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AFOSR-TR. 89-0328

Annual Report
Grant No. AFOSR-87-0321
September 1, 1987 - August 31, 1988

INCREASING THE MARGIN OF STABILITY OF ARBITRARILY FINITE
MODES OF FLEXIBLE LARGE SPACE STRUCTURES WITH DAMPING

Submitted to:

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Report No. UVA/525683/AM89/102
December 1988

Copy No. 4

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified		1b. RESTRICTIVE MARKINGS None	
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT Distribution Unlimited, approved for public release	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			
4. PERFORMING ORGANIZATION REPORT NUMBER(S) UVA/525683/AM89/102		5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 89-0328	
6a. NAME OF PERFORMING ORGANIZATION University of Virginia Dept. of Applied Mathematics	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research	
6c. ADDRESS (City, State, and ZIP Code) Thornton Hall Charlottesville, VA		7b. ADDRESS (City, State, and ZIP Code) Building 410 Bolling Air Force Base Washington, DC 20332	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Air Force Office of Scientific Research	8b. OFFICE SYMBOL (If applicable) NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-87-0321	
8c. ADDRESS (City, State, and ZIP Code) Building 410 Bolling Air Force Base Washington, DC 20332		10. SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO.	PROJECT NO.
		TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) Increasing the Margin of Stability of Arbitrarily Finite Modes of Flexible Large Space Structures with Damping			
12. PERSONAL AUTHOR(S) I. Lasiecka, R. Triggiani			
13a. TYPE OF REPORT Annual	13b. TIME COVERED FROM 9/1/87 TO 8/31/88	14. DATE OF REPORT (Year, Month, Day) 1988 December	15. PAGE COUNT
16. SUPPLEMENTARY NOTATION			

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number).

This annual report summarizes the principal investigators' achievements on the research project during the period September 1, 1987 through August 31, 1988. These include new results for wave equations and plate equations, linear and nonlinear, on the following problems: exact controllability, strong and uniform stabilization, structural damping, quadratic optimal control problem, Riccati equations, and numerical aspects thereof.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS		21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL Col. Crowley		22b. TELEPHONE (Include Area Code) (202) 767-5025 22c. OFFICE SYMBOL NM	

DD FORM 1473, 84 MAR

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Unclassified

INCREASING THE MARGIN OF STABILITY OF ARBITRARILY FINITE MODES OF FLEXIBLE LARGE SPACE STRUCTURES WITH DAMPING

During the period in question, the research activities of the principal investigators have continued along the lines summarized in Section I of the research proposal (continuation) submitted to the AFOSR in January 1987. Research efforts have centered on boundary control problems for waves and plates equations (in any dimension) with various boundary conditions, their abstract unifying representation introduced by the principal investigators, as well as related problems of structural damping for elastic systems. More precisely, major themes of research performed under the grant include the following problems:

- (i) exact boundary controllability for waves and plates;
- (ii) uniform stabilization (linear case) and strong stabilization (nonlinear case) by a-priori, explicit boundary feedback for waves and plates;
- (iii) study of the optimal quadratic cost problem for waves and plates, in particular of the associated Algebraic Riccati Equation, which produces a boundary feedback based on the Riccati operator which uniformly stabilizes the system [compare with (ii)];
- (iv) Structural damping for elastic systems under a natural, broad class of damping operators; and
- (v) numerical aspects related to some of the topics above.

Specific contributions in each of the above areas (i)-(v) will be separately reviewed below.

- (i) Exact boundary controllability for waves and plates
- Waves

It was already noted in Section I of the research proposal (continuation) submitted to the AFOSR in January 1987, that the first time that exact controllability of the wave equation with Dirichlet boundary control was obtained in its natural state

space was in the principal investigators' paper [L-T.1], as a corollary of a related uniform stabilization result. Further contributions followed, by J. L. Lions and his associates, and by the principal investigators, see [T.2]. More recent contributions by the principal investigators include a rather thorough study of exact boundary controllability of wave equations with Neumann boundary control [L-T.3], and a study of these questions when the equation contains damping [T.5]. New results by the principal investigators on the nonlinear case are being written up at present.

Plate equations

In a recent paper [L-T.7], first announced in [L-T.6], the principal investigators have solved an open problem, raised by J. L. Lions, on exact controllability of a plate equation with control action only in the Dirichlet boundary conditions, while the corresponding Neumann boundary conditions are homogeneous. Further work by the principal investigators on plates with different boundary conditions is included in [L-T.9] (maximal, optimal regularity) and in [L-T.8] (exact controllability). Further results on the case where the control acts only in the bending moment were recently obtained by the principal investigators in [L-T.10].

(ii) Boundary feedback stabilization

Linear case: uniform stabilization

The principal investigators results on uniform stabilization for the wave equation with feedback in the Dirichlet boundary conditions was already noted above [L-T.1]. In addition we cite a new proof which provides further insight into J. Lagnese's result on uniform stabilization with feedback in the Neumann boundary conditions given by the principal investigators in [T.1]. Uniform stabilization results were recently obtained for plates under a variety of boundary conditions including the case

where only one feedback control is used out of the two available ones: see [B-T.1] (which is based on J. Bartolomeo's Ph.D. dissertation* written under R.T.) and [L-T.10], the latter reference covering the case where the feedback control acts only in the bending moment. A sharp result on the lack of uniform stabilization was also achieved in [T.3].

Nonlinear case: strong stabilization

Strong stabilization results by the principal investigators for waves and plates with nonlinear boundary damping defined by monotone functions are given in [L.1-L.3]. These results extend, by means of a general technique applicable to spatial domains of any dimension, recent results of the literature obtained only in the one-dimensional case.

Nonlinear case: uniform exponential stabilization

The following question is addressed and solved in [L.2]: does a linear stabilizing feedback operator provide a robust mechanism (at least locally) in the presence of nonlinear undesirable perturbations? An affirmative answer is given and applied to waves and plates in feedback form. Other nonlinear problems are solved in [L.4-L.7].

(iii) Optimal quadratic cost problem over an infinite horizon

The principal investigators have recently completed, in collaboration with F. Flandoli, a rather thorough and comprehensive study on the optimal quadratic cost problem over an infinite horizon for an abstract unifying model which includes waves and plates [L-T.5]. This work extends prior work by the principal investigators on the same problem for wave equations with Dirichlet boundary control (SIAM J. Control, 1985).

*J. Bartolomeo receives his Ph.D. degree in Mathematics from the University of Florida, in December 1988.

(iv) Structural damping for elastic systems

As already noted in the proposal (continuation) submitted to the AFOSR in January 1987, work by the principal investigators in collaboration with S. Chen, has succeeded in proving two conjectures, raised by G. Chen and D. Russell in 1982, that if the damping operator behaves like $A^{\frac{1}{2}}$ (in a technical sense), where A is the elastic operator, then the elastic system displays structural damping, see [C-T.1]. Further work by the same authors shows that the same conclusion holds if the damping operator behaves like A^{α} , $\frac{1}{2} \leq \alpha \leq 1$ see [C-T.2]. Indeed, even a more general result holds true. The result is false if, instead, $0 \leq \alpha < \frac{1}{2}$. However, in this range $0 < \alpha < \frac{1}{2}$, the semigroup is differentiable for all positive times [C-T.3].

(v) Numerical aspects

Work in the area of numerical analysis can be divided into two parts, reviewed separately below.

Numerical approximations of Riccati equations.

The ultimate goal of proving exponential stability, which is uniform with respect to the parameter of approximation, of stabilizing feedbacks based on the algebraic Riccati equations was achieved by the principal investigators in the case of arbitrary analytic semigroups with (strongly) unbounded feedbacks, see [L-T.11]. Examples include parabolic equations with Dirichlet boundary controls and strongly damped wave equations with boundary control.

Numerical schemes for the computation of the solutions of wave equations with rough (nonsmooth) data

Convergence and stability of the algorithm, along with numerical illustrations and examples are obtained in [L-S.1-3]. Numerical analysis for delay differential equations are given in [L-M.1].

SECTION II

SUMMARY OF SCIENTIFIC PROGRESS

List of Recent Work by the Co-Principal Investigators

(I. Lasiecka and R. Triggiani) related to AFOSR Grant 87-0321

A. Work by I. Lasiecka and J. Sokolowski:

1. Finite element approximations of wave equation with Dirichlet boundary data defined on a bounded domain in R^2 (jointly with P. Neittaanmaki). Lecture notes in Control and Information Sciences, 102. Distributed Parameter Systems, ed. by F. Kappel et al., Springer-Verlag, 1987, pp. 216-234.
2. Regularity and strong convergence of variational approximations to nonhomogeneous Dirichlet hyperbolic boundary value problems, in press, SIAM Journal of Mathematical Analysis. Vol. 19, No. 3, 1988.
3. Semidiscrete approximations of hyperbolic boundary value problems with nonhomogeneous Dirichlet boundary conditions, SIAM Journal of Mathematical Analysis, to appear.

B. Work by I. Lasiecka and A. Manitius

Differentiability and convergence rates at approximating semigroups for related functional differential equations. SIAM J. Numer. Anal. Vol 25, No. 4, 1988.

C. Work by I. Lasiecka

1. Strong stabilization of a nonlinear wave equation with dissipation on the boundary and related problems, Recent Advances in Communication and Theory, ed. by R. E. Kalman, et al., Optimization Software Inc., Publication Division, New York, 1987, pp. 194-213.

2. Boundary stabilization of hyperbolic and parabolic equations with nonlinearly perturbed boundary conditions, J. Differential Equations. Vol. 75, No. 1, 1988.
3. Exponential local stability of first order strictly hyperbolic systems with nonlinear perturbations on the boundary. To appear in Lecture Notes in Control Sciences, Springer-Verlag, ed. J. Zolesio 100, pp. 212-235, 1988.
4. Stability of wave equations with nonlinear damping in the Dirichlet and Neumann boundary conditions. To appear in Lecture Notes in Control and Information Sciences, Springer-Verlag.
5. Boundary stabilization of hyperbolic and parabolic equations with nonlinearly perturbed boundary conditions. J. Diff. Eq.
6. Controllability of viscoelastic plates, Vorau Proceedings, 1988, Springer-Verlag Lectures Notes.
7. Asymptotic behavior of the solutions of the Kirchhoff plate with nonlinear dissipation in the bending moment, presented at the IFIP Conference at Clermont-Fernand. To appear, Proceedings Springer-Verlag.

D. Work by I. Lasiecka and R. Triggiani

1. Exponential uniform energy decay rates of the wave equation in a bounded region with $L_2(0, \infty; L_2(\Gamma))$ -boundary feedback in the Dirichlet Boundary Conditions, J. Diff. Eqs., 66 (1987), 340-390.
2. Trace regularity of the solutions of the wave equation with homogeneous Neumann boundary conditions and data supported away from the boundary, J. Mathem. Anal. and Applications, to appear.

3. Exact boundary controllability for the wave equation with Neumann boundary control, Applied Mathem. and Optimization, to appear.
4. A lifting theorem for the time regularity of solutions to abstract equations with unbounded operators and applications to hyperbolic equations, Proc. Amer. Math. Soc., to appear November 1988.
5. Algebraic Riccati equations with non-smoothing observation arising in hyperbolic and Euler-Bernoulli boundary control problems (jointly with F. Flandoli), Annali Mathem. Pura e Appl., to appear.
6. Exact controllability of the Euler-Bernoulli equation with $L_2(\Sigma)$ - control only in the Dirichlet boundary condition, Rendiconti Accademia Nazionale dei Lincei Classe di Scienze Fisiche e Matematiche, Vol. LXXXI, 1987.
7. Exact controllability of the Euler-Bernoulli Equation with Controls in the Dirichlet and Neumann boundary conditions: a non-conservative case, SIAM J. Control, to appear.
8. Exact controllability of the Euler-Bernoulli equation with boundary controls for displacement and moments, J. Math. Anal. and Applications, to appear.
9. Infinite horizon quadratic cost problems for boundary control problems, Proceedings CDC Conference, Los Angeles, Calif., December 1987.
10. Exact controllability and uniform energy decay rates for Euler-Bernoulli equations with control only in the bending moment, presented at Workshops in Vorau (Austria) July 1988; Pisa (Italy) July 1988, at Amer. Math. Soc.

Meeting, University of Kansas, October 1988; at Southeast Conference in Differ. Equations, University of Georgia, Athens, November 1988; at CDC Conference, Austin, Texas, December 1988.

E. Work by Shuping Chen and R. Triggiani

1. Proof of two conjectures of G. Chen and D. L. Russell on structural damping for elastic systems: The case $\alpha = \frac{1}{2}$. Springer-Verlag Lecture Notes in Mathematics, to appear. Also, Proceedings of First Communication Control Conference, Washington D. C., June 1987, to appear.
2. Proof of extensions of two conjectures on structural damping for elastic systems: the case $\frac{1}{2} \leq \alpha \leq 1$, Pacific J. of Mathematics, to appear.
3. Differentiable semigroups arising from elastic systems with gentle perturbation: the case $0 < \alpha < \frac{1}{2}$, to appear. Presented at CDC Conference Austin, Texas, December 1988.

F. Work by Jerry Bartolomeo and R. Triggiani

1. Uniform energy decay rates for Euler-Bernoulli equations with feedback operators in the Dirichlet/Neumann B.C., to appear. Presented at Workshops in Vorau (Austria) July 88; Pisa (Italy) July 88; at Amer. Math Soc. Meeting (Specialized Session) University of Kansas, October 1988; at Southeast Conference in Differential Equations, University of Georgia, Athens, Georgia, November 1988; at CDC Conference Austin, Texas, December 1988.

G. Work by R. Triggiani

1. Wave equation on a bounded domain with boundary dissipation: an operator approach, J. Mathem. Anal. and

Applications, to appear.

Also, Recent Advances in Communication and Control Theory, volume in honor of A. V. Balakrishnan's 60th birthday, ed. by R. E. Kalman, G. I. Marchuck, etc. Optimization Software, New York, 1987, pp. 262-286.

Also, Operator Methods for Optimal Control Problems, ed. by S. Lee, Marcel Dekker, 1987, Vol. 108, pp. 283-310.

2. Exact boundary controllability on $L_2(\Omega) \times H^1(\Omega)$ of the wave equation with Dirichlet boundary control acting on a portion of the boundary and related problems, Applied Mathem. and Optimiz., Appl Math & Optimiz 18 (1988) 241-277. Also: Lecture Notes in Control Sciences, Springer-Verlag, No. 102, pp. 292-332, 1987.
3. Finite rank, relatively bounded perturbations of semigroup generators. Part III: A sharp result of the lack of uniform stabilization, Springer-Verlag Lecture Notes in Control, Proceedings, INRIA Conference, Antibes (France) June 1988. Also: Proceedings of First Conference on Communication and Control Theory, Washington DC, June 1987, Optimization Software, pp. 123-146.
4. Lack of uniform stabilization of non-contractive semigroups under compact perturbation, Proc. Amer. Math. Soc. to appear January 1989.
5. Exact controllability of wave and Euler-Bernoulli equations in the presence of damping, Proceedings International Conference on Differential Equations Columbus, Ohio, March 21-25, 1988; and Proceedings of Conference "Thirty years of modern optimal control theory," University of Rhode Island, Kingston, Rhode Island, June 1988.

6. Constructive exact controllability and finite rank conditions, to appear.
7. A counter example to strong stability for dissipative operators, to appear.

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