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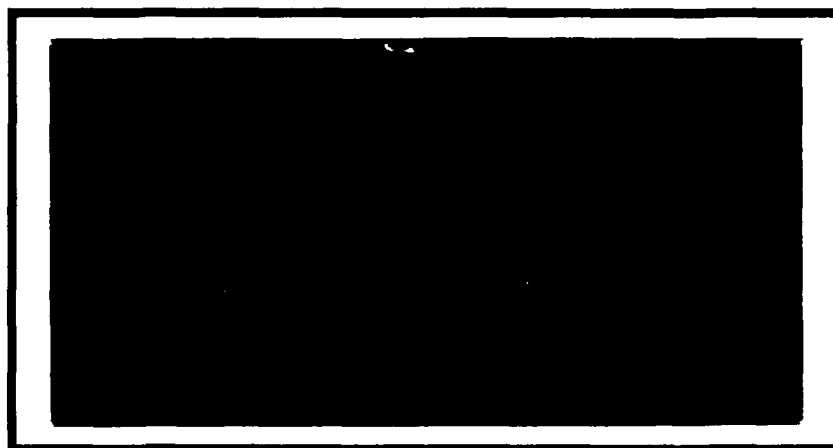MAR 3 0 1989
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OPTIMIZATION OF INVENTORY LEVELS

FOR THE AIR FORCE COMMISSARY SERVICE

THESIS

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FOR THE AIR FORCE COMMISSARY SERVICE

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

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Preface

The purpose for this study was to find an optimal procedure for distributing a fixed allocation of inventory dollars among competing inventory items to provide the greatest constant aggregate availability possible. To this end, techniques of regression were implemented in order to develop an equation which accurately predicts inventory item availabilities.

Items were classified by their review period (7, 14, or 30 days) and an equation was developed from a simulation model for each of these review period values. These equations were used to optimize aggregate availability using the marginal analysis technique.

A great deal of thanks is owed to many people who have assisted me in this endeavor. I wish to offer my deepest appreciation to my faculty advisor, Major Joseph R. Litko, for his guidance and selfless devotion of time in offering insight to this research effort. I also wish to thank Dr. James Chrissis for his involvement and for providing valuable comments with regard to preparation of the final draft. Additionally, I wish to thank my wife Janet for her understanding and patience during this time. And finally, I thank the sovereign Lord, by whose providence I have been kept to see the completion of this work.

<div style="text-align:right">Richard J. Britt</div>

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Abstract

The purpose for this study was to find an optimal procedure for distributing a fixed allocation of inventory dollars among competing inventory items to provide the greatest constant aggregate availability possible. Regression techniques were employed to develop a set of equations which accurately predict inventory item availabilities.

Items were classified by their review period (7, 14, or 30 days) and a response surface was fit from a simulation model for each of these review period values. These surface equations were used to optimize aggregate availability using the marginal analysis technique.

An algorithm was created that generates a shopping list which prioritizes items according to their contribution to increasing the aggregate availability measure. It was generally observed that items having characteristics which produced unstable stock levels appeared nearer to the top of the list than those items having stable stock level characteristics.

This study contributes to the achievement of the primary mission of the Air Force Commissary Service, which is to provide a benefit to military members that promotes a positive attitude towards the Air Force and enhances the quality of life for Air Force members. This in turn

improves morale and retention rates of quality personnel, providing contribution to the overall Air Force mission.

OPTIMIZATION OF INVENTORY LEVELS FOR THE

AIR FORCE COMMISSARY SERVICE

I. Introduction

Background

The Air Force Commissary Service (AFCOMS) is
responsible for the administration and operation of over 130
retail stores worldwide. In 1987, Captain Michael B. Stark,
who at that time was a graduate student at the Air Force
Institute of Technology School of Engineering, conducted a
thesis research effort which compared the performance of
the current inventory control strategy used by AFCOMS with
two alternative strategies that he chose from the body of
literature that existed at that time. (15) The study
investigated the current policies for managing inventories
of selected goods sold at these stores, with particular
attention focused on the Wright-Patterson Air Force Base
commissary as a representative store. The specific problems
which concerned inventory managers were excessive stockouts
(unavailability to the customer of items in demand), an
over-inflated inventory-to-sales ratio, and occasional
inventory losses of goods due to events other than sales
(such as spoilage and damage due to careless handling). The
study evaluated the possible causes of these problems under

the current inventory policy and presented analysis which compared attractive alternatives for consideration as possible replacement policies.

The study provided an initial foundation upon which more in-depth analyses could be based, yielding conclusions and recommendations for further investigation. One of the products which emerged from Stark's thesis was a simulation language (SLAM II) code that modelled the operation of the current inventory system implemented by AFCOMS. This model could be implemented to provide inventory managers a means of setting approximately optimal inventory objectives at the single-item level. Captain Stark presently works for the Air Force Commissary Service (HQ AFCOMS) at Kelly AFB, Texas. Since the submission of his thesis in 1987, he has devoted a great deal of time and effort to the embellishment of the model, incorporating features which were not within the scope of the earlier study. Among these additions are considerations for surges in demand due to military pay days, loss of goods from the inventory due to events other than sales, reorder quantities incremented by cases instead of individual items, and other minor alterations which provide a more realistic representation of the actual store environment. The focus of the study presented herein is to use the current version of the simulation model as a tool in the development of a model which will provide inventory managers a means for managing their inventory systems at the aggregate level.

Problem and Scope

The problem is to find an optimal procedure for distributing a fixed allocation of inventory dollars among competing inventory items to provide the greatest constant aggregate availability possible. The goal of this thesis is to provide a set of guidelines which will enable AFCOMS to manage their industrial inventory fund effectively in the face of increasing budget constraints, while providing the customer with the highest quality service possible.

For the purposes of this study, the single greatest measure of customer satisfaction is having available at the time of purchase as many products in the store as possible which customers might desire. To this end, AFCOMS is unique in that it does not seek to maximize profits, but rather product availability. It is not feasible within the scope of this study to model inventory levels for every product sold, nor to evaluate the hundreds of possible inventory policies which might be adopted. Therefore, 90 products sold in the Wright-Patterson Air Force Base commissary were chosen to represent the inventory at large and to serve in an illustrative capacity for presenting the results of this study. The Wright-Patterson Air Force Base commissary will be used as a representative store because of its convenient location and sufficient size. Data for these 90 products were obtained from AFCOMS.

Objective

A primary use of mathematical models is to establish a way of relating input variables to some output of interest. The output of interest in this study is inventory product availability. This research effort was initiated to provide a means of predicting inventory item availabilities without the actual use of the AFCOMS inventory simulation model. There exists a variety of statistical methods through which experiments can be conducted to this end. The method used in this analysis employs techniques of Response Surface Methodology and Design of Experiments in order to derive an equation that closely approximates the simulation process.

This offers a variety of advantages. The first advantage is a reduction in the time required to obtain an output. To run the simulation for any significant run length (several years of simulated time), requires approximately 17 minutes on an office personal computer. An equation could closely approximate this simulation virtually instantaneously. When thousands of products are being analyzed, the time savings achieved from the equation is rather substantial when compared with the simulation, which can only incorporate a few items at a time per simulation run.

The second advantage is the ease at which sensitivity analysis for the input variables can be conducted. With a simulation, the input parameters for multiple runs must be adjusted until the desired output results through trial-and-

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error. With an equation, however, the output can simply be set at the value desired, and then the equation may be used to find the value at which a particular input must be set in order to achieve that output. An example of this might be to determine the minimum level of safety stock or buffer stock required to maintain an availability rate of, say, 98%.

After the regression equation has been developed, this study seeks to implement the equation in an algorithm that will identify which products should be given priority when allocating funds to safety stock. Safety stock is that portion of the inventory intended as a buffer, or reserve stock, in case the regular inventory supply runs out. Priority will be given to those products that provide the greatest increases in the overall availability of products in the store. Products are to be rank ordered on a "shopping list" that will give inventory managers an efficient means of optimizing aggregate availability. This study also seeks to predict the behavior of an item as characteristics peculiar to that item are changed. The effect of an item's peculiar characteristics upon its position on the shopping list will also be evaluated.

Motivation

The Air Force has much concern for the quality of life of its members. As a result of this concern, a great effort is made to provide benefits to Air Force families which enhance their lifestyle and which promote a positive

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attitude towards Air Force life. Such attitudes will affect personnel retention and job performance, which ultimately will impact mission accomplishment.

The commissary exists to provide an indirect pay compensation for military members by providing food items and other household goods for sale at prices significantly lower than those offered by non-military stores. Profit-maximization is therefore not a factor in establishing an inventory management system for the commissary, but rather the quality of service is of greater importance. To be an effective enhancement to the quality of Air Force life, the commissary must be perceived by its military patrons to be a benefit. This perception will be shaped, to be sure, by the savings gained from using the commissary rather than a non-military store; yet certain annoyances peculiar to the commissary could dampen an otherwise positive perception. Among these annoyances are long lines at the registers, poor quality of service by commissary personnel, and the subject of the thesis presented herein, frequent non-availability of desired items. Efforts to remove these dampening effects will help increase the effectiveness and the quality of the commissary, which in turn will offer greater satisfaction to commissary patrons. It is to this end that this study proceeds.

Conclusion

To effectively develop a means for improving the Air Force Commissary inventory management system, an

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understanding of inventory management theory is appropriate to this study. This understanding will enable the development of an experimental design suitable to the AFCOMS inventory simulation model. An appreciation of experimental design techniques in general will provide great aid in achieving this objective. An adequate review of the literature in both areas is therefore pertinent to this thesis, and will be the focus of Chapter II. Chapter III provides discussion of the particular means which will be implemented towards achieving the stated objectives, with the results of this effort being presented in Chapter IV. Chapter V presents a concluding overview of the entire thesis presented, offering recommendations for future study.

II. Background and Literature Review

Review of Inventory Management Literature

Introduction. Inventory management policies are the procedures and regulations by which store managers maintain a specific level of stock (items waiting to be sold) for each product sold by the store. The development of these procedures is driven by the demand for each item, hence inventory policies are reactive to the level of sales for each item considered.

The broad topic of inventory management has been widely studied for many years; consequently, there exists a rather extensive library of knowledge with regard to the subject. The objective of this study is to summarize the body of knowledge available with regard to the inventory control procedures within the Air Force Commissary Service (AFCOMS). The review will first consider the environment in which the commissary operates and its impact on inventory policies. From an understanding of the environment, four possible methods of inventory control emerge and are discussed. From there, the discussion will center upon the various costs associated with inventory management and the unique approach that AFCOMS must take towards costs. Finally, a critical item of concern stemming from this unique approach will be addressed, namely, the optimal level of reserve items, or

safety stock, which must be kept on hand in order to ensure product availability to meet demands.

Most of the literature available deals with inventory controls in a profit-maximizing firm. This literature review, therefore, tends to focus on the features common to every inventory system, whether in a profit-maximizing firm or non-profit organization. The objective of the literature search and review is to gain insight into inventory systems in general and to develop an appreciation for the contrasts that exist between classical inventory control procedures and the unique procedures of the commissary services. The specific goal in mind is to pursue the best means of tailoring a mathematical model to incorporate those contrasts.

Among the general inventory literature is a study by Eliezer Naddor in 1979 (12) which deals with single and multi-item inventory systems, followed by a study in 1980 by J. L. Millar (9) which was concerned with inventory control as it relates to the supermarket industry. In 1981, Edward A. Silver (13) contributed to the literature with a critique and review of the use of operations research techniques applied to inventory management. With regard to the study of safety stock, notable contributions were made by James Krupp in 1982 (5), and B. van der Veen in 1981. (17)

Discussion of the Literature. Edward A. Silver wrote in 1981 that there are three key questions that inventory managers face:

1. How often should the inventory status be determined; that is, what is the review interval?
2. When should a replenishment order be placed?
3. How large should the replenishment order be?
                                                    (13:629)

These three issues are key elements critical to the objectives of the AFCOMS inventory control study. In order to address these issues, it is necessary to first gain insight into the environment in which the Air Force Commissary Systems (AFCOMS) operates.

The Commissary Environment. Silver points out several aspects of the inventory environment that will act as drivers in establishing inventory control policies. Among them are:

1. Deterministic versus probabilistic demand.
2. Continuous versus periodic review.
3. Backorders versus lost sales.
4. Single versus multiple items.
5. Single versus multiple periods. (13:632-634)

Silver provides an overview of these various environmental aspects. Each of these aspects will be addressed briefly.

Deterministic versus Probabilistic Demand.
Deterministic demand is a very rare state of nature where no uncertainty exists concerning the number of items that will be sold over a period of time. Deterministic demand is more often an assumption to simplify an inventory model than it is an actual occurrence. The idea is that with deterministic demand the exact demand for a product over a specified time period is known.

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On the other hand, probabilistic demand exists when the number of items that will be sold cannot be predicted exactly. This is usually the case in most inventory systems. In the case where supermarket items are being evaluated, such as in the commissary, Millar points out that "demand is random, fluctuations are immediate and trends can be short-lived." (9:109) Therefore, the commissary inventory model will be approached from the perspective of probabilistic demand.

Continuous versus Periodic Review Cycle. Millar defines continuous review as the practice where the level of stock is always known because it is being monitored constantly. (9:109) Periodic review is the practice where stock levels are counted only at discrete time intervals. Silver, in collaborating on a textbook with R. Peterson in 1979, points out that "the main advantage of continuous review is that, to provide the same level of customer service, it requires less safety stock (hence, lower carrying costs) than does periodic review." (14:255) One means of conducting continuous review in the commissary has been implemented in recent years- the Automated Commissary Operations System (ACOS). This is a computerized scanning system located at the checkout aisle that reads a universal product code symbol, registers the sale, and, among other things, keeps a count of how many of each item is sold. According to Millar, the purpose for continuous review is that it enables an inventory manager to place a

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replenishment order for an item as soon as inventories are depleted to a specified level. (9:109) The relationship between ACOS and the feasibility of continuous review was studied in 1987. In his thesis, Stark states that "the Air Force Commissary Service commissioned Bytronic Technologies Corporation to conduct a research study to ensure that stores are receiving the maximum possible benefits of data automation." (15:12) In commenting on the relationship between a continuous review system and ACOS, Bytronic Technologies Corporation states:

> The commissary environment does not quite match the requirements for such a system to work correctly. The scanning system provides the means to have a perpetual (or continuous) inventory count, but orders are only placed when the vendor representatives call. This periodic stock replenishment violates the requirements of a continuous review system. (2:79)

Therefore, the conclusion was made that it is reasonable to model the commissary inventory with periodic review.

Backorders versus Lost Sales. In their text, Silver and Peterson address these two issues:

1. Complete Backordering. Any demand, when out of stock, is backordered and filled as soon as an adequately sized replenishment arrives.

2. Complete Lost Sales. Any demand, when out of stock, is lost; the customer goes elsewhere to satisfy his or her need. (14:253)

Stark noted in his thesis that "although commercial food retailers frequently issue 'rain checks' for out-of-stock specials and in effect 'backorder' a specific item, the

Commissary Service does not . . . backorder any unsatisfied demand." (15:13) He did not incorporate into his model the contingency where a customer will wait until an out-of-stock item is replenished.

Single versus Multiple Items. Silver acknowledges that most inventory modelling deals with "single items in isolation from all other items." (13:632) However, he cites some of the interrelationships which can exist between items in a multiple item model:

1. Overall constraint on budget or space used by a group of items.

2. Coordinated control to save on replenishment costs.

3. Substitutable items - when a particular item is not in stock, the customer may be willing to accept a substitute product.

4. Complementary demand - certain products tend to be demanded together; in fact, the customer may not accept one without the other. (13:632)

Stark excluded consideration of these interdependencies both in the development of his model and in subsequent embellishments. These are reasonable exclusions because the demand data for each individual product incorporates these factors implicitly, and the simulation model deals with only one item at a time.

Single versus Multiple Periods. A multiple period item is one that is sold year-round and can be expected to continue to be sold for an extended period of outyears. Most commissary items fit in this category. However, as

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Silver points out, "in some situations (e.g. style goods, newspapers) there is a relatively short selling season (or period) and remaining stock cannot be used to satisfy demand in the next season (or period)." (13:632) Stark did not give consideration to the single-period case, since it is infrequent in the commissary environment. The current study concurs and will also only address the multiple period case.

Now that a foundation has been laid regarding the environmental influences that are imposed upon an inventory manager when establishing inventory control policies, discussion of the various ways in which inventory reorder algorithms can be modeled can proceed. Naddor of The Johns Hopkins University, discusses three of the most commonly used policies:

> The decision 'when to replenish inventory' is made
> by using one of two kinds of policies. Replenish-
> ments are determined every scheduling period t,
> and only then, or whenever the amount on hand and
> on order is at or below the reorder point s. The
> amount ordered for replenishment can also be deter-
> mined in two ways. It is always a lot-size q (or an
> integer multiple of q), or it is the amount to bring
> the total on hand and on order to a level Z. One can
> thus have three different inventory policies: tZ, sq,
> and sZ. (12:1235)

Silver and Peterson, however, point out a fourth policy, the (tsZ). They discuss each of these four:

1.   (sq). This system involves continuous review
      (that is, (t)(review period)=0). A fixed quantity
      (q) is ordered whenever the inventory position

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(stock on hand minus backorders plus stock on order) drops to the reorder point s or lower.

2. (sZ). This system again involves continuous review and a replenishment is made whenever the inventory position drops to the reorder point s or lower . . . enough being ordered to raise the inventory position to the order-up-to-level (Z).

3. (tZ). This system, also known as a replenishment cycle system, is in common use, particularly in companies not utilizing computer control. The control procedure is that every (t) units of time, enough stock is ordered to raise the inventory position to (Z).

4. (tsZ). This is a combination of (sZ) and (tZ) systems. The idea is that every (t) units of time we check the inventory position. If it is at or below the reorder point s, we order enough to raise it to (Z). If the position is above s, nothing is done until at least the next review instant. The sZ system is the special case where t=0, and the tZ is the special case where s=Z-1. (14:256-258)

Of these four, it must be determined which is the most suitable to the AFCOMS environment. But before this can be done, it must be determined which costs are relevant to inventory control in order to establish some measure of performance for each policy.

Silver defines four categories of costs that are relevant to measuring performance:

1. Replenishment Costs. These are the costs incurred each time a replenishment action is taken. It is convenient to express the costs as the sum of two parts: (i) a fixed component, often called the setup cost, independent of the size of the replenishment; and (ii) a component that depends on the size of the replenishment, in particular, including the cost of the material itself.

2. Carrying Costs. Having material in stock incurs a number of costs including: (i) the cost

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of borrowing the capital tied up or foregoing its use in some other investment, (ii) warehouse operation costs, (iii) insurance, (iv) taxes, and (v) potential spoilage or obsolescence.

3. Costs of Insufficient Supply in the Short Run. When inventory levels are insufficient to routinely satisfy customer demand, costs are incurred, whether or not they are explicitly measured. Unsatisfied demand leads to immediate costs of backordering and/or lost profit on sales. In addition, such poor service can have a longer range cost impact through loss of good will.

4. System Control Costs. This crucial category of costs has largely been ignored in the inventory theory literature. It includes the cost of acquiring the data necessary for the adopted decision rules, the computational costs, and other costs of implementation (including possible adverse behavioral effects of a new system). (13:630-631)

Naddor, in comparing the performances of the tZ, sq, and sZ policies, uses these cost categories to develop an objective function that defines and measures the performances for each policy:

$$C = C_1 + C_2 + C_3 = c_1 I_1 + c_2 I_2 + c_3 I_3$$

where
C_1 = Carrying costs
C_2 = Shortage costs
C_3 = Replenishment costs
c_1 = dollars/unit in inventory/unit time
c_2 = dollars/unit short/unit time
c_3 = dollars/replenishment (disregards amount of order)
I_1 = average inventory carried
I_2 = average shortage
I_3 = average number of replenishments/unit time

(12:234)

After developing numerical examples for each of the three inventory policies, Naddor concludes:

a. The minimum cost for the tZ policy is equal or greater than that for the sq policy which, in turn, is equal or larger than that for the sZ policy.

b. The minimum cost for the sZ policy is about 10% less than that for the tZ policy.

c. The optimal scheduling period in a tZ policy is the same as that in a corresponding deterministic system.

d. The optimal order levels Z* in the tZ and sZ policies are about the same.

e. The optimal reorder points s* in the sq and sZ policies are about the same.

f. The optimal lot size q* in a sq system is about the same as that in a corresponding deterministic system. (12:1238)

Interestingly enough, strong evidence exists that the tsZ policy, which Naddor did not consider, is likely to be optimal. In his thesis, Stark cites two previous studies, namely a study in 1960 by Scarf and a study in 1968 by Eilon and Elmaleh, which both demonstrate why the tsZ policy produces a lower total cost than any other type of system. (15:27) The conclusions from these studies were that the tsZ policy is optimal because it does not incorporate the expense of continuous review, it places a ceiling on the inventory level allowed, and it allows for adequate depletion of stock to take place before reordering. Reorders are conditional, not automatic as in the other methods. The Bytronic Technologies Corporation study which was previously mentioned issued a report which states that a tsZ policy is the best choice for the commissary

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situation. (2:79) The AFCOMS simulation model reflects a tsZ policy.

The Unique Mission of AFCOMS. Since AFCOMS is not oriented towards profit-maximization, but rather to meeting customer demand, particular emphasis must be placed on the subject of safety stock, that is, inventory items kept on hand to ensure that demand will be met. It is not desired to keep a high level of safety stock, because that needlessly drives holding costs upwards. B. van der Veen of the ISA-Research Philips' Industries, Netherlands, addressed this topic in 1980 when he published a practical algorithm for the calculation of an appropriate optimal level of safety stock for a single item. His solution is dependent upon the probability distribution of demand for the item and will vary for different probability distributions. Although the algorithm is simplistic, he asserts that "the results at this stage are not appropriate for application in a computerized inventory control system. Explicit formulae, and not too many, have to be found for handling thousands of products." (17:370) James A. G. Krupp of the Carlyle Johnson Machine Company of Manchester, Connecticut, addressed the subject of safety stock, offering an alternative to the classical statistical techniques, such as used by van der Veen. He proposed "a statistical method for calculation of safety stocks based on units of time (rather than quantity) to provide a safety stock technique which will be reactive to future trends." (5:35) He addresses the

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problems that exists with current statistical methods,
citing:

>A significant phenomenon becomes apparent in reviewing
>the classic statistical safety stock approach: the
>calculated safety stock is a fixed quantity applied to
>all future periods. As forecasts of future demand
>trend upward or downward, the fixed quantity does not
>react to the change. On the upside, therefore, the
>safety stock does not increase proportionately to the
>demand, thus providing the potential of inadequate
>coverage of demand variability. On the downside,
>failure to react can create an exposure to inventory
>excess, especially in the case of a product approaching
>the end of its product life cycle. (5:36)

As he states, "to counteract these problems, we can express
statistical variance of demand in units of time rather than
quantity by converting the deviation to a decimal factor of
the incremental forecast per period." (5:36)

With this goal stated, he generates what he calls a Time
Increment Contingency Factor (TICF) noting that "(with
TICF), as future forecasted demand levels change, the
planned safety stock will change proportionately." (5:37-38)

Summary of Inventory Literature Discussion. The basic
questions that inventory control managers must address in
establishing inventory policies have been observed and
discussed. The answer to these questions, it has been seen,
will be driven by the environment in which the inventory
system operates. Emerging from the environment are the four
basic inventory policies which have been discussed in
detail. Examination has been made of the use of costs as a
measure of performance for each of these policies and these

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costs have been related to the Air Force Commissary Service environment. Lastly, the unique mission of AFCOMS has been examined and an overview of the special attention required for calculating a reasonable, if not optimal, level of safety stock has been provided.

To summarize the AFCOMS environment, AFCOMS incorporates probabilistic demand, periodic review, no backorders, single-item only consideration, and multiple period item only consideration. Stark configured the simulation to reflect these environmental characteristics. The AFCOMS environment necessitates a tsZ policy, which is also modelled by the simulation.

An understanding of this environment shapes the experimental design which is implemented in this thesis. Among the elements affected are selection of factors to be included in the design, the number of equations necessary to adequately capture the simulation's performance for varying review periods, and the reasonable settings of parameters not included as factors, but relevant to the outcome of the experiment.

Review of Experimental Design Literature

Scope. The subject of experimental design has been widely studied and there exists a vast library of literature relating to the topic. This review of the literature is intended to focus on the peculiarities of regression analysis and experimental design as it relates to simulation, particularly emphasizing those techniques used

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in the development of the inventory item availability equations.

This chapter is intended to provide a general overview of the mathematical theory involved in the regression technique and the assumptions under which the experimental design procedure is conducted and implemented.

Several notable textbooks concerning the field of experimental design are available. (1) (8) (10) (11) Two texts which deal with experimental design as it relates to simulation are <u>Statistical Tools for Simulation Practitioners</u> by Jack P.C. Kleijnen and <u>Simulation Modeling and Analysis</u> by Averill M. Law and W. David Kelton. (4) (6) One of these texts should be consulted for a more in-depth discussion than presented herein.

<u>Experimental Design Terminology</u>. Kleijnen provides the following definition of regression analysis: "Regression analysis is a statistical technique that we may use to summarize the simulation output's reaction to changes in the simulation input." (4:133) In his discussion of experimental design, he expands upon this discussion: ". . . we purposefully make changes in the inputs in order to learn as much as possible about the reactions of the outputs." (4:259)

In his text Montgomery gives a brief history of the origins of experimental design terminology:

> Many of the early applications of experimental
> design methodology were in the agricultural and

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biological sciences. As a result, much of the terminology of the discipline is derived from this agricultural background. For example, an agricultural scientist may plant a variety of a crop in several plots [of land], then apply different fertilizers or _treatments_ [factors] to the plots, and observe the effect of the fertilizers on crop yield . . . However, much experimental design, such as "treatment","plot", and "block" have lost their strictly agricultural connotation and are in wide use in many fields of application. (10:6)

Basic Terms. A concise summary of some of the terms encountered in experimental design is provided by Law and Kelton:

In the terminology of experimental design, the (input) parameters and particular structural assumptions composing a model are called _factors_, and the (output) measure of performance is called the _response_. Factors can be either quantitative or qualitative. Quantitative factors are those which naturally assume numerical values. . . Qualitative factors typically represent structural assumptions in a model which do not have a natural numerical meaning. . .(6:370)

In terms of the inventory environment, an example of a quantitative factor would be the average number of a particular item demanded on a daily basis. A qualitative factor might be the review policy or inventory reorder policy. Kleijnen adds further comment with regard to factors. "Two basic concepts that we need now are 'factor and level.' The input of the simulation program corresponds to the factors of the experiment;. . . The _levels_ of a factor are the values of the corresponding input."(4:259) Law and Kelton identify a few more terms common to

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experimental design. They tell us that the average change in the response due to moving a factor j from one level to the next while holding all other factors fixed is the _main effect_ of factor j. (6:373) When the effect of a factor depends on the level of other factors, each factor involved is said to _interact_ with the other factors. (6:372)

Factors in Simulation. Law and Kelton expand their discussion of factors, giving further categories of identification.

> In simulation experiments we can also classify factors as being controllable or uncontrollable. Controllable factors represent policy options available to the managers of the real-world system being modeled. . . Uncontrollable factors, while still being inputs to the model and thus generally having an effect on the response, cannot be manipulated in the real-world system at will by management. (6:370-371)

One of the unique advantages offered from simulation which is not available with other experimental techniques is that any type of environment desired can be created in which to conduct the experiment. Law and Kelton point out that "we have much more control over the experimental conditions than is usually possible in physical experiments. Thus, we can actually control 'uncontrollable' factors ..." (6:371)

Factorial Designs. Ideally, it is desirable to vary the values of the inputs (factors) over as many values (levels) as possible, running the simulation with as many factor-level combinations as allowed. This would give the

most information regarding the response of the output variable to changes among inputs as they go from one level to the next. This is not feasible in most cases, however, since each factor-level setting requires a single observation from each run of the simulation. As the number of factors increases, the number of runs required to incorporate all factor-level combinations is increasing exponentially. For example, if a simulation has 6 input factors with only two levels for each factor, 64 runs of the simulation are required. Every time a factor is added, the number of runs required is doubled. When the experiment is conducted implementing all factor-level combinations, it is called a _full factorial design_. For simulations which involve a great amount of computer system time to produce just one observation, or run of the experiment (or which for other reasons are extremely expensive to run) to implement full factorial designs can be rather costly. Methods exist where the number of factor-level combinations can be reduced and yet a fairly precise measure of response behaviour can be obtained. These methods are called fractional factorial designs. The inventory model used for analysis in this thesis has a short run time, and can incorporate multiple design points per run; therefore, it implements a full factorial design, so discussion of other design methods will be foregone.

Regression. Every time the simulation is run at some setting of the factors, a unique output for that

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factor-level combination is obtained. If there are n
observations, each observation can be expressed in the form
of an equation involving the mean of the observations, the
values of the input factors, interactions between the input
factors, and a parameter which is a measure of error. Each
of the elements of these equations has a coefficient
associated with it. (The coefficient of the mean is always
1.) The values for each of the observations are known, as
well as the values for each of the factors, because these
were set when the simulation was run. The error is not
known directly, nor can the associated coefficients be known
from direct observation. It is the goal of regression
analysis to implement mathematical techniques to find the
values for these parameters. The method most commonly
implemented is the least-squares method of estimation. This
was the method used for the analysis currently under
consideration.

The approach taken with least-squares estimation is to
first organize all of the observations into an array of
length n, which is normally designated y. The factors are
organized into an n x p matrix where n = number of
observations, and p = the number of equation parameters, to
include mean effects. This matrix is called the X matrix.
The first column vector of the X matrix is a vector of the
observation means. The coefficients associated with each of
the X_i values constitute a matrix called the B matrix.
Hence, the first column vector of the B matrix is a vector

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of 1's. The error terms are constitute an array e of
length n. The system can now be expressed in the form of an
equation:

$$y = BX + e. \tag{1}$$

The values observed for y are known, and the values for the
elements of the X matrix have been set. The expected value
of each of the parameters in array e, E(e), is zero. To
find estimated values for B, the following equation standard
to most regression texts is implemented:

$$\hat{B} = (X^TX)^{-1}X^Ty. \tag{2}$$

Once this has been solved and the values for the $\hat{B}$ matrix are
known, these values can be substituted back into Equation 1
and estimates of y_i for each of the n equations can be
obtained, which we call $\hat{y}_i$. Having these two values,
residuals, e_i, can now be calculated. Residuals are
estimates of the differences between the observed values y
and the estimators y.

$$\hat{e}_i = y_i - \hat{y}_i. \tag{3}$$

The sum of the squares of each of the $\hat{e}_i = y_i - \hat{y}_i$ is
called the sum of the squares for error, or SSE. (8:444)
The best estimators for the elements of the B matrix occur
when SSE is minimized. It is upon this principle that
Equation (2) is based. Residuals are assumed to be normally
distributed with a mean of zero and a constant variance.
Box and Draper provide a thorough and sufficient discussion
of residuals in their text (1), which should be referred to

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for more detail. The main use of residuals is in checking

the adequacy of the fitted model.

Assumptions. Kleijnen identifies that the

estimators obtained from Equation 2 provide the best

unbiased linear estimator if the following assumptions hold:

1. The X matrix is not collinear.
2. The regression model is linear in its parameters B.
3. The simulation responses y have constant variance.
4. The responses y are independent.
5. The responses y are normally distributed.
6. The regression model is valid. (4:158)

Most of the time, experiments must be conducted under these

assumptions, with tests following to see whether or not they

do in fact hold true. At this point, brief discussion of

the implications of each assumption is warranted.

If the X matrix is collinear, then the columns are not

linearly independent, and hence no unique solution exists.

If X is collinear, then the inverse $(X^TX)^{-1}$ does not

exist, and therefore the $\widehat{B}$ matrix cannot be found. With

regard to this inverse, Kleijnen notes:

> The numerical properties of the inverse correspond
> to the following statistical properties. An orthogonal
> X minimizes the variances of the least squares
> estimators $\widehat{B}_i$. An ill-conditioned X results in
> large standard errors for the estimators. . . To
> improve numerical accuracy the software may use
> standardized variables, as long as the results are
> presented in the original scales. (4:158-159)

In other words, the values of the factors at each of their

levels can be "coded" to reflect collinearity (and to ensure

orthogonality). The model presented herein reflects such a designed experiment.

With regard to the linearity assumption, Kleijnen points out that this assumption "does not mean that the metamodel is necessarily linear in the simulation parameters [X]. For instance the regression model may be a second-degree polynomial in [X]." (4:160) What this assumption does mean is that the parameters, $\widehat{B}_i$, are coefficients of the X_i, and not exponents.

The constant variance assumption allows the validity of the $\widehat{B}$ matrix as a best unbiased linear estimator to be proven mathematically. Box and Draper confirm that ". . . least-squares estimators of the elements of [B] are the maximum likelihood estimated when the errors are normally distributed with a mean of zero and constant variance." (1:79) Since the error terms are derived from Equation 3 above, the relationship between the response variable constant variance assumption and the maximum likelihood estimator theorem expressed by Box and Draper is apparent. This relationship also provides the key for validating the use of residuals as measure of the constant variance assumption for y. A plot of the residuals versus the fitted values of y is one of the more common methods for testing this assumption. For the constant variance assumption to hold, such a plot should reflect a structureless shape. (10:90-91) Often, the case where the observed values of y do not reflect constant variance is encountered. One of the

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more common ways in which this problem is corrected is to perform a transformation on the observed values y to produce a set of new values which do reflect constant variance. A common transformation used is to take the log of the observations, and use these as the new "observations".

The assumptions for independence and normal distribution of the responses y are related to the constant variance assumption and are required to prove the validity of the $\hat{B}$ matrix as a best linear unbiased estimator. A popular method for testing the normality assumption, the use of normal probability plots, was developed by Filliben:

> A normal probability plot, as here used, is defined as plot of the ith order statistic, X_i, versus some measure of location $loc(X_i)$ [called M_i] of the ith order statistic from a standardized normal distribution . . . Now if the sample was, in fact, generated from a hypothesized normal distribution, then the plot of X_i versus M_i will be approximately linear. (3:112)

A plot can be obtained of the residuals (which are reflections of the observed responses y, having been derived from them) against their position on a standardized normal distribution through the use of specially designed normal probability paper developed specifically for this task. If visual examination reveals that the plot is roughly linear, the normality assumption can be concluded to be valid. Kleijnen points out that "if the responses (or equivalently the errors) are normally distributed, then we can derive confidence intervals for the least squares estimator of the

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regression parameters B_i using standard t and F

statistics." (4:175) This is a key advantage offered by

normality. These test statistics were used to determine the

significance of the various effects of the regression model.

The test for the final assumption, that of model

validity, is perhaps one of the most vital. In the summary

to his discussion of validity, Kleijnen states:

> The specification of a regression model depends
> on common sense, deduction, and induction
> specific to the system being studied. . . Our
> approach applies to the validation of any
> model: we compare the model's forecast to the
> actual response. In case of a regression
> metamodel of a simulation model, our approach
> results in the Studentized forecast error. . .
> We study individual parameters only after we
> have validated the regression model as a whole.
> We test these parameters either individually or
> jointly depending on the context; we do not
> automatically replace a non-significant
> individual B_i by 0. (4:196)

A further method of model validation under concern for

purposes of this thesis is the traditional R-Square

criterion. The R-Square Statistic is found by the equation

$$R\text{-Square} = \sum_{1}^{n} (y_i - \widehat{y}_i)^2 / (y_i - \overline{y})^2$$

As Kleijnen notes, "this criterion suggests that the

regression model gives an adequate explanation whenever

R-Square approaches the value 1. However R-Square always

improves if we add more explanatory variables." (4:193)

Recognizing that this statistic can be deceiving, the

adjusted R-Square statistic, which takes account of the

number of parameters in order to provide a more accurate
measure of fit, can be substituted. This thesis seeks to
obtain R-Square values near 1.

Replications. Law and Kelton point out that

> Since the [responses y_i] are random variables,
> the effects are also random. To ascertain whether
> the effects are 'real,' as opposed to being
> explainable by random fluctuation, we need to
> estimate their variances. . . A very simple approach
> is to replicate independently the whole design n
> times and obtain n independent values of [the
> effects]. (6:376)

The thesis incorporates the use of replications to provide
greater precision in measuring the degree to which error is
affected by variability among the responses.

Second-Order Models. It is often the case that a
strictly linear equation cannot adequately model the true
behavior of a simulation, and an equation reflecting
curvature (i.e. quadratic terms) is more appropriate.
Montgomery notes that "the preferred class of second-order
response surface designs is the class of rotatable designs."
(10:462) He defines a design to be rotatable if "the
variance of the predicted response y at some point X is a
function only of the distance of the point from the design
center, and not a function of direction." (10:462) The
experimental design of the thesis presented herein
anticipated curvature to be present, so a set consisting of
several different rotatable central composite designs was
evaluated to determine an appropriate quadratic technique

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with which to model the AFCOMS inventory simulation. The uniform-precision design offered attractive features sought, and was chosen from among the alternatives. With regard to the features offered by this design, Montgomery points out:

> In a uniform-precision design the variance of y at the origin is equal to the variance of y at unit distance from the origin. A uniform-precision design affords more protection against bias in the regression coefficients because of the presence of third-order and higher terms in the true surface than does an orthogonal design. (10:462)

A feature of this design common to other central composite designs is that the design matrix consists of a 2^K factorial design (where K = number of factors) augmented by 2K axial points and multiple runs of the simulation performed at the center point. The greek letter alpha is used to designate the distance of the axial points from the center. The appropriate setting of alpha in uniform precision designs is determined by the number of factors K. The value for K will also determine the number of center point replications required. Montgomery (10:463) cites an earlier work by Box and Hunter which presents a table of the various appropriate settings for alpha and the number of center points to be used for different values of K in various rotatable central composite designs. This table was used to develop the design presented herein. The uniform precision design implemented for this study is provided in Figure 2.

Summary of Experimental Design Discussion. An overview
of the various terms peculiar to experimental design have
been provided to increase understanding of its processes.
Factorial designs were evaluated and found suitable to the
objectives of this study. Assumptions regarding the
conditions which produce the best linear unbiased estimator
were discussed, with the various tests of the validity of
these assumptions included. The literature offered a
variety of designed experiments which could be used to
derive the regression equations sought by this study.
Anticipating curvature, a uniform-precision rotatable
central composite design was chosen as the design for the
experiment being considered.

Summary of Literature Review

With a sufficient understanding of inventory theory,
combined with an appreciation for experimental design and
regression techniques, a proper approach can be taken
towards designing an experiment that builds a metamodel of
the AFCOMS inventory simulation. Insight has been gained
from the literature in both areas. A good foundation has
been laid from which beneficial analysis can be conducted,
and a designed experiment can be developed to reflect the
AFCOMS inventory simulation model.

III. Methodology

Objective

The methods employed in this thesis are divided into two phases, each of which seeks to fulfill one of two primary objectives of the study. Phase I seeks to employ the experimental design techniques from the literature to derive regression equations which accurately duplicate the process of the AFCOMS inventory simulation model, translating input parameters to an output measurement of interest.

The second phase of this thesis uses the regression equations developed in Phase I in an algorithm which enables inventory managers to prioritize products as they allocate a fixed inventory budget in the building of a reserve inventory of safety stock. The chief aim of this prioritization is to maximize the aggregate availability of the entire inventory.

Data

The data used for both phases of the experiment consists of the characteristics of 90 products which are actually sold in the Wright-Patterson Air Force Base Commissary. The parameters associated with each product were obtained from HQ AFCOMS. For Phase I, the data was used to determine the region of the experimental design variables. In Phase II, the 90 products were regarded as

the inventory population under evaluation to determine which of the 90 products should be given priority over the others when ordering safety stock in an effort to maximize aggregate availability.

Regression Procedure for the First-Order Model

For this analysis, Captain Stark's simulation model was implemented using the output value for Not-in-Stock (NIS) rate as the response variable. There were a variety of potential factors, both controllable and uncontrollable, which could have been used in the experiment. Four factors were chosen for the experiment. They are: (1) Average Daily Demand for a product (DEMAND); (2) Standard Deviation of Daily Demand (SDDMD); (3) the Leadtime between the time an order is placed and the time it arrives to the inventory (LEADTIME); and (4) the amount of safety stock carried in the inventory (SAFE). The last factor, safety stock, is the only input that is controllable in the real-world system. The other three factors are determined by the products themselves, and if not for the features offered by simulation, would otherwise be uncontrollable.

The other "inputs" were set throughout the experiment at values typical of the Wright-Patterson commissary. Among these are vendor fill rate and the shape of the triangular distribution for leadtime. One final factor, review period, was considered critical to the value of the response variable, but rather than incorporating this as an additional factor to the experiment, three separate

regression equations were developed for the three typical values which review period might have. These values are 7, 14, and 30 days, respectively. Developing three equations as opposed to incorporating an additional factor enabled retention of a 2^K factorial design, rather than expanding to a three-level factorial design for the first-order model.

As mentioned earlier, since the run time for the simulation under consideration is short and the simulation can incorporate several design points per run, a full-factorial design can be implemented when only four factors are incorporated. With 4 replications, 64 independent observations are required for each equation. Each of the four factors was assigned a high and a low level for the first-order design.

Recognizing that DEMAND has a tremendous range, the experiment concerned itself only with the region where DEMAND ranges from 20 to 44 items demanded per day. This range was not selected arbitrarily, but was based rather upon the ranges associated with the 90 products themselves. These values were chosen as the respective high and low settings for the factor DEMAND.

When determining the high and low settings for the second variable, SDDMD, it is noted that the value for this variable is dependent upon the value for the first variable, DEMAND. When DEMAND is at its low value, SDDMD has two settings, depending on whether or not SDDMD is at its high

or low value. Likewise, when DEMAND is at its high value, SDDMD again has two values, a high and a low.

So while there are two levels for DEMAND, there are actually four values that SDDMD might have. This situation was resolved by re-stating SDDMD as the product of DEMAND and a fractional constant, C, where SDDMD = DEMAND*C. C was then set at a high or low value, which was a measurement of the degree of variability of demand. For the low value, C was set at 0.25 to reflect mild variability in demand, and for the high value, C was set at 0.75 to reflect a greater variability in demand. With regard to the fourth variable, SAFE, which is a measurement of the number of days the current value of DEMAND can be satisfied at the present level of safety stock, it is recognized that the following equation holds true:

No. Items of Safety Stock = DEMAND * SAFE

Thus, Table 1 was built to reflect the variables and their coded values.

Table 1

Coded Values for First-Order Full-Factorial Design

	LOW	CODED VALUE	HIGH	CODED VALUE
DEMAND	20	-1	44	+1
C = SDDMD/DEMAND	0.25	-1	0.75	+1
LEADTIME	10	-1	20	+1
SAFE	2	-1	6	+1

Notice that the settings for the variable SAFE has the smaller number for the low setting. In view of the fact that the value for the response variable, NIS, is inversely proportional to the value for the amount of safety stock, it is observed that the greater the amount of safety stock that is in the inventory, the lower the Not-in-Stock rate will be. (i.e. The more safety stock there is, the less likely it is that the inventory will sell out.) With this in mind, the observed responses will reflect higher values for low settings of safety stock.

Thus, with 4 replications of a 2^4 design, 64 total observations were required for each equation with varying review periods of 7, 14, and 30 days. For the remainder of this discussion, however, the 14-day review period model is used for illustration.

In consideration of the fact that the response variable, NIS, is a percentage, and therefore bounded on both sides at 0 and 1, a transformation was anticipated in order to obtain constant variance for the observed responses. The standard transformation LY = LOG(Y) did not produce a non-structured residual plot as hoped, so a slight variation of the LOG transformation was implemented, using LY = LOG (Y + 0.05). The addition of the small constant resulted in the non-structured residual plot required for constant variance assumptions to hold. The residual plot for the 14-day review period first-order experiment is provided as Appendix A. Appendix B is the residual plot

which checks for normality. Figure 1 displays the structure
of the design matrix implemented for the first- order
experiment.

Figure 1

Design Matrix For First-Order Model

DEMAND	C	LEADTIME	SAFE
-1	-1	-1	-1
+1	-1	-1	-1
-1	+1	-1	-1
+1	+1	-1	-1
-1	-1	+1	-1
+1	-1	+1	-1
-1	+1	+1	-1
+1	+1	+1	-1
-1	-1	-1	+1
+1	-1	-1	+1
-1	+1	-1	+1
+1	+1	-1	+1
-1	-1	+1	+1
+1	-1	+1	+1
-1	+1	+1	+1
+1	+1	+1	+1

The design matrix of Figure 1 was encoded into a SAS
file implementing the PROC GLM procedure, and checks were
made for significant main effects and interactions. The
first run of the SAS PROC GLM procedure revealed that the
main effects of SDDMD, LEADTIME, and SAFE were all
significant. Significant interaction effects included a
3-way interaction between these same three effects. Main
effects of DEMAND were not significant. The R^2 value was
0.988446.

Having anticipated a higher-order model from the
outset, further analysis of the first-order results were

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overlooked, and the next step was to investigate the model for possible second-order terms. Implementing techniques of R. H. Myer's text (11), to fit a uniform precision second-order model, the simulation was run at the center points in order to conduct the F-test for curvature. The number of center points was obtained from a table in Montgomery's text, citing G.E.P. Box and J.S. Huiter's article "Multifactor Experimental Designs for Exploring Response Surfaces." For a uniform precision design with 4 factors, 7 center points should be implemented. (10:463) With 4 replications, the total number of center points required was 28, and the total number of design points is 64. The mean of the 28 responses at the center points was 0.0342179 with a variance of 0.0000294. The mean of the 64 responses at the corner points was 0.0395 with variance of 0.000408. Using the equation from Myers (11):

$$\frac{(64 \times 28)*(0.0395 - 0.0342179)^2}{64 + 28}$$

the value 0.0005435 is obtained. This value reflects SS_{quad}. The value for the F-statistic is measured by:

$$F = SS_{quad} / \text{variance for center points}$$

This F-statistic is found to be (0.0005435 / 0.0000294) which equals 18.4864. The F Table value is less than 3.0, so it is determined that the statistic is very significant, and curvature is present. Therefore, the second-order uniform precision design was implemented. These results are assumed to be conclusive for the 7-day and 30-day review

period models as well, so first-order models were passed over and second-order models were implemented directly for these other experiments.

Regression Procedure for the Second-Order Model

The simulation experiment was run again to gain observations for axial points. Box and Hunter recommend a coded axial setting at alpha = 2.000 for a uniform precision rotatable central composite design. (10:195) With center points included, Table 2 provides the settings and coded values for the additional second-order design points.

Table 2

Coded Values for Second Order Uniform Precision Design

	CENTER	CODED	- ALPHA	CODED	+ ALPHA	CODED
DEMAND	32	0	8	-2	56	+2
C = SDDMD/DEMAND	0.5	0	0.0	-2	1.0	+2
LEADTIME	15	0	5	-2	25	+2
SAFE	4	0	0	-2	8	+2

Figure 2 is provided to display the design structure for the second-order uniform precision rotatable central composite design. Note that additional center points are incorporated in order to round-out the uniform precision design.

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Figure 2

Design Matrix for Second-Order Uniform Precision Design

DEMAND	C	LEADTIME	SAFE
-1	-1	-1	-1
+1	-1	-1	-1
-1	+1	-1	-1
+1	+1	-1	-1
-1	-1	+1	-1
+1	-1	+1	-1
-1	+1	+1	-1
+1	+1	+1	-1
-1	-1	-1	+1
+1	-1	-1	+1
-1	+1	-1	+1
+1	+1	-1	+1
-1	-1	+1	+1
+1	-1	+1	+1
-1	+1	+1	+1
+1	+1	+1	+1

SEVEN CENTER POINT REPLICATIONS

DEMAND	C	LEADTIME	SAFE
-2	0	0	0
2	0	0	0
0	-2	0	0
0	2	0	0
0	0	-2	0
0	0	2	0
0	0	0	-2
0	0	0	2

FIVE CENTER POINT REPLICATIONS

Phase I Conclusion

The methodology of Phase I offered the best
conditions under which to conduct an experimental design
procedure to estimate the effect of different combinations
of the simulation input parameters upon the response
variable of inventory Not-in-Stock rate. Evaluation of the
first-order design revealed that curvature was present, and
therefore a uniform precision rotatable central composite
design was implemented to evaluate second-order terms and

interactions. The uniform precision design was used for three different experiments involving inventory review periods of 7, 14, and 30 days, respectively, with the 14-day review period model used as the initial test-case.

Phase II Methodology

Another recent study in 1986 at the Air Force Institute of Technology by Major Rui M. Pereira Lopes of the Portuguese Air Force examined a similar situation to what is being evaluated with this thesis. His study sought to maximize the availability of aircraft for the Portuguese Air Force through properly managing inventory levels of spare parts for the planes in the face of limited budget allocations for spare parts. The similarity to the AFCOMS inventory availability problem is striking, with commissary items closely paralleling the spare parts of the other study. Major Lopes, in the development of his model, implemented the following strategy in order to achieve his objectives:

> The next step in the development of the model is to address the optimization question: given a budget constraint, what spares [inventory items] should be procured to attain the best possible availability rate? The marginal analysis technique is the optimization procedure used . . .The last step and the ultimate goal for the model is the discussion of the algorithm used to generate the aircraft availability curve and the shop lists. (7:43)

Major Lopes' model for the Portuguese Air Force was based upon a similar model developed by the Logistics Management Institute (LMI) for the United States Air Force, the

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Aircraft Availability Model. (16) This study proposes to apply the concepts and methodology of the Aircraft Availability Model (AAM) to the AFCOMS inventory system. The problem statement can be expressed:

$$\text{Maximize} \quad A = \prod_{i=1}^{90} A(n)_i \qquad (4)$$

s.t. ($ Fixed Inventory Budget)

where

A = Measure of Overall Availability (for entire system)

$A(n)_i$ = Individual Item Availability

n = number of casepacks of safety stock for product i

The fixed inventory budget constraint is actually an imposition resulting from maximum inventory-to-sales ratio goals. These goals indirectly place a ceiling on the amount of money that can be allocated towards inventory procurements. These procurements are further divided into safety and non-safety stock portions of the total inventory. It is the safety stock portion of the inventroy that is reflected by Equation (4).

The Measure of Effectiveness (MOE) for $A(n)_i$ is the item availability rate, derived from the regression equations obtained in Phase I: $A(n)_i = 1 - NIS_i$, where NIS is the response variable for the regression equations. The marginal analysis technique can be easily implemented using these regression equations. To understand this technique, consider Equation (4). The goal is to determine

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which is the next inventory item that will be sought for procurement if given additional inventory dollars. This will be the item that provides the greatest increase in overall availability, A, as measured by Equation (4).

Keep in mind that additional items aι not procured one item at a time, but rather must be ordered in casepack quantities at a time. After the inventory level has been adjusted and a new A has been recalculated, the next item yielding the greatest increase in availability when an additional casepack of that item is brought into the inventory can be sought. The procedure is repeated until a "shopping list" which rank orders items in terms of decreasing benefit per unit cost has been generated, where benefit is defined as the increase in A which would occur if the next inventory dollar allotment were to be given towards the item. The inventory manager would then know which items should be given priority under budget constraints which do not permit everything on the "shopping list" to be procured.

The Optimization Procedure. The regression equations developed in Phase I can be expressed as functions of some known constants (these constants being obtained from the product input parameters and the regression coefficient estimates) and the controllable variable factor, SAFE. Let the amount of safety stock be denoted as $SAFE(n)_i$, where n = the number of casepacks of safety stock, and i = the product identifier from 1 to 90. $SAFE(n)_i$ is found from the relationship between safety days and average daily

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demand.   The regression equations can be written as:

$$NIS_i = \{K_1 + K_2 * SAFE(n)_i\}$$

and the corresponding individual item availability equation as:

$$A(n)_i = 1 - NIS_i = 1 - \{K_1 + K_2 * SAFE(n)_i\}$$

If a starting inventory level with no safety stock, (i.e. n = 0) is projected, then the aggregate availability for the inventory with no safety stock can be measured by the equation:

$$A = \prod_{i=1}^{90} [1-NIS]_i = \prod_{i=1}^{90} A(0)_i$$

This equation can alternately be written as:

$$A = \prod_{\substack{i=1 \\ i \neq j}}^{90} A(0)_i * A(0)_j \qquad\qquad (5)$$

The aggregate availability rate after the first additional casepack of item j is procured is:

$$A' = \prod_{\substack{i=1 \\ i \neq j}}^{90} A(0)_i * A(1)_j$$

Thus, in general, the aggregate availability rate after n additional casepacks of item j is added can be stated as:

$$A' = \prod_{\substack{i=1 \\ i \neq j}}^{90} A(0)_i * A(n)_j \qquad\qquad (6)$$

To calculate the ratio of the new availability rate to the old availability rate, A'/A, Equation (6) is divided by

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Equation (5). When this is done, it is found that the term common to both the numerator and denominator:

$$\prod_{\substack{i=1 \\ i \neq j}}^{90} A(0)_i$$

cancels out, and the ratio can be found simply by:

$$A'/A = A(n+1)_j/A(n)_j \qquad (7)$$

This ratio is called the improvement factor, $I(n)_i$. In general, $I(n)_i = A(n)_i/A(n-1)_i$. The cost component of product i is defined to be $Cost_i$, and is merely the product of the unit price for item i and the number of items per casepack of product i. The sort value of the nth casepack of safety stock for item i, $S(n)_i$, can now be defined by:

$$S(n)_i = LOG[I(n)_i]/Cost_i \qquad (8)$$

$$S(n)_i = LOG[A(n)_i/A(n-1)_i]/Cost_i$$

The sort value is the measure of benefit per cost that is used to sort the candidate items for procurement. As pointed out in the AAM Analyst's Manual:

> The natural logarithm function in the expression
> of benefit arises because of the multiplicative
> nature of availability, as expressed in the product
> formula [Equation (4)]. It ensures the
> mathematical accuracy of the optimization." (16:9)

The "shopping list" is generated by ranking the sort values from greatest to least, realizing that n casepacks of a

product must be in safety stock before the $n + 1$ casepack can be procured.

Summary of Phase II Methodology

The result of this effort is a model which acts as a management tool, enabling inventory managers to distribute a fixed allocation of inventory dollars among competing items, by rank-ordering items on a "shopping list" which lists the subsequent item to which the next inventory dollar should be allocated in order to provide the greatest increase in the aggregate availability measure of the system.

In Chapter IV, the methodology presented in this chapter is implemented, accompanying discussion of the results obtained from using these procedures.

IV. Results

Introduction

The results of the Experimental Design (Phase I) and the Availability Optimization (Phase II) portions will each be addressed in the order in which the respective analyses were performed. For discussion of Phase I, the ANOVA tables, one for each of the three experiments involving 7, 14, and 30 day review periods, will be presented first, with the significant effects for each experiment. The resulting regression equations for each experiment will then be evaluated for accuracy in portraying the real-world expectations of inventory system behavior. This will provide a means for assessing the validity of the AFCOMS simulation model.

The discussion of Phase II will center upon the "shopping list" and the relationship between inventory dollars spent and the associated aggregate availability.

ANOVA Tables

The 7-Day Review Period Case. Table 3 reflects the ANOVA Table for the experiment involving inventory items having a 7 day review period. This table is obtained from the final SAS processing of the simulation data, and reflects values for the final fitted model with only significant parameters included. The student t-statistic was the basis used for testing the significance of

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parameters, with a 0.05 alpha level. The experiment was conducted using 10,000 days of simulated time. The resulting R-Square statistic of 0.976663 reflects a high degree of accuracy and goodness-of-fit.

Table 3

ANOVA Table for 7-Day Review Period Experiment

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	6	2.68278064	0.44713011	202.28
ERROR	29	0.06410402	0.00221048	PR > F
CORRECTED TOTAL	35	2.74688466		0.0001

R-SQUARE	C.V.	ROOT MSE	LOG(Y+.05) MEAN
0.976663	1.8507	0.04701578	-2.54038241

The significant effects for this experiment are the mean effect; the main effects of C, LEADTIME, and SAFE; the interaction effect between C and SAFE (C*SAFE); and the second-order main effects of C^2 and $SAFE^2$. Let the following variables represent the respective factors.

$$Y = NIS$$
$$X1 = DEMAND$$
$$X2 = C = SDDMD/DEMAND$$
$$X3 = LEADTIME$$
$$X4 = SAFE \text{ (days)}$$
$$X1S = DEMAND^2$$
$$X2S = SDDMD^2$$
$$X3S = LEADTIME^2$$
$$X4S = SAFE^2$$

The regression equation for the 7-day review period experiment is therefore:

Not-in-Stock = EXP[-2.6039 + 0.17989*X2 + 0.09725*X3
- .24936*X4 - 0.05095*X2*X4 + .04571*X2S + .04949*X4S]
- 0.05

Note that this equation applies to _coded_ values of the factors. For example, an increase in the amount of safety stock causes an increase in the coded variable X4 = SAFE, resulting in a corresponding decrease in the Not-in-Stock (NIS) rate. This will be an important point to remember throughout this discussion.

The standard error measurements are as follows:

Estimate	Std Error
INTERCEPT	0.01108173
X2	0.00959706
X3	"
X4	"
X2*X4	0.01175395
X2S	0.00831129
X4S	"

The 14-Day Review Period Case. Table 4 reflects the ANOVA Table for the final fitted model of the experiment involving inventory items with a review period of 14 days. This experiment also simulated 10,000 days. The R-Square statistic of 0.970437 is almost as high as in the 7-day case, and also reflects a high degree of accuracy and goodness of fit.

Table 4

ANOVA Table for 14-Day Review Period Experiment

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	6	1.26477958	0.21079660	158.66
ERROR	29	0.03852985	0.00132862	PR > F
CORRECTED TOTAL	35	1.30330943		0.0001

R-SQUARE	C.V.	ROOT MSE	LOG(Y+.05) MEAN
0.970437	1.3955	0.03645018	-2.61188786

The significant effects of this experiment are the same as in the 7-day experiment: the mean effect; the main effects of C (X2), LEADTIME (X3), and SAFE (X4); the interaction effect of C (X2) and SAFE (X4); and the second-order main effects of C^2 and $SAFE^2$. The regression equation for the coded values of the factors for the 14-day review period experiment are:

Not-in-Stock = EXP[-2.66917 + 0.12877*X2 + 0.05896*X3
 - 0.15939*X4 - .05711*X2*X4 + .05024*X2S + .03568*X4S]
 - 0.05

The standard error measurements are as follows:

Estimate	Std Error
INTERCEPT	0.00859139
X2	0.00744036
X3	"
X4	"
X2*X4	0.00911255
X2S	0.00644354
X4S	"

The 30-Day Review Period Case. Table 5 reflects the ANOVA Table for the final fitted model of the experiment

involving inventory items with review periods of 30 days. The length of simulated time for this experiment was also set at 10,000 days. While the R-Square statistic of 0.845821 was not as high as for the other experiments, it is still high enough that the model can be used with reasonable confidence.

Table 5

ANOVA Table for 30-Day Review Period Experiment

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	6	0.31878164	0.0531302	26.52
ERROR	29	0.5810850	0.0020037	PR > F
CORRECTED TOTAL	35	0.37689014		0.0001

R-SQUARE	C.V.	ROOT MSE	LOG(Y+.05) MEAN
0.845821	1.7019	0.04476317	-2.63025791

The significant effects of this experiment are almost the same as the other two experiments: the mean effect; the main effects of C (X2) and SAFE (X4); the interaction effects of DEMAND and LEADTIME (X1*X3) and C and SAFE (X2*X4); and the second-order main effects of C^2 (X2S) and $SAFE^2$ (X4S). The difference between the significant effects of this experiment and the other experiments is that X3 no longer is significant as a main effect, but is significantly interacting with X1, the DEMAND term. The regression equation for the coded values for the factors of the 30-day review period experiment are:

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```
Not-in-Stock = EXP[-2.66813 + 0.07168 X2 + 0.04332 X1*X3
   - 0.06388 X4 - 0.02758 X2*X4 + .03603 X2S + .02078 X4S]
   - 0.05
```

The standard error measurements are as follows:

Estimate	Std Error
INTERCEPT	0.01055078
X2	0.00913724
X1*X3	0.01119079
X4	0.00913724
X2*X4	0.01119079
X2S	0.00791308
X4S	"

Model Validity

One observation which is noted when comparing the three
fitted models is that nearly all of the same effects are
significant from one model to the next. This lends
credibility to the accuracy of the range of the experiments.
The fact that the magnitudes of the coefficients for each of
the significant effects are different from one model to the
next reflects that the review period itself affects the
parameters of the model. Each of the significant effects
will be individually evaluated to gain an understanding of
the processes being modelled and to determine if these
processes are logical and consistent with what classical
inventory theory would predict.

Significant Factors.

First-Order Terms. The discussion proceeds with
evaluation of the first-order factors which might generally
be expected to be significant, dealing with them one at a
time in order.

The simulation incorporates a reorder algorithm that computes the amount of stock to be ordered. This amount is based primarily upon the product's demand, as well as other significant but less influencing factors. Hence, if the value of the DEMAND parameter is changed from one product to the next, the simulation merely makes appropriate adjustments to incorporate these changes. Therefore, the value for the DEMAND parameter would be expected to have little influence on the outcome of the experiment. And, as each of the experiments has shown, this is in fact the case, as the main effects of DEMAND were not significant.

The variability of DEMAND, SDDMD, will however impact the liklihood of running out of inventory stock. A high variability, for example, allows unstable surges in demand which might cause a stockout before the stock can be replenished. Thus the ratio, $C = SDDMD/DEMAND$ should have an effect on the value of the response. The expectations are once again demonstrated to be correct, as it is found that C is a significant factor in all three experiments. The positive coefficient for C is logical since increasing variability should yield increasing Not-in-Stock (NIS) rates.

The value of LEADTIME is a measure of how quickly the inventory can be replenished. Intuitively, this value would be expected to affect the response variable of Not-in Stock rate. If demand remains consistent throughout a reorder cycle, and the LEADTIME value falls on the high end of the

distribution (i.e. the inventory is unable to be replenished as quickly as usual), it should come as no surprise that a shortage would be experienced and the item would sell out. And it is in fact found that the main effects due to LEADTIME are significant in all three of the models.

A factor which obviously exerts a great influence on the response is that of safety stock, represented by the variable SAFE = SAFETY STOCK / DEMAND. SAFETY STOCK is actually driven by the value for DEMAND and SAFE (the number of "safety days", or days which the expected demand could continue to be met with just the safety stock and no reorders.) Since DEMAND is determined by the product, any changes to the level of safety stock must come from a change in the value of SAFE, or safety days. Recall that this variable is the controllable variable of the experiment. It is expected that SAFE would be a significant factor throughout the analysis, and indeed it is found to be so. Keep in mind, once again, that as the value of SAFETY STOCK is increased, the coded value for SAFE increases, causing the response, NIS, to decrease. Thus, _increases_ in SAFETY STOCK result in _decreases_ in NIS, as they should. As a result of this, the coefficient for the variable SAFE is negative in all three experiments.

Interaction Terms. It is expected that some characteristics of a product do not act independently of other characteristics with regard to their effect on the NIS rate. Each of the three experiments reflects two

factors which interact with one another in their effect upon NIS. These factors are the variability factor, C, and the amount of safety stock, reflected by SAFE. It should be noted that the main effects of each of these factors contributes _positively_ to the response, while the interaction term has a _negative_ coefficient. The insight offered by this observation is clear. Sudden and unexpected surges in DEMAND (reflected by C) act in a manner which increases the liklihood of a stockout. Likewise, a low value of safety stock (reflected by SAFE), also acts in such a way as to increase the liklihood of a stockout. But not all of the contribution of either factor can be attributed solely to the factors individually. For example, a sudden surge in demand coupled with a low level of safety stock could cause an increase in NIS that would not have been observed with just the sudden surge in demand or simply the low level of safety stock alone. Therefore, the equation reflects a positive contribution to the NIS rate from each factor, then subtracts that portion of the contribution which was caused by overlap from the two factors working together.

There is another interaction effect observed, occurring only in the 30-day review period experiment, which is the effect due to interaction between DEMAND and LEADTIME. It is made even more peculiar by the fact that it is the only significant term in any of the experiments which involves DEMAND. An explanation for the presence of this term is not

easily derived, but it is likely a reflection of the relationship between DEMAND and LEADTIME as they contribute to the establishment of the Stock Control Level (Order-up-to level), which in turn impacts the response.

Second-Order Terms. Since DEMAND is not an expected significant factor, it should not be expected that its second-order term would be either. And indeed it is not found to be significant. Yet, second-order terms for two of the other factors are found to be present in all three experiments.

The variable factor C^2 = (SDDMD/DEMAND)^2 is significant in all three of the experiments. The impact of this second-order term is that it causes a decreasing marginal effect of the first-order term, C, on the response variable. To illustrate, observe the graph of Figure 3. As the value of the ratio C increases with other factors set at their mid-points, the variability of demand becomes greater, resulting in frequent surges of demand which increase the liklihood of a stockout. Likewise, as C becomes smaller, the opposite is true. Demand approaches a nearly deterministic value as the variability of demand approaches zero, and the curve levels off, indicating that inventory levels stabilize in this region of the graph with stockouts less likely.

Hence, as demand approaches nearly deterministic characteristics, then small incremental changes in the

Figure 3

**EFFECT OF INCREASES IN DEMAND
VARIABILITY ON ITEM AVAILABILITY**
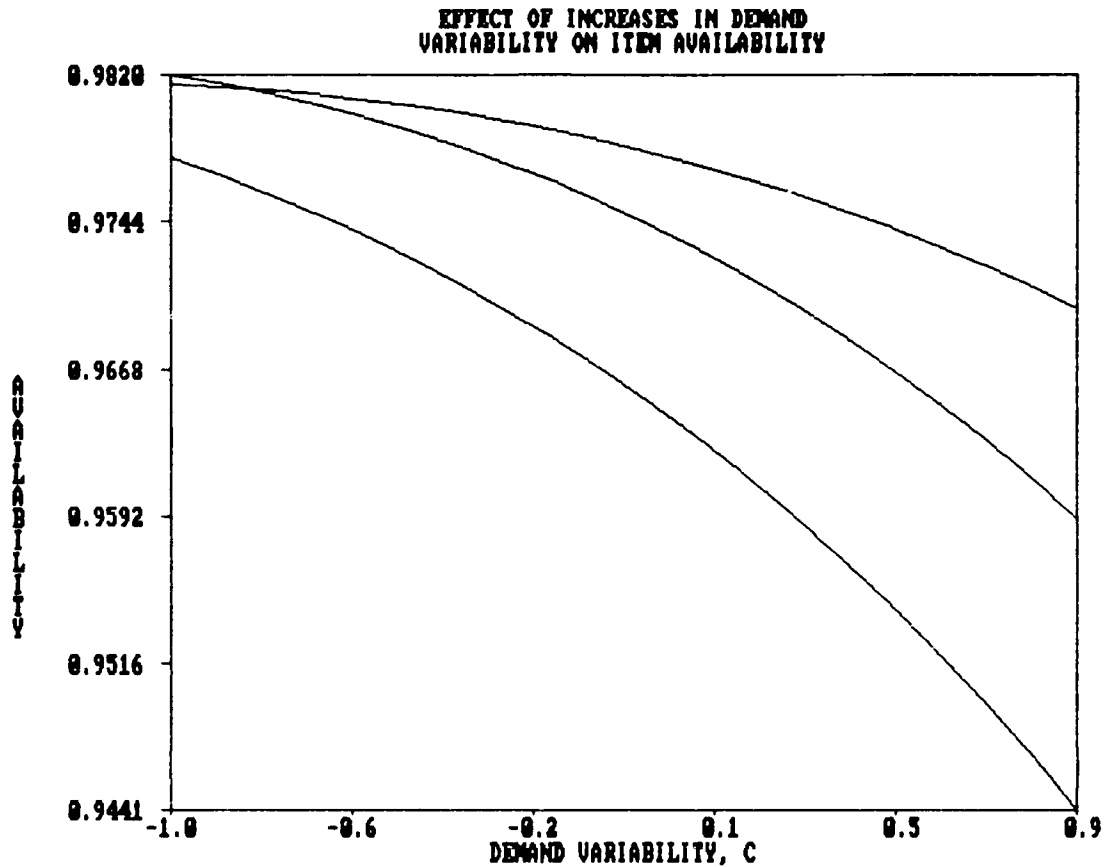
variability of demand will have less and less of an effect on changing the response.

The other second-order term encountered in each of the three experiments is the squared term for SAFE (X4S). The effect of safety stock on the response has been discussed previously. Now consider the case where the squared term with a positive coefficient is present. Figure 4 illustrates the resulting effect of increases in safety stock levels on NIS, displayed in terms of availability (1 - NIS) versus dollars spent for sa´^ty stock

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Figure 4

EFFECT OF INCREASES IN DOLLARS SPENT FOR SAFETY STOCK ON ITEM AVAILABILITY
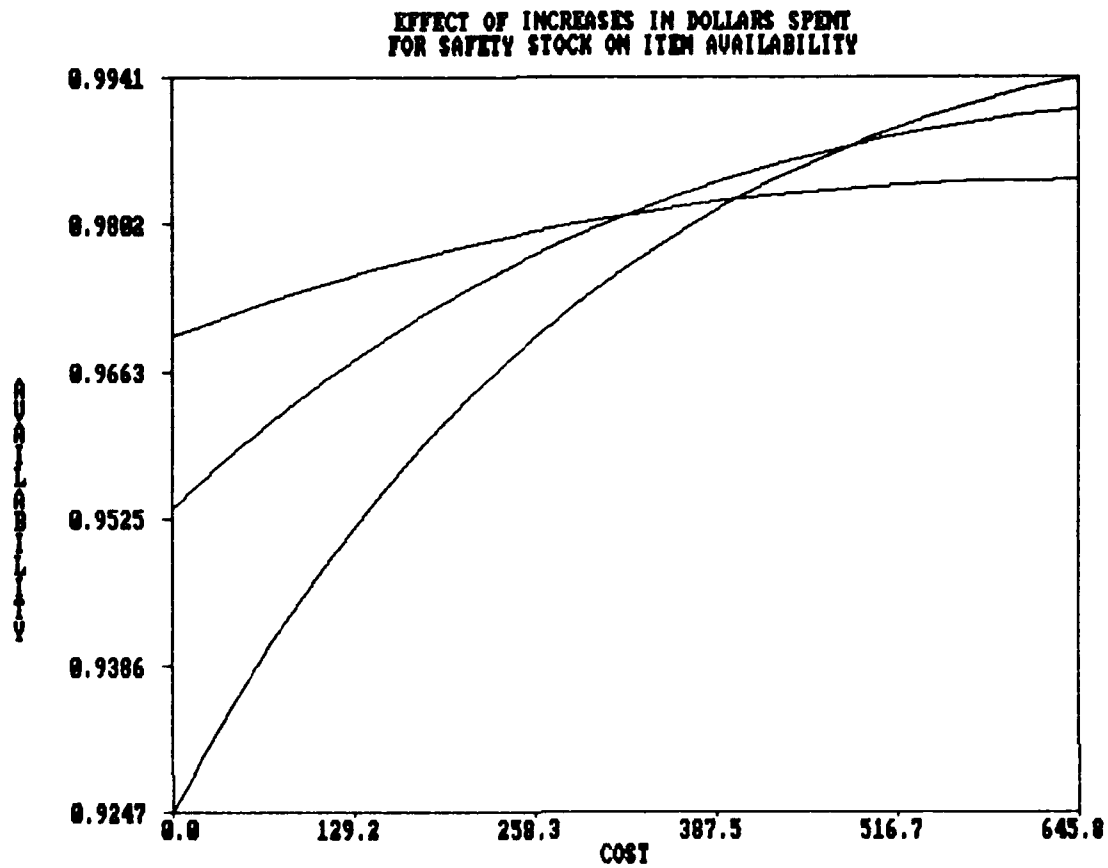
procurements. The other significant parameters were set at their mid-points. The graph reflects that as increases in the amount of safety stock continue, the NIS rate is reduced, but at a decreasing rate of reduction. This is again what is expected to be seen in the real-world environment. As SAFE becomes very large, eventually there will be so much safety stock that additional items tend to provide only extremely small decreases in the NIS rate than they did earlier.

Graphical Support of Validity. Eight hypothetical items with parameters generated at random (in the experimental region) were created as a means of testing the accuracy of the equations. The parameters of each of these items were used as inputs for both the original simulation and for the three equations. Appendices C, D, and E display plots of the simulation outputs against the regresson equation predictors for NIS. If both axes are similarly scaled for values of NIS, an accurate regression equation would be expected to plot a straight line of positive slope 1. Note that the plots of Appendix C, a plot of the 7-day review period model, reflects a slope of approximately 1. As a result, it is concluded that the 7-day review period equation predicts a highly reliable NIS rate when compared to the simulation. The plot of the 14-day review period model of Appendix D may at first appear to have a somewhat scattered structure. Yet notice that the observations all lie within about two percent of one another. The same comments can be made concerning the 30-day review period model of Appendix E. Tables 6 and 7 provide more accurate interpretations of these plots. These tables reflect the actual data points plotted.

These tables reflect a high degree of accuracy, with no tendency observed for the equation to predict either a higher or lower value than the simulation.

Table 6

A Comparison Between the Outputs of the Simulation
and the Regression Equation for the 14-day Model

Simulation Output	Equation Predictor
0.0168	0.0128
0.0243	0.0234
0.0182	0.0213
0.0247	0.0225
0.0120	0.0153
0.0155	0.0177
0.0178	0.0146
0.0240	0.0258

Table 7

A Comparison Between the Outputs of the Simulation
and the Regression Equation for the 30-day Model

Simulation Output	Equation Predictor
0.0188	0.0157
0.0198	0.0244
0.0237	0.0210
0.0153	0.0187
0.0132	0.0164
0.0190	0.0154
0.0215	0.0233
0.0328	0.0266

Conclusion. Evaluation of the three experiments and
the respective regression equations gives confidence that
the simulation has adequately modelled the real-world
system. We can also conclude from the graphical plots that
the regression equations have adequately duplicated the
process of the simulation. Therefore, the values generated
by the equations can confidently be used to implement the
optimization procedure discussed in Chapter III.

The Optimization Procedure.

Appendix F is the FORTRAN code which was implemented to conduct the optimization. A convenient feature of this code is that no values are calculated until they are required by the algorithm. The algorithm produces two results of primary interest. The first of these results is a graph of aggregate availability versus dollars spent on safety stock. This graph is presented as Figure 6. As in the individual item availability graphs, aggregate availability increases marginally as more safety stock is added to the inventory, and the curve levels off once a sufficient amount of money has been spent. The other product of the algorithm is the aforementioned "shopping list." Figure 5 displays a portion of the shopping list produced by the algorithm given a population of 90 products and a safety stock budget of $ 100,000.00. The algorithm terminates when the amount of funds allocated to safety stock has expired, or when no further improvement to aggregate availability is possible. The latter situation occurs when the bounds of the experimental region have been exceeded by all 90 of the products, but it would of course not occur in the real-world system.

Sensitivity Analysis. It is desirable to have a means of predicting an item's position on the shopping list, given a set of specified parameters similar to those used in the simulation, in order to evaluate the item's behavior on the list as these parameters vary. For the shopping list with

90 products, it is noticed that 10 different products appear

for the first time on the list in the top 20 positions. It

is also noted that the last item to appear for the first

time on the list is not acquired until priority 1517. The

Figure 5

Sample Shopping List Generated by the Optimization Model

```
PRODUCT    44212. SHOULD BE ORDERED AT PRIORITY       1
PRODUCT      287. SHOULD BE ORDERED AT PRIORITY       2
PRODUCT      287. SHOULD BE ORDERED AT PRIORITY       3
PRODUCT      411. SHOULD BE ORDERED AT PRIORITY       4
PRODUCT    44212. SHOULD BE ORDERED AT PRIORITY       5
PRODUCT      287. SHOULD BE ORDERED AT PRIORITY       6
PRODUCT      735. SHOULD BE ORDERED AT PRIORITY       7
    .          .      .      .      .      .      .        .
    .          .      .      .      .      .      .        .
PRODUCT      555. SHOULD BE ORDERED AT PRIORITY    2115
PRODUCT      717. SHOULD BE ORDERED AT PRIORITY    2116
PRODUCT    61612. SHOULD BE ORDERED AT PRIORITY    2117
PRODUCT    62351. SHOULD BE ORDERED AT PRIORITY    2118
PRODUCT    60712. SHOULD BE ORDERED AT PRIORITY    2119
```
NO FURTHER IMPROVEMENT IS POSSIBLE WITHIN THE BOUNDS OF
THE EXPERIMENT.

characteristics of the items which cause them to appear

early or late on the list are not apparent from merely

looking at each item's parameters.

As a result of this, a subsequent experimental design

was developed to predict the position at which an item would

appear for the first time on the shopping list. The results

of this subsequent experiment indicate which parameters

cause the greatest effect upon shopping list position. Six

factors were selected, and a 2^6 full factorial design was

implemented. Since the equations are deterministic,

replications were not required for this experiment. This

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experiment was conducted not to absolutely predict an item's position on the list, but rather to evaluate which parameter effects impact the shopping list position, and the manner in which they impact it. In view of this limited objective, consideration was given only to a first-order design.

The six factors selected were:

X1 = Review Period
X2 = Unit Price
X3 = Average Daily Demand (DMD)
X4 = the variability constant, C
X5 = The Casepack Quantity
X6 = The Leadtime

Table 8 displays the various high and low factor settings used for the experiment.

Table 8

Factor	Low Value	Coded Value	High Value	Coded Value
X1	7	-1	14	+1
X2	1.99	-1	5.99	+1
X3	20	-1	44	+1
X4	0.25	-1	0.75	+1
X5	6	-1	12	+1
X6	8	-1	16	+1

The procedure of the experiment was centered around the addition of one additional product to the list of 90. This 91st product was the test product. The six parameters of this product were the factors of the experiment, these factors being changed from one run to the next while the parameters for the other 90 products were held constant. Each combination of factor-level settings required one run of the optimization algorithm. Therefore, a 2^6

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Figure 6

**EFFECT OF INCREASES IN DOLLARS SPENT FOR
SAFETY STOCK ON AGGREGATE AVAILABILITY**
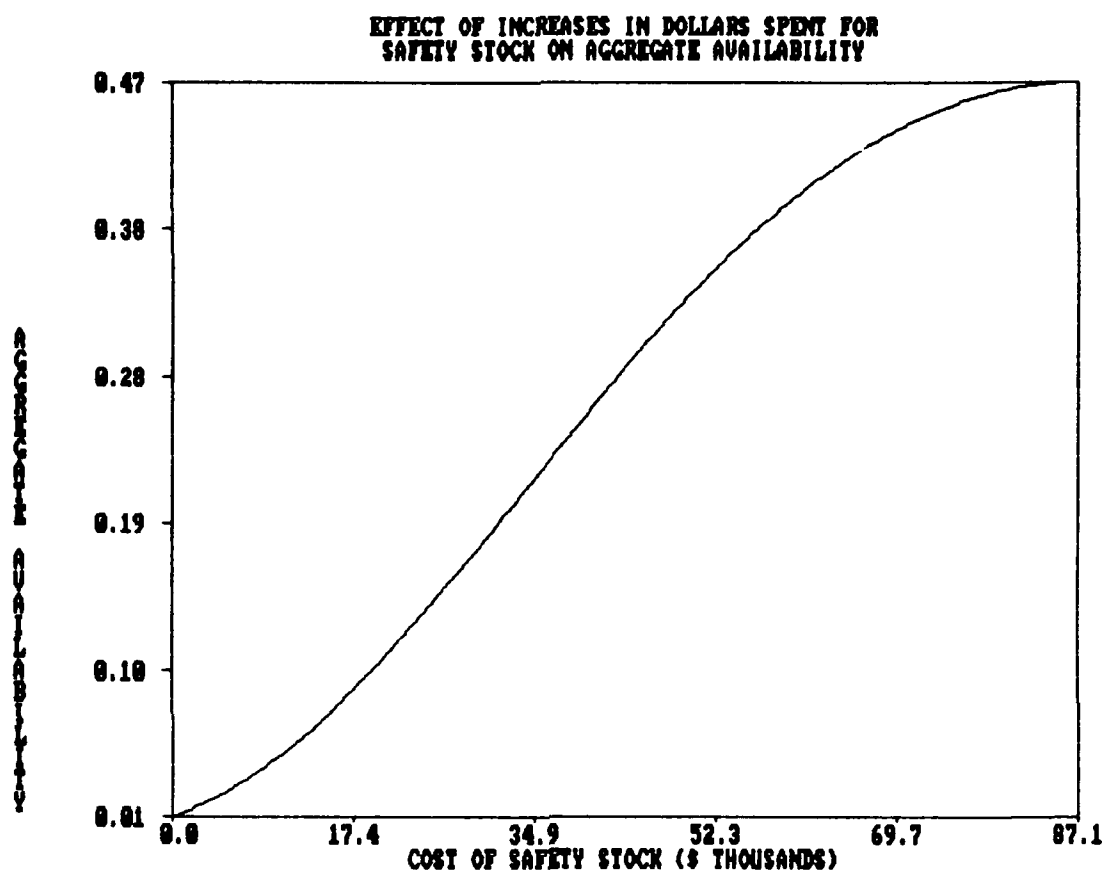
full-factorial design necessitated 64 runs of the

optimization. The code was slightly altered to include the

recording of the position where Product 91 first appears on

the shopping list in each run. This position was used as

the response variable. With all of the other products held

constant, the experiment enables observations to be made

regarding how the test product "jumps around" on the list as

various product characteristics are altered.

The resulting R-Square statistic was 0.986553, which reflects an accurate model. The significant main effects and interaction terms of this model are:

> Mean effect
> Review Period (X1)
> Unit Price (X2)
> Demand (X3)
> Variability of Demand, C (X4)
> Leadtime (X6)
> Review Period with Price (X1*X2)
> Review Period with C (X1*X4)
> Review Period with Leadtime (X1*X6)
> Unit Price with Demand (X2*X3)
> Unit Price with C (X2*X4)
> Demand with C (X3*X4)

The regression equation for the coded values of the factors of the shopping list position experiment is therefore:

List Position = EXP[4.08128 + 0.71918*X1 + 1.18885*X2 + 0.79484*X3 - 0.97139*X4 - 0.17037*X6 - 0.13030*X1*X2 + 0.07713*X1*X4 + 0.06982*X1*X6 - 0.16865*X2*X3 + 0.21939*X2*X4 + 0.13309*X3*X4]

Note that as the response <u>increases</u>, the item's position on the shopping list moves <u>down</u>. Likewise, a decrease in the response indicates that a product moves up on the shopping list. It is difficult to make conclusions by simple inspection of this rather long and complex equation, so an item was chosen at random from the data, and its parameters were applied to the above equation. The equation was used to generate graphs of shopping list position versus increases in each of the item's parameters, while other parameters were held constant at their mid-points. Appendices G through K display the various plots which reflect the behaviour of the item as it moves up

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and down on the shopping list. From these plots, Table 9 was developed to summarize the effects of the various item parameters on the item's position on the shopping list.

Table 9

Effect of Item Parameters on Shopping List Position

Parameter	Adjustment Made	Effect on List Position
Demand	Increase	Moves Further Down
Review Period	Increase	Moves Further Down
Unit Price	Increase	Moves Further Down
DMD Variability	Increase	Moves Further Up
Leadtime	Increase	Moves Further Up
Casepack	Increase	No Effect

From these observations, the following general comments can be made regarding aggregate availability and shopping list position. Additional units of safety stock do not provide as great an incremental increase in aggregate availability for items with large demand as they do for items with a smaller demand. Hence, as an item's demand increases, its priority diminishes. This is the behavior observed from the Position model.

Increases in Unit Price impact the cost elements of the availability model. Hence, as Unit Price increases, the denominator of the sort value increases, causing the sort value to generally decrease. Reductions in the sort value are what cause an item to move further down on the shopping list. Therefore, as the price increases, the incremental aggregate availability obtained from an additional casepack

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of safety stock is smaller, and an item does not emerge as high on the list.

Increases in Demand Variability, the ratio C, cause an item's availability rate to be more unstable. Hence, additional casepacks of safety stock will contribute more to increasing the item's availability rate, as well as the aggregate availability rate, than they would for more stable items. The same phenomenon is observed for increases in Leadtime. The result of increasing these two parameters is that the products become more valuable in terms of increasing the aggregate availability and, hence, move up in the shopping list.

Conclusion. The general conclusion, then, is that any adjustment which tends to decrease stability in terms of item availability, also tends to move an item further up on the shopping list. This occurs because incremental increases in the level of safety stock contributes more to increasing availability for unstable products than they do for stable products.

Summary

The results presented herein support the validity of the AFCOMS inventory simulation model. Both the simulation and the regression equations accurately model the expected performance of inventory systems in general, and the Wright-Patterson Air Force Base Commissary in particular. The optimization model provides a logical means of prioritizing items for safety stock purchases, as well as

providing a measure of aggregate system availability. The
effects of an item's parameters on its availability rate
have been discussed, with subsequent analysis of those
parameters given to assess their impact on the item's
position in the generated shopping list.

V. Survey and Recommendations

Conclusive Overview

The objectives set forth in Chapter I have been met. A regression model which accurately approximates the inventory simulation has been developed. This process produced three equations, one for each of the three typical review periods encountered for commissary products. The equations were then implemented to produce an aggregate Inventory Availability Model (IAM) which accomplishes two goals: (1) The IAM calculates aggregate inventory availability and projects the amount of money required to achieve high levels of aggregate availability; (2) The IAM generates a shopping list which prioritizes items according to the degree to which an incremental addition to inventory safety stock contributes to increasing the aggregate availability.

A subsequent experiment was designed to perform sensitivity analysis for the IAM shopping list. The general observation was that items having characteristics which tended to produce unstable stock levels appeared nearer to the top of the list than items with stable characteristics.

A series of plots (See Appendices) was generated to illustrate the effects of changes in the different item parameters on the responses of the various experiments.

Recommendations for Further Analysis

An obvious goal of future research could be to expand upon what is presented herein. If more data were to be gathered on additional products and their parameters, it is likely that several products would exceed the range of the current experiment. A subsequent experiment could be designed to consider products with parameters which lie outside the bounds of the presented model. Yet, since demand does not appear to be a significant factor, and variability is reflected as the ratio of standard deviation of demand to demand, the fruitfulness of such a subsequent model is uncertain, unless the other factors, such as leadtime or safety stock behave differently for items with larger or smaller demands than considered here.

A more reasonable goal of a future research effort in the area of AFCOMS inventory issues would be to investigate alternative inventory management policies using the results of this thesis effort. This research provides AFCOMS with an effective means of developing inventory management strategies. The regression equations can be implemented, for example, to determine the minimum amount of safety stock of a product required to achieve a desired availability goal for the product. Thus, a subsequent analysis could consider availability objectives for each product and seek to develop a strategy for determining optimal safety stock levels.

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Another issue, key to inventory management, and related to safety stock, is the inventory-to-sales ratio. If there is an established availability goal, then the IAM can be used to calculate the minimum inventory-to-sales ratio required to meet that goal. This will enable AFCOMS to assess the feasibility of desired aggregate availability levels given imposed inventory-to-sales ratio maximums.

Further, given the multiplicative nature of the current method used to calculate aggregate availability, and the vast number of inventory items, a more meaningful measure of aggregate availability needs to be developed. This could be a challenging task. For example, consider a similar problem related to electrical systems reliability. If the reliability goal is 0.98 for a system with 90 identical components wired in series, then what must the individual components' reliabilities be in order to achieve the goal? This problem is identical to that addressed in this study, only in terms of electrical components, not inventories. An aggregate availability goal of 0.98 would cost a fortune if measured in the terms of this study.

But to desire, for example, to be able to take a shopping list of 100 items chosen at random into a commissary store on any given day, and find 98 of those hundred items in the store, might be feasible. The electrical system counterpart to this problem is that there are 100 different components wired in a system in such a way that so long as no more than any two components fail, the

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system will work. The system will not work if three or more components fail. Assuming that such a system could be designed, and if each of the individual components' reliabilities were known, what is the liklihood that the sytem will work? Further analysis using this type of measure could prove very beneficial.

Another issue which needs to be addressed is the way in which the Not-in-Stock (NIS) rate is calculated. The current method has the advantage in that it is simplistic, dividing the number of customers with unsatisfied demand by the total number of customers over a 20-year period to obtain a measure of NIS. Yet this method can be misleading. It is conceivable, for instance, that an item could consistently sell out every month, but only for one or two days per cycle, still maintaining a high availability rate. The current simulation has no way of detecting this trend.

An alternative approach to the measurement of NIS in order to discover these types of trends is to divide the simulation run length into equal blocks of time, perhaps the typical length of a complete reorder cycle, and then count the number of blocks where an item sold out, measuring this number against the simulation run length. Although some of the precision afforded by the current method will be foregone with this technique, certain undesirable trends can be spotted which might be overlooked using the current method. Implementation of this technique would simply require slight modifications to the simulation code.

Finally, a more practical means for implementing the system should be considered. The IAM is not a practical tool for daily use by inventory managers. However, the IAM could be used as a means of identifying products which have similar behavior patterns. Products which display the same characteristics could be grouped together. This categorization could enable a store manger to make on-the-spot decisions regarding prioritization of products according to which class a product belongs. Some classes of products could be given priorities over other classes, making these decisions easier for store managers.

Closing Remarks

The ultimate purpose and motivation for improving AFCOMS store operations transcends mere economic advantages. AFCOMS' primary mission is to provide a benefit to military members that promotes a positive attitude towards the Air Force and the Air Force lifestyle. This attitude impacts retention, job performance, and, ultimately, mission accomplishment in general. Any subsequent research efforts in this area should be performed with this purpose in mind.

Appendix A: Plot of Residuals for the 14-Day
Review Period Experiment


```
PLOT OF RESID*YHAT  LEGEND: A = 1 OBS, B = 2 OBS, ETC.

RESID |
      |
0.100 +
      |
      |
      |                       A
      |
0.075 +
      |
      |                       A
      |
      |
0.050 +                        A
      |        A                                        A
      |
      |        A               A
      |                        A        A
0.025 +
      |                         A
      |                        A
      |              A
      |          A             A
0.000 +                        B
      |                        A
      |                        A                 B
      |              A             A
      |
-0.025 +        A               A    A
      |            A           A
      |            A           A                     A
      |              A         A
      |                        A
-0.050 +                        A
      |
      ---+---------+---------+---------+---------+---------+------
         -3.0      -2.8      -2.6      -2.4      -2.2

                              YHAT
```
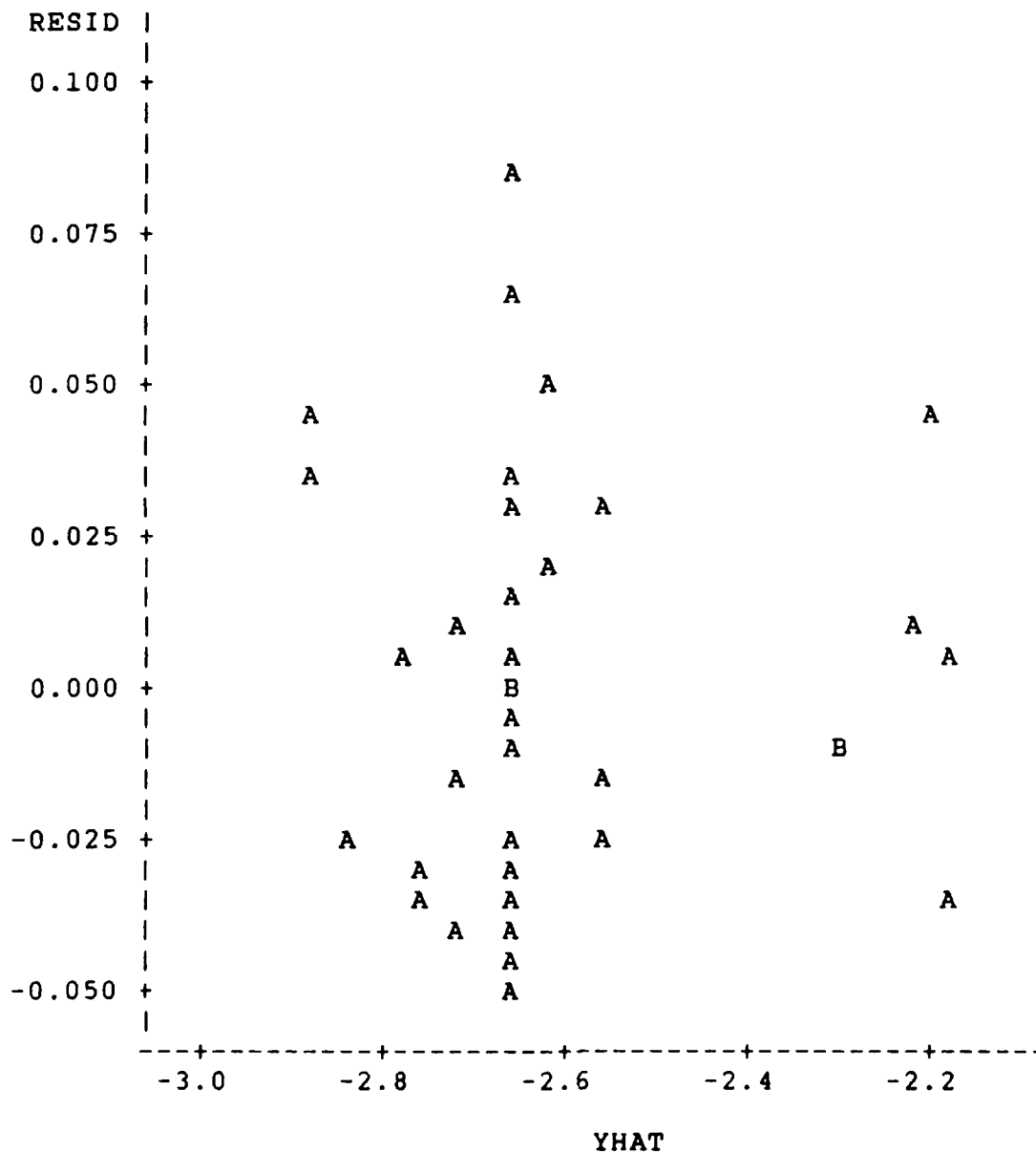
Appendix B: <u>Residual Plot for Normality</u> <u>for the 14-Day Review Period Experiment</u>

```
        PLOT OF RESRANK*RESID     LEGEND: A = 1 OBS, B = 2 OBS, ETC.
                    |
            2.0  +
                    |                                                        A
                    |                                                  A
        R   1.5  +
        A           |                                             A
        N           |                                           A
        K           |                                           A
            1.0  +                                         A
        F           |                                       A
        O           |                                   AA
        R           |                               A
            0.5  +                             B
        V           |                       A
        A           |                     AA
        R           |                     BA
        I   0.0  +                   A
        A           |                   A
        B           |                 AB
        L           |               A
        E  -0.5  +             AA
                    |             A
        R           |           B
        E           |           A
        S  -1.0  +           A
        I           |         A
        D           |       A
                    |     A
            -1.5  +
                    |       A
                    |
                    |   A
            -2.0  +
            ------+--------------+--------------+--------------+-----
                -0.08          -0.03           0.02            0.07

                                  RESID
```
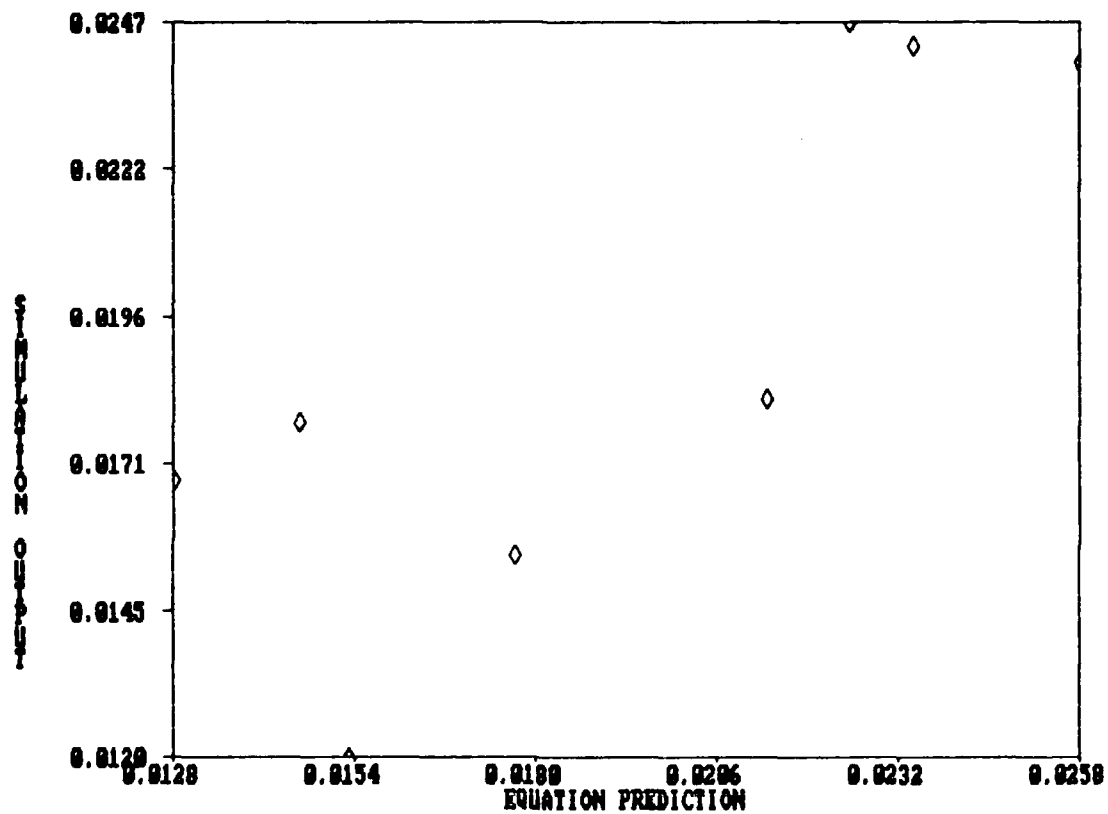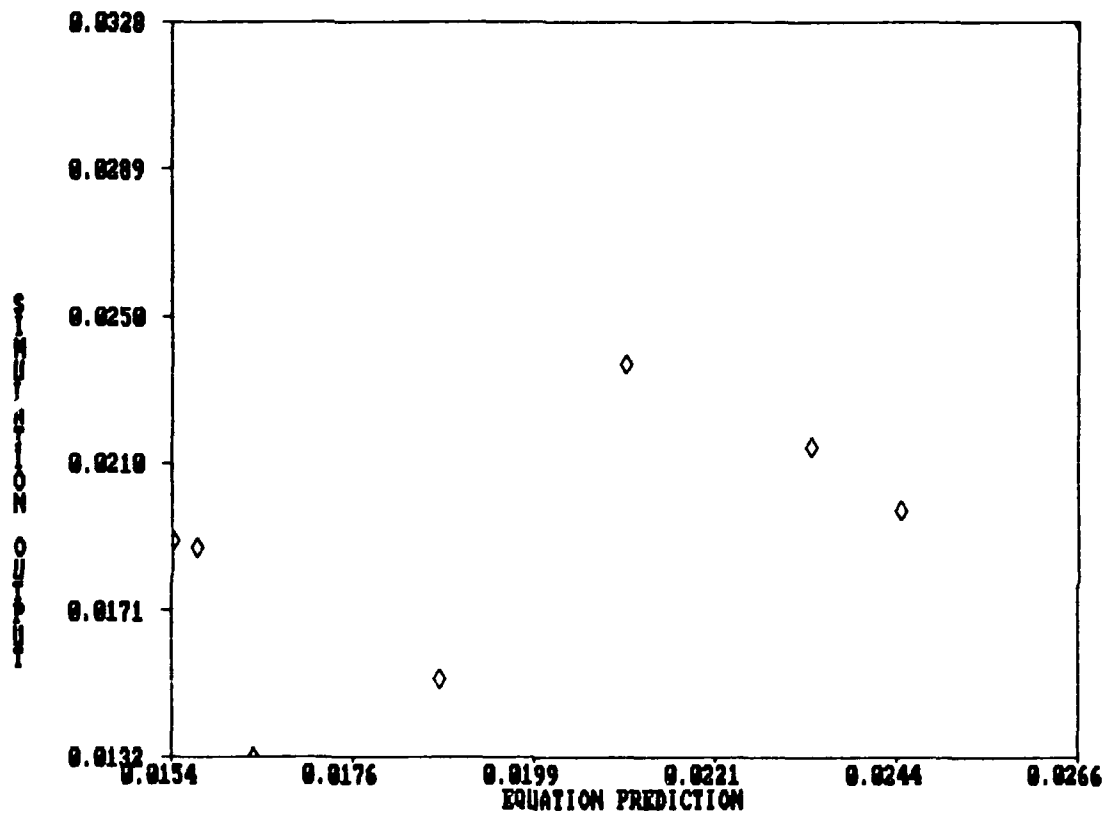
77
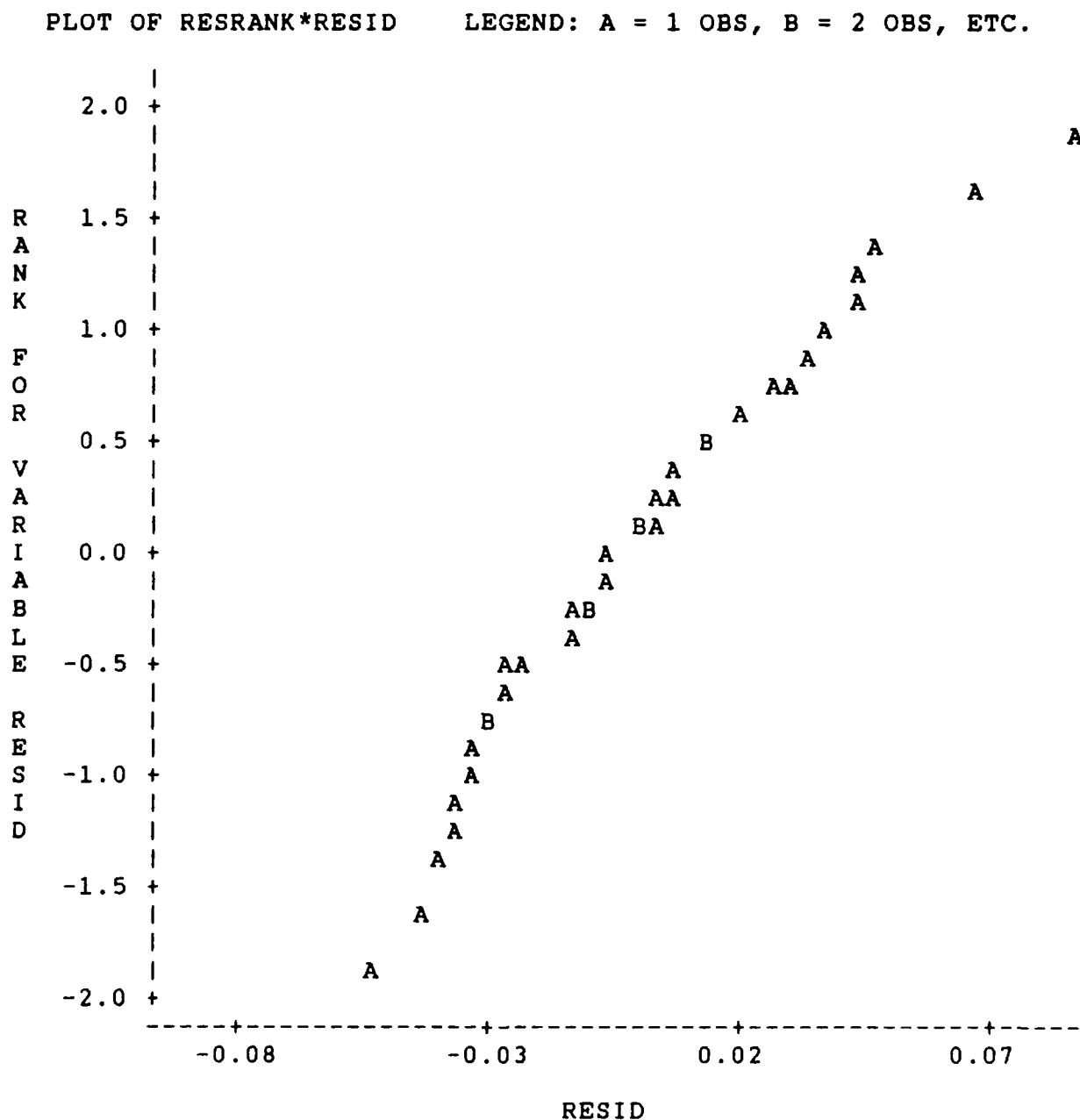
Appendix C: Plot of Simulation Outputs Versus
Equation Predictions for the 7-Day Model
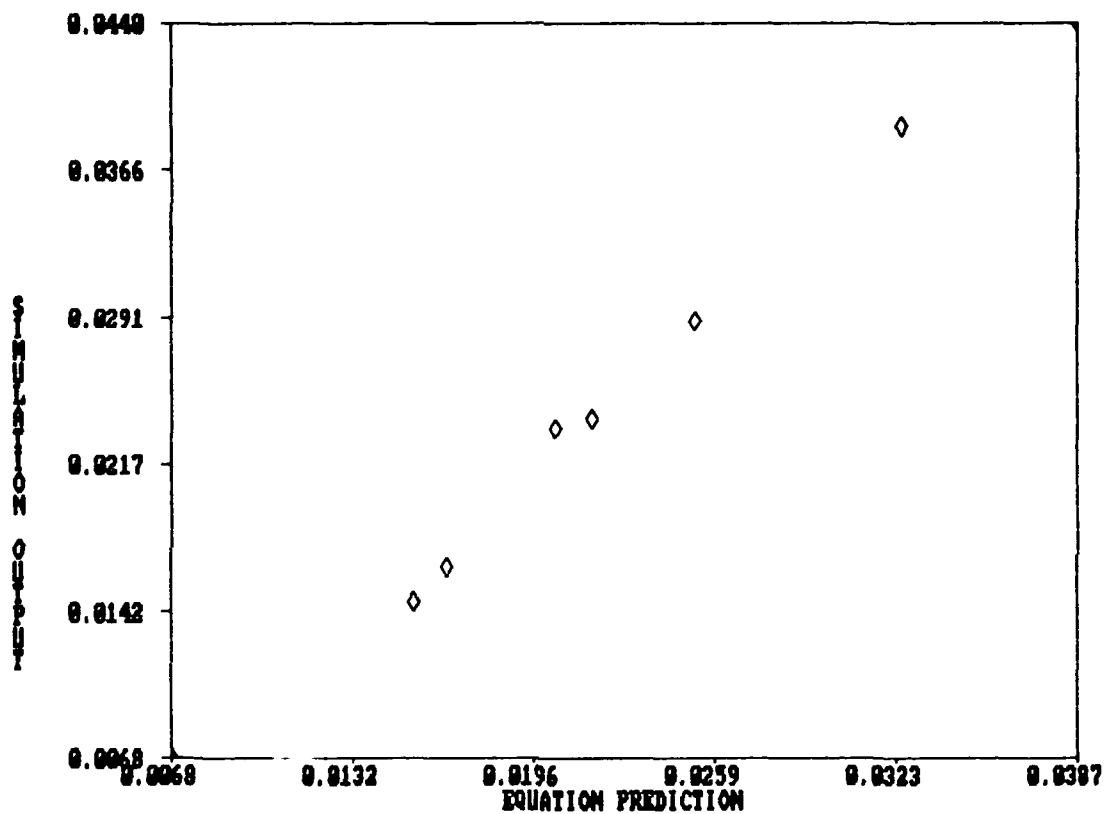
Appendix D: Plot of Simulation Outputs Versus
Equation Predictions for the 14-Day Model

Appendix E: Plot of Simulation Outputs Versus
Equation Predictions for the 30-Day Model

```
      REAL SAFESTOCK,IMPRV,AVAIL,SORT,UPC,TEMP,BASE
      REAL VALMAX,COST,RMAX,SAFDAYS,COL,C,LEAD,STOCK
      REAL ANEW,SPENT,DMD
      DIMENSION DMD(90)
      DIMENSION SAFDAYS(90)
      DIMENSION LEAD(90)
      DIMENSION C(90)
      DIMENSION COL(90)
      DIMENSION BASE(90)
      DIMENSION STOCK(90)
      DIMENSION SAFESTOCK(90)
      DIMENSION IMPRV(90,150)
      DIMENSION TEMP(90)
      DIMENSION AVAIL(90,150)
      DIMENSION SORT(90,150)
      DIMENSION UPC(90,7)
      OPEN(UNIT=11,FILE='LINV80.DAT',STATUS='OLD')
      OPEN(UNIT=12,FILE='SHOPLIST.DAT',STATUS='NEW')
      OPEN(UNIT=13,FILE='RATES.DAT',STATUS='NEW')
      OPEN(UNIT=14,FILE='TRACE.DAT',STATUS='NEW')
C
C     SET UP EACH PRODUCT'S INPUT PARAMETERS IN ARRAY
C     UPC(I,J)
C
C     READ DATA IN ORDER:  UPC NO.,REVIEW PRD,UNIT PRICE,
C     AVERAGE DAILY DEMAND,STD DEVIATION OF DEMAND,
C     CASEPACK,LEADTIME
C
      DO 10 I=1,90
      READ(11,*) UPC(I,1),UPC(I,2),UPC(I,3),UPC(I,4),
     1           UPC(I,5),UPC(I,6),UPC(I,7)
10    CONTINUE
C
C     CREATE ARRAY FOR BASE INVENTORY LEVELS WITH NO SAFETY
C     STOCK
C
      DO 15 I=1,90
         DMD(I)=(UPC(I,4)-32.0)/12.0
         C(I)=((UPC(I,5)/UPC(I,4))-0.5)/0.25
         LEAD(I)=(UPC(I,7)-15.)/5.0
         SAFESTOCK(I)=0.0
         SAFDAYS(I)= ((SAFESTOCK(I)/UPC(I,4))-4.0)/2.0
15    CONTINUE
      DO 20 I=1,90
         IF (UPC(I,2).EQ.7) THEN
```

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```fortran
          BASE(I) = 1.05-EXP(-2.60385146 + 0.17989127*
     1C(I) + 0.09724780*LEAD(I) - 0.24936345*SAFDAYS(I)
     2 - 0.05094705*C(I)*SAFDAYS(I) +  0.04571362*(C(I)**2)
     3 + 0.04948995*(SAFDAYS(I)**2))
          ELSE IF (UPC(I,2).EQ.14) THEN
            BASE(I) = 1.05-EXP(-2.66916583 + 0.12876624*
     1C(I) + 0.05895797*LEAD(I) - 0.15939171*SAFDAYS(I)
     2 - 0.05710741*C(I)*SAFDAYS(I) + 0.05023833*(C(I)**2)
     3 + 0.03567863*(SAFDAYS(I)**2))
          ELSE IF (UPC(I,2).EQ.30) THEN
           BASE(I) = 1.05-EXP(-2.66813000 + 0.07167810*C(I)
     1 + 0.04331506*DMD(I)*LEAD(I) - 0.06387629*SAFDAYS(I)
     2 - 0.02758407*C(I)*SAFDAYS(I) + 0.03603161*(C(I)**2)
     3 + 0.02077653*(SAFDAYS(I)**2))
          END IF
20      CONTINUE
C
C       CALCULATE THE INITIAL AGGREGATE AVAILABILITY FOR
C       THE BASE INVENTORY WITH NO SAFETY STOCK
C
        ANEW = 1.0
        DO 55 I = 1,90
          ANEW = ANEW*BASE(I)
55      CONTINUE
C       CREATE, THE FIRST COLUMN OF THE AVAILABILITY MATRIX
C
C
C                   NO. CASEPACKS OF SAFETY STOCK
C
C       PRODUCT NUMBER        1       2       3       4   . . .
C
C           1                a11     a12     a13     a14  . . .
C           2                a21     a22     a23     a24  . . .
C           3                a31     a32     a33     a34  . . .
C           .                 .       .       .       .   . . .
C           .                 .       .       .       .   . . .
C           90               a90,1   a90,2   a90,3   a90,4 . . .
C
C       INCREMENT SAFETY STOCK FOR EACH PRODUCT BY CASEPACK
C
        DO 25 I=1,90
          SAFESTOCK(I)=SAFESTOCK(I)+UPC(I,6)
          SAFDAYS(I)=((SAFESTOCK(I)/UPC(I,4))-4.0)/2.0
25      CONTINUE
        DO 30 I=1,90
          IF (UPC(I,2).EQ.7) THEN
           AVAIL(I,1) = 1.05-EXP(-2.60385146 + 0.17989127*
     1C(I) + 0.09724780*LEAD(I) - 0.24936345*SAFDAYS(I)
     2 - 0.05094705*C(I)*SAFDAYS(I) +  0.04571362*(C(I)**2)
     3 + 0.04948995*(SAFDAYS(I)**2))
          ELSE IF (UPC(I,2).EQ.14) THEN
            AVAIL(I,1) = 1.05-EXP(-2.66916583 + 0.12876624*
     1C(I) + 0.05895797*LEAD(I) - 0.15939171*SAFDAYS(I)
     2 - 0.05710741*C(I)*SAFDAYS(I) + 0.05023833*(C(I)**2)
```

```
      3 + 0.03567863*(SAFDAYS(I)**2))
          ELSE IF (UPC(I,2).EQ.30) THEN
           AVAIL(I,1)=1.05-EXP(-2.66813000 + 0.07167810*C(I)
      1 + 0.04331506*DMD(I)*LEAD(I) - 0.06387629*SAFDAYS(I)
      2 - 0.02758407*C(I)*SAFDAYS(I) + 0.03603161*(C(I)**2)
      3 + 0.02077653*(SAFDAYS(I)**2))
          END IF
C     CREATE FIRST COLUMN OF THE IMPROVEMENT MATRIX
          IMPRV(I,1)=AVAIL(I,1)/BASE(I)
C     CREATE FIRST COLUMN OF THE SORT MATRIX
          SORT(I,1)=LOG(IMPRV(I,1))/(UPC(I,3)*UPC(I,6))
C     CREATE THE INITIAL ARRAY TO COMPARE SORT VALUES
          TEMP(I)=SORT(I,1)
C     REGISTER THE NUMBER OF COLUMNS CALCUALTED FOR EACH
C     PRODUCT IN THE SORT MATRIX
          COL(I)=1.0
          STOCK(I)=SAFESTOCK(I)
30    CONTINUE
C
C**********IMPLEMENT THE OPTIMIZATION PROCEDURE ********
C
      COST =100000
      M=0
C     FIND THE LARGEST SORT VALUE IN THE TEMP ARRAY
100   M=M+1
      VALMAX=0.0
      DO 35 I = 1,90
         IF (TEMP(I).GE.VALMAX) THEN
            VALMAX=TEMP(I)
            RMAX=I
         ENDIF
35    CONTINUE
C
C     CHECK TO SEE IF THE BOUNDS OF THE EXPERIMENT HAVE
C     BEEN EXCEEDED.  IF SO, THEN NO FURTHER IMPROVEMENT
C     TO AGGREGATE AVAILABILITY IS POSSIBLE.  TERMINATE
C     THE ALGORITHM.
C
         IF (VALMAX .LE. 0.0) THEN
         WRITE(12,5)
         DO 65 L = 1,90
         LL=COL(L)
         MM=COL(L)-1.0
C
C     RECORD THE INITIAL AVAILABILITY, THE HIGHEST
C     AVAILABILITY THAT WAS ATTAINED, THE NUMBER OF
C     CASEPACKS REQUIRED, AND THE FINAL NUMBER OF
C     SAFETY DAYS OF STOCK REQUIRED FOR MAX AVAILABILITY.
C
         WRITE(14,4)BASE(L),AVAIL(L,MM),MM,SAFDAYS(L)
65    CONTINUE
      STOP
      ENDIF
```

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```
C
C      INCREMENT THE LEVEL OF SAFETY STOCK FOR PRODUCT IMAX
C      BY ITS CASEPACK VALUE. FIND THE NEW AVAILABILITY RATE
C      FOR THIS LEVEL, AND CALCULATE NEW IMPROVEMENT AND SORT
C      FACTORS, RECORDING THEM IN THE NEXT COLUMN OUT IN THE
C      APPROPRIATE MATRIX. INCREMENT THE NUMBER OF COLUMNS
C      USED FOR PRODUCT IMAX BY 1.  ENTER THE NEW SORT RATE
C      FOR PRODUCT IMAX INTO ARRAY TEMP SO THAT IT CAN
C      COMPETE WITH THE OTHER SORT VALUES.  SUBTRACT COST OF
C      NEW CASEPACK ORDER FROM TOTAL COST ALLOTTED TO SAFETY
C      STOCK
C
       J = RMAX
       JJ = COL(J)
       ANEW = ANEW*IMPRV(J,JJ)
       STOCK(J) = STOCK(J) + UPC(J,6)
       SAFDAYS(J)=((STOCK(J)/UPC(J,4))-4.0)/2.0
       COL(J) = COL(J)+1
       K = COL(J)
       L = K-1
          IF (UPC(J,2).EQ.7) THEN
          AVAIL(J,K) = 1.05-EXP(-2.60385146 + 0.17989127*
1C(J) + 0.09724780*LEAD(J) - 0.24936345*SAFDAYS(J)
2 - 0.05094705*C(J)*SAFDAYS(J) +  0.04571362*(C(J)**2)
3 + 0.04948995*(SAFDAYS(J)**2))
          ELSE IF (UPC(J,2).EQ.14) THEN
          AVAIL(J,K) = 1.05-EXP(-2.66916583 + 0.12876624*
1C(J) + 0.05895797*LEAD(J) - 0.15939171*SAFDAYS(J)
2 - 0.05710741*C(J)*SAFDAYS(J) + 0.05023833*(C(J)**2)
3 + 0.03567863*(SAFDAYS(J)**2))
          ELSE IF (UPC(J,2).EQ.30) THEN
          AVAIL(J,K)=1.05-EXP(-2.66813000 + 0.07167810*C(J)
1 + 0.04331506*DMD(J)*LEAD(J) - 0.06387629*SAFDAYS(J)
2 - 0.02758407*C(J)*SAFDAYS(J) + 0.03603161*(C(J)**2)
3 + 0.02077653*(SAFDAYS(J)**2))
          END IF
             IMPRV(J,K) = AVAIL(J,K)/AVAIL(J,L)
             IF (IMPRV(J,K).LT.1.0) THEN
               IMPRV(J,K)=1.0
             END IF
             SORT(J,K)=LOG(IMPRV(J,K))/(UPC(J,3)*UPC(J,6))
             TEMP(J)=SORT(J,K)
       COST = COST - (UPC(J,3)*UPC(J,6))
          IF ((COST.GT.0.0).AND.(ANEW.LT.1.0)) THEN
             WRITE(12,1) UPC(J,1), M
             SPENT = 100000 - COST
             WRITE(13,*)ANEW,SPENT
             GO TO 100
          ELSE
             WRITE(12,2) UPC(J,1), M
          ENDIF
       STOP
1      FORMAT(1X, PRODUCT',F9.0,' SHOULD BE ORDERED AT
```

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```
       1PRIORITY',I6)
  2      FORMAT(1X,'SAFETY STOCK FUNDS WILL BE EXHAUSTED IN AN
       1 ATTEMPT',/,' TO ORDER ONE ADDITIONAL CASEPACK OF
       2 PRODUCT ', 2F6.0,' AT PRIORITY 'I6)
  3      FORMAT(1X,'NO FURTHER IMPROVEMENT IS POSSIBLE',/,
       1' WITHIN THE BOUNDS OF THE EXPERIMENT.')
  4      FORMAT(1X,F9.7,2X,F9.7,2X,I3,2X,F9.7)
         END
```
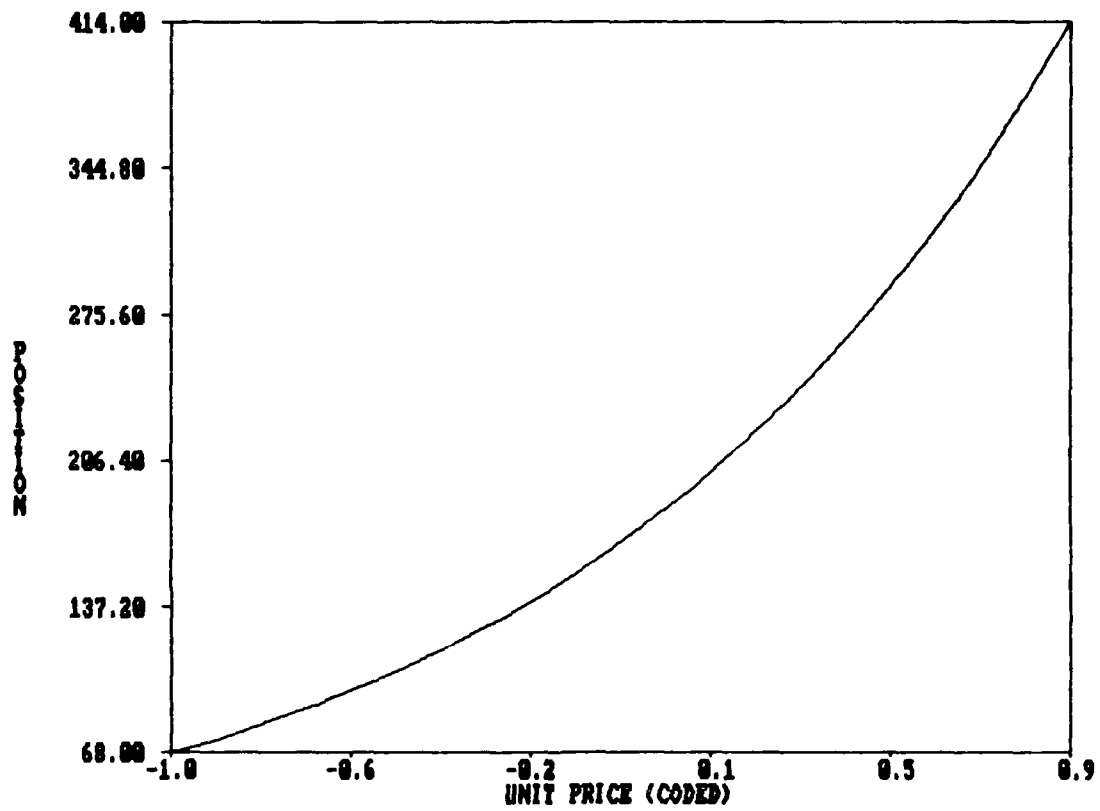
Appendix G: Graph of the Effect of Increases In
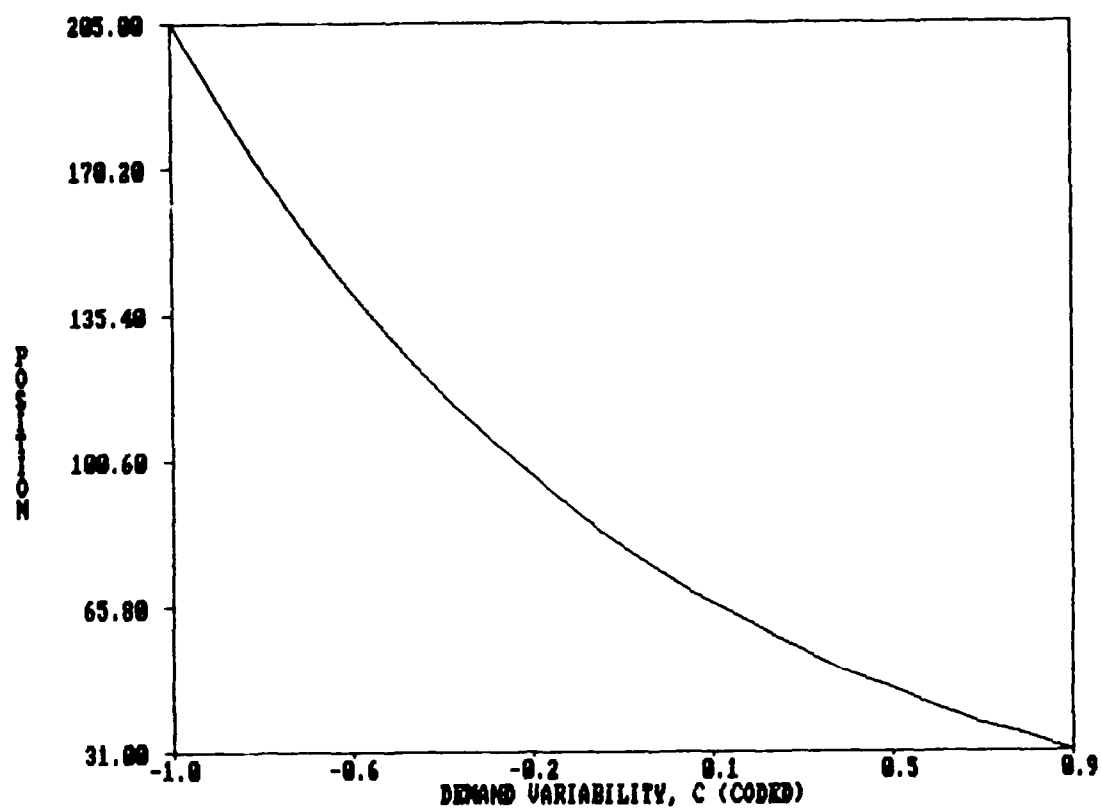   Demand on Shopping List Position
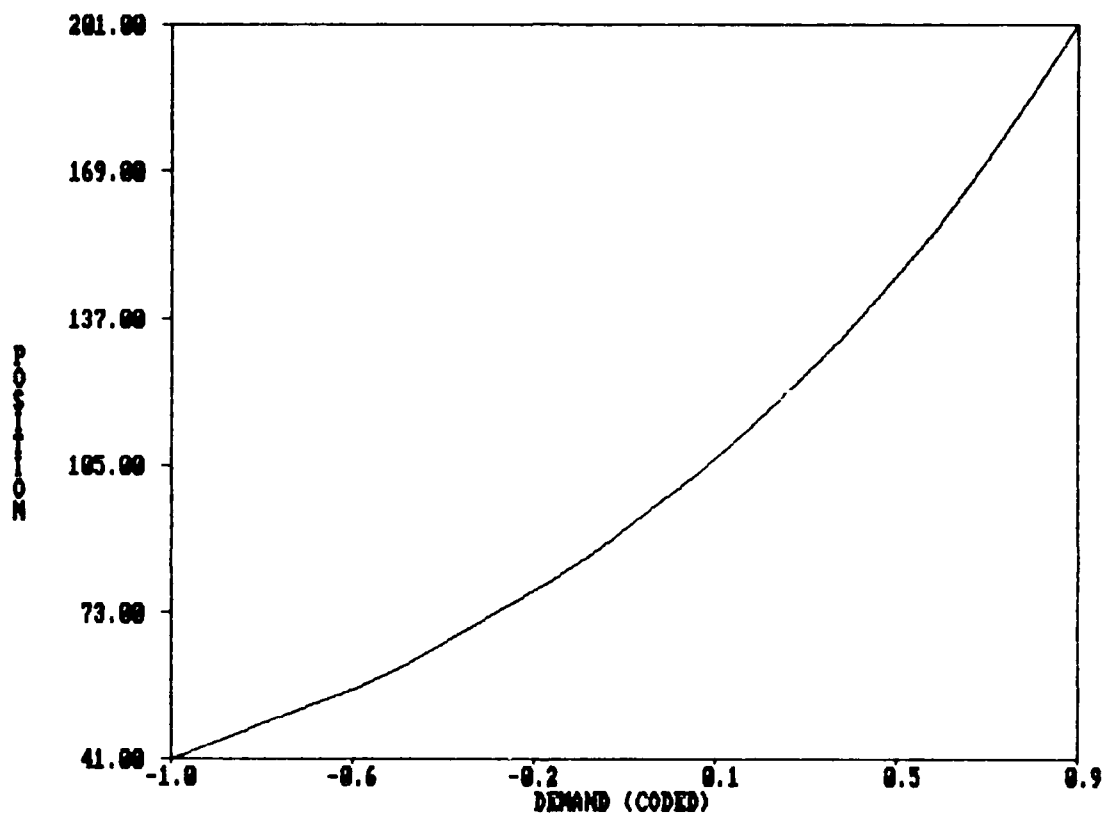
Appendix H: <u>Graph of the Effect of Increases In Review</u>
<u>Period on Shopping List Position</u>
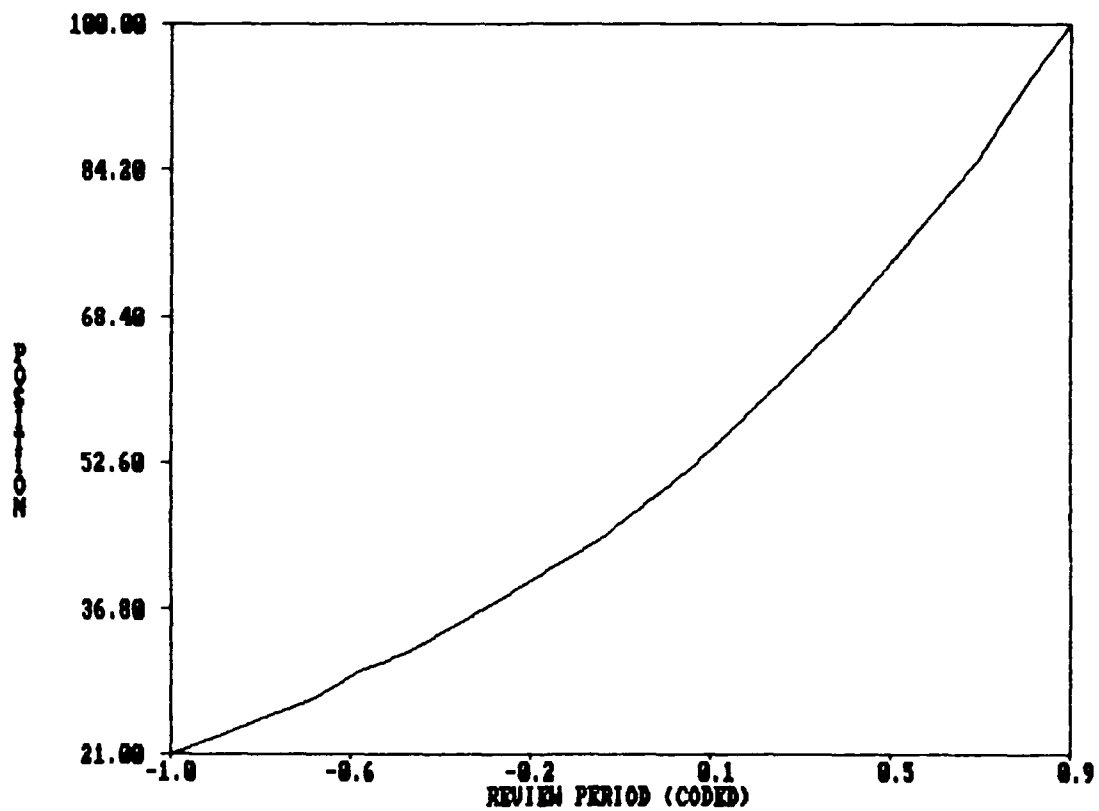
Appendix I: Graph of the Effect of Increases In
              Unit Price on Shopping List Position

Appendix J: Graph of the Effects of Increases In Demand
              Variability on Shopping List Position

Appendix K: Graph of the Effect of Increases In
              Leadtime on Shopping List Position

90
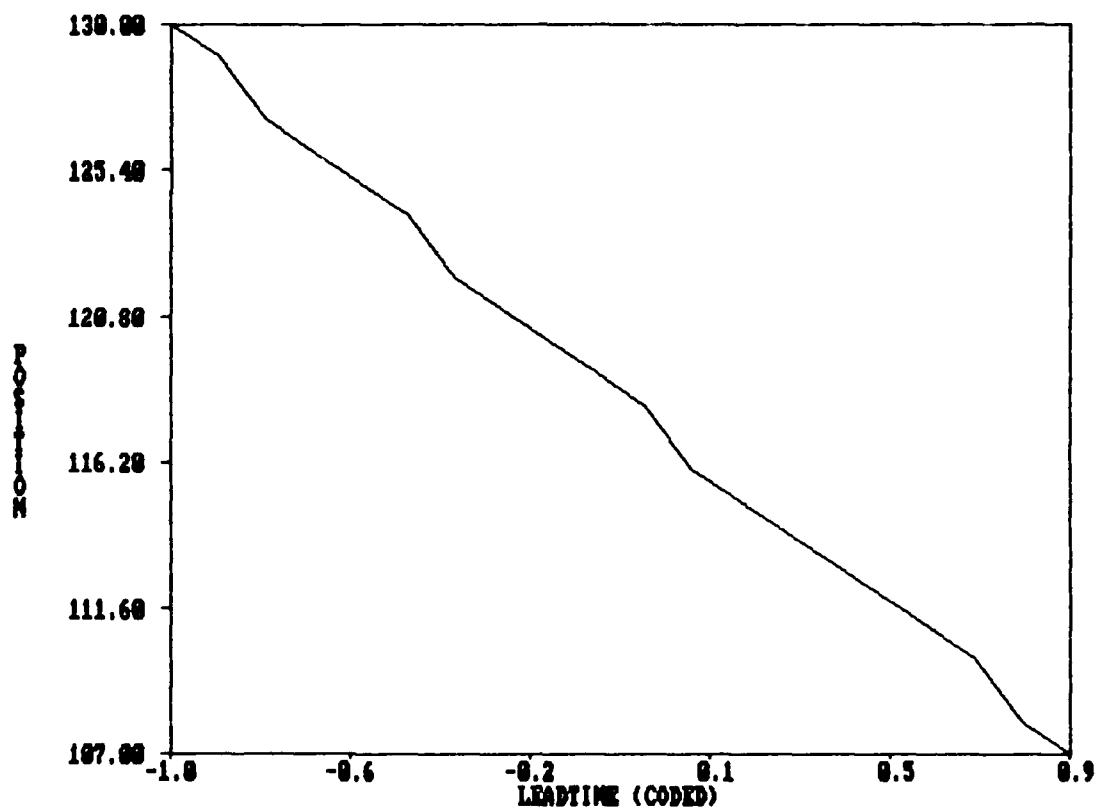
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REPORT DOCUMENTATION PAGE

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17.	COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	AFCOMS, Inventory Modelling, Safety Stock, Aggregate
12	03		Availability

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

See reverse

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DD Form 1473, JUN 86   *Previous editions are obsolete.*   SECURITY CLASSIFICATION OF THIS PAGE

The purpose for this study was to find an optimal procedure for distributing a fixed allocation of inventory dollars among competing inventory items to provide the greatest constant aggregate availability possible. Regression techniques were employed to develop a set of equations which accurately predict inventory item availabilities.

Items were classified by their review period (7, 14, or 30 days) and a response surface was fit from a simulation model for each of these review period values. These surface equations were used to optimize aggregate availability using the marginal analysis technique.

An algorithm was created that generates a shopping list which prioritizes items according to their contribution to increasing the aggregate availability measure. It was generally observed that items having characteristics which produced unstable stock levels appeared nearer to the top of the list than those items having stable stock level characteristics.

This study contributes to the achievement of the primary mission of the Air Force Commissary Service, which is to provide a benefit to military members that promotes a positive attitude towards the Air Force and enhances the quality of life for Air Force members. This in turn improves morale and retention rates of quality personnel, providing contribution to the overall Air Force mission.

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12	03		

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