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An Improved Strategy for Determining Earth Satellite Orbits by Radio

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For satellite orbit determination, the most accurate observable available today is microwave phase, differenced between observing stations and between satellites to cancel both transmitter and receiver related errors. For maximum accuracy, the integer cycle ambiguities of such observations must be resolved. To perform this ambiguity resolution, a bootstrapping strategy is effective. The tracking stations must have a wide ranging progression of spacings. Then, by conventional "integrated Doppler" processing of the observations from the most widely spaced stations, the orbits can be determined well enough to permit resolution of the ambiguities of the observations from the most closely spaced stations. The resolution of these ambiguities can reduce the uncertainty of the orbit determination enough to enable ambiguity resolution for more widely spaced stations, which will reduce the orbital uncertainty further, and enable ambiguity resolution for still more widely spaced stations, and so on. In a test of this strategy with a total of six tracking stations, both the formal and the actual errors of determining NAVSTAR Global Positioning System satellite orbits were reduced by a factor of two.

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INTRODUCTION

Since October 1957, when ground based observations of the Doppler-shifted frequencies of signals transmitted by Sputnik were used to determine the orbit of the first artificial earth satellite [*Brown et al.*, 1957; *Peterson*, 1957], most satellite orbits have been determined by radio. Radio observations may be "one-way" or "round-trip," the latter involving transmission of a signal from the observer's location (usually fixed on the ground) to the satellite, and reflection or re-transmission of this signal back to the observer. Time delay may be observed in addition to, or instead of, Doppler shift. Time delay is sensitive to range, which is related to orbital position; Doppler shift is sensitive to range-rate, which is related to velocity.

One-way, as opposed to round-trip, delay and Doppler observations are biased by clock epoch and oscillator frequency offsets between the transmitter (usually aboard the satellite) and the receiver (usually on the ground). However, effects of a satellite-related offset may be canceled by taking differences between simultaneous one-way observations at different tracking stations, which preferably are far apart in order to retain sensitivity to the satellite orbit. Similarly, an offset associated with a tracking station is canceled in the difference between one-way observations of different satellites. In the technique known as *double differencing*, one-way observations of two or more satellites made simultaneously at two or more tracking stations are differenced both between satellites and between stations, to cancel both station- and satellite-related offsets [*Counselman et al.*, 1972].

Double Differencing

The immunity to clock-epoch and oscillator-frequency errors obtainable by double differencing is imperfect because of the inevitable inequalities among the individual satellite-to-ground signal propagation time delays and rates. However, double differencing is so effective that, in one form or another, it is the preferred method for determining orbits of Navstar Global Positioning System (GPS) satellites [*Abbot et al.*, 1985, 1986a; *Beutler et al.*, 1986; *Lichten and*

Border, 1987]. Doubly differenced observations of GPS satellites are also used for the determination of unknown ground station positions, either simultaneously with the satellite orbits, or when the latter are known *a priori*. Precision of the order of 1 part in 10^8 has been reported for such determinations of relative-position, or "baseline" vectors [*Bertiger and Lichten*, 1987].

Carrier Phase

Double-differencing is useful with delay and with Doppler observations, but it is most useful with *carrier phase* observations. "Carrier phase" is the instantaneous phase of a periodic "carrier" wave implicit in the signal being received from a satellite, and is measured with respect to a similar wave generated by a local oscillator at the tracking station. This oscillator has the same nominal frequency as the transmitter in the satellite. If its frequency were exactly the same, the time derivative of the carrier phase observable would be just the one-way Doppler shift. The carrier phase varies, continuously and cumulatively, at a rate of one cycle per wavelength of variation in the satellite-to-ground-station range.

A one-way carrier phase observation (not differenced between stations or satellites) includes the phase difference between the transmitting and receiving oscillators. This difference is random and varies with time. However, a time series of doubly differenced carrier phase observations has only a constant bias, equal to an integer number of cycles, as long as the observations are continuous. That is,

$$\Delta\Delta\phi_{kq}{}^{ij} = -(1/\lambda)\,\Delta\Delta r_{kq}{}^{ij} + N_{kq}{}^{ij}, \qquad (1)$$

where $\Delta\Delta\phi_{kq}{}^{ij}$ and $\Delta\Delta r_{kq}{}^{ij}$ are the doubly differenced phase and range, respectively, between satellites i, j and stations k, q; λ is the carrier wavelength; and $N_{kq}{}^{ij}$, known as the "ambiguity parameter", is an integer number of cycles.

Ambiguity Resolution

Sometimes the integer value of the ambiguity parameter can be determined, so that it may be subtracted out, or otherwise accounted for. In either case it is said that the ambiguity of the series of observations is resolved, or that the bias is "fixed."

Fixing the bias enhances the utility of the observations, as may be appreciated by considering that, in general, a series of observed values has some mean value, or average, plus a variation about the mean. Both the mean value and the variation about the mean contain potentially useful information about the orbits of the satellites and the positions of the tracking stations. Except for a factor of λ, the average doubly-differenced carrier phase equals the average doubly-differenced range. The variation of the phase equals the variation of this distance.

If the average of a series of phase observations includes an unknown bias, in other words if the bias has not been fixed, then the position- or range-related part of the average phase is unknown, and it is difficult to derive position information from it. However, when the bias is fixed, the position-related part of the average phase is known and can contribute to determining the satellite orbits and receiver positions.

In a method of bias-fixing known as the "geometric" method [*Counselman et al.*, 1979; *Bossler et al.*, 1980; *Wübbena et al.*, 1986], ambiguity parameters and unknown position coordinates are estimated simultaneously by least-squares fitting to a collection of doubly-differenced phase observations. In effect, the variation of each series of observations about its mean serves to determine the position-related parameters. From these parameters, the satellite-to-station ranges may be computed. The doubly differenced phase may be computed from these ranges, and subtracted from the observed value. The average of the difference between the observed and the computed phase is an estimate of the bias. If this estimate is near an integer value and has sufficiently small uncertainty, then the correct integer value of the bias can be identified with confidence. The bias can be fixed.

In an extension of this method due to Sergei Gourevitch [personal communication, 1982], every integer value in a finite interval surrounding the estimate of each ambiguity parameter is tested. A least-squares fit of all the non-ambiguity parameters to the observations is made for each set of ambiguity-parameter integers within these ranges. For each set, the sum of the squares of the post-fit differences between the observed and computed values of doubly differenced phase is computed. This sum, known as χ^2, which the fitting process minimizes, indicates the badness of the fit. The particular set of integer ambiguity-parameter values having the smallest χ^2 is identified. Confidence in the correctness of this identification is determined by the contrast between the associated value of χ^2, and the next-smallest value found.

For reliable bias fixing, the errors in the theoretically computed values of the phase observables must be small in comparison with one cycle of phase. This requirement can be met in practice if the distance between the receiving stations is small enough, and if the orbits of the satellites are known well enough.

Effect of Orbital Uncertainty

An error in the assumed knowledge of a satellite's orbit causes an error in the theoretically computed value of a between-stations satellite range difference, such as $\Delta r_{kq}{}^i$ for satellite i and stations k and q, which is approximately proportional to the distance between stations k and q. The magnitude of the error in $\Delta r_{kq}{}^i$ is about equal to the inter-station baseline length multiplied by the orbital error measured in radians of arc subtended at the midpoint of the baseline, and projected onto the sky in a direction parallel to the projected baseline.

As Abbot *et al.* [1985, 1986a], Beutler *et al.* [1986], Lichten and Border [1987], and others have described, knowledge of satellite orbits may be improved by processing doubly differenced phase observations. Until recently, no attempt was made to fix biases in such processing. The biases were estimated as real-number (*i.e.* continuous, as opposed to integer or discrete-valued) unknowns along with the unknown orbital parameters, or they were pre-eliminated algebraically.

The processing of doubly differenced phase data without bias fixing is sometimes called "integrated Doppler" processing because only the time-variation of the phase, equal to the integral of the Doppler shift, is contributing to the determination of the orbital (or other) parameters.

The essential reason for historically not attempting to fix biases in orbit determination is that bias parameters are difficult to separate from orbital parameters. Certain combinations of orbit-parameter adjustments can change the doubly differenced phase observable by a nearly constant amount, like a bias. Thus, determination of bias parameters is difficult without *a priori* knowledge of the orbit parameters. By the same token, orbit determination accuracy can be enhanced by constraining the bias parameters.

Fixing Biases in Orbit Determination

A direct method of constraining the doubly differenced carrier phase biases when the orbits are uncertain is to use doubly differenced observations of group delay, such as GPS pseudoranges [*Counselman et al.*, 1979; *Hatch*, 1982; *Wübbena et al.*, 1986; *Lichten and Border*, 1987]. Another method, which does not rely on group delay observations but still enables biases to be fixed in the process of orbit determination, is to arrange the tracking stations to form a wide range of interstation baseline lengths, from short to long. Ideally, a closely spaced progression of intermediate lengths is also provided [*Counselman*, 1987].

If the baseline lengths span a range of about 30 to 1, then the time-variation of the doubly differenced carrier phases from the longest baselines is sufficient to determine the orbits accurately enough that the biases of the observations from the shortest baselines can be fixed. With these biases fixed, the uncertainty of the orbit determination is smaller, so that longer baseline biases may now be fixed. The process may be continued until no more biases can be fixed.

In theory, it is best to combine all observations from all baselines and to minimize χ^2 with respect to all parameters simultaneously. However, when a small computer is being used, as in the present work, a baseline-by-baseline method is more practical.

A Test of the Progressive-Spacing Method

To test the progressive-spacing method of determining satellite orbits, we have performed a series of data-processing experiments using dual-frequency carrier phase observations of five GPS satellites (Navstar numbers 3, 4, 6, 8, and 9), from six tracking stations whose spacings ranged from 71 km to 4000 km. (See Fig. 1.) Observations of these satellites were available from these stations on three days. We treated each day separately in order to obtain three somewhat independent test results. On each day we determined the five satellites' orbits, first without fixing biases, then applying the stepwise procedure to fix the doubly differenced phase biases.

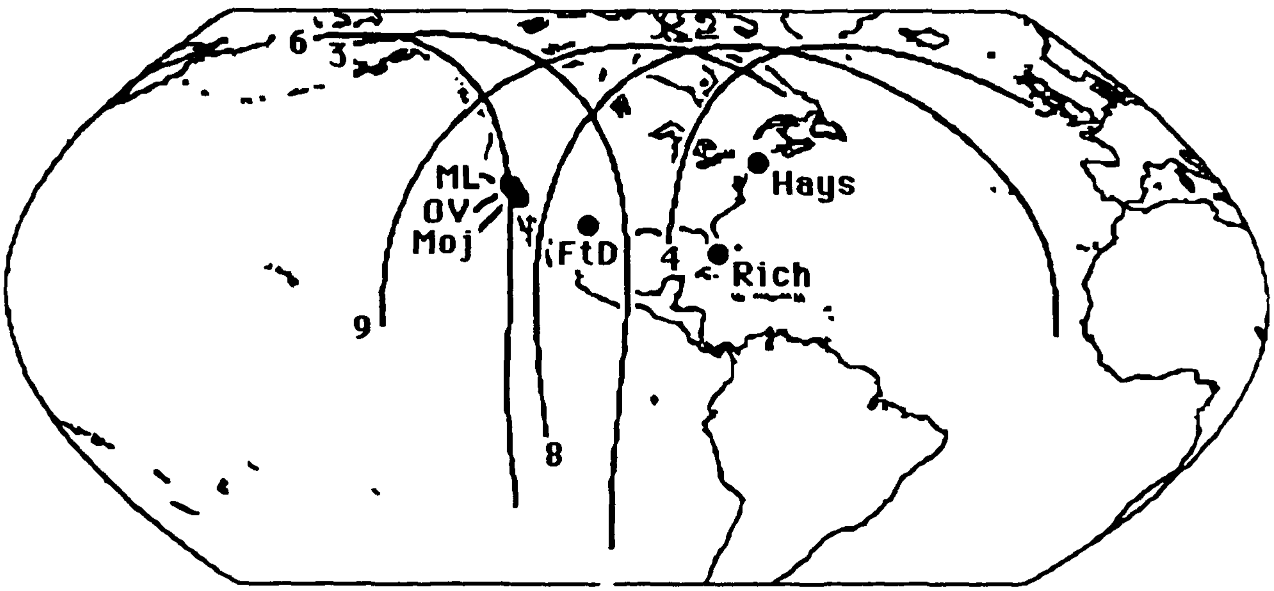

Fig. 1. The ground tracks of the five Navstar GPS satellites and the locations of the six tracking stations used in the test are shown. The portion of each satellite's orbit which was observed is shown with the Navstar number (3, 4, 6, 8, 9) marking the beginning of the track. ML = the Mammoth Lakes station; OV = Owens Valley; Moj = Mojave; FtD = Ft. Davis; Rich = Richmond; and Hays = Haystack. The ML- OV distance was 71 km; OV- Hays, about 4000 km.

Our goals were: (1) generally, to show that fixing biases substantially improves orbit-determination accuracy; and (2) more specifically, to show that fixing biases just for the observations from a very short baseline so improves the orbits, that it becomes possible to fix biases for a significantly longer baseline.

A preliminary report of these experiments and their results was made by Abbot and Counselman [1987]. More detailed descriptions of the test procedure, the observations, the data analysis, and results are given here, with additional discussion.

Test Procedure

We began by determining the satellite orbits by the usual method of least-squares fitting to the doubly differenced phases with all bias parameters free (unconstrained). Using these orbits, we fixed the biases for the shortest baseline, which was 71 km long. With these biases fixed, we re-determined the orbits and fixed the biases for the next-longer baseline, which was 245 km long. With the biases of these two baselines fixed, we made a final determination of the orbits. The remaining baselines in our six-station network were all too long (over 1500 km) for their biases to be fixed.

Simultaneously with the orbital parameters and those biases which had not been fixed, we adjusted an atmospheric delay parameter for each station, the position coordinates of three stations, and clock epoch offsets for three stations. No external constraint was ever placed on any adjusted parameter.

We weighted all observations equally in these least-squares adjustments. Since some systematic errors (*e.g.*, those due to unmodeled atmospheric refraction) are known to increase with increasing baseline length, it could be argued that too little weight was given to the observations from the shortest baselines. However, the fact that the shorter baselines were not more heavily weighted does tend to strengthen our conclusion from the test results, that satellite orbit determination can be enhanced substantially by including quite a short baseline in a tracking network of very long baselines.

We compared the formal standard errors of the orbital position determinations with and without bias fixing. We also compared the actual errors, which we determined by reference to more accurate orbit-determinations based on additional observations, and an additional type of observation. These comparisons showed that the short-baseline bias-fixing had reduced both the formal and the actual errors of the orbital position determinations, consistently, by about a factor of two.

To measure the effect of fixing biases for the 71-km baseline on the ability to fix biases for the 245-km baseline, we used χ^2 statistics as described above. We found a substantial improvement on each of the three days.

Finally, we considered the day-to-day scatter of determinations of the 245-km baseline vector, using orbits which had been determined each day with, and without, bias fixing. There was no significant difference in the scatter along a north-south axis, or along a vertical axis. However, the east-west scatter was much smaller with the bias-fixed orbits.

Observations and Data Analysis

Of the six observing sites, four were widely spaced: the Haystack Observatory of the Northeast Radio Observatory Corporation in Westford, Massachusetts; the U. S. Naval Observatory Time Service Alternate Station, at a site known as "Richmond" in Miami, Florida; the G. R. Agassiz Station of the Harvard College Observatory in Ft. Davis, Texas; and the Owens Valley Radio Observatory (OVRO) in California. Among these stations, the smallest spacing (between Ft. Davis and OVRO) was about 1500 km and the largest (between OVRO and Haystack) was about 4000 km. The geocentric position coordinates of these stations were accurately known from prior analyses of very-long-baseline radio interferometry (VLBI) observations of quasars [*Carter et al.*, 1985] and satellite laser ranging (SLR) observations [*Smith et al.*, 1985].

Since our method of fixing biases in orbit determination requires closely- as well as widely-spaced stations, we added observations from two sites relatively close to OVRO: one just 71 km to the north, at Mammoth Lakes, California; the other 245 km to the south at Mojave, California. The OVRO-Mammoth Lakes distance is less than 2 percent, and the OVRO-Mojave distance is only
6 percent, of the OVRO-Haystack distance.

The GPS receivers at Haystack, Richmond, and Ft. Davis were dual-band Air Force Geophysics Laboratory receivers whose carrier-frequency oscillators were stabilized by hydrogen maser frequency standards and whose clocks had been synchronized with UTC. The receivers at the three California sites were Texas Instruments model TI-4100's with relatively unstable oscillators and unsynchronized clocks. For the latter sites, clock epoch offsets were included among the parameters estimated in our least-squares fits.

In these fits we also adjusted three position coordinates for each of the Ft. Davis, Mammoth Lakes, and Mojave sites. No *a priori* constraints were applied. The coordinates of the Haystack, Richmond, and OVRO sites were always held fixed at the VLBI- and SLR-determined values given by Bock *et al.* [1986a].

From each receiver, observations of the L1 and L2 band center frequency (1575.42 and 1227.60 MHz, respectively) carrier phases were obtained at 6-minute intervals, for at least two satellites simultaneously, from about 4^h to 10^h UTC on each of three days: 1985 April 1, 2, and 3. A given satellite was observed for 2 to 4 hours daily. No pseudorange observations were used.

Further descriptions of the observing sites, receivers, schedules, etc., are given by Davidson *et al.* [1985], Bock *et al.* [1986a], and Lichten and Border [1987]. The algorithm which we used for least-squares fitting of the doubly differenced, dual band, carrier phase observations is described in detail by Bock *et al.* [1986b]. In the process of fixing dual-band biases by this algorithm, *a priori* constraints are applied temporarily to the ionospheric effects on the observations. However, the final adjustment of the orbital and other parameters is based on the linear combination of the L1 and L2 observations which is free of ionospheric effects, whether or not biases are fixed in the solution.

TEST RESULTS

Standard Deviations of Orbital Position Estimates

All observations, from all six stations, were included with equal weighting in the least-squares parameter estimation, whether or not any biases were fixed. We computed the scaled "formal" standard errors, or estimated standard deviations of the errors of the parameter estimates, in each case by taking the square roots of the diagonal elements of the inverse of the coefficient matrix of the normal equations, and multiplying by the square root of the (equally) weighted sum of the postfit residuals, divided by the number of degrees of freedom.

The estimated standard deviations of the orbital positions of all five satellites, determined with and without fixing the biases of the observations from the two shortest baselines, are shown in Figure 2. The greatest of the standard deviations (σ_x, σ_y, σ_z) of the three Cartesian coordinates of each satellite, at a time near the middle of the observing span on April 1, is shown. Very similar results were obtained for April 2 and 3.

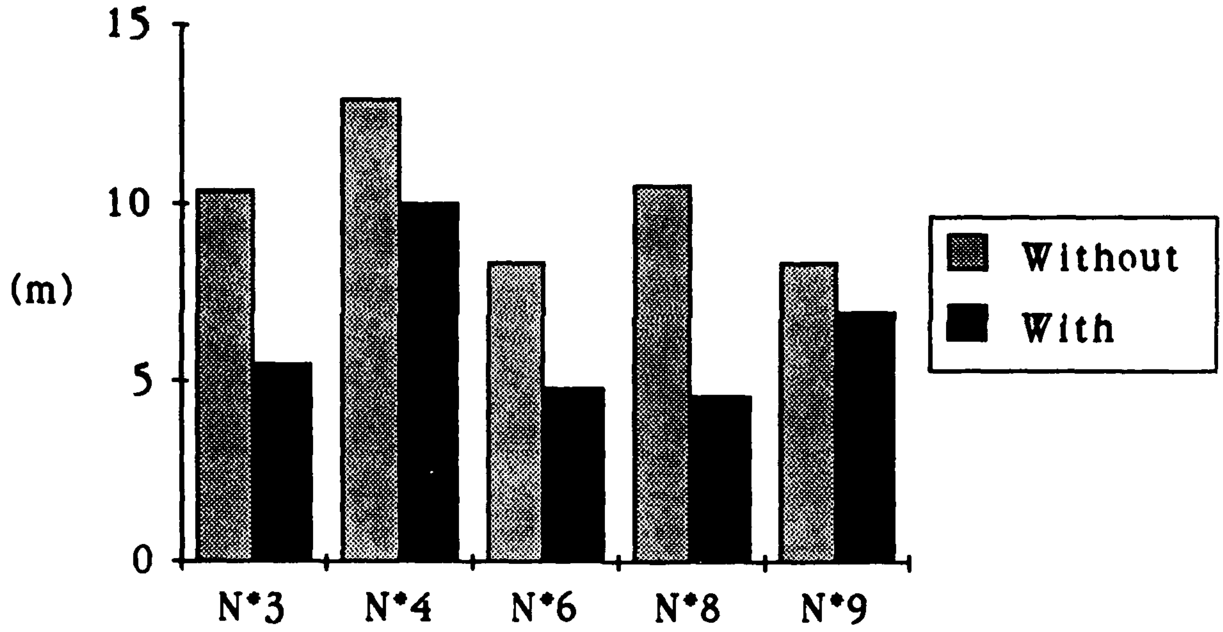

Fig. 2. Formal standard error of orbital position (greatest of σ_x, σ_y, σ_z) for each satellite (Navstar 3, ..., 9), estimated **without**, and **with**, fixing the short-baseline (71- & 245-km) doubly-differenced phase biases at integer-cycle values. In either case, the longer-baseline biases were free and all observations from all six stations were included with equal weights in the least-squares estimation.

Actual Errors of Orbital Position Estimates

To assess the actual — as opposed to the formal — errors of our orbit determinations, we compared them with a determination by King *et al.* [1986], which we were confident was more accurate than any of the present determinations. The comparison orbits were derived from a weighted fit to a combination of singly-differenced (between stations, not between satellites) phase observations from the three hydrogen-maser equipped stations (Haystack, Richmond, and Ft. Davis), and doubly-differenced observations from each of these stations and OVRO, from all three days (April 1, 2, and 3) simultaneously. No biases were fixed in this determination, but the inclusion of the singly-differenced observations and the much greater arc lengths of the observations gave it much greater strength than the present determinations.

Naturally, the comparison determination, being based on observations from an overlapping time span and mostly the same stations as the present orbit determinations, and in which the same values were assumed for station coordinates, earth rotation parameters, etc., has systematic errors in common with the present orbit determinations. Thus, our claim of assessing "actual errors" must be interpreted cautiously. However, there is independent evidence [*Abbot et al.*, 1986b; *Bock et al.*, 1986a] that the errors of the comparison orbits are smaller than 2 parts in 10^7 — *i.e.*, under 5 meters in orbital position.

The orbital position differences between the comparison determination and the two test determinations, one done without, and one done with fixing biases for the short baselines, are shown in Figure 3. For each satellite, the greatest magnitude of the vector position difference occurring at any time during the observations on April 2, is plotted. (Only for April 2, the middle day of the comp. rison determination, are we confident that this determination is much more accurate than the test determinations.) The differences, which we interpret as errors in the test determinations, are consistent with the formal error estimates which were plotted in Figure 2.

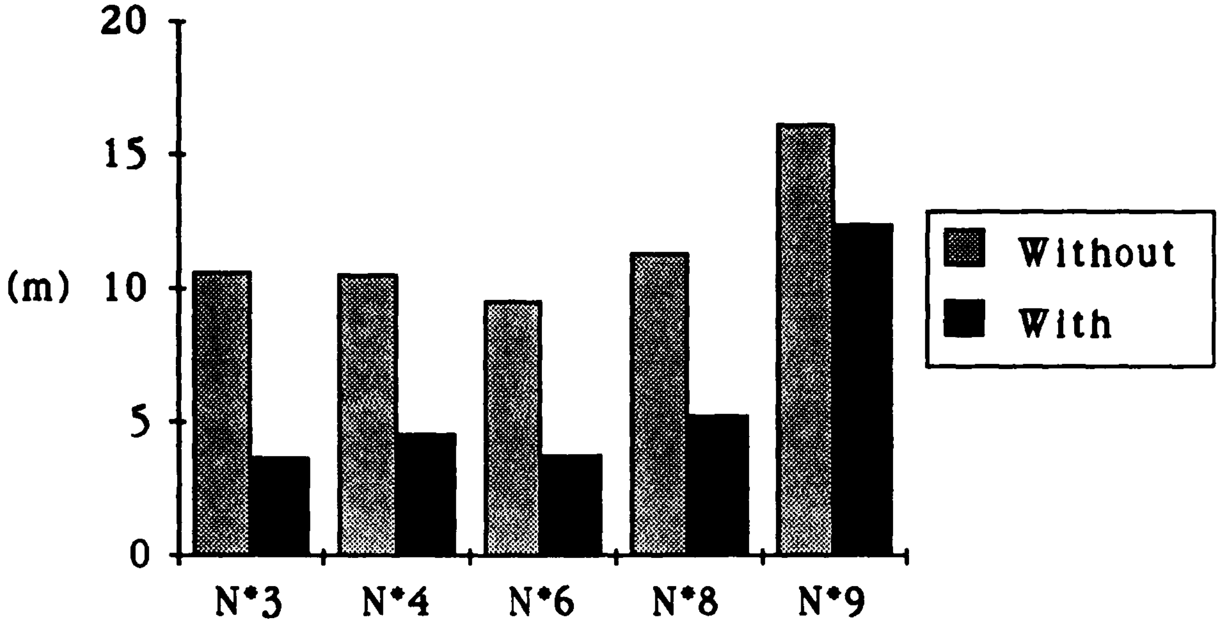

Fig. 3. Actual error of orbital position (peak magnitude of vector error) for each satellite (Navstar 3, ..., 9) determined **without**, and **with**, fixing the short-baseline (71- & 245-km) doubly-differenced phase biases at integer-cycle values. In either case, the longer-baseline biases were free and all observations from all six stations were included with equal weights in the least-squares estimation.

Enhanced Ability to R solve Ambiguity

Fixing the biases of the observations from one baseline makes it easier to fix the biases of the observations from another baseline. This "bootstrap" principle is the key to the progressively-spaced-station method of orbit determination. Ability to determine uniquely the integer values of bias parameters, and thus to fix them, has customarily been measured by Gourevitch's χ^2 contrast statistic. This statistic is given by

$$\text{Contrast} = [\,(\chi_1^{\,2}/\chi_0^{\,2}) - 1\,] \cdot [\,N_{d.o.f.}^{\;1/2}\,], \tag{2}$$

where $\chi_0^{\,2}$ is the smallest and $\chi_1^{\,2}$ is the next-smallest value of χ^2 found in the integer bias search, and $N_{d.o.f.}^{\;1/2}$ is the number of degrees of freedom of χ^2. In our experience, contrast less than 2 is definitely not sufficient for reliable bias-fixing; a value of 3 indicates that the biases are probably, but not certainly fixable; and a value greater than 4 implies a high level of confidence.

On each of the three days of our six-station test, all the biases (both L1 and L2) for the shortest baseline, the 71-km line between OVRO and Mammoth Lakes, could be fixed with high confidence even when no biases were fixed for any other baseline. The same could not be said for any longer baseline. However, if the biases for the 71-km line were fixed, then all the biases of the 245-km (OVRO-Mojave) baseline could be fixed. The χ^2 contrast statistics for the 245-km baseline, with and without bias-fixing on the 71-km line, are shown for each day in Figure 4.

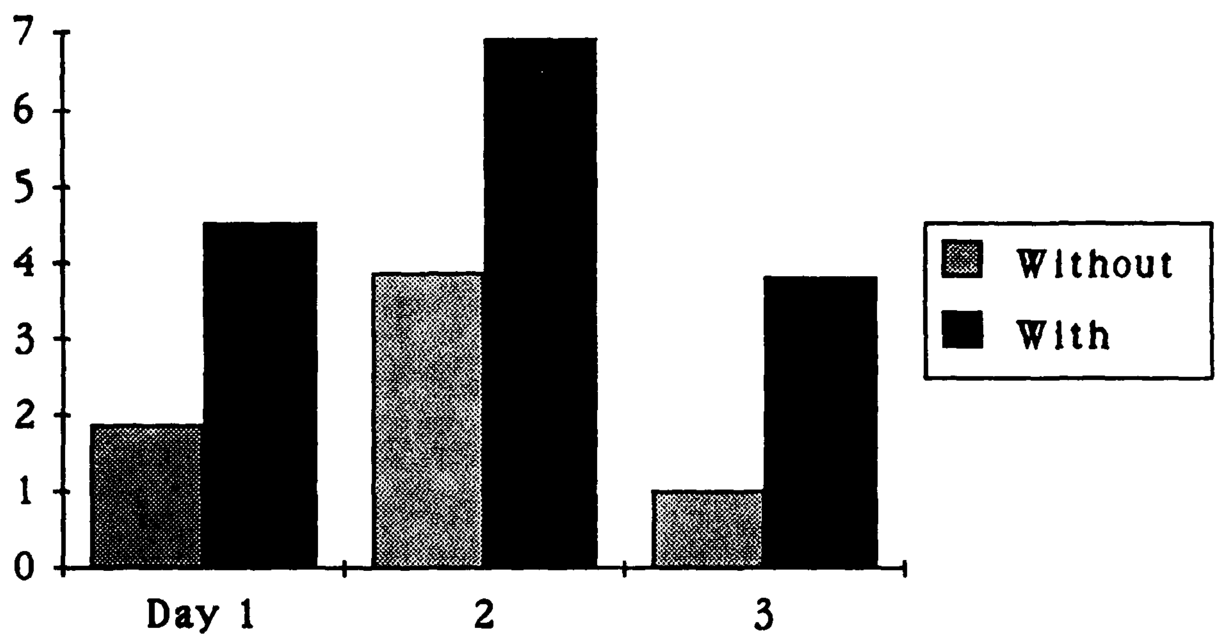

Fig. 4. For each day of the test, χ^2 contrast statistics measuring the ability to determine integer bias values for the 245-km baseline are shown, computed **without**, and **with**, fixed-integer biases for the 71-km baseline.

Improved Baseline Repeatability

The effect of bias-fixing on the accuracy and precision of baseline vector determinations is well known [see, for example, *Bock et al.*, 1985]. Given a fixed representation of the satellite orbits, bias-fixing improves the baseline determination most strongly if the time span of the observations is very short. (Recall our previous discussion of the information content of the mean, and the variation about the mean, of a series of observations.) With a span as long as six hours, as was obtained on each of the days used in the present investigation, bias-fixing is unlikely to have much effect — again assuming a fixed representation of the satellite orbits. On the other hand, if the orbit determination is improved by bias-fixing, the baseline-vector determination should improve. The improvement might or might not be apparent, because effects of site-specific errors, such as unmodeled atmospheric refraction, might dominate and mask the effects of orbital errors.

Figure 5 shows the r.m.s. scatter about the mean of our three daily determinations of each of the Mojave station coordinates — in other words, of our determinations of the 245-km baseline vector, since the coordinates of OVRO were held fixed.

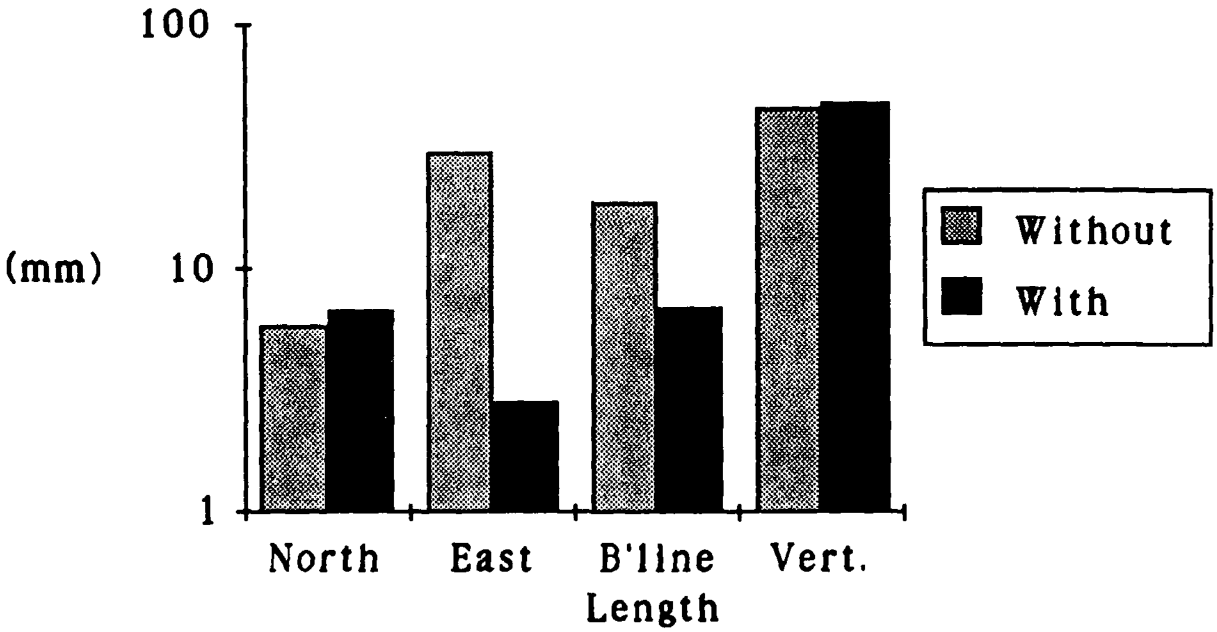

Fig. 5. R.m.s. scatter of the daily determinations of the OVRO-Mojave baseline vector. As many biases as possible were fixed in each baseline determination, given fixed orbits which had been determined **without**, and **with**, bias fixing. Note the logarithmic scale.

Only in the longitude (east) coordinate is any significant effect of bias-fixing seen. Latitude (north) and vertical scatters were virtually unchanged. The baseline vector had enough of an eastward projection, that the reduction of the longitude scatter caused the baseline-length scatter to drop. Consistent changes occurred in the formal standard errors of the coordinate determinations when the biases were fixed. Only the longitude error changed significantly, being reduced by about one-third.

Summary of Results

We have shown that orbit-determination accuracy can be enhanced by including very short baselines in a tracking network of long baselines. In our test network, the length of one baseline was less than 2 percent of the overall network size. We were able to resolve the ambiguities of the doubly differenced carrier phase observations from this baseline because it was so short. Resolving these ambiguities improved the satellite orbit determinations, and made possible the resolution of the ambiguities of a 3.5-times-longer baseline. As a result, orbit-determination accuracy was improved by a factor of two.

This result is remarkable, in view of how short our ambiguity-resolved baselines were. We believe that, given a denser and more regularly spaced progression of tracking-station spacings, we could have carried the ambiguity-resolution further, and would have improved the orbit-determination accuracy further.

DISCUSSION

Use of Group Delay Observations

In these experiments we used no group delay observations although, as Lichten and Border [1987] have shown, simply adding group delay observations to carrier phase observations can improve orbit-determination accuracy substantially. Why this is so may be understood from the paper of Hatch [1982], who showed that the effect of adding group delay observations is qualitatively the same as that of fixing biases. The essential difference is in the error distributions of the bias parameters. If the biases are fixed at integer values, the error distribution is discrete, with (hopefully) only a very small probability of a nonzero error. Simply adding group delay observations without fixing biases yields a continuous error distribution with nonzero width.

Given a choice between fixing phase biases (by any means, but correctly), or simply adding group delay observations, one would prefer to fix biases because the errors would be smaller. Multipath error especially, but other errors as well, are generally much larger in group delay than in carrier phase observations. Since we have shown that carrier phase biases can be fixed without using any group delay data, why use the latter? The answer is that group delay observations can help to fix carrier phase biases. Most importantly, group delay observations from one baseline can help to fix carrier phase biases for other baselines. The leverage gained by going from a long to a short baseline is particularly important, as can be appreciated from our discussion of the bootstrap principle, above.

In the group delay observations from a particular baseline, multipath and other errors may be so great in comparison with the carrier wavelength, or ambiguity spacing, that these ambiguities cannot be resolved directly. But, by constraining the orbit determination, these group delay observations can enable resolution of the ambiguities for another, shorter, baseline.

It follows that group delay observations from the longest baselines in a network are the most valuable, and group delay observations from the shortest baselines in the same network are redundant.

Further Confirmation

Since the original suggestion of the progressively-spaced-baseline method by Counselman [1987] and the preliminary report of test results by Abbot and Counselman [1987], independent reports have further confirmed the utility of the method. Dong and Bock [1988] analyzed observations from a network of baselines with lengths ranging from 30 to 4200 km. For each of three days of observation, they were able to fix all L1 and L2 carrier phase biases for baseline lengths up to 480 km, and most of the biases for baselines up to 1200 km long. Both the day-to-day reproducibility of their baseline determinations, and the agreement with independent determinations by VLBI, were improved when the biases were fixed.

Blewitt [1988] analyzed data from a 12-station network including a wide range of baseline lengths up to 1933 km. Pseudorange (group delay) observations were used to aid ambiguity resolution, which was done by bootstrapping. Some 94 percent of the ambiguities in this network could be resolved. As a result, both the formal errors and the mean repeatability of determining the horizontal components of the baseline vectors were improved by a factor of three. Nearly identical results were obtained, whether the "bias optimizing" method [*Blewitt et al.*, 1987] or the "bias fixing" method of Gourevitch was used to resolve ambiguity.

None of these experiments was designed to answer questions such as, what network configuration is optimal in the sense of yielding the smallest orbital uncertainty, or minimum baseline-estimation uncertainty, with a given number or with the smallest possible number of tracking stations. To investigate such questions, during the recent Global Orbit Tracking Experiment (GOTEX) [*International Association of Geodesy*, 1988] we observed the GPS satellites for 14 days with a "Nautilus" network comprising twelve stations in a logarithmic spiral array. The interstation baseline lengths increased in geometric progression from 10 to 320 km, and each baseline was orthogonal to its shorter and longer neighbors. With these observations and those available from the random, but global, GOTEX network, we are investigating the sensitivity of the bootstrapping to baseline orientation, limits on the baseline lengths and length ratios for reliable bias-fixing under different atmospheric conditions, the use of group delay observations, etc.

CONCLUSIONS

The accuracy of satellite orbit determination from doubly differenced phase observations is enhanced substantially if the integer-cycle ambiguities of the observations are resolved, that is, if the biases are fixed at the proper integer values. An effective method of fixing biases, usable when the tracking stations have a wide ranging progression of spacings, is bootstrapping. In this method, conventional "integrated Doppler" processing of the observations from the most widely spaced stations determines the orbits well enough that the ambiguities for the most closely spaced stations can be resolved. Resolving these ambiguities reduces the uncertainty of the orbit determination enough to enable ambiguity resolution for more widely spaced stations, which further reduces the orbital uncertainty. In a limited test of this bootstrapping strategy with six tracking stations, both the formal and the true errors of determining GPS satellite orbits were reduced by a factor of two. Further investigation is needed to determine what tracking network configurations make best use of the bootstrapping principle.

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