1.465	-.340667
49	-1.645	-.496	1.175	-.322
46	-1.881	-.706	.412	-.725
12	-1.175	-.44	1.282	-.111
39	-1.751	-.025	1.751	-.008333
13	-1.751	-.878	.675	-.651333
14	-1.881	-.706	1.175	-.470667
25	-1.037	.358	1.881	.4006667
55	-1.881	-.202	1.475	-.202667
50	-1.645	.05	1.645	.0166667
52	-.995	.05	1.881	.312

Col Avg -1.43976 -.163714 1.439
Grand Avg-.054825 B = 4.16142

Normalize Group AB

No.	A	B	Row Avg
129	-.675	.583	-.046
126	0	1.34	.67
135	-.412	1.282	.435
134	-.176	.995	.4095
108	-.126	1.282	.578
71	-1.08	.553	-.2635
142	-.412	.995	.2915

Col Avg -.411571 1.004286
Grand Avg.2963571 B = 1.002326

Normalize Group BC

No.	B	C	Row Avg
41	-1.881	-.253	-1.067
59	-1.037	.772	-.1325
88	-.613	.613	0
26	-.706	.706	0
31	-.643	.995	.176
Col Avg	-.976	.5666	
Grand Avg	-.2047	B =	1.189807

Normalize Group CD

No.	C	D	Row Avg
40	-.739	1.226	.2435
23	-.126	1.645	.7595
10	-2.052	-.496	-1.274
Col Avg	-.972333	.7916667	
Grand Avg	-.090333	B =	1.555848

Square of Differences : Group ABCD

No.	A	B	C	D	Ai
1	.0832322	.0832322	.0832322	.0832322	.332929
91	3.327888	.6548856	.1251391	5.19726	9.305173
53	3.407716	.595984	.281961	4.355569	8.64123
100	3.897663	.6500391	.9058281	3.344327	8.797857
43	2.422692	.7233503	.2772023	3.53628	6.959525
37	.7898766	.3454501	.1542526	3.494096	4.783675
73	1.535741	.2758876	.0160656	2.682225	4.509919
45	3.149738	.3991081	.4280431	3.07038	7.047269
19	.9555063	.9555063	.0715562	2.847656	4.830225
57	2.618733	1.801635	.6017881	4.773133	9.795289
63	3.538161	.251001	.649636	2.483776	6.922574
118	2.390116	.173056	.141376	2.515396	5.219944
61	2.644689	1.823175	.6396001	4.738241	9.845705
65	3.35073	.3642123	.5814063	2.793912	7.090261
58	2.484564	.4215256	.2077081	3.132015	6.245813
56	2.186702	1.71021	.0260016	6.891938	10.81485
27	4.072324	.351649	1.214404	2.277081	7.915458
82	1.619893	.5651281	.0314176	3.412333	5.628771
5	.9452701	.9452701	.0158131	3.307852	5.214205
78	3.783998	.3997401	.6818631	3.068628	7.934229
7	.7512556	.3489856	.1167931	3.237301	4.454335
2	.7961101	.0993826	.0150676	1.176683	2.087243
144	.6589381	.0040641	.0191131	.5435376	1.225653
4	.6621891	.4131276	.0278056	2.634941	3.738063
81	1.708903	.5058766	.2517531	2.300531	4.767063
74	3.032822	.6360063	.2280062	4.249782	8.146617
35	3.42435	.2074803	1.139556	1.533882	6.305269
16	1.134758	.3934426	.0245706	2.358528	3.911299
15	2.969591	1.022627	.1251391	5.667971	9.785327
83	3.936256	.717409	.505521	4.4944	9.653586
9	3.771364	.438244	.195364	4.674244	9.079216
115	3.135556	.0857026	.6859981	1.525843	5.433099
11	1.115136	.421201	.081796	3.964081	5.582214

Square of Differences : Group ABC

No.	A	B	C	Ai
102	2.409739	.0029884	2.243005	4.655733
103	1.23284	.0093444	1.02752	2.269705
99	2.275069	.0024668	2.127708	4.405245
101	3.117579	.0000284	3.098773	6.216381
89	2.278087	.0083418	2.562134	4.848563
90	2.668867	.0217071	2.209187	4.899761
92	1.462487	.0050884	1.640107	3.107683
84	1.568339	.0485468	2.168747	3.785633
85	1.202678	.1067111	2.025878	3.335267
86	1.319435	.0075111	1.526048	2.852995
96	2.0449	.029929	2.569609	4.644438
97	1.224711	.0019951	1.325568	2.552275
98	1.63158	.0516804	2.264022	3.947283
93	1.217344	.0672538	1.85686	3.141459
94	1.637547	.0111654	1.919148	3.567861
95	.9241618	.0667361	1.487587	2.478485
104	2.125764	.034225	1.620529	3.780518
130	1.51208	.0009201	1.4384	2.951401
131	1.242482	.0689938	1.897047	3.208523
132	1.658944	.025281	1.274641	2.958866
128	1.816205	.0101338	2.097669	3.924009
124	1.484336	.0842934	2.276075	3.844705
125	1.749447	.0034028	1.598539	3.351389
127	1.515361	.0625	.962361	2.540222
133	2.635211	.1080218	1.676162	4.419395
140	1.195378	.0107468	.9794401	2.185565
141	1.258884	.168921	.505521	1.933326
143	.7679601	.0315654	.4881351	1.287661
139	1.314844	.0126188	1.069845	2.397309
136	1.332485	.0001284	1.358779	2.691393
137	1.791136	.0101338	1.531819	3.333089
138	1.125014	.0019654	1.032933	2.159913
123	2.015453	.0577601	1.390827	3.464041
110	1.267876	0	1.267876	2.535752
111	1.925619	.0025334	1.78846	3.716613
112	1.897965	.0005921	1.831511	3.730069
109	.893025	.020736	1.185921	2.099682
105	1.80544	.0730801	1.152044	3.030565
106	3.271275	.0489884	2.519627	5.839891
107	1.489213	.0021468	1.604444	3.095805
113	1.448011	.0019068	1.344827	2.794745
120	1.831511	.0060321	1.627325	3.464869
121	1.092722	.0101338	.8923951	1.995251
122	1.320967	.0004694	1.271632	2.593069
119	1.112322	.0056751	.9590938	2.077091
114	1.706507	.0002668	1.749447	3.456221
116	1.721344	.001369	1.625625	3.348338
117	1.08924	.0277778	1.464907	2.581925

No.	A	B	C	A
29	1.651225	.000289	1.695204	3.346718
51	1.523579	.0185868	1.878727	3.420893
30	2.640625	.000841	2.547216	5.188682
54	1.7161	.076176	2.515396	4.307672
60	1.598539	.0147218	1.920072	3.533333
24	1.200485	.2466778	2.535525	3.982689
28	1.247689	.001681	1.340964	2.590334
32	2.615767	.0105404	2.294215	4.920523
44	.7557404	.4876694	2.457579	3.700989
42	1.049259	.0191361	1.351794	2.420189
38	.1411254	.0055751	.2028001	.3495007
36	2.629803	.0238188	2.153067	4.806689
48	.9241618	.0257068	1.258136	2.208005
33	2.189413	.0022721	2.332747	4.524433
34	4.210704	0	4.210704	8.421408
62	1.792921	.006241	1.5876	3.386762
75	1.096907	.0055254	1.258136	2.360569
72	1.967474	.0476694	1.402645	3.417789
18	1.162803	.0110951	1.401067	2.574965
80	1.880555	.0078028	2.130627	4.018985
79	.7855868	.0244401	1.087154	1.897181
77	1.162803	.0769138	1.837832	3.077549
68	1.738882	.0074534	1.518645	3.264981
67	1.620529	.010816	1.896129	3.527474
66	1.715227	.0162988	2.065927	3.797453
69	2.518569	.015376	2.140369	4.674314
70	2.215136	.0245444	1.773336	4.013017

Square of Differences : Group BCD

No.	B	C	D	Ai
22	1.947885	.0036401	1.783115	3.734641
8	1.282934	.0608444	1.90256	3.246339
64	1.739761	.000009	1.731856	3.471626
3	.9164871	.1104454	1.66324	2.690173
21	2.832489	.0064	2.569609	5.408498
87	2.118965	.0012484	2.017347	4.137561
76	1.721344	.042849	2.307361	4.071554
17	2.54828	.0103361	2.234028	4.792645
20	2.227059	.0013201	2.336822	4.565201
6	2.418025	.0001	2.449225	4.86735
47	1.701285	.2513351	3.260432	5.213053
49	1.750329	.030276	2.241009	4.021614
46	1.336336	.000361	1.292769	2.629466
12	1.132096	.108241	1.940449	3.180786
39	3.036887	.0002778	3.095254	6.132419
13	1.209267	.0513778	1.75916	3.019805
14	1.98904	.0553818	2.708219	4.752641
25	2.066885	.0018204	2.191387	4.260093
55	2.816803	.0000004	2.814565	5.631369
50	2.761136	.0011111	2.651469	5.413717
52	1.708249	.068644	2.461761	4.238654

Square of Differences : Group AB

No.	A	B	Ai
129	.395641	.395641	.791282
126	.4489	.4489	.8978
135	.717409	.717409	1.434818
134	.3428103	.3428102	.6856205
108	.495616	.495616	.991232
71	.6666723	.6666723	1.333345
142	.4949122	.4949122	.9898245

Square of Differences :Group BC

No.	B	C	Ai
41	.662596	.662596	1.325192
59	.8181202	.8181202	1.63624
88	.375769	.375769	.751538
26	.498436	.498436	.996872
31	.670761	.670761	1.341522

Square of Differences : Group CD

No.	C	D	Ai
40	.9653062	.9653062	1.930612
23	.7841102	.7841103	1.568221
10	.605284	.605284	1.210568

Scale Value of Instances : Group ABCD

No.	S Value	Grand Avg	Row Avg	(B/Ai)[s]
1	4.87499	-.212879	-1.7635	2.885097
91	-.08859	-.212879	-.22775	.5457252
53	-.193058	-.212879	-.035	.5663025
100	-.338175	-.212879	.22325	.561239
43	-.008111	-.212879	-.3245	.6310249
37	.6724988	-.212879	-1.16325	.761124
73	.1882739	-.212879	-.51175	.7838842
45	-.400847	-.212879	.29975	.6270842
19	.6009986	-.212879	-1.0745	.7574476
57	-.142269	-.212879	-.13275	.5318971
63	-.514047	-.212879	.476	.6327067
118	-.552417	-.212879	.466	.7286238
61	-.146695	-.212879	-.12475	.5305336
65	-.262581	-.212879	.0795	.6251801
58	-.009884	-.212879	-.30475	.6661036
56	.0773029	-.212879	-.57325	.5062045
27	-.534169	-.212879	.543	.5916955
82	-.070967	-.212879	-.20225	.7016645
5	.5742856	-.212879	-1.07975	.7290247
78	-.390325	-.212879	.30025	.5909951
7	.4845822	-.212879	-.88425	.7887599
2	1.123453	-.212879	-1.15975	1.152258
144	-1.9327	-.212879	1.14375	1.50367
4	.8532786	-.212879	-1.23825	.8610195
81	-.62098	-.212879	.53525	.7624491
74	-.207338	-.212879	-.0095	.5832404
35	-.752193	-.212879	.8135	.6629556
16	.1320224	-.212879	-.40975	.8417357
15	-.037929	-.212879	-.32875	.5321678
83	-.176445	-.212879	-.068	.5357867
9	-.152107	-.212879	-.11	.5524743
115	-.796192	-.212879	.81675	.7141879
11	.4888873	-.212879	-.996	.7045844

Scale Value of Instances : Group ABC

No.	S Value	Grand Avg	Row Avg	(B/Ai)[s]
102	-.116228	.0068578	.1473333	.8354248
103	.2122588	.0068578	-.171667	1.196511
99	-.502725	.0068578	.5933333	.8588481
101	-.080383	.0068578	.1206667	.7229905
89	.1179204	.0068578	-.135667	.8186436
90	-.122353	.0068578	.1586667	.8143554
92	.5607371	.0068578	-.541667	1.022547
84	.3706523	.0068578	-.392667	.9264717
85	.949813	.0068578	-.955333	.9870431
86	.5365509	.0068578	-.496333	1.067213
96	-.116935	.0068578	.148	.83644
97	.6142776	.0068578	-.538333	1.128334
98	-.039717	.0068578	.0513333	.9073029
93	.247556	.0068578	-.236667	1.017035
94	.7439167	.0068578	-.772333	.9543275
95	.5950099	.0068578	-.513667	1.145007
104	-.255511	.0068578	.283	.9270982
130	-.282391	.0068578	.2756667	1.04927
131	-.672093	.0068578	.6746667	1.00635
132	-.533882	.0068578	.516	1.047946
128	-.386864	.0068578	.4326667	.9099896
124	-.492643	.0068578	.5433333	.9193268
125	-.600353	.0068578	.6166667	.9846661
127	-.056479	.0068578	.056	1.131008
133	-.495907	.0068578	.5863333	.8574721
140	-1.07997	.0068578	.8913333	1.219325
141	-.983614	.0068578	.764	1.296429
143	-1.22639	.0068578	.7763333	1.588549
139	-.50618	.0068578	.4406667	1.164232
136	-.519827	.0068578	.4793333	1.098785
137	-.797187	.0068578	.8143333	.9873655
138	-1.26343	.0068578	1.035667	1.226544
123	-.444151	.0068578	.4656667	.9685229
110	.0068578	.0068578	0	1.132004
111	-.280821	.0068578	.3076667	.9350347
112	-.1823	.0068578	.2026667	.9333467
109	.1747994	.0068578	-.135	1.244012
105	-.585089	.0068578	.5716667	1.035475
106	-.115226	.0068578	.1636667	.7459317
107	-.380747	.0068578	.3783333	1.024506
113	-.420499	.0068578	.3963333	1.078277
120	-.453458	.0068578	.4753333	.9684072
121	-.057375	.0068578	.0503333	1.276153
122	-.165907	.0068578	.1543333	1.119423
119	-.119052	.0068578	.1006667	1.250759
114	-.408462	.0068578	.4283333	.969618
116	-.590122	.0068578	.606	.9851146
117	-.937355	.0068578	.8416667	1.121837
29	.5941282	.0068578	-.596	.985353
51	.8037654	.0068578	-.817667	.9746118

No.	S Value	Grand Avg	Row Avg	$(B/Ai)^s$
30	.2094454	.0068578	-.256	.7913578
54	.0329134	.0068578	-.03	.8685205
60	.2088827	.0068578	-.210667	.9589791
24	.7162187	.0068578	-.785333	.9032609
28	.7169468	.0068578	-.634	1.120014
32	.3600833	.0068578	-.434667	.8126355
44	1.115024	.0068578	-1.18267	.9370063
42	.9994907	.0068578	-.856667	1.158715
38	5.118233	.0068578	-1.67633	3.04914
36	-.473856	.0068578	.5846667	.8222018
48	1.122518	.0068578	-.919667	1.213113
33	-.292861	.0068578	.3536667	.8474602
34	.0068578	.0068578	0	.621168
62	.306588	.0068578	-.306	.9795104
75	.6036544	.0068578	-.508667	1.173257
72	-.67243	.0068578	.6966667	.9750543
18	.5434455	.0068578	-.477667	1.123352
80	.3482438	.0068578	-.379667	.8991729
79	.6856477	.0068578	-.518667	1.308721
77	.8316303	.0068578	-.802667	1.02754
68	.243624	.0068578	-.237333	.9976107
67	.2007324	.0068578	-.202	.9597751
66	-.018735	.0068578	.0276667	.9250287
69	-.144887	.0068578	.182	.8337627
70	.1478329	.0068578	-.156667	.8998413

Scale Value of Instances : Group BCD

No.	S Value	Grand Avg	Row Avg	$(B/Ai)^{.5}$
22	.1144213	-.054825	-.160333	1.055593
8	-.163139	-.054825	.0956667	1.132202
64	-.860634	-.054825	.736	1.094849
3	.9322921	-.054825	-.793667	1.243743
21	-.298678	-.054825	.278	.8771673
87	-.516818	-.054825	.4606667	1.002879
76	-.289372	-.054825	.232	1.010976
17	-.293683	-.054825	.2563333	.9318225
20	.0059605	-.054825	-.063667	.9547526
6	.0283926	-.054825	-.09	.9246439
47	.2495464	-.054825	-.340667	.8934592
49	.2727237	-.054825	-.322	1.017233
46	.857238	-.054825	-.725	1.258018
12	.0721374	-.054825	-.111	1.143809
39	-.047961	-.054825	-.008333	.8237679
13	.7097753	-.054825	-.651333	1.173901
14	.3855943	-.054825	-.470667	.9357359
25	-.450825	-.054825	.4006667	.9883511
55	.1193939	-.054825	-.202667	.8596344
50	-.069438	-.054825	.0166667	.8767445
52	-.36397	-.054825	.312	.9908475

Scale Value of Instances : Group AB

No.	S Value	Grand Avg	Row Avg	$(B/Ai)^{.5}$
129	.3481293	.2963571	-.046	1.125483
126	-.411571	.2963571	.67	1.05661
135	-.067219	.2963571	.435	.8358071
134	-.19877	.2963571	.4095	1.209101
108	-.284868	.2963571	.578	1.00558
71	.5248191	.2963571	-.2635	.8670283
142	.0030221	.2963571	.2915	1.006295

Scale Value of Instances : Group BC

No.	S Value	Grand Avg	Row Avg	$(B/Ai)^{.5}$
41	.8063284	-.2047	-1.067	.947543
59	-.091712	-.2047	-.1325	.8527363
88	-.2047	-.2047	0	1.258238
26	-.2047	-.2047	0	1.092493
31	-.370449	-.2047	.176	.9417582

Scale Value of Instances : Group CD

No.	S Value	Grand Avg	Row Avg	$(B/Ai)^{.5}$
40	-.308926	-.090333	.2435	.8977099
23	-.846831	-.090333	.7595	.9960474
10	1.35397	-.090333	-1.274	1.133676

Transform Values : Group ABCD

No.	S Value	α	β	T Value
1	4.87499	68.7055	20.5357	168.8168
91	-.08859	68.7055	20.5357	66.88625
53	-.193058	68.7055	20.5357	64.74091
100	-.338175	68.7055	20.5357	61.76083
43	-.008111	68.7055	20.5357	68.53893
37	.6724988	68.7055	20.5357	82.51573
73	.1882739	68.7055	20.5357	72.57184
45	-.400847	68.7055	20.5357	60.47382
19	.6009986	68.7055	20.5357	81.04743
57	-.142269	68.7055	20.5357	65.7839
63	-.514047	68.7055	20.5357	58.14918
118	-.552417	68.7055	20.5357	57.36122
61	-.146695	68.7055	20.5357	65.69302
65	-.262581	68.7055	20.5357	63.31322
58	-.009884	68.7055	20.5357	68.50253
56	.0773029	68.7055	20.5357	70.29297
27	-.534169	68.7055	20.5357	57.73596
82	-.070967	68.7055	20.5357	67.24814
5	.5742856	68.7055	20.5357	80.49886
78	-.390325	68.7055	20.5357	60.6899
7	.4845822	68.7055	20.5357	78.65673
2	1.123453	68.7055	20.5357	91.77639
144	-1.9327	68.7055	20.5357	29.01612
4	.8532786	68.7055	20.5357	86.22817
81	-.62098	68.7055	20.5357	55.95325
74	-.207338	68.7055	20.5357	64.44767
35	-.752193	68.7055	20.5357	53.25869
16	.1320224	68.7055	20.5357	71.41667
15	-.037929	68.7055	20.5357	67.92661
83	-.176445	68.7055	20.5357	65.08207
9	-.152107	68.7055	20.5357	65.58188
115	-.796192	68.7055	20.5357	52.35515
11	.4888873	68.7055	20.5357	78.74514

Transform Values : Group ABC

No.	S Value	α	β	T Value
102	-.116228	50.2492	16.9595	48.27803
103	.2122588	50.2492	16.9595	53.849
99	-.502725	50.2492	16.9595	41.72323
101	-.080383	50.2492	16.9595	48.88594
89	.1179204	50.2492	16.9595	52.24907
90	-.122353	50.2492	16.9595	48.17415
92	.5607371	50.2492	16.9595	59.75902
84	.3706523	50.2492	16.9595	56.53528
85	.949813	50.2492	16.9595	66.35755
86	.5365509	50.2492	16.9595	59.34884
96	-.116935	50.2492	16.9595	48.26603
97	.6142776	50.2492	16.9595	60.66704
98	-.039717	50.2492	16.9595	49.57562
93	.247556	50.2492	16.9595	54.44763
94	.7439167	50.2492	16.9595	62.86566
95	.5950099	50.2492	16.9595	60.34027
104	-.255511	50.2492	16.9595	45.91586
130	-.282391	50.2492	16.9595	45.45999
131	-.672093	50.2492	16.9595	38.85084
132	-.533882	50.2492	16.9595	41.19483
128	-.386864	50.2492	16.9595	43.68817
124	-.492643	50.2492	16.9595	41.89422
125	-.600353	50.2492	16.9595	40.06751
127	-.056479	50.2492	16.9595	49.29135
133	-.495907	50.2492	16.9595	41.83887
140	-1.07997	50.2492	16.9595	31.93349
141	-.983614	50.2492	16.9595	33.56759
143	-1.22639	50.2492	16.9595	29.45031
139	-.50618	50.2492	16.9595	41.66463
136	-.519827	50.2492	16.9595	41.4332
137	-.797187	50.2492	16.9595	36.72931
138	-1.26343	50.2492	16.9595	28.822
123	-.444151	50.2492	16.9595	42.71662
110	.0068578	50.2492	16.9595	50.3655
111	-.280821	50.2492	16.9595	45.48661
112	-.1823	50.2492	16.9595	47.15747
109	.1747994	50.2492	16.9595	53.21371
105	-.585089	50.2492	16.9595	40.32639
106	-.115226	50.2492	16.9595	48.29502
107	-.380747	50.2492	16.9595	43.79192
113	-.420499	50.2492	16.9595	43.11774
120	-.453458	50.2492	16.9595	42.55877
121	-.057375	50.2492	16.9595	49.27614
122	-.165907	50.2492	16.9595	47.43551
119	-.119052	50.2492	16.9595	48.23014
114	-.408462	50.2492	16.9595	43.32189
116	-.590122	50.2492	16.9595	40.24103
117	-.937355	50.2492	16.9595	34.35213
29	.5941282	50.2492	16.9595	60.32532
51	.8037654	50.2492	16.9595	63.88066

No.	S Value	α	β	T Value
30	.2094454	50.2492	16.9595	53.80129
54	.0329134	50.2492	16.9595	50.80739
60	.2088827	50.2492	16.9595	53.79175
24	.7162187	50.2492	16.9595	62.39591
28	.7169468	50.2492	16.9595	62.40826
32	.3600833	50.2492	16.9595	56.35603
44	1.115024	50.2492	16.9595	69.15945
42	.9994907	50.2492	16.9595	67.20006
38	5.118233	50.2492	16.9595	137.0519
36	-.473856	50.2492	16.9595	42.21284
48	1.122518	50.2492	16.9595	69.28654
33	-.292861	50.2492	16.9595	45.28243
34	.0068578	50.2492	16.9595	50.3655
62	.306588	50.2492	16.9595	55.44878
75	.6036544	50.2492	16.9595	60.48688
72	-.67243	50.2492	16.9595	38.84512
18	.5434455	50.2492	16.9595	59.46576
80	.3482438	50.2492	16.9595	56.15524
79	.6856477	50.2492	16.9595	61.87744
77	.8316303	50.2492	16.9595	64.35323
68	.243624	50.2492	16.9595	54.38094
67	.2007324	50.2492	16.9595	53.65352
66	-.018735	50.2492	16.9595	49.93147
69	-.144887	50.2492	16.9595	47.79199
70	.1478329	50.2492	16.9595	52.75637

Transform Values : Group BCD

No.	S Value	α	β	T Value
22	.1144213	75.0066	17.3686	76.99394
8	-.163139	75.0066	17.3686	72.1731
64	-.860634	75.0066	17.3686	60.05858
3	.9322921	75.0066	17.3686	91.19921
21	-.298678	75.0066	17.3686	69.81898
87	-.516818	75.0066	17.3686	66.03019
76	-.289372	75.0066	17.3686	69.98062
17	-.293683	75.0066	17.3686	69.90575
20	.0059605	75.0066	17.3686	75.11013
6	.0283926	75.0066	17.3686	75.49974
47	.2495464	75.0066	17.3686	79.34087
49	.2727237	75.0066	17.3686	79.74343
46	.857238	75.0066	17.3686	89.89562
12	.0721374	75.0066	17.3686	76.25953
39	-.047961	75.0066	17.3686	74.17359
13	.7097753	75.0066	17.3686	87.3344
14	.3855943	75.0066	17.3686	81.70383
25	-.450825	75.0066	17.3686	67.17641
55	.1193939	75.0066	17.3686	77.0803
50	-.069438	75.0066	17.3686	73.80056
52	-.36397	75.0066	17.3686	68.68495

Transform Values : Group AB

No.	S Value	α	β	T Value
129	.3481293	35.1074	14.829	40.26981
126	-.411571	35.1074	14.829	29.00421
135	-.067219	35.1074	14.829	34.11061
134	-.19877	35.1074	14.829	32.15984
108	-.284868	35.1074	14.829	30.88309
71	.5248191	35.1074	14.829	42.88994
142	.0030221	35.1074	14.829	35.15222

Transform Values : Group BC

No.	S Value	α	β	T Value
41	.8063284	64.0226	14.3674	75.60744
59	-.091712	64.0226	14.3674	62.70493
88	-.2047	64.0226	14.3674	61.08159
26	-.2047	64.0226	14.3674	61.08159
31	-.370449	64.0226	14.3674	58.7002

Transform Values : Group CD

No.	S Value	α	β	T Value
40	-.308926	87.507	15.7805	82.632
23	-.846831	87.507	15.7805	74.14358
10	1.35397	87.507	15.7805	108.8733

Appendix E

Independent Variables with Transform Values

No.	PER	ESS	CS	TIME	T Value
1	100	100	100	100	168.8168
2	100	100	100	60	91.77639
3	100	100	100	20	91.19921
4	100	100	60	100	86.22817
5	100	100	60	60	80.49886
6	100	100	60	20	75.49974
7	100	100	20	100	78.65673
8	100	100	20	60	72.1731
9	100	100	20	20	65.58188
10	100	80	100	100	108.8733
11	100	80	100	60	78.74514
12	100	80	100	20	76.25953
13	100	80	60	100	87.3344
14	100	80	60	60	81.70383
15	100	80	60	20	67.92661
16	100	80	20	100	71.41667
17	100	80	20	60	69.90575
18	100	80	20	20	59.46576
19	100	60	100	100	81.04743
20	100	60	100	60	75.11013
21	100	60	100	20	69.81898
22	100	60	60	100	76.99394
23	100	60	60	60	74.14358
24	100	60	60	20	62.39591
25	100	60	20	100	67.17641
26	100	60	20	60	61.08159
27	100	60	20	20	57.73596
28	100	40	100	100	62.40826
29	100	40	100	60	60.32532
30	100	40	100	20	53.80129
31	100	40	60	100	58.7002
32	100	40	60	60	56.35603
33	100	40	60	20	45.28243
34	100	40	20	100	50.3655
35	100	40	20	60	53.25869
36	100	40	20	20	42.21284
37	75	100	100	100	82.51573
38	75	100	100	60	137.0519
39	75	100	100	20	74.17359
40	75	100	60	100	82.632
41	75	100	60	60	75.60744
42	75	100	60	20	67.20006
43	75	100	20	100	68.53893
44	75	100	20	60	69.15945
45	75	100	20	20	60.47382
46	75	80	100	100	89.89562
47	75	80	100	60	79.34087
48	75	80	100	20	69.28654

No.	PER	ESS	CS	TIME	T Value
49	75	80	60	100	79.74343
50	75	80	60	60	73.80056
51	75	80	60	20	63.88066
52	75	80	20	100	68.68495
53	75	80	20	60	64.74091
54	75	80	20	20	50.80739
55	75	60	100	100	77.0803
56	75	60	100	60	70.29297
57	75	60	100	20	65.7839
58	75	60	60	100	68.50253
59	75	60	60	60	62.70493
60	75	60	60	20	53.79175
61	75	60	20	100	65.69302
62	75	60	20	60	55.44878
63	75	60	20	20	58.14918
64	75	40	100	100	60.05858
65	75	40	100	60	63.31322
66	75	40	100	20	49.93147
67	75	40	60	100	53.65352
68	75	40	60	60	54.38094
69	75	40	60	20	47.79199
70	75	40	20	100	52.75637
71	75	40	20	60	42.88994
72	75	40	20	20	38.84512
73	60	100	100	100	72.57184
74	60	100	100	60	64.44767
75	60	100	100	20	60.48688
76	60	100	60	100	69.98062
77	60	100	60	60	64.35323
78	60	100	60	20	60.6899
79	60	100	20	100	61.87744
80	60	100	20	60	56.15524
81	60	100	20	20	55.95325
82	60	80	100	100	67.24814
83	60	80	100	60	65.08207
84	60	80	100	20	56.53528
85	60	80	60	100	66.35755
86	60	80	60	60	59.34884
87	60	80	60	20	66.03019
88	60	80	20	100	61.08159
89	60	80	20	60	52.24907
90	60	80	20	20	48.17415
91	60	60	100	100	66.88625
92	60	60	100	60	59.75902
93	60	60	100	20	54.44763
94	60	60	60	100	62.86566
95	60	60	60	60	60.34027
96	60	60	60	20	48.26603
97	60	60	20	100	60.66704
98	60	60	20	60	49.57562
99	60	60	20	20	41.72323

No.	PER	ESS	CS	TIME	T Value
100	60	40	100	100	61.76083
101	60	40	100	60	48.88594
102	60	40	100	20	48.27803
103	60	40	60	100	53.849
104	60	40	60	60	45.91586
105	60	40	60	20	40.32639
106	60	40	20	100	48.29502
107	60	40	20	60	43.79192
108	60	40	20	20	30.88309
109	40	100	100	100	53.21371
110	40	100	100	60	50.3655
111	40	100	100	20	45.48661
112	40	100	60	100	47.15747
113	40	100	60	60	43.11774
114	40	100	60	20	43.32189
115	40	100	20	100	52.35515
116	40	100	20	60	40.24103
117	40	100	20	20	34.35213
118	40	80	100	100	57.36122
119	40	80	100	60	48.23014
120	40	80	100	20	42.55877
121	40	80	60	100	49.27614
122	40	80	60	60	47.43551
123	40	80	60	20	42.71662
124	40	80	20	100	41.89422
125	40	80	20	60	40.06751
126	40	80	20	20	29.00421
127	40	60	100	100	49.29135
128	40	60	100	60	43.68817
129	40	60	100	20	40.26981
130	40	60	60	100	45.45999
131	40	60	60	60	38.85084
132	40	60	60	20	41.19483
133	40	60	20	100	41.83887
134	40	60	20	60	32.15984
135	40	60	20	20	34.11061
136	40	40	100	100	41.4332
137	40	40	100	60	36.72931
138	40	40	100	20	28.822
139	40	40	60	100	41.66463
140	40	40	60	60	31.93349
141	40	40	60	20	33.56759
142	40	40	20	100	35.15222
143	40	40	20	60	29.45031
144	40	40	20	20	29.01612

Appendix F : Regression Analysis

Y = PER ESS CS TIME

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	4	38651.26463	9662.81616	102.093	0.0001
ERROR	139	13156.00301	94.64750365		
C TOTAL	143	51807.26763			

ROOT MSE	9.728695	R-SQUARE	0.7461
DEP MEAN	59.28338	ADJ R-SQ	0.7388
C.V.	16.41049		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-24.0766	4.24400338	-5.673	0.0001
PER	1	0.52750273	0.0370164	14.251	0.0001
ESS	1	0.37383196	0.03625671	10.311	0.0001
CS	1	0.18371308	0.02482327	7.401	0.0001
TIME	1	0.16505237	0.02482327	6.649	0.0001

$Y = PER^{.5}\ ESS^{.5}\ CS^{.5}\ TIME^{.5}$

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	4	39334.27497	9833.56874	109.586	0.0001
ERROR	139	12472.99267	89.73376020		
C TOTAL	143	51807.26763			

ROOT MSE	9.472791	R-SQUARE	0.7592
DEP MEAN	59.28338	ADJ R-SQ	0.7523
C.V.	15.97883		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-100.187	7.73187109	-12.958	0.0001
$PER^{.5}$	1	8.76339187	0.58944560	14.867	0.0001
$ESS^{.5}$	1	6.15138874	0.57627114	10.674	0.0001
$CS^{.5}$	1	2.62557710	0.34782866	7.548	0.0001
$TIME^{.5}$	1	2.36900314	0.34782866	6.811	0.0001

Y = Ln(PER) Ln(ESS) Ln(CS) Ln(TIME)

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	4	39532.83990	9883.20997	111.921	0.0001
ERROR	139	12274.42774	88.30523551		
C TOTAL	143	51807.26763			

ROOT MSE	9.397087	R-SQUARE	0.7631
DEP MEAN	59.28338	ADJ R-SQ	0.7563
C.V.	15.85113		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-256.949	15.14136285	-16.970	0.0001
LnPER	1	35.41797105	2.33951521	15.139	0.0001
LnESS	1	24.68773111	2.28576225	10.801	0.0001
LnCS	1	8.72121552	1.16618504	7.478	0.0001
LnTIME	1	7.90198270	1.16618504	6.776	0.0001

Y = PER$^{.5}$ ESS$^{.5}$ CS TIME

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	4	39427.25806	9856.81451	110.670	0.0001
ERROR	139	12380.00958	89.06481712		
C TOTAL	143	51807.26763			

ROOT MSE	9.437416	R-SQUARE	0.7611
DEP MEAN	59.28338	ADJ R-SQ	0.7542
C.V.	15.91916		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-84.1224	7.09492852	-11.857	0.0001
PER$^{.5}$	1	8.76339187	0.58724441	14.923	0.0001
ESS$^{.5}$	1	6.15138874	0.57411914	10.714	0.0001
CS	1	0.18371308	0.02408006	7.629	0.0001
TIME	1	0.16505277	0.02408006	6.854	0.0001

Appendix G : Stepwise Regression with Maximized R Square

Y = PER ESS CS TIME X1 X2 X3 X4 X5 X6 X7 X8 X9 X10

where X1 = PER*ESS
X2 = PER*CS
X3 = PER*TIME
X4 = ESS*CS
X5 = ESS*TIME
X6 = CS*TIME
X7 = PER^2
X8 = ESS^2
X9 = CS^2
X10= $TIME^2$

MAXIMUM R-SQUARE IMPROVEMENT FOR DEPENDENT VARIABLE

STEP 1 VARIABLE X1 ENTERED R SQUARE = 0.58395220
C(P) = 205.35800836

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	1	30252.96783106	30252.96783106	199.31	0.0001
ERROR	142	21554.29980352	151.79084369		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	27.94235146				
X1	0.00651242	0.00046130	30252.96783106	199.31	0.0001

BOUNDS ON CONDITION NUMBER: 1, 1
THE ABOVE MODEL IS THE BEST 1 VARIABLE MODEL FOUND.

STEP 2 VARIABLE X6 ENTERED R SQUARE = 0.75430726
C(P) = 65.94761063

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	2	39078.59814551	19539.29907276	216.44	0.0001
ERROR	141	12728.66948907	90.27425170		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	18.45129524				
X1	0.00651242	0.00035575	30252.96783106	335.12	0.0001
X6	0.00263640	0.00026664	8825.63031445	97.76	0.0001

BOUNDS ON CONDITION NUMBER: 1, 4

THE ABOVE MODEL IS THE BEST 2 VARIABLE MODEL FOUND.

STEP 3 VARIABLE X2 ENTERED R SQUARE = 0.77469678
C(P) = 51.02243390

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	40134.92324786	13378.30774929	160.46	0.0001
ERROR	140	11672.34438671	83.37388848		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	17.96432659				
X1	0.00597905	0.00037328	21390.96284068	256.57	0.0001
X2	0.00131143	0.00036844	1056.32510235	12.67	0.0005
X6	0.00198201	0.00031537	3292.99278481	39.50	0.0001

BOUNDS ON CONDITION NUMBER: 1.706867, 13.2412

STEP 3 X6 REPLACED BY TIME R SQUARE = 0.79190328
C(P) = 36.73944917

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	3	41026.34527227	13675.44842409	177.59	0.0001
ERROR	140	10780.92236230	77.00658830		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	12.27045070				
TIME	0.16505237	0.02239073	4184.41480922	54.34	0.0001
X1	0.00543006	0.00034878	18665.34099329	242.39	0.0001
X2	0.00266124	0.00028770	6588.96263199	85.56	0.0001

BOUNDS ON CONDITION NUMBER: 1.126826, 9.760956
THE ABOVE MODEL IS THE BEST 3 VARIABLE MODEL FOUND.

STEP 4 VARIABLE X7 ENTERED R SQUARE = 0.79461181
C(P) = 36.49111899

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	4	41166.66685635	10291.66671409	134.44	0.0001
ERROR	139	10640.60077823	76.55108474		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	12.223155[illegible]				
TIME	0.16505237	0.02232441	4184.41480922	54.66	0.0001
X1	0.00581304	0.00044827	12873.11591044	168.16	0.0001
X2	0.00281511	0.00030854	6372.54273559	83.25	0.0001
X7	-0.00046685	0.00034482	140.32158408	1.83	0.1780

BOUNDS ON CONDITION NUMBER: 2.166187, 25.36938

STEP 4 TIME REPLACED BY X3 R SQUARE = 0.80635048
C(P) = 26.74694763

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	4	41774.81488157	10443.70372039	144.70	0.0001
ERROR	139	10032.45275301	72.17591909		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	17.47625710				
X1	0.00577950	0.00043529	12723.82009850	176.29	0.0001
X2	0.00280164	0.00029960	6311.47115466	87.45	0.0001
X3	0.00244135	0.00029960	4792.56283444	66.40	0.0001
X7	-0.00146632	0.00035658	1220.50308158	16.91	0.0001

BOUNDS ON CONDITION NUMBER: 2.456881, 27.74804
THE ABOVE MODEL IS THE BEST 4 VARIABLE MODEL FOUND.

STEP 5 VARIABLE PER ENTERED R SQUARE = 0.82291993
C(P) = 14.99277437

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	5	42633.23300644	8526.64660129	128.26	0.0001
ERROR	138	9174.03462813	66.47851180		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	-9.00909821				
PER	0.83892233	0.23346026	858.41812488	12.91	0.0005
X1	0.00558841	0.00042113	11706.70677439	176.10	0.0001
X2	0.00272486	0.00028833	5937.52000985	89.31	0.0001
X3	0.00236458	0.00028833	4471.19067232	67.26	0.0001
X7	-0.00715901	0.00162074	1297.06583067	19.51	0.0001Press

BOUNDS ON CONDITION NUMBER: 56.63247, 581.3247
THE ABOVE MODEL IS THE BEST 5 VARIABLE MODEL FOUND.

STEP 6 VARIABLE X8 ENTERED R SQUARE = 0.82679386
C(P) = 13.77705544

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	6	42833.93074156	7138.98845693	108.99	0.0001
ERROR	137	8973.33689302	65.49880944		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	-2.82567269Press RETURN to continue				
PER	0.69073566	0.24671238	513.42295133	7.84	0.0059
X1	0.00770537	0.00127957	2375.16648694	36.26	0.0001
X2	0.00272486	0.00028619	5937.52000985	90.65	0.0001
X3	0.00236458	0.00028619	4471.19067232	68.26	0.0001
X7	-0.00715901	0.00160875	1297.06583067	19.80	0.0001
X8	-0.00114508	0.00065416	200.69773511	3.06	0.0823

BOUNDS ON CONDITION NUMBER: 64.1903, 894.7263
THE ABOVE MODEL IS THE BEST 6 VARIABLE MODEL FOUND.

STEP 7 VARIABLE X4 ENTERED R SQUARE = 0.83149159
C(P) = 11.87750735

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	7	43077.30725661	6153.90103666	95.87	0.0001
ERROR	136	8729.96037796	64.19088513		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	-5.67384920				
PER	0.77208662	0.24778431	623.24356424	9.71	0.0022
X1	0.00755332	0.00126913	2273.71441659	35.42	0.0001
X2	0.00154640	0.00066825	343.74555911	5.36	0.0222
X3	0.00236458	0.00028332	4471.19067232	69.65	0.0001
X4	0.00127488	0.00065474	243.37651505	3.79	0.0536
X7	-0.00715901	0.00159261	1297.06583067	20.21	0.0001
X8	-0.00160921	0.00069007	349.07475904	5.44	0.0212

BOUNDS ON CONDITION NUMBER: 66.06861, 1159.122

THE ABOVE MODEL IS THE BEST 7 VARIABLE MODEL FOUND.

STEP 8 VARIABLE CS ENTERED R SQUARE = 0.83989868

C(P) = 6.89885063

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	8	43512.85545759	5439.10693220	88.53	0.0001
ERROR	135	8294.41217699	61.44009020		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	5.15270737				
PER	0.68346315	0.24469150	479.34069822	7.80	0.0060
CS	-0.23942070	0.08992273	435.54820098	7.09	0.0087
X1	0.00736441	0.00124367	2154.36996611	35.06	0.0001
X2	0.00324385	0.00091317	775.30396635	12.62	0.0005
X3	0.00236458	0.00027718	4471.19067232	72.77	0.0001
X4	0.00285884	0.00087420	657.06471614	10.69	0.0014
X7	-0.00715901	0.00155811	1297.06583067	21.11	0.0001
X8	-0.00218587	0.00070901	583.97880788	9.50	0.0025

BOUNDS ON CONDITION NUMBER: 67.31422, 1611.276

THE ABOVE MODEL IS THE BEST 8 VARIABLE MODEL FOUND.

STEP 9 VARIABLE X6 ENTERED R SQUARE = 0.84209424

C(P) = 7.07632974

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	9	43626.60165902	4847.40018434	79.40	0.0001
ERROR	134	8180.66597555	61.04974609		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	5.15270737				
PER	0.71805817	0.24522620	523.44300199	8.57	0.0040
CS	-0.28308381	0.09517333	540.11149072	8.85	0.0035
X1	0.00736441	0.00123971	2154.36996611	35.29	0.0001
X2	0.00324385	0.00091026	775.30396635	12.70	0.0005
X3	0.00178799	0.00050475	766.05516881	12.55	0.0005
X4	0.00285884	0.00087142	657.06471614	10.76	0.0013
X6	0.00072772	0.00053313	113.74620143	1.86	0.1745
X7	-0.00715901	0.00155315	1297.06583067	21.25	0.0001
X8	-0.00218587	0.00070675	583.97880788	9.57	0.0024

BOUNDS ON CONDITION NUMBER: 68.04101, 1923.178

THE ABOVE MODEL IS THE BEST 9 VARIABLE MODEL FOUND.

STEP 10 VARIABLE X5 ENTERED R SQUARE = 0.84300016

C(P) = 8.32433553

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	10	43673.53472076	4367.35347208	71.41	0.0001
ERROR	133	8133.73291381	61.15588657		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	3.77404408				
PER	0.74791265	0.24779393	557.13277915	9.11	0.0030
CS	-0.27136678	0.09619046	486.72947276	7.96	0.0055
X1	0.00729676	0.00124319	2106.81389912	34.45	0.0001
X2	0.00324385	0.00091105	775.30396635	12.68	0.0005
X3	0.00136934	0.00069542	237.12140433	3.88	0.0510
X4	0.00283229	0.00087270	644.13974214	10.53	0.0015
X5	0.00059374	0.00067777	46.93306174	0.77	0.3826
X6	0.00056341	0.00056560	60.68242734	0.99	0.3210
X7	-0.00715901	0.00155450	1297.06583067	21.21	0.0001
X8	-0.00239236	0.00074561	629.61169078	10.30	0.0017

BOUNDS ON CONDITION NUMBER: 69.35279, 2296.592

THE ABOVE MODEL IS THE BEST 10 VARIABLE MODEL FOUND.

STEP 11 VARIABLE TIME ENTERED R SQUARE = 0.84439023

C(P) = 9.17044843

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	11	43745.55050696	3976.86822791	65.12	0.0001
ERROR	132	8061.71712762	61.07361460		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	8.69998543				
PER	0.71435358	0.24954823	500.46196819	8.19	0.0049
CS	-0.28453776	0.09688794	526.73717497	8.62	0.0039
TIME	-0.10520994	0.09688794	72.01578620	1.18	0.2795
X1	0.00722873	0.00124393	2062.46137031	33.77	0.0001
X2	0.00324385	0.00091044	775.30396635	12.69	0.0005
X3	0.00200803	0.00091044	297.09280677	4.86	0.0291
X4	0.00280559	0.00087246	631.54888656	10.34	0.0016
X5	0.00119093	0.00087246	113.79740741	1.86	0.1746
X6	0.00081408	0.00061054	108.58069829	1.78	0.1847
X7	-0.00715901	0.00155346	1297.06583067	21.24	0.0001
X8	-0.00260005	0.00076926	697.70288199	11.42	0.0010Press

BOUNDS ON CONDITION NUMBER: 70.43301, 2948.786

THE ABOVE MODEL IS THE BEST 11 VARIABLE MODEL FOUND.

STEP 12 VARIABLE ESS ENTERED R SQUARE = 0.84451648

C(P) = 11.06564422

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	12	43752.09149146	3646.00762429	59.29	0.0001
ERROR	131	8055.17614311	61.48989422		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	5.31113376				
PER	0.72512213	0.25256466	506.85170454	8.24	0.0048
ESS	0.08463804	0.25950510	6.54098450	0.11	0.7448
CS	-0.28031141	0.09807738	502.28191635	8.17	0.0050
TIME	-0.10098359	0.09807738	65.18799166	1.06	0.3051
X1	0.00707489	0.00133431	1728.74196674	28.11	0.0001
X2	0.00324385	0.00091354	775.30396635	12.61	0.0005
X3	0.00200803	0.00091354	297.09280677	4.83	0.0297
X4	0.00274521	0.00089479	578.77943242	9.41	0.0026
X5	0.00113055	0.00089479	98.16224676	1.60	0.2087
X6	0.00081408	0.00061262	108.58069829	1.77	0.1862
X7	-0.00715901	0.00155874	1297.06583067	21.09	0.0001
X8	-0.00306965	0.00163366	217.09991573	3.53	0.0625

BOUNDS ON CONDITION NUMBER: 78.85342, 4813.629

THE ABOVE MODEL IS THE BEST 12 VARIABLE MODEL FOUND.

STEP 13 VARIABLE X10 ENTERED R SQUARE = 0.84458150
C(P) = 13.01167435

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	13	43755.45982971	3365.80460229	54.34	0.0001
ERROR	130	8051.80780487	61.93698311		
TOTAL	143	51807.26763458			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	4.79743891				
PER	0.72512213	0.25348119	506.85170454	8.18	0.0049
ESS	0.08463804	0.26044681	6.54098450	0.11	0.7457
CS	-0.28031141	0.09843330	502.28191635	8.11	0.0051
TIME	-0.07665067	0.14344502	17.68524415	0.29	0.5940
X1	0.00707489	0.00133915	1728.74196674	27.91	0.0001
X2	0.00324385	0.00091685	775.30396635	12.52	0.0006
X3	0.00200803	0.00091685	297.09280677	4.80	0.0303
X4	0.00274521	0.00089804	578.77943242	9.34	0.0027
X5	0.00113055	0.00089804	98.16224676	1.58	0.2103
X6	0.00081408	0.00061484	108.58069829	1.75	0.1878
X7	-0.00715901	0.00156440	1297.06583067	20.94	0.0001
X8	-0.00306965	0.00163958	217.09991573	3.51	0.0634
X10	-0.00020277	0.00086952	3.36833824	0.05	0.8160

BOUNDS ON CONDITION NUMBER: 78.85342, 5929.765

THE ABOVE MODEL IS THE BEST 13 VARIABLE MODEL FOUND.

STEP 14 VARIABLE X9 ENTERED R SQUARE = 0.84459556
C(P) = 15.00000000

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	14	43756.18844331	3125.44203167	50.08	0.0001
ERROR	129	8051.07919126	62.41146660		
TOTAL	143	51807.26763458			

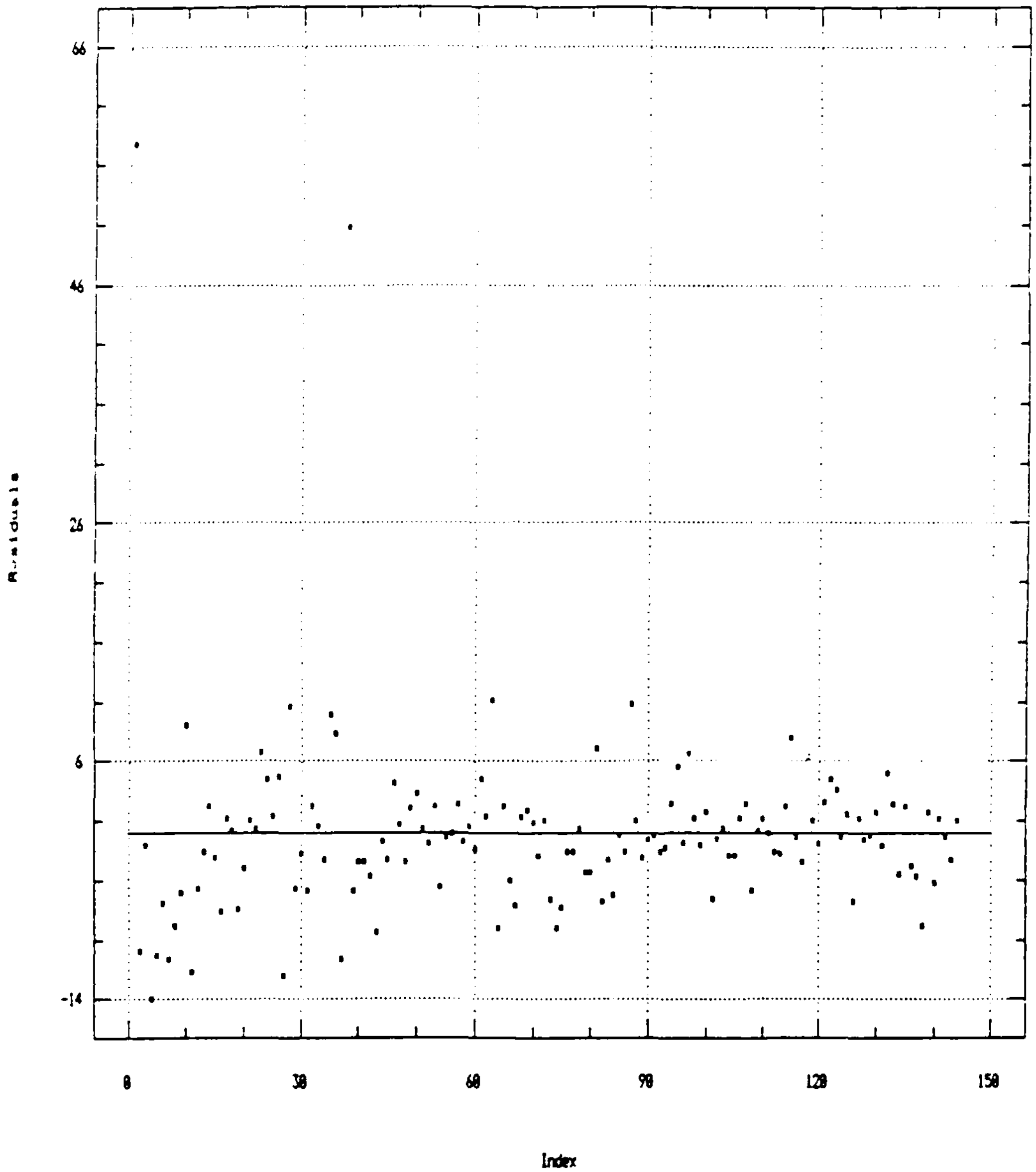
Press RETURN to continue

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTERCEPT	5.03635533				
PER	0.72512213	0.25445026	506.85170454	8.12	0.0051
ESS	0.08463804	0.26144252	6.54098450	0.10	0.7467
CS	-0.29162850	0.14399342	255.99944579	4.10	0.0449
TIME	-0.07665067	0.14399342	17.68524415	0.28	0.5954
X1	0.00707489	0.00134427	1728.74196674	27.70	0.0001
X2	0.00324385	0.00092036	775.30396635	12.42	0.0006
X3	0.00200803	0.00092036	297.09280677	4.76	0.0309
X4	0.00274521	0.00090147	578.77943242	9.27	0.0028
X5	0.00113055	0.00090147	98.16224676	1.57	0.2121
X6	0.00081408	0.00061719	108.58069829	1.74	0.1895
X7	-0.00715901	0.00157038	1297.06583067	20.78	0.0001
X8	-0.00306965	0.00164585	217.09991573	3.48	0.0644
X9	0.00009431	0.00087285	0.72861361	0.01	0.9141
X10	-0.00020277	0.00087265	3.36833824	0.05	0.8167

BOUNDS ON CONDITION NUMBER: 78.85342, 7155.9

THE ABOVE MODEL IS THE BEST 14 VARIABLE MODEL FOUND.

Appendix H : Residual Plot for 6 Variable Model



Bibliography

1. Crawford, Capt Paul M. Dynamic Study of Factors Impacting on Combat Power. MS thesis, Naval Post Graduate School, Monterey CA, March 1988.

2. Department of the Army. Operations. FM 100-5. Washington: HQ US Army, 20 August 1982.

3. Draper,N. R. and H. Smith. Applied Linear Regression. New York: John Wiley and Sons, 1981.

4. Etheridge, Elizabeth W. and Michael R. Anderson. Criteria for Reconstitution of Forces, September 1981. US Army Combined Arms Combat Developments Activity. Report Number TR 7-81. (AD A111803).

5. Kidder, Louise H. Research Methods in Social Relations. New York: Holt, Rhinehart and Wilson, 1981.

6. Kilmer, Capt Robert A. The Generalized Value System and Future State Decision Making. MS thesis, Naval Post Graduate School, Monterey CA, March 1986 (AD-A168446).

7. Lindsay, Glen F. On Constructing Interval Scales Using Data Resulting From Categorical Judgments. Unpublished report. Naval Post Graduate School, Monterey CA, September 1981.

8. VP-PLANNER. Paperback Software International. Berkeley, CA: 1986.

9. Schoenstadt, Arthur L. and Samuel H. Parry. Toward an Axiomatic Generalized Value System. Technical Report NPS-53-86-008. Naval Post Graduate School, Monterey CA, June 1985 - March 1986.

10. WordPerfect 5.0. WordPerfect Corporation. Orem, Utah: 1988.

11. Zikmund, William G. Business Research Methods. Chicago: Dryden Press, 1984.

VITA

Captain J. Marc Le Gare [redacted] attended the United States Military Academy, from which he received the degree of Bachelor of Science in Operations Research in June 1981. Upon graduation, he received a commission in the United States Army. His first assignment was in the 1st Battalion 36th Infantry, Federal Republic Germany. During the period of January 1982 to December 1984, he held the positions of rifle platoon leader, rifle company executive officer, battalion S1, and headquarters company executive officer. In July 1985, Captain Le Gare was assigned to the 1st Battalion 31st Infantry, Republic of South Korea. During this tour of duty, he held the positions of battalion maintenance officer and rifle company commander. His subsequent assignment was to the School of Engineering, Air Force Institute of Technology, in August 1987.

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Abstract

This thesis attempts to develop an equation, based on expert opinion, that models the appraisal of combat power. The motivation came from identifying a shortcoming in the assignment of combat power in the AirLand Research Model (ALARM). A link is needed to join the Basic Inherent Power (BIP) of a unit and its Adjusted Basic Inherent Power (ABIP). The ABIP is discounted portion of the BIP based on the current mission and status of the unit.

The unit and mission explored in this thesis was a mechanized infantry task force in the attack. The survey required combat arms officers to give categorical judgments on unit effectiveness, based on the state of the unit. Four state variables were used. 144 different unit profiles were generated and divided into four versions of the survey. Surveys were completed by students at the Army Combined Arms and Services Staff School and the Army War College. Responserate was approximately 80 %.

Survey responses were transformed to numerical values using an interval scaling technique. These values and the variable settings were used in regression analysis. The best fit model was used to develop the Unit Appraisal Function (UAF). The UAF can now link the BIP to the ABIP, based on the mission and status of the unit.