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CONTINUOUS GLOBAL N-TUPLE COVERAGE WITH (2N + 2) SATELLITES

STA Final Report - 028

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Sponsor: U.S. Air Force Space Division SD/XRX
Los Angeles, CA 90009-2960

Contract Number: F04701-88-C-0080

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DESCRIPTION OF STUDY

- Six-month study
  - — SBIR Phase One Effort
  - — Principal investigator, plus associate investigator
  - — Computer intensive
  - — Emphasis on innovation
  - — Sponsor: USAF Space Division

CONTINUOUS GLOBAL N-TUPLE COVERAGE WITH (2N + 2) SATELLITES -- DESCRIPTION OF STUDY

This study was carried out under the auspices of the Small Business Innovation Research (SBIR) Program. The Study Sponsor was the U.S. Air Force Space Division, Los Angeles, California; Capt. James H. Sloan, USAF, SD/XRX, (213) 336-4625. Duration of the study was six months, beginning on September 22, 1988. The principal investigator, Mr. John E. Draim, and an associate investigator, Mr. Henry M. Bowers, both of Science and Technology Associates, performed the research. The study was computer intensive and emphasized innovation in the conception and development of elliptic-orbit, multi-satellite arrays.

PURPOSE OF STUDY

- Explore whether triple and quadruple coverage can be obtained with 8- and 10-satellite arrays
  - — Answer is yes, but only at very high, super-synchronous altitudes
- Develop a generalized, or unified theory for all minimum-satellite continuous-coverage constellations
  - — Accomplished, with an indexed matrix of orbital parameters

PURPOSE OF STUDY

The purpose of the study was to investigate the underlying principles of continuous, redundant coverage of a spherical planet, and to determine the minimum number of satellites required to provide this coverage. Previous research by the principal investigator had already determined that continuous global single coverage is possible with four satellites, and that continuous global double coverage is possible with six satellites. By extension, it appeared reasonable to assume that continuous global triple and quadruple coverage should be feasible using eight, and ten satellites, respectively. A further objective was the determination of the actual orbital parameters for such constellations.

ACCOMPLISHMENTS OF STUDY

- Major objectives accomplished
  - Developed additional theorems and corollaries
  - Developed computer models
  - Proved stated coverage is possible
  - Determined actual orbital parameters for arrays

ACCOMPLISHMENTS OF STUDY

Several new theorems and corollaries addressing satellite coverage were developed. These proved helpful in the statement of a generalized, or unified, theory for continuous-coverage, minimum-satellite constellations. Some new computer models were developed which verified that the minimum desired redundant coverage was in fact obtained throughout the constellation period. The optimized orbital parameters of the constellations are defined for single through quadruple continuous coverage of a spherical earth. The generalized, or unified, theory can be concisely presented by means of an indexed table of orbital parameters for minimum satellite constellations of any desired degree of redundant coverage.

SATELLITE REQUIREMENTS FOR N-TUPLE COVERAGE
(MINIMUM NUMBER)

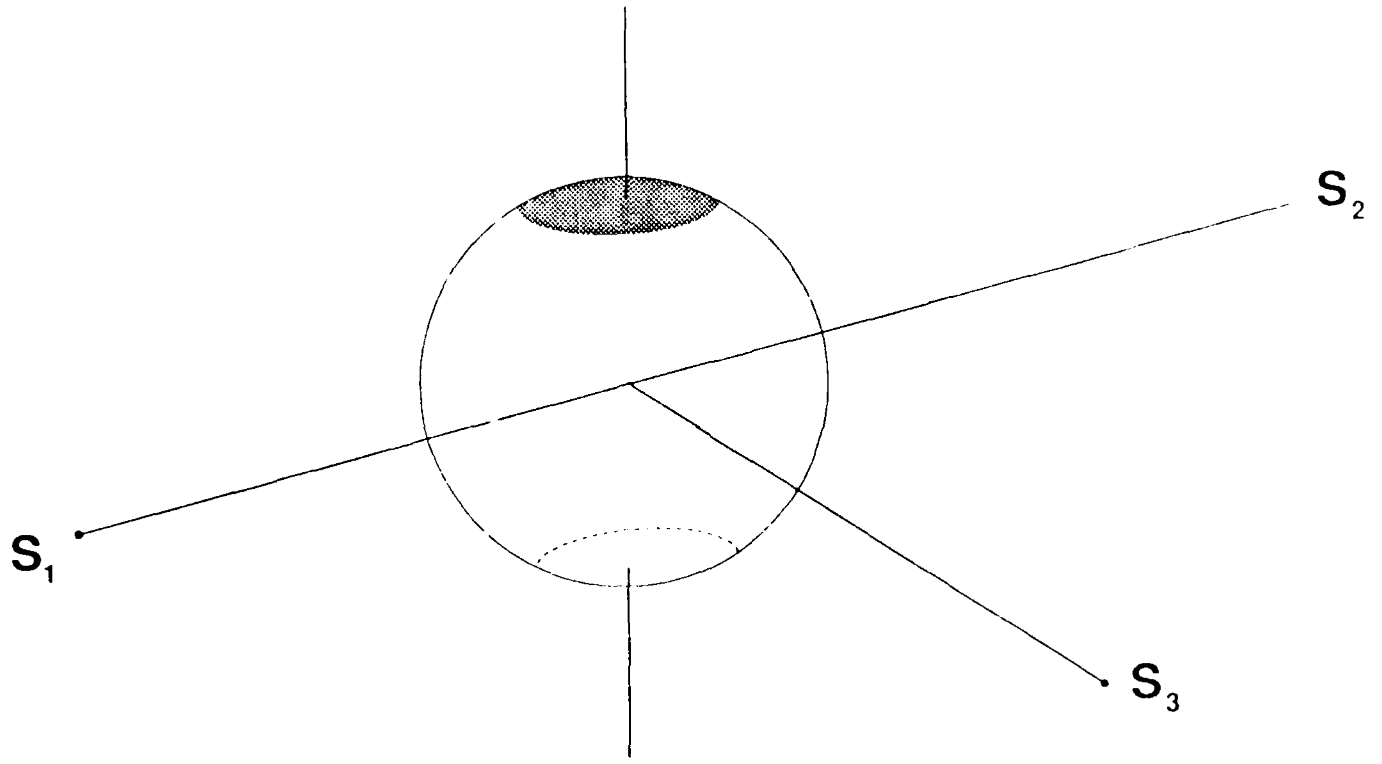
science and technolo​gy associates, Inc.

Coverage Level / Developer	Gobetz/Easton Constellations 1963 - 1969	Walker Constellations 1970 - 1982	Draim Constellations 1984 - 1989
Continuous Single Coverage	6	5	4
Continuous Double Coverage	—	7	6
Continuous Triple Coverage	—	9	8
Continuous Quadruple Coverage	—	11	10

SATELLITE REQUIREMENTS FOR N-TUPLE COVERAGE (MINIMUM NUMBER)

The minimum number of satellites required for continuous global single coverage was commonly thought to be six in the first decade of the space age. As more sophisticated approaches to constellation design were proposed and developed by Mr. John Walker, of Britain's Royal Aeronautical Establishment, five satellite arrays giving this type of coverage were developed. The first two four satellite continuous global single-coverage arrays were developed by the principal investigator in 1984 and 1985. The first six-satellite continuous global double-coverage array was developed by the principal investigator in 1986. Continuous triple- and quadruple-coverage arrays using eight and ten satellites, respectively, were developed in 1988.

THEOREM III



If two satellites (S_1 and S_2) are on diametrically opposite sides of a planet, a minimum of (2n + 3) satellites is required to obtain instantaneous global n-tuple coverage.

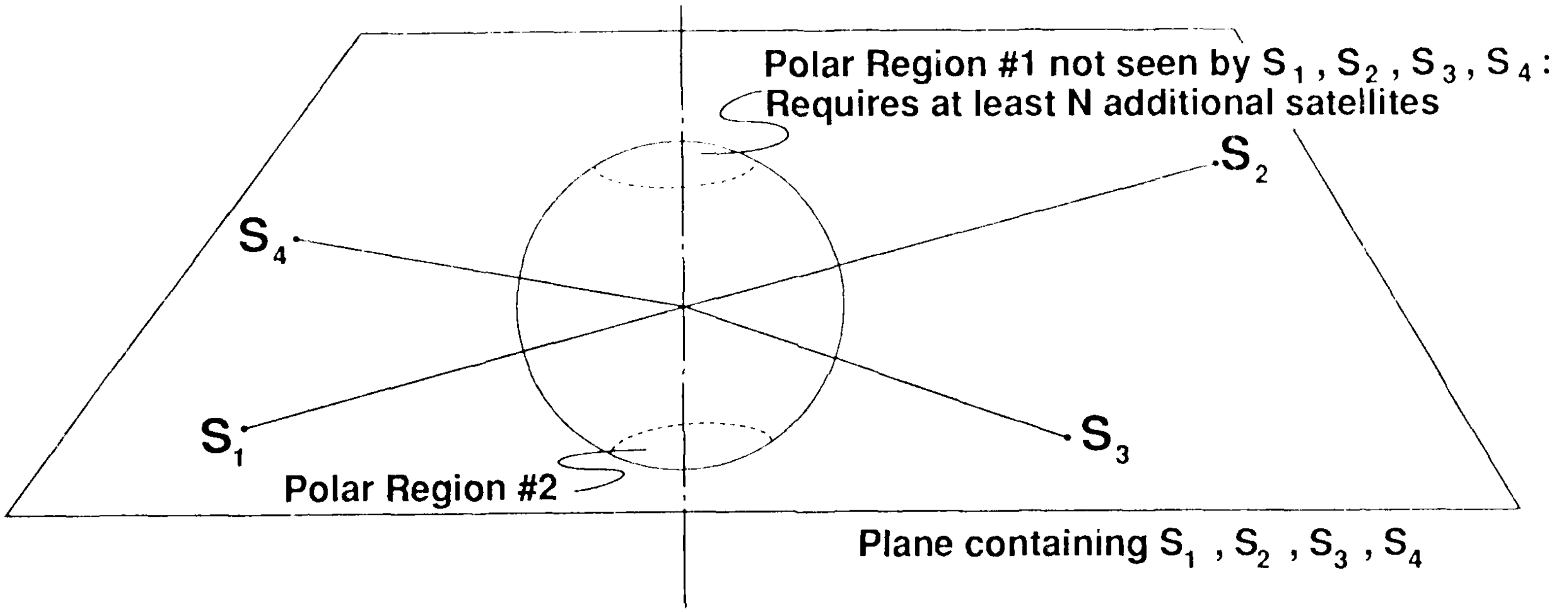
THEOREM III

Several theorems and corollaries were developed by the principal investigator to assist in understanding redundant-coverage phenomena. Theorem I and II were previously published by the principal investigator in AIAA Technical papers. Theorem III is important, since it shows that n-tuple coverage cannot be obtained with (2n + 2) satellites if any two satellites occupy positions diametrically opposite each other. This theorem, when extended to all instants of time in the constellation period, will address the problem of continuous coverage.

Theorem III: If two satellites in a multi-satellite constellation are diametrically opposite one another, a minimum of (2n + 3) satellites is required to obtain instantaneous global n-tuple coverage. (Satellites are assumed to be at a finite altitude.)

Proof: A great circle may be passed through the two satellites which are diametrically opposite and any third satellite. An additional 2n satellites, at least, are then needed in order to obtain n-tuple coverage of the two poles normal to the great circle plane.

COROLLARY III



If m satellites are coplanar with a planet's center, then a minimum of (m + 2n) satellites is required to obtain instantaneous n-tuple coverage.

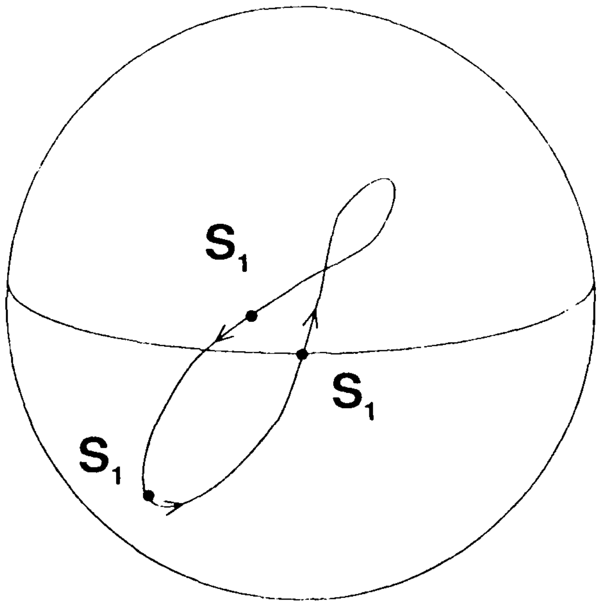
COROLLARY III

A corollary to Theorem III which extends the locus of base satellites from a linear (diameter) to a planar arrangement is shown in this diagram.

Corollary III: If m satellites are coplanar with a planet's center, then a minimum of (m + 2n) satellites are required to obtain instantaneous n-tuple coverage.

Proof: A great circle may be passed through the m satellites. The two poles to the great circle require an additional 2n satellites, at a minimum, to obtain n-tuple coverage.

THEOREM IV



For a system of synchronous elliptic orbit satellites to have identical ground tracks, the satellites must have the same inclination, eccentricity and argument of perigee, and the sum of the right ascension of the ascending node and the satellite mean anomaly must be a constant, for each satellite in the system, for any instant of time.

THEOREM IV

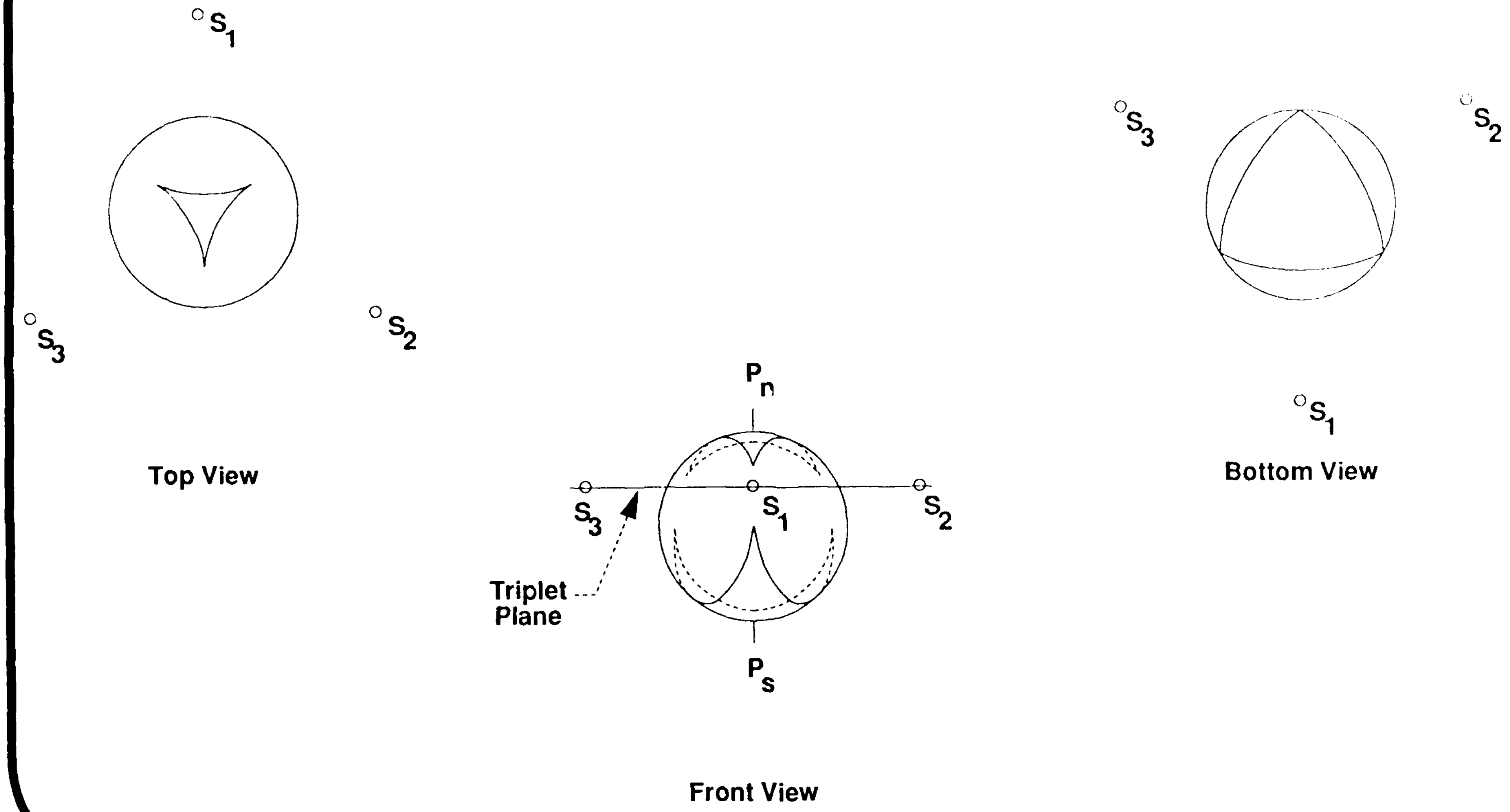
Theorem IV: For a system of synchronous elliptic orbit satellites to have identical ground tracks, the satellites must have the same period, inclination, eccentricity and argument of perigee, and the sum of the right ascension of the ascending node and the mean anomaly for each and every satellite in the system must be a constant, for any instant of time.

Proof: Since all of the satellites' parameters except right ascension of the ascending nodes and mean anomalies are identical, the shape of all of the orbital ground tracks are identical, save for a possible longitudinal displacement. Equatorial crossings will occur at the same longitude, resulting in overlying ground tracks, provided any angular increase in the right ascension of a given satellite's line of nodes is offset by an equal but opposite angular decrease of the mean anomaly for that same satellite.

THEOREM IV (cont.)

Subtracting the mean anomaly for a synchronous system effectively subtracts out the planetary rotation rate and converts the inertial reference frame to a planetary reference frame. Constraining the sum of the right ascension of ascending node plus the mean anomaly to a unique and constant value ensures that any increase in right ascension of the ascending node will occasion a corresponding decrease in mean anomaly. Then, at earlier or later points in time where the other satellites in the system cross the equator at their respective ascending nodes, their mean anomalies will be the same as that of the original reference satellite. The crossing longitudes will thus be the same, since the shape of every satellite's ground track is identical (with corresponding mean anomalies matching with the latitude crossings).

THEOREM V


S_1
S_3
S_2
Top View
P_n
S_3
S_1
S_2
Triplet Plane
P_s
Front View
S_3
S_2
S_1
Bottom View

THEOREM V

If three satellites form a plane which intersects a spherical planet, then a minimum of (2n + 3) satellites is required to obtain instantaneous global n-tuple coverage.

Proof: Erect a perpendicular to the triplet plane through the planet's center. We will define the "poles" as the intersections of this perpendicular diameter with the surface of the planet. It may be easily seen that at each pole there is no visibility by any of the triplet satellites (assuming these satellites are at finite altitudes). Thus, n additional satellites are needed to provide instantaneous n-tuple coverage of each of these poles. The minimum total number of satellites required is, therefore, (2n + 3). That is, the three satellites in the intersecting plane, plus 2n, for the two poles.

Note: this theorem represents an extension of Theorem III and Corollary III for a plane which intersects the planet, but does not necessarily pass through its center.

COMPUTER PROGRAMS USED IN CONSTELLATION ANALYSIS

- LIMIT PLOTTER
- MACPERP3
- LIMIT ANIMATOR
- MOVIE MAKER
- COVSTAT
- VIEW PLOTTER

COMPUTER PROGRAMS USED IN CONSTELLATION ANALYSIS

A number of computer programs were developed by STA for use in the Phase I SBIR Study. It is helpful to briefly review the purpose of these programs, in order to understand the family of tetrahedral/prismoidal continuous-coverage arrays. Succeeding charts will address each of the listed programs.

LIMIT PLOTTER

- Calculates ground-track coordinates in latitude and longitude for all satellites
- Rotates coordinate frame to shift satellite groups from equator to poles
- Calculates visibility-limit curves in latitude and longitude for all satellites, centered on equatorial band
- Plots visibility-limit curves for all satellites

LIMIT PLOTTER

LIMIT PLOTTER creates a plot of the overlapping visibility limits of all satellites in a given constellation. This graph can then be used to check for gaps in coverage that may fall below the minimum level of coverage demanded of the constellation.

LIMIT PLOTTER receives the orbital parameters of each satellite in a constellation, the period of the constellation, and the instant in time during the constellation's orbit that is to be examined in detail. This information is run through a sequence of three processes. First, the ground-track coordinates are calculated for each satellite in the constellation. In order to facilitate ground-track calculations the computerized version of the Earth is forced to rotate at the same period as the constellation, producing a "pseudo-synchronous" constellation orbit.

LIMIT PLOTTER (cont.)

Because the visibility limit curve plot is rectangular and plotted in latitudinal and longitudinal coordinates, there is a large distortion in higher latitudes near the poles. To avoid this distortion, the satellite position points and visibility limit curves are both rotated ninety degrees so that the visibility limits lie in the relatively undistorted equatorial region. The satellite ground tracks are not plotted.

After the ground-track coordinates are calculated for each satellite, the visibility limit curves for these ground-track coordinates are calculated. The limit curves represent the line along the globe that separates the portion of the world that the satellite can see from the portion of the world that they cannot see.

A plot can be created for any instant in time during the orbit of a constellation. By analyzing the plot, we are able to verify that minimum coverage has been maintained, at that instant.

MACPERP3

- Surveys satellite orbits for minimum look angle and minimum distance from earth for all satellites in the constellation
- Pinpoints minimum orbital period for constellation that provides the minimum required coverage
- Calculates inclination and eccentricity for constellation of minimum orbital period

MACPERP3

MACPERP3 finds the minimum period of the constellation that satisfies maintenance of at least n-tuple coverage. In the process, other crucial parameters, namely inclination and eccentricity of the constellation, are calculated.

The main process of the program puts Theorem V to use by calculating the orthogonal distance from the plane formed by three satellites in the constellation to the surface of the Earth. If during an orbital period no instances occur where the plane intersects the Earth at more than a point, then that period is acceptable. The smallest acceptable period is the minimum which we are trying to find.

The program actually works by surveying a user-selected realm of inclination, eccentricity, and period, and prints out the look angles for each combination being examined. A look angle of zero corresponds to a minimum for the search space. By increasing the resolution and decreasing the area of the search space, the global minimum period can be located.

LIMIT ANIMATOR AND MOVIE MAKER

- Creates frame sequences of visibility limit graphs for n satellites
- Displays "movie" on screen of changing visibility limits for the duration of orbit for any constellation

LIMIT ANIMATOR AND MOVIE MAKER

LIMIT ANIMATOR is a spin-off of LIMIT PLOTTER. Its primary purpose is to store a sequence of visibility limit plots for the duration of a constellation's orbital period. The plots are generated in the same manner as described for LIMIT PLOTTER.

Once the sequence has been created, another program, MOVIE MAKER, plays back the plots on the computer screen, producing a moving effect to the visibility limit curves. The resulting "movie" illustrates the changing coverage patterns over a period, and helps to locate the existence or non-existence of gaps in coverage.

As in LIMIT PLOTTER, the visibility limit curves have been reoriented to avoid major distortions. In the real world the identical pattern shown around the equatorial region would exist, but it would be aligned along a great circle passing through the north and south poles.

COVSTAT

- Calculates average percentage of global coverage for a given constellation
- Reveals minimum and maximum levels of global coverage for a given constellation
- Produces distribution of levels of coverage from minimum to maximum for a given constellation

COVSTAT

COVSTAT is another spin-off of LIMIT PLOTTER. The purpose of COVSTAT is to calculate the average percentage of each existent level of coverage over an orbital period for a constellation.

In this program, instead of plotting the overlying visibility limit curves for a constellation, the main process stores an internal "picture" of the plot. The picture is then divided into a grid of degrees in latitude and longitude. Each square degree is then assigned a level of coverage. The number of squares containing a particular level are then compared to the total number of squares being surveyed to obtain the instantaneous percentage of coverage. This instantaneous percentage is calculated over a full period, and then averaged to obtain the average percentage values for each level of coverage.

COVSTAT (cont.)

COVSTAT provides statistical information which can be used to substantiate the maintenance of a minimum level of coverage over an orbital period. Additionally, the average percentage of coverage is a very useful measure with which two differing constellation designs may be compared. The statistics may be presented as a frequency histogram (for each individual level of coverage) or as a cumulative histogram (for all levels of coverage equal to or greater than a particular level.

VIEW PLOTTER

- Prints side view of locus of all satellite points in XZ plane for any given constellation

- Prints frontal view of locus of all satellite points in YZ plane for any given satellite

- Depicts earth in correct size and position with respect to locus to provide scale

VIEW PLOTTER

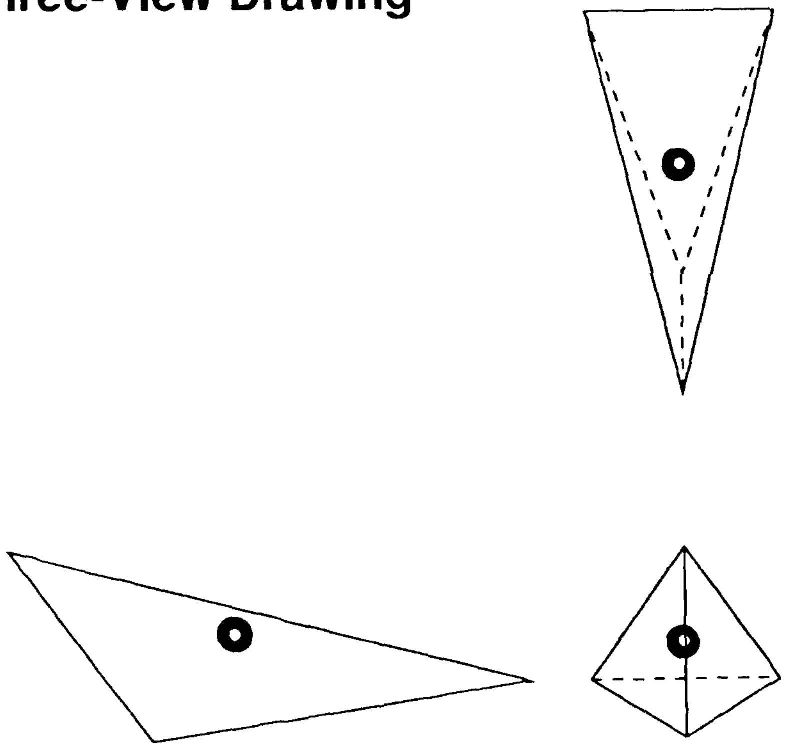
VIEW PLOTTER produces two views of a constellation as it rotates around the Earth. The first view is the locus of a constellation's satellite points over an orbital period that lie in the celestial XZ plane. This produces an edge-on view of the constellation over time that shows a mostly linear relationship among the satellite points in the view. The second view is the locus of a constellation's satellite points over an orbital period that lie in the celestial YZ plane. This view reveals the elliptical nature of the satellites' paths.

VIEW PLOTTER calculates the two views by converting the satellite positions into cartesian coordinates in a pseudo-planet reference frame with the center of the Earth located at the origin. The appropriate two values (X and Z or Y and Z, depending on the view) for each point are then plotted over the duration of the period, to complete the loci plot.

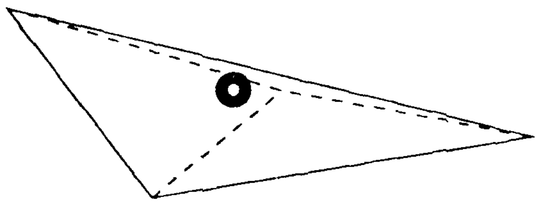
FOUR-SATELLITE TETRAHEDRAL CONSTELLATION FOR CONTINUOUS SINGLE COVERAGE *

science and technology associates, inc.

Three-View Drawing



Isometric View

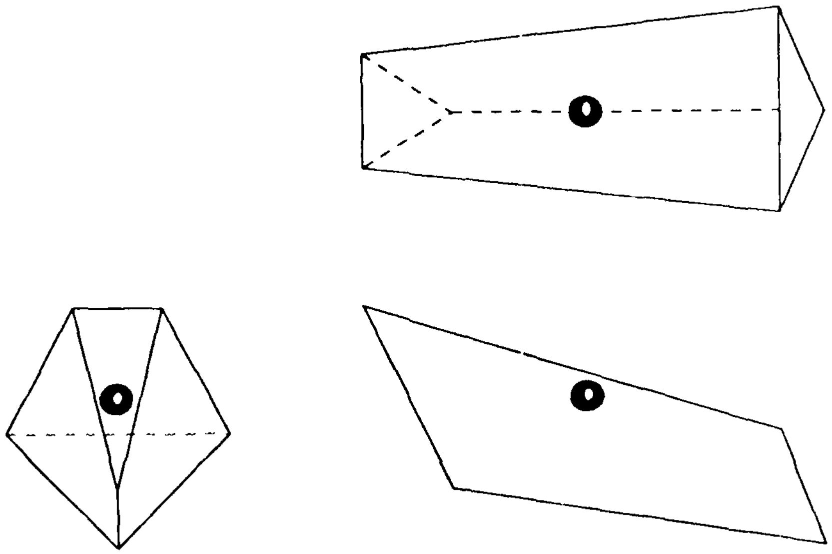


* U.S. Patent Pending

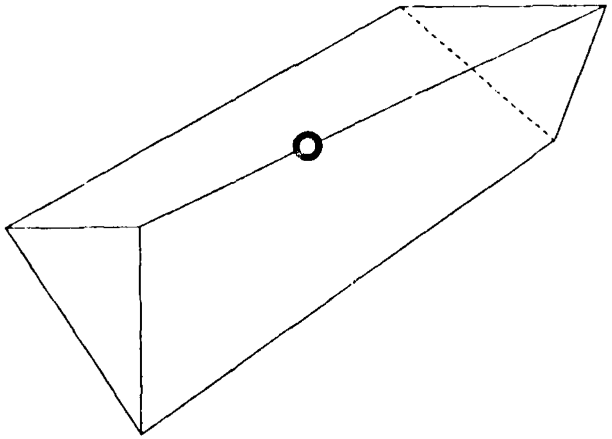
FOUR-SATELLITE TETRAHEDRAL CONSTELLATION FOR CONTINUOUS SINGLE COVERAGE

This figure shows a three view and an isometric view of the four-satellite tetrahedral continuous-coverage constellation at the starting position. This constellation was developed in 1985 solely by Science and Technology Associates, Inc., and for which there is a U.S. Patent Pending. It has been fully described in the article "A Common-Period Four-Satellite Continuous Global Coverage Constellation" by John E. Draim, which appeared in the Sept-Oct 1987 issue of the Journal of Guidance, Control and Dynamics. Unlike all of the other arrays for double- and higher-order redundancy of coverage, which are prismoids, this array is purely tetrahedral. It is included in this study for completeness, since it does provide continuous single coverage with 4 (i.e., $2n + 2$) satellites. It is evident that the array is not a regular tetrahedron, but rather a warped, elongated tetrahedron. The shape and size of the tetrahedron changes constantly as the satellites (at the corners) move in their orbits. However, the sides of the tetrahedron always enclose, and never make contact with, the spherical planet. This ensures the continuity of the coverage by at least one satellite at any point on the planet's surface.

SIX-SATELLITE TRIANGULAR PRISMOID CONSTELLATION FOR CONTINUOUS DOUBLE COVERAGE



Three-View Drawing

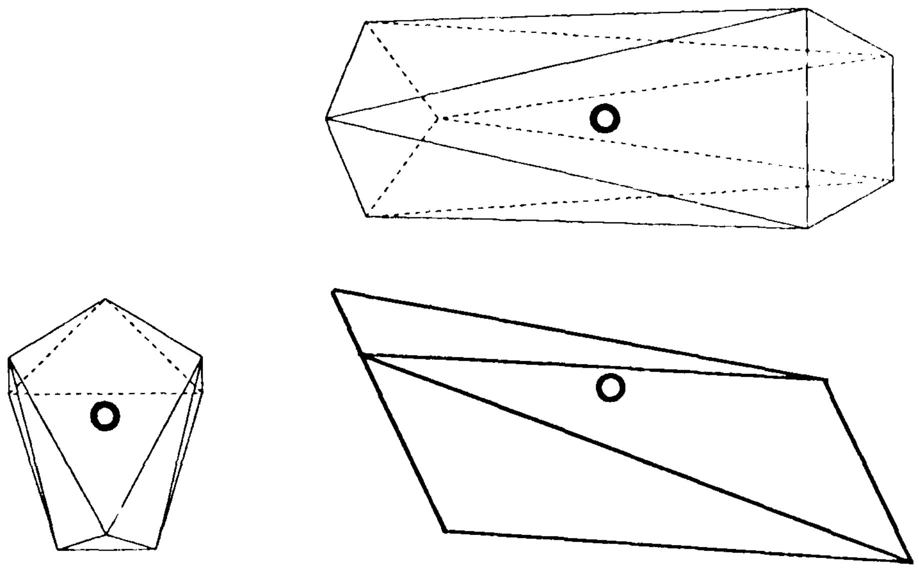


Isometric View

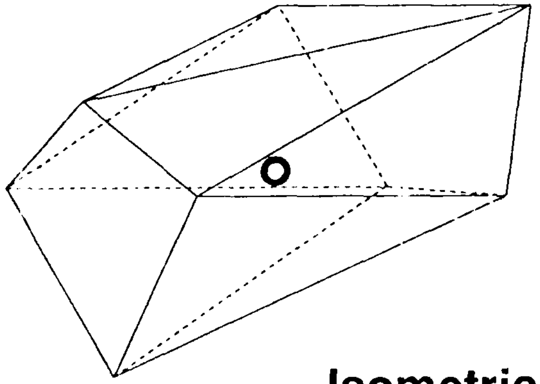
SIX-SATELLITE TRIANGULAR PRISMOID CONSTELLATION FOR CONTINUOUS DOUBLE COVERAGE

This figure shows a warped, elongated, triangular prismoid which provides continuous double coverage of the enclosed planet's surface. This constellation was developed by STA in 1986 and was completely described in AAS 87-497 "A Six-Satellite Continuous Global Double Coverage Constellation" (August 10, 1987). It represents the first of the prismoidal class of constellations for continuous, redundant coverage. The array is shown at the starting position (the beginning of a constellation period). Unlike the tetrahedral single-coverage array, this array must satisfy more complicated, additional constraints, due to the larger number of satellites, to ensure the desired double coverage. A basic requirement which must be met, however, is that no plane determined by a satellite triplet can ever be permitted to intersect the sphere representing the planet (Theorem V). Another feature shown in this figure is that of congruent alignment of the triangular prismoid bases (at the top of the figure). This congruent arrangement is typical for the even levels of redundant coverage (i.e., 2x, 4x, 6x, etc).

EIGHT-SATELLITE QUADRANGULAR PRISMOID FOR CONTINUOUS TRIPLE COVERAGE



Three-View Drawing



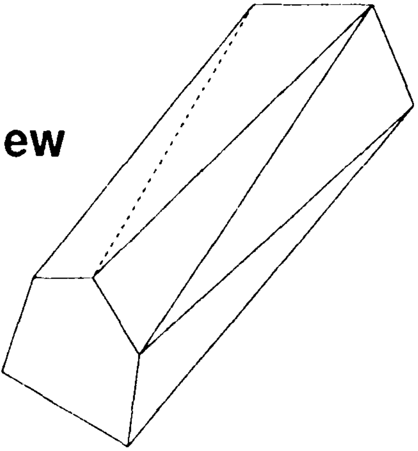
Isometric View

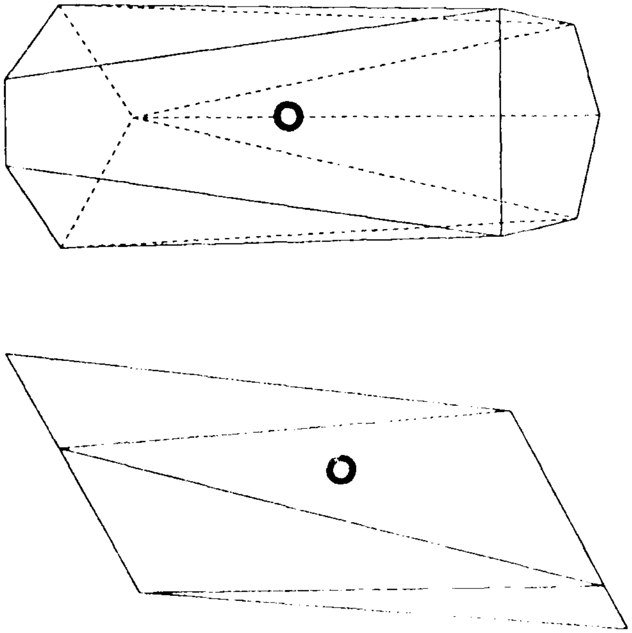
EIGHT-SATELLITE QUADRANGULAR PRISMOID FOR CONTINUOUS TRIPLE COVERAGE

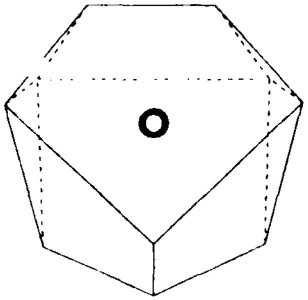
Shown here is a quadrangular prismoid (eight satellites) giving continuous global triple coverage. As is typical for odd levels of redundant coverage, the array features a skewed alignment of the prismoid bases. Here again, the considerations of visibility limit lines impose the constraint that no plane determined by any satellite triplet can be allowed to intersect the surface of the spherical planet.

TEN-SATELLITE PENTAGONAL PRISMOID FOR CONTINUOUS QUADRUPLE COVERAGE

Isometric View





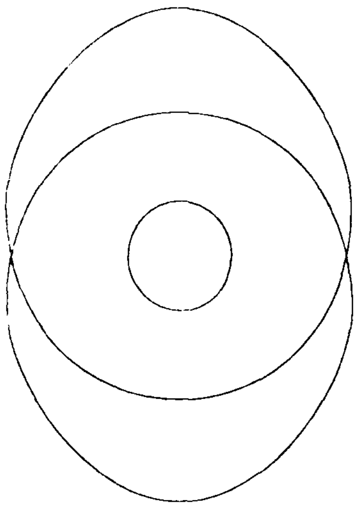


Three-View Drawing

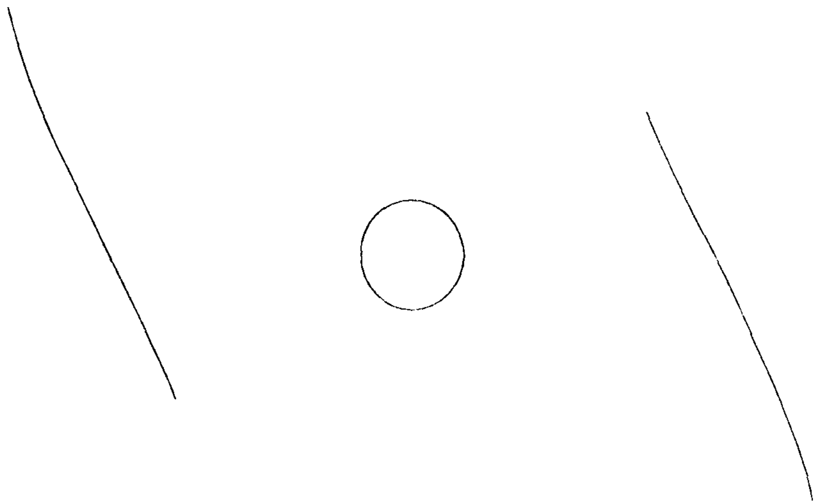
TEN-SATELLITE PENTAGONAL PRISMOID FOR CONTINUOUS QUADRUPLE COVERAGE

This figure shows a three view and an isometric view of the pentagonal prismoid for continuous quadruple coverage. As in the six-satellite double-coverage array, the bases exhibit congruent alignment.

FOUR-SATELLITE TETRAHEDRAL ARRAY *
FRONTAL AND SIDE VIEWS IN PSEUDO-PLANET REFERENCE FRAME



Frontal View



Side View

* U.S. Patent Pending

FOUR-SATELLITE TETRAHEDRAL ARRAY *

FRONTAL AND SIDE VIEWS IN PSEUDO-PLANET REFERENCE FRAME

This figure shows the loci of satellite positions, over a complete constellation period, as seen by an observer rotating in a reference frame attached to the pseudo-planet. It is interesting to note that the side view shows an almost perfect linearity for satellite motion. The frontal view shows clearly the front hemisphere satellite pair loci (perigees in southern hemisphere), and the back hemisphere satellite pair loci (perigees in northern hemisphere). Each pair appears as a single oval shape. The planet is shown to scale lying entirely within the ovals in the frontal view, and between the two straight lines in the side view.

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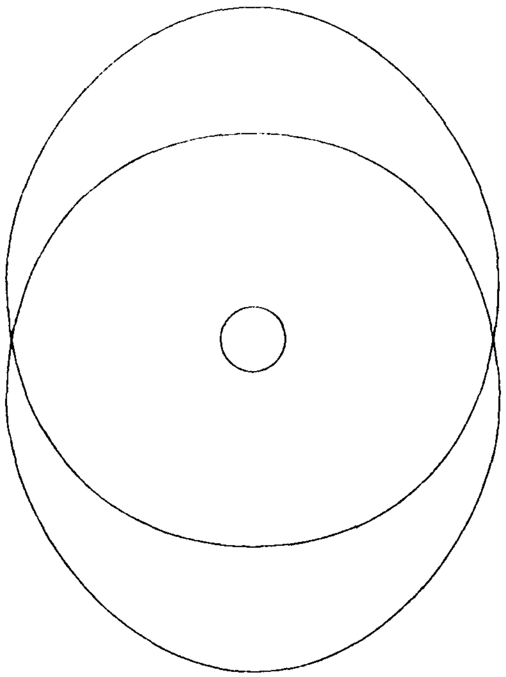
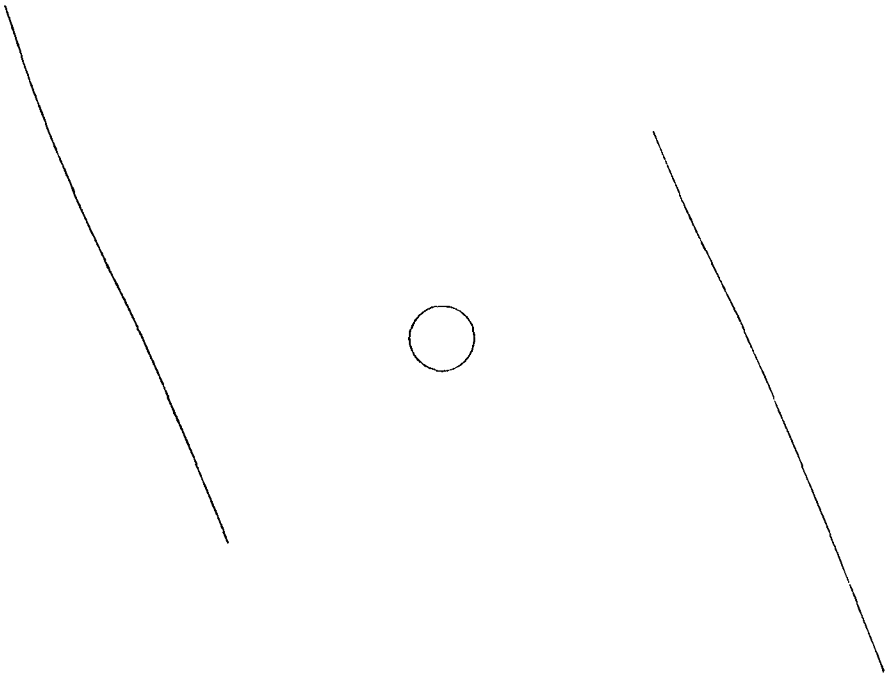
* **U.S. Patent Pending**

SIX-SATELLITE TRIANGULAR PRISMOIDAL ARRAY
FRONTAL AND SIDE VIEWS IN PSEUDO-PLANET REFERENCE FRAME

science and technology associates, Inc.



Frontal View



Side View

SIX-SATELLITE TRIANGULAR PRISMOIDAL ARRAY

FRONTAL AND SIDE VIEWS IN PSEUDO-PLANET REFERENCE FRAME

As in the previous figure, the loci of satellite positions in a rotating pseudo-planet reference frame are shown in front and side views. Again the planet is shown in scale to the satellite positions. Since the planet appears smaller, it is evident that the satellite altitudes are considerably higher than in the four satellite array.

EIGHT-SATELLITE QUADRANGULAR PRISMOIDAL ARRAY
FRONTAL AND SIDE VIEWS IN PSEUDO-PLANET REFERENCE FRAME

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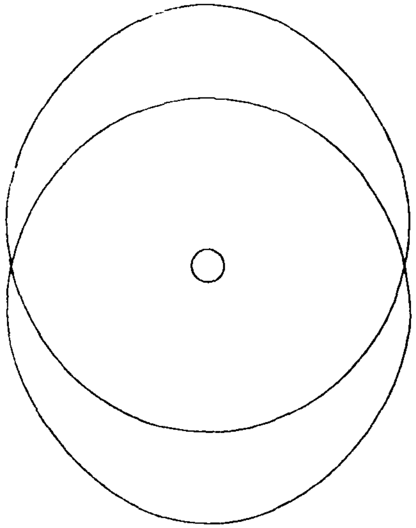
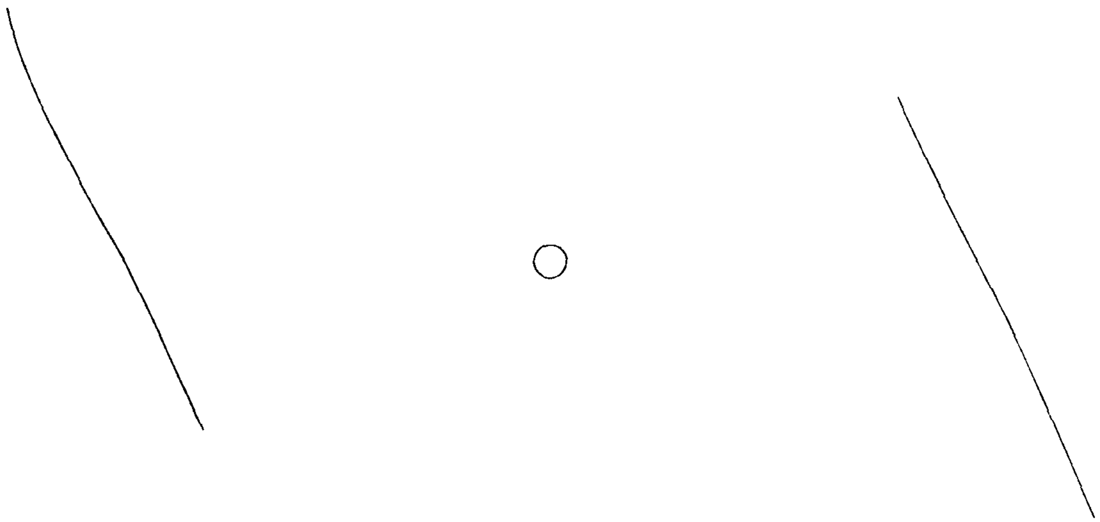


Frontal View



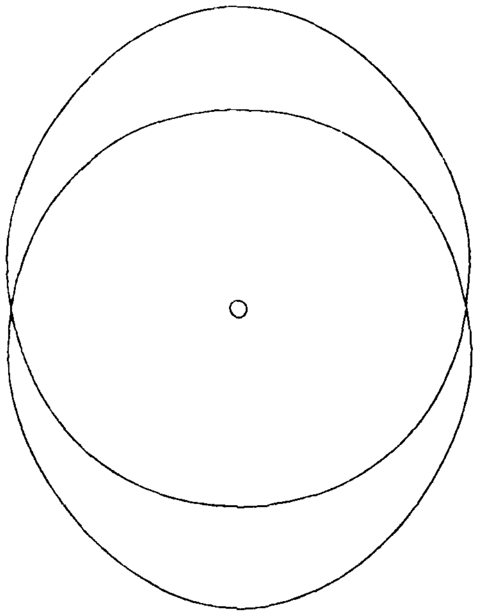
Side View

EIGHT-SATELLITE QUADRANGULAR PRISMOIDAL ARRAY

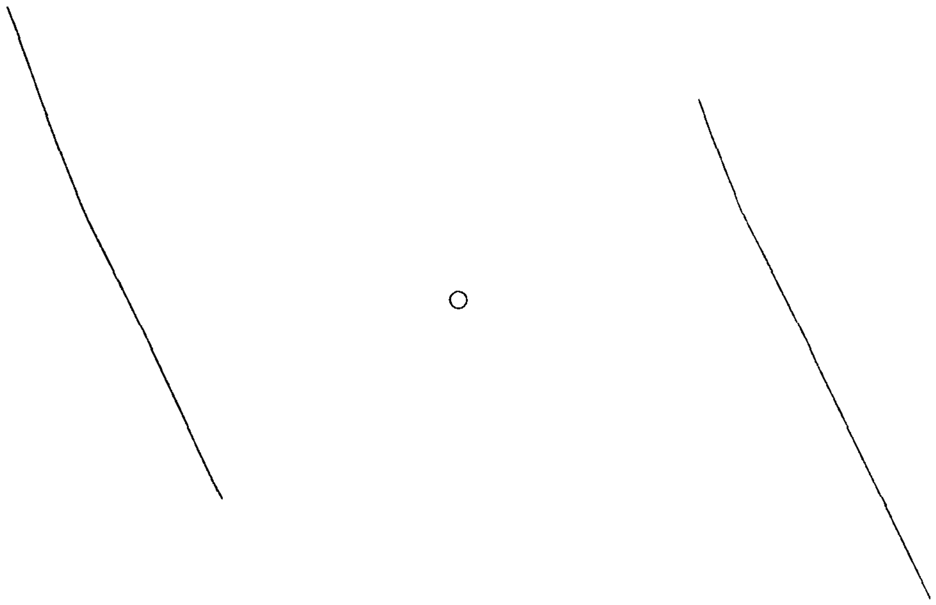
FRONTAL AND SIDE VIEWS IN PSEUDO-PLANET REFERENCE FRAME

Same as before, for the continuous triple-coverage eight-satellite array. Note the skewed arrangement of the prismoid bases, which is characteristic of odd-levels-of-redundancy constellations.

TEN-SATELLITE PENTAGONAL PRISMOIDAL ARRAY
FRONTAL AND SIDE VIEWS IN PSEUDO-PLANET REFERENCE FRAME



Frontal View



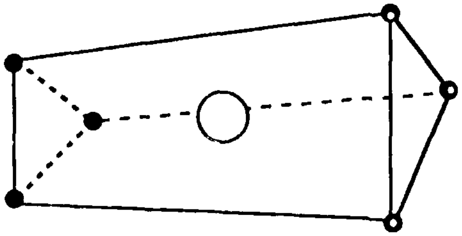
Side View

TEN-SATELLITE PENTAGONAL PRISMOIDAL ARRAY

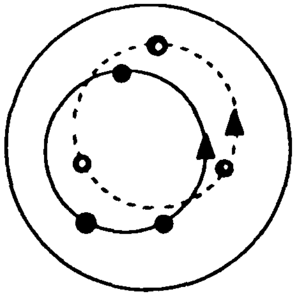
FRONTAL AND SIDE VIEWS IN PSEUDO-PLANET REFERENCE FRAME

This figure shows a side view and a front view of the pentagonal prismoid for continuous quadruple coverage. As in the six-satellite double-coverage array, the bases exhibit congruent alignment.

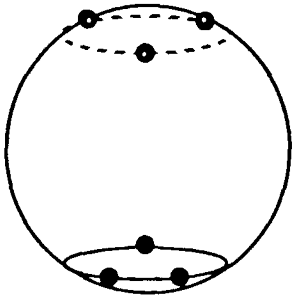
PROCEDURE FOR GENERATING CONSTELLATION VISIBILITY LIMITS



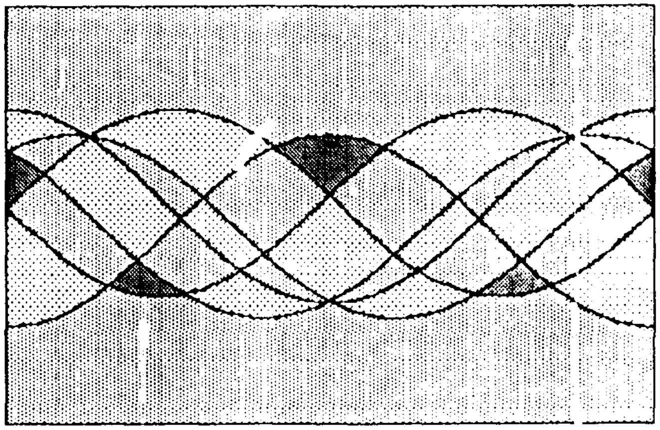
(1) OBTAIN CONSTELLATION ORBITAL PARAMETERS.



(2) CALCULATE GROUND TRACKS USING SATELLITE SUBORBITAL POINTS.



(3) ROTATE GROUND TRACKS 90 DEGREES.

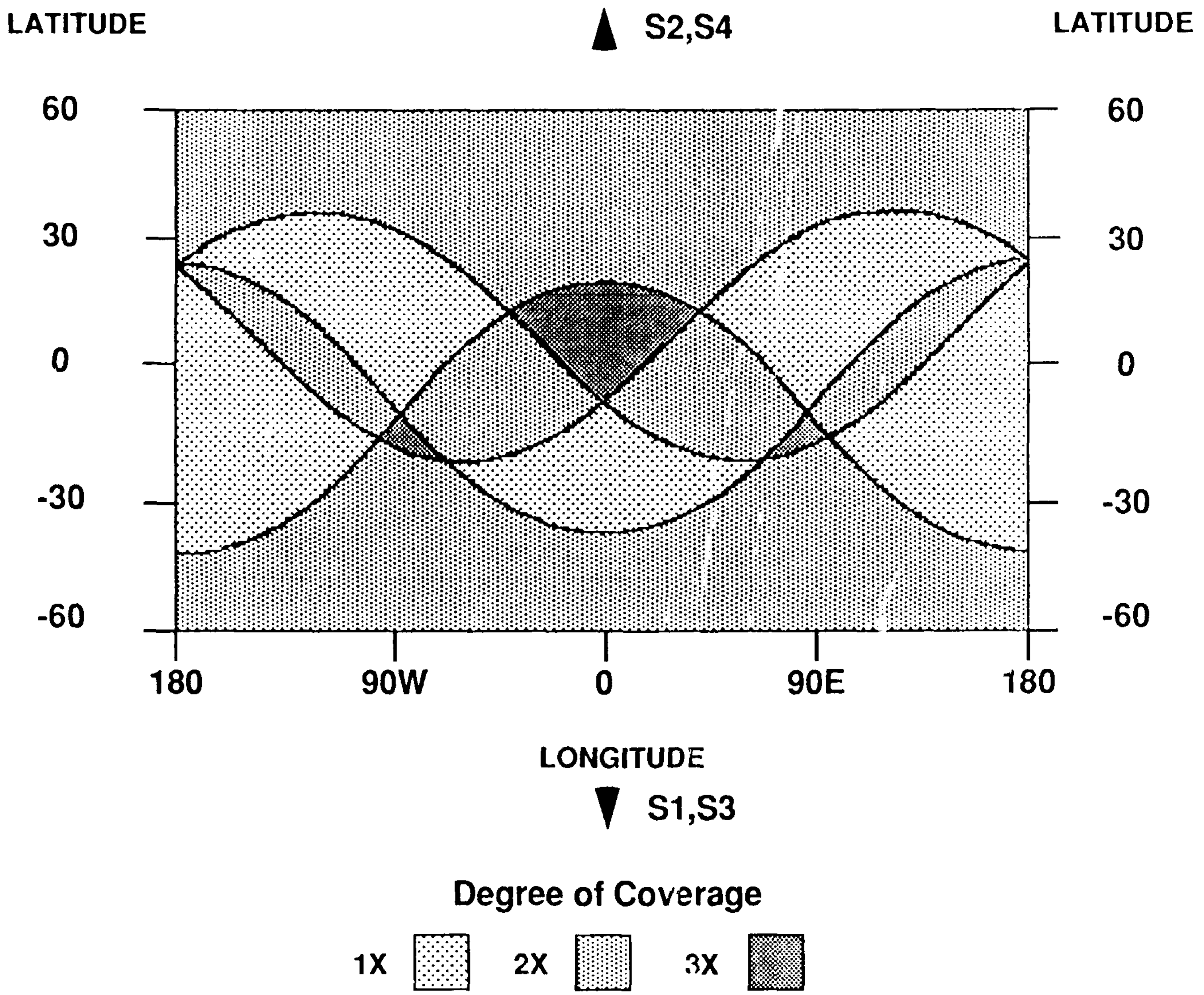


(4) GRAPH VISIBILITY LIMITS FOR EACH SATELLITE ILLUSTRATING LEVEL OF COVERAGE.

PROCEDURE FOR GENERATING CONSTELLATION VISIBILITY LIMITS

The usual orientation of a tetrahedral or prismoidal continuous-coverage constellation is with the centers of the circular pseudo-ground tracks lying on the equator. This results in a relatively narrow rope-like pattern oriented along an orthogonal pseudo-meridian. To avoid distortion, we rotate the ground tracks 90 degree (step 3) so that they are centered at the north and south poles, so that the rope-yarn pattern lies along the equator - thus minimizing distortion on the usual map projections. We have used a rectangular projection, but a Mercator or equal area projection might also be employed. From each instantaneous satellite position we can plot a visibility limit line, giving us the pattern shown in step (4).

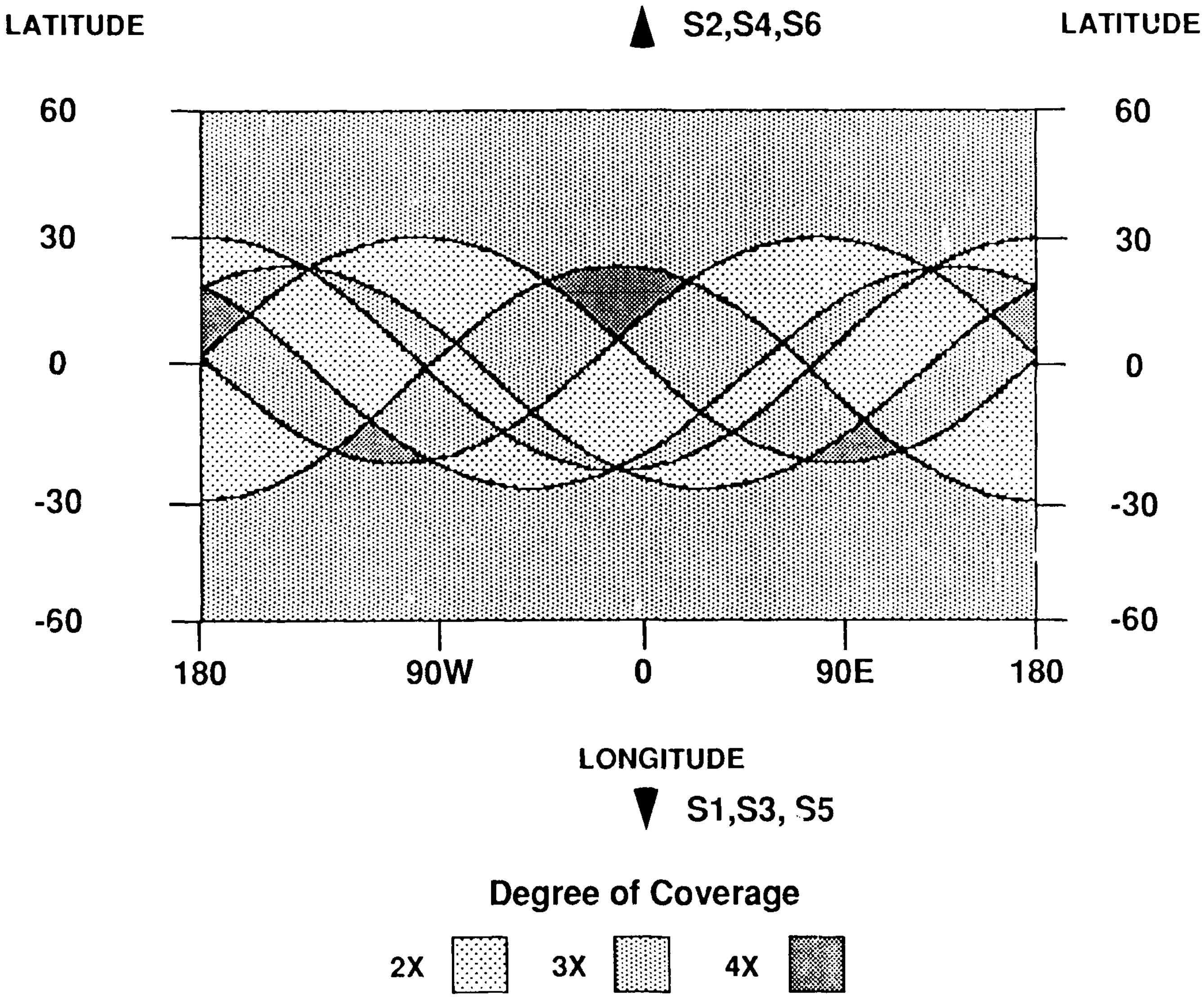
FOUR-SATELLITE VISIBILITY LIMITS STARTING POSITON OF CONSTELLATION



FOUR-SATELLITE ARRAY VISIBILITY LIMITS
(SINGLE COVERAGE MINIMUM)

This figure, compiled from the computer program LIMIT PLOTTER portrays a picture of the areas of coverage for specified levels of redundancy. This example represents coverage of the four-satellite tetrahedral, single-coverage array at the constellation starting position. In order to depict the visibility limits on a rectangular lat-long grid, the satellite groups have been rotated 90° so that their ground tracks appear to encircle the north and south poles rather than two points 180° apart on the equator. Note that the large areas lying nearest a particular satellite group have a visibility level of (n + 1) — half the number of satellites in the constellation. If the period of the satellite is at the minimum value to sustain coverage there will be three visibility lines converging to a point at some time during the constellation period (e.g., the near convergence at the upper right and upper left of the pattern shown in this figure).

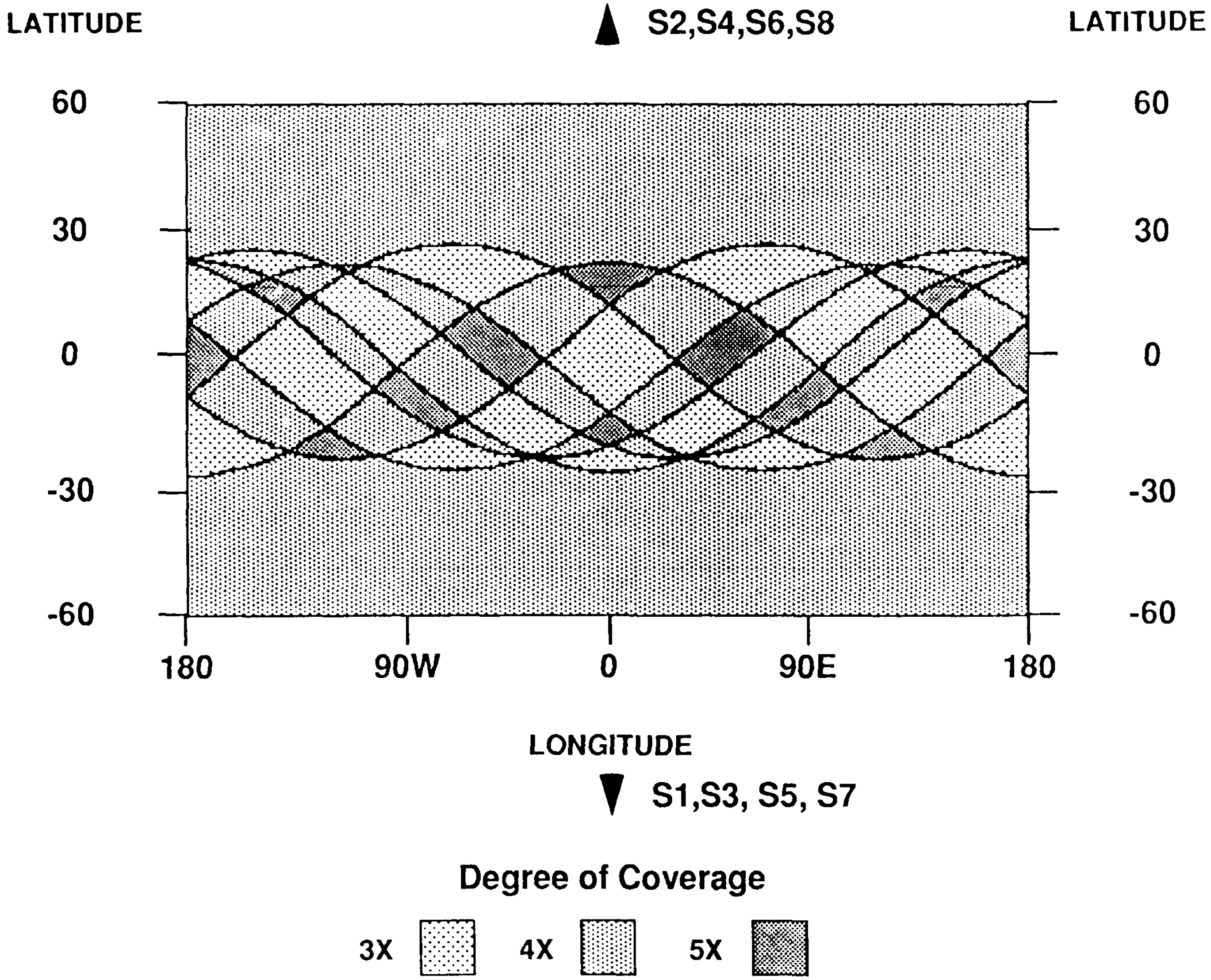
SIX-SATELLITE VISIBILITY LIMITS
STARTING POSITION OF CONSTELLATION



SIX-SATELLITE ARRAY VISIBILITY LIMITS
(DOUBLE COVERAGE MINIMUM)

This figure shows the constellation starting position of the six-satellite triangular prismoid for obtaining continuous double coverage. The triangular areas, by their shading, show that everywhere on the planet double coverage or better has been obtained.

EIGHT-SATELLITE VISIBILITY LIMITS
STARTING POSITION OF CONSTELLATION



EIGHT-SATELLITE ARRAY VISIBILITY LIMITS
(TRIPLE CONTINUOUS COVERAGE MINIMUM)

As in the previous two figures, this chart shows the existence of triple or better coverage at the start position using the eight-satellite prismoidal constellation. If the program **Limit Animator** is used, a continuing picture of the changing pattern as time advances can be seen. The triangular and diamond-shaped areas of the pattern change size and move from right to left as time increases.

TEN-SATELLITE VISIBILITY LIMITS
STARTING POSITION OF CONSTELLATION

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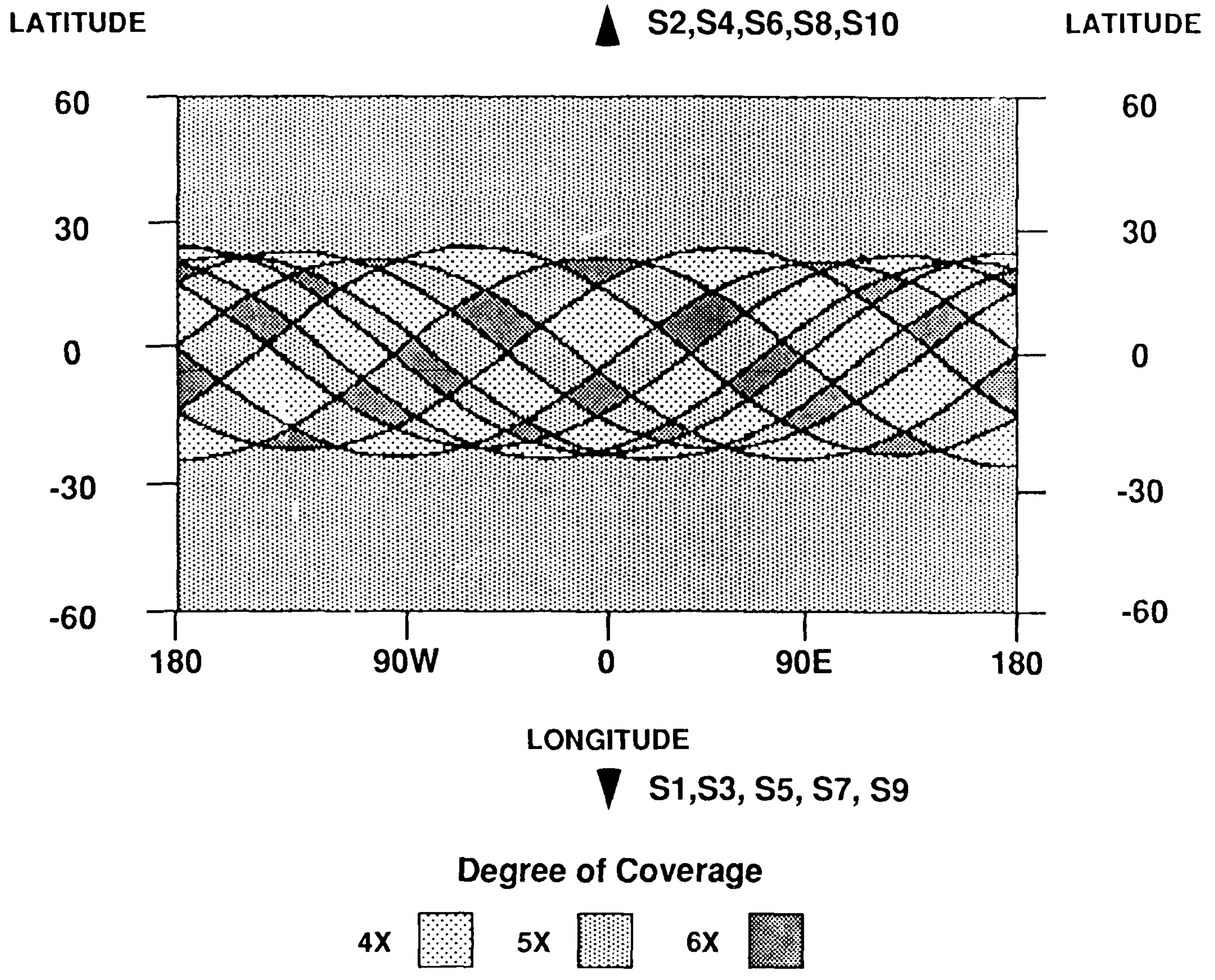

TEN-SATELLITE ARRAY VISIBILITY LIMITS
(QUADRUPLE CONTINUOUS VISIBILITY MINIMUM)

This figure shows, at the start position, at least 4x coverage everywhere, from the ten-satellite (pentagonal prismoid) constellation.

STARTING POSITIONS
SINGLE-COVERAGE CONSTELLATION; n = 1*

$T_{c_1} => 26.49$ h

Satellite No.	i (deg)	e	ω (deg)	Ω (deg)	M (deg)
1	31.3	0.263	-90	0	0
2	↓	↓	+90	-90	+90
3	↓	↓	-90	-180	+180
4	↓	↓	+90	-270	+270

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* U.S. Patent Pending

STARTING POSITIONS
SINGLE-COVERAGE CONSTELLATION; n = 1 *

This table lists the start position orbital parameters of the four-satellite continuous single-coverage constellation. Note that the minimum period for which continuous coverage is obtainable is 26.49 hours, making it a slightly supersynchronous array. If the period is some multiple of 23 hours 56 minutes (siderial day) then a repeating ground track will be obtained. Otherwise, the successive ground tracks will smear longitudinally, eventually covering the area up to and including the latitude limits equivalent to the indicated orbital inclination angle. Note that the satellites at apogee and perigee (No. 1 and 3) lie in the same (near) hemisphere.

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* **U.S. Patent Pending**

STARTING POSITIONS
DOUBLE-COVERAGE CONSTELLATION; n = 2

$T_{c_2} = > 102\ h$

Satellite No.	i (deg)	e	ω (deg)	Ω (deg)	M (deg)
1	27.5	0.233	-90	0	0
2	↓	↓	+90	-60	+60
3			-90	-120	+120
4			+90	-180	+180
5			-90	-240	+240
6			+90	-300	+300

STARTING POSITIONS
(DOUBLE-COVERAGE CONSTELLATION; n = 2)

This table shows the start position orbital parameters for the triangular prismoid, six-satellite, double-coverage array. Note that the minimum operable period for this constellation is 102 hours. Apogee and perigee satellites now lie in opposite hemispheres (No. 1 in near hemisphere; No. 4 in far hemisphere).

STARTING POSITIONS
TRIPLE-COVERAGE CONSTELLATION; n = 3

$T_{c_3} = > 272$ h

Satellite No.	i (deg)	e	ω (deg)	Ω (deg)	M (deg)
1	25	0.218	-90	0	0
2	↓	↓	+90	-45	+45
3	↓	↓	-90	-90	+90
4	↓	↓	+90	-135	+135
5	↓	↓	-90	-180	+180
6	↓	↓	+90	-225	+225
7	↓	↓	-90	-270	+270
8	↓	↓	+90	-315	+315

STARTING POSITIONS
TRIPLE-COVERAGE CONSTELLATION; n = 3

This table gives the start position orbital parameters for the triple-coverage constellation. Note that the minimum period for continuous triple coverage is 272 hours.

STARTING POSITIONS
QUADRUPLE-COVERAGE CONSTELLATION; n = 4

$T_{c_4} => 568$ h

Satellite No.	i (deg)	e	ω (deg)	Ω (deg)	M (deg)
1	24	0.205	-90	0	0
2	↓	↓	+90	-36	+36
3	↓	↓	-90	-72	+72
4	↓	↓	+90	-108	+108
5	↓	↓	-90	-144	+144
6	↓	↓	+90	-180	+180
7	↓	↓	-90	-216	+216
8	↓	↓	+90	-252	+252
9	↓	↓	-90	-288	+288
10	↓	↓	+90	-324	+324

STARTING POSITIONS
QUADRUPLE-COVERAGE CONSTELLATION; n = 4

The ten-satellite 4x coverage array has the listed starting position orbital parameters. Note the extremely long minimum period, 568 hours (or 23.7 days). This array would probably be too strongly influenced by lunar gravitational attraction to be practical.

STARTING POSITIONS

GENERALIZED TABLE OF ORBITAL PARAMETERS FOR N-TUPLE CONTINUOUS-COVERAGE CONSTELLATIONS

Satellite No.	i (deg)	e	ω	Ω	M
1	f (n)	g (n)	-90°	0°	0°
$\vdots$	↓	↓	$\vdots$	$\vdots$	$\vdots$
k	↓	↓	$(-1)^k (90°)$	$-\left(\frac{k-1}{n+1}\right) 180°$	$+\left(\frac{k-1}{n+1}\right) 180°$
$\vdots$	↓	↓	$\vdots$	$\vdots$	$\vdots$
$k = n + 2$	↓	↓	$(-1)^{n+2} (90°)$	-180°	+180°
$\vdots$	↓	↓	$\vdots$	$\vdots$	$\vdots$
$k = 2n + 2$	↓	↓	+90°	$-\left(\frac{2n+1}{n+1}\right) 180°$	$+\left(\frac{2n+1}{n+1}\right) 180°$

Note: f (n) and g (n) are optimal values which minimize $T_{c_{(n)}}$

STARTING POSITIONS

GENERALIZED TABLE OF ORBITAL PARAMETERS FOR N-TUPLE CONTINUOUS-COVERAGE CONSTELLATIONS

This table allows the determination of orbital parameters for any level of redundancy, except for inclination and eccentricity. These latter two values will decrease as the level of redundancy (n) increases.

SATELLITE PERIODS, ECCENTRICITIES, APOGEES AND PERIGEES

n =	T_{c_n} => (h)	e =	a = (nm)	r_a = (nm)	r_p = (nm)	h_a = (nm)	h_p = (nm)
< 1 (synch)	24	0.263	22767	28755	16779	25313	13337
1	26.49	0.263	24316	30711	17921	27269	14479
2	102	0.233	59735	73653	45817	70211	42375
3	272	0.218	114871	139913	89829	136471	86387
4	568	0.205	187671	226144	149198	222702	145756

SATELLITE PERIODS, ECCENTRICITIES, APOGEES, AND PERIGEES

This table summarizes, for the minimum period cases, the values of the continuous-coverage constellation periods, eccentricities, semi-major axes, apogees and perigees. The special case of a (non-continuous) tetrahedral array at synchronous altitude is given to show that the apogee and perigee differences from the slightly longer period (26.49 hour) continuous coverage case are not too large. Again, it is evident that as the redundancy level increases, so do the periods, and apogee and perigee distances. The distances are expressed in nautical miles.

BENEFITS/PAYOFFS FOR TETRAHEDRAL/PRISMOIDAL CONSTELLATIONS

- Cost Reduction
  - Reduce numbers of satellites per system required
  - Reduce numbers of boosters per system required
  - Reduce satellite control/support requirements
- Coverage Improvements
  - Eliminate coverage gaps
  - Enhance coverage in critical geographic areas
  - Provide more "graceful degradation"
  - Cross link connectivity equals 100%
- Survivability Improvements
  - Unique, distinct orbits provide unambiguous attack warning
  - Slightly to moderately supersynchronous orbits
- Interference Reduction (with other satellites)
  - Avoidance of synch-eq belt
  - Maintain velocity differentials in crossing situations

BENEFITS/PAYOFFS FOR TETRAHEDRAL/PRISMOIDAL CONSTELLATIONS

The special, continuous-coverage constellations display a number of important benefits. The first, and most obvious, is that continuous earth coverage, at any level of redundancy, can be achieved using fewer satellites than any other system (including Walker arrays). Fewer satellites also implies fewer boosters, and less ground control/support, compounding the cost savings. Continuous coverage implies no coverage gaps. For surveillance systems, this is critical, particularly in the polar regions. A high degree of "bias" control can be exerted, to emphasize performance in selected critical areas of the world. A more graceful degradation of coverage may be achieved, in some cases. Cross-link connectivity is maximized, since no satellite-to-satellite LOS is ever eclipsed by the earth. Survivability is enhanced through use of unique, slightly elliptic, super-synchronous orbits. Further, the satellites do not reside in the equatorial regions, thus avoid the synchronous equatorial belt which is presently vastly overcrowded. In crossing situations with other satellites, large crossing velocities insure minimum interference intervals.

SIGNIFICANCE OF RESULTS

- Mean, continuous coverage is greater than minimum coverage would indicate
- Single-coverage (4-satellite) and double-coverage (6-satellite) arrays appear to be of more practical value
- Triple-coverage (8-satellite) and quadruple-coverage (10-satellite) arrays are probably not practical or useful due to extremely high altitudes and resulting lunar and solar perturbations
- Reduced number of satellites/boosters
- More survivable against enemy attack than synchronous arrays

SIGNIFICANCE OF RESULTS

The mean level of redundancy obtained is generally higher than the minimum level required. It has been shown that there is much more double-coverage than single-coverage, in a continuous single-coverage (tetrahedral) constellation, for example. Since the 8-satellite and 10-satellite triple and quadruple-coverage arrays, in order to obtain minimum satellite coverage, must operate at such high altitudes that lunar perturbations become unacceptable, they do not appear to be of practical value for space systems. On the other hand, the four-satellite single-coverage and (possibly) the six-satellite double-coverage arrays do appear to have advantages, as to reduction in satellite numbers and increased survivability against hostile attack.

POSSIBLE APPLICATIONS FOR CONSTELLATIONS

- 4-Satellite single-coverage
  - MILSTAR
  - DSCS
  - DSP
  - METSAT
  - Data relay
  - BSTS
  - General reconnaissance/surveillance
- 6-Satellite double-coverage
  - DSP
  - Data relay
  - BSTS

POSSIBLE APPLICATIONS FOR CONSTELLATIONS

We foresee a number of possible applications for minimum satellite constellations. In general, systems which involve sensors for surveillance and reconnaissance, and systems for communications or data relay, appear prime candidates for satellite constellations of this type.

RECOMMENDATIONS FOR PHASE 2 SBIR STUDY

- Continue invention/innovation process
  - Control over satellite constellation geometry
  - "Synergistic Overlay" of two or more tetrahedral constellations
- Develop statistics of coverage
  - Baseline tetrahedral/prismoidal constellations
  - "Satellite Out" constellations
- Preservation of constellation integrity
  - Passive survivability of baseline constellations
  - Active survivability by maneuvering one or all satellites
- Develop preliminary application constellations
  - Pseudo-icosahedral NAVSTAR array (global continuous PDOP ≤ 7)

RECOMMENDATIONS FOR PHASE 2 SBIR STUDY

The primary objective of this SBIR effort should remain the invention/innovation type achievements which have characterized earlier work by the principal investigator (including Phase 1 SBIR). It has been shown that significant coverage improvements can be achieved using these new techniques; but, these concepts and techniques are still in their infancy, and further improvements may be expected. Overlaying several tetrahedral arrays holds great promise for multiple-coverage at sub-synchronous, synchronous, and super-synchronous periods. The statistics of baseline and degraded ("satellite-out") constellations deserves more precise definition. The inherent advantages of these constellations as to survivability in both passive and active senses should be quantified. Development of a proprietary pseudo icosahedral NAVSTAR type constellation using 15-18 satellites and having maximum continuous global PDOP values of 7 or less is proposed in Phase II.

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This report is an annotated briefing covering research on arrays or constellations of satellites giving n-tuple continuous, global line-of-sight coverage of a planet using the minimum number of satellites. A generalized theory was developed which appears to fit cases for any level of redundancy. Except for the case for single coverage, which is a tetrahedral array, all of the constellations can be characterized as either skew- or congruent-prismoidal arrays. The constellation altitudes increase with increasing levels of redundancy.

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