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PREDICTION OF
ACOUSTIC CAVITY MODES BY
FINITE ELEMENT METHODS

Thomas A. Vernon - S. Tang

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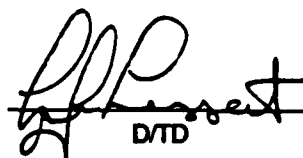
**PREDICTION OF
ACOUSTIC CAVITY MODES BY
FINITE ELEMENT METHODS**

Thomas A. Vernon - S. Tang

March 1989

Approved by W.C.E. Nethercote
H/Hydronautics Section

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Abstract

A finite element method for acoustic cavity modal analysis is presented. The approach utilizes the VAST finite element program eigensolution routines operating on equivalent acoustic mass and stiffness matrices generated from a program ACOUSTIC. Both closed and open cavities can be treated, within the assumption of rigid boundaries. Two elements have been implemented in ACOUSTIC: a 20-noded isoparametric brick element and an 8-noded isoparametric axisymmetric element. The performance of the analysis system has been evaluated via comparison to both analytical and experimental results, and good agreement has been found.

Ce texte décrit une méthode à éléments finis pour l'analyse modale de cavités acoustiques. Cette approche exploite les routines de solutions propres du programme à éléments finis VAST, appliquées aux matrices de masse et de rigidité produites par le programme ACOUSTIC. Cette méthode permet de traiter des cavités ouvertes ou fermées, en se fondant sur l'hypothèse de limites rigides. Deux éléments ont été implantés dans ACOUSTIC: un élément de brique isoparamétrique à 20 noeuds et un élément asymétrique isoparamétrique à 8 noeuds. Le performance du système d'analyse a été évaluée par comparaison aux prévisions analytiques et aux résultats expérimentaux et on a constaté une bonne corrélation.

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1 Introduction

The prediction of natural modes of vibration of acoustic cavities is an area of interest to those involved in acoustical design or measurement. Cavity resonances can lead to annoying human environmental conditions, interfere with acoustical measurements, and can in some cases produce significant acoustic radiation. A familiar example of the last phenomenon is the Helmholtz resonance tonals produced by blowing over the open end of an empty bottle. For some simple shapes, natural modes can be easily estimated from the linear frequency-wavelength relation. For more complex geometry, or for non-homogeneous fluids, numerical methods must be used for accurate modal prediction. The finite element method is well-suited for the prediction of cavity modes, since the governing differential equation is easily cast in a weighted residual form.

This communication presents a brief description of the development and verification of a system for finite element prediction of acoustic cavity modes. Familiarity with the VAST¹ finite element program and finite element methods in general is assumed.

The approach utilizes two programs, ACOUSTIC and VAST. The two programs have been made as compatible as is practical at this time; the ACOUSTIC program reads VAST format input files, and forms and assembles the element 'stiffness' and 'mass' matrices in the format required for a natural frequency analysis using VAST. The VAST bandwidth minimization routines have been imbedded in the ACOUSTIC program to increase the overall solution efficiency.

Within the program ACOUSTIC, two common structural solid elements have been adapted to model the acoustic mode problem: a brick element and an axisymmetric element. The former allows accurate modelling of general geometries, while the axisymmetric element is limited to domains meeting the requirement of circumferential invariance, in both the geometry and the pressure distribution. The restriction on the pressure distribution could be removed by the inclusion of higher order harmonics in the formulation.

A number of analyses have been undertaken to verify the implementation of the acoustic modal analysis capability, several of which are reported here. Numerical predictions are compared to analytical values for the case of a rectangular cavity, and to experimental results for an empty bottle. The latter example also highlights the effectiveness of the axisymmetric formulation for geometries which satisfy that restriction. As an illustration of a more practical analysis, the natural frequencies of a complex tank are calculated.

The assumption of acoustically hard boundaries inherent in this formulation is in general quite realistic when the cavity fluid is air. For high density fluids such as water, this assumption may not be as reliable, and work is progressing on the inclusion of the structural compliance in the prediction of the natural modes, both of the structure and cavity. In the limit of very low frequency, the added fluid mass approach can be utilized to obtain the effect of the fluid on the structure. The current work thus represents a further step in the

development of a capability to solve the general fluid/structure interaction problem.

2 Theory

Linear cavity acoustics are governed by the simple wave equation which reduces to a Helmholtz form for the spatial distribution of pressure,

$$\nabla^2 p + \left(\frac{\omega}{c}\right)^2 p = 0 \quad (1)$$

At acoustically hard boundaries, the normal pressure gradient vanishes,

$$\frac{\partial p}{\partial n} = 0 \quad (2)$$

A variational form of equation (1), suitable for finite element solution, can be obtained by defining a weighted residual statement with a virtual pressure as a weight function and integrating over the cavity volume,

$$\int_v [\nabla^2 p \delta p + \left(\frac{\omega}{c}\right)^2 p \delta p] dV = 0 \quad (3)$$

Integrating by parts the first term of equation (3) and using the boundary condition of equation (2), the equivalent integral form becomes

$$\delta \int_v [(\nabla p)^2 - \left(\frac{\omega}{c}\right)^2 p^2] dV = 0 \quad (4)$$

The first term involving the pressure gradient can be interpreted as a generalized stiffness associated with the potential energy of the fluid. The second term is a generalized mass term associated with the kinetic energy of the fluid.

Following a standard finite element development (see Reference 2), the pressure distribution in the cavity is approximated by a set of nodal values $\{p\}_i$ and interpolation functions $[N]_i$. Within volume elements, the pressure field is defined as

$$p = [N]_i \{p\}_i \quad (5)$$

Within each element, the energy integrals of equation (4) become

$$\int_v \{p\}_i^T [k]_i \{p\}_i dv \quad (6)$$

$$\int_v \{p\}_i^T [m]_i \{p\}_i dv \quad (7)$$

with

$$[k]_i = [B]_i^T [B]_i \quad (8)$$

$$[m]_i = \frac{1}{c^2} [N]_i^T [N]_i \quad (9)$$

and

$$[B]_i = [J]^{-1} \nabla \cdot [N]_i \quad (10)$$

where $[J]$ is the Jacobian matrix relating local and global coordinate derivatives. The program ACOUSTIC is used to form and assemble the individual element matrices into the global stiffness and mass matrices $[K]$ and $[M]$ respectively. The VAST system is then used to solve the eigenvalue problem

$$([K] - \omega^2 [M])\{p\} = 0 \quad (11)$$

To minimize roundoff error, the c^2 factor in equation (9) is not explicitly used in the actual element matrix formulation for homogeneous fluids. In its place, a non-dimensional sound speed parameter $(c_{ref}/c)^2$ is used, in which the reference speed is input by the program user. The eigenvalues calculated with the VAST program then must be multiplied by that factor to obtain the actual frequency predictions. Note that this option is also applicable to models which include fluid of variable density.

The two elements implemented to date in the ACOUSTIC program are a 20-noded isoparametric brick element and an 8-noded isoparametric axisymmetric element, Figure 1. Both of these elements allow quadratic variation of both geometry and pressure within an element, providing quite a high degree of modelling versatility. The element connectivity must follow that defined for these elements (types 2 and 15) in the VAST analysis system. A 27-point Gauss integration is used for the brick element; a 9-point surface integration with explicit circumferential integration is used for the axisymmetric elements. The implementation of the 20-noded element and the assembly routine has drawn largely from the routines developed for added fluid mass calculation within VAST.³

For completely closed cavities for which the boundary condition of equation (2) is enforced everywhere, the stiffness matrix $[K]$ in equation (11) is singular. This singularity can be expected, since the one boundary condition is insufficient to provide a unique solution to the second order governing differential equation. To obtain natural modes of closed cavities, the shifting technique must be invoked in the VAST decomposition module. The choice of an appropriate shifting parameter is important from the point of view of both accuracy and computational efficiency. Since the shift should be of the same order as the calculated eigenvalues, it is suggested that an estimate of the fundamental frequency, in Hz, divided by the reference sound speed, be used. This will in general be much lower than the default value used in the VAST program.

To model openings or planes of antisymmetry in a cavity, the nodal pressures can be set to zero by imposing boundary conditions. The imposition of one or more such constraints will render the stiffness matrix non-singular, and shifting will not be required. Numerically, these constraints are imposed by placing large stiffness terms on the diagonal term associated with the node in the assembled stiffness matrix. While it is computationally more efficient to remove these nodes from the assembled system, the modal displacement vectors produced by VAST are then incomplete, and interpretation of modes becomes quite difficult. It should be noted that acoustic stiffness terms are in general much smaller than structural stiffness terms, and a lower spring stiffness value can be used in VAST.

For detailed models, particularly those modelled with the brick elements, bandwidth minimization can be very effective in reducing both storage and computing time in the VAST frequency analysis. For this purpose, the bandwidth minimization module from the VAST program has been adapted for use in the ACOUSTIC program. This module provides both the GPS and NSA algorithms¹ for nodal reordering. The reordering remains transparent to the user.

3 Verification and Applications

The performance of the 20-noded element was evaluated via comparison to results presented in Reference 3 for a rectangular cavity. An identical element was used in that study. The geometry of the cavity is shown in Figure 2. A half-model was used, which provides xz-plane symmetric modes for the closed cavity, and antisymmetric xz-plane modes when the nodal pressures in that plane are set to zero. The analytical values, in Hz, can be obtained from the relation

$$f = \frac{c}{2} \sqrt{\left(\frac{l}{l_x}\right)^2 + \left(\frac{m}{l_y}\right)^2 + \left(\frac{n}{l_z}\right)^2} \quad (12)$$

where $l, m, n = 0, 1, 2, \dots$ and subscripted variables are actual lengths. A comparison of the results is presented in Table 1, for an 8 element model. In accordance with standard practice, the mode description refers to the number of half-wavelengths in the x, y, and z directions respectively. For the symmetric modes ($m=0$), the finite element results agree exactly with those of Reference 3, but the antisymmetric modes in that reference were apparently calculated for a slightly different geometry ($y=0.132$ m in place of 0.128 m). The sound speed used in all analyses was 330 m/sec.

The results of this analysis indicate that the 20-noded element gives quite accurate results for discretizations down to about 2 elements per wavelength for this simple geometry.

A second example considered in the application of the acoustic elements was the prediction of the cavity modes of an empty bottle. For this case, experimentally determined

Mode	Predicted	Theory	Ref 3
1,0,0	699	699	699
0,1,0	1294	1289	1255
2,0,0	1404	1398	1404
1,1,0	1471	1466	1437
0,0,1	1505	1500	1506
1,0,1	1660	1654	1660

Table 1: Comparison of numerically predicted and analytical frequencies of a rectangular cavity.

natural frequencies could be compared to predictions from the finite element analysis. Various discretization levels were employed, and an exterior endpiece was added to all but one model, as an approximation to the open end.⁴ As suggested earlier, an alternative to the endpiece is simply to define the pressure at the opening to be zero, and it was part of the intent of this study to determine if that model would provide sufficient accuracy.

The results obtained from the brick models indicated that all of the lowest modes of interest were in fact axisymmetric, which could probably be anticipated given the axisymmetric nature of the volume. A number of axisymmetric models were thus developed, and the results of those analyses compared to the brick model results. Three of the brick models are shown in Figure 3. The corresponding axisymmetric models are shown in Figure 4. Model 1 represents the actual bottle volume. In the analyses, the end nodes in this model were assigned zero pressure.

A summary of the results of the numerical analyses is presented in Table 2. The first line for each model refers to the brick element model, the second line to the axisymmetric model. Included also is the number of elements and nodes in the model and the computer time demands for both the matrix formulation (program ACOUSTIC) and total analysis time for three modes. As is evident, all of the models produce essentially the same results, although the axisymmetric analyses are an order of magnitude more efficient than those which utilize the brick element. The addition of the endpiece does little if anything to improve the overall accuracy of the modal predictions. Other geometries tested during the course of the program development showed similar results. The analysis times are for bandwidth-minimized models. The axial pressure distributions for the first 3 modes are shown in Figure 5; these were obtained using the axisymmetric model of only the bottle volume (Model 1).

To obtain results for comparison, two experiments were carried out using the same

Model	Frequencies (Hz)			Elems	Nodes	CPU (sec)	
	F1	F2	F3			[K],[M]	Total
Model 1	212	1361	2080	54	296	121	290
	212	1357	2078	6	33	4	12
Model 2	209	1353	2050	93	488	248	645
	209	1350	2054	9	59	5	17
Model 3	209	1354	2053	119	608	411	1076
	209	1351	2056	11	75	5	20
Exp	202	1381	2123				

Table 2: Comparison of predicted and experimentally obtained natural frequencies of an empty bottle.

bottle as was modelled. First, the fundamental cavity mode was determined by measuring the predominant frequencies emitted when air was blown over the open end. To excite the higher cavity modes, the set-up shown in Figure 6 was used. A typical power spectrum obtained by blowing over the bottle is shown in Figure 7. Note the strong harmonics of the fundamental frequency, approximately 202 Hz, which are present in the signature. A portion of the frequency response plot obtained from the loudspeaker set-up is shown in Figure 8. Note that the scale on the frequency axis is not the same as for Figure 7. The first mode identified using the loudspeaker system was the Helmholtz fundamental obtained in the first test. The experimentally obtained results are given in the final line of Table 2.

The numerical results compare well with those measured experimentally. The largest discrepancy of about 4 percent occurs for the lowest mode. The exact sound speed was not determined during the experiment, and it is probable that some of the discrepancy is due to the potential difference in that value, which was again taken as 330 m/sec in the numerical calculations. The harmonics observed in the spectrum obtained from blowing over the bottle do not appear in the results of the second experiment. This is apparently a result of the different forms of excitation mechanisms in the two tests. Blowing air over the bottle leads to a nonlinear interaction of shear layer instability and cavity oscillation⁵⁻⁸, which can support harmonics of the fundamental which are not true cavity modes. The excitation of the cavity mode by the loudspeaker is a linear process, and only true cavity modes should be identified through this approach, within the limits of the experiment.

As a final example, the acoustic analysis system has been utilized to predict the cavity modes of a large tank of complex geometry. The finite element model, with hidden lines removed, is shown in Figure 9. The model contains 172 20-noded elements, and a total of 1220 nodes. The three circular structures in the model represent internal detail in the main

tank.

The results of three acoustic modal analyses of this model are summarized in Table 3. The initial analysis was of the model, filled with water, with no openings. The effect of including surface openings and the introduction of a small volume of air into the model were then investigated. The natural frequency predictions have been non-dimensionalized against the fundamental mode prediction for the closed model.

Mode	Model		
	Closed	Open	Open with air
1	1.000	0.557	0.528
2	1.165	1.116	1.104
3	1.671	1.635	1.420
4	2.112	2.076	1.945
5	2.315	2.208	2.064

Table 3: Non-dimensionalized frequency predictions for the tank model.

These results show quite a wide separation of frequencies for the tank. The introduction of the surface openings appears to have a large effect on the fundamental frequency, while the addition of air into the model also decreases the fundamental frequency, but to a much lesser extent.

4 Conclusions

A capability for predicting acoustic cavity modes by finite element analysis has been implemented. The two elements developed provide both a general purpose and axisymmetric modelling capability. The element formulation, bandwidth reduction and assembly algorithms have been included in the program ACOUSTIC, which reads and writes data files in a format compatible with the VAST finite element analysis system. Several sample verification analyses have been reported on here, and the system appears to perform its intended function.

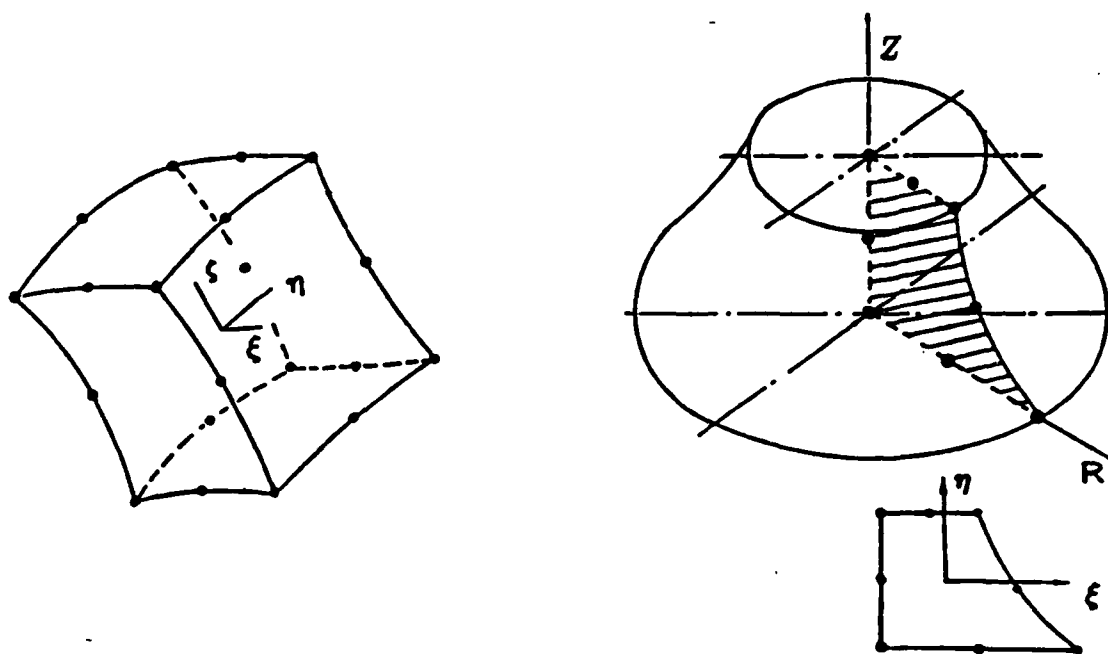


Figure 1: Quadratic brick and axisymmetric finite elements.

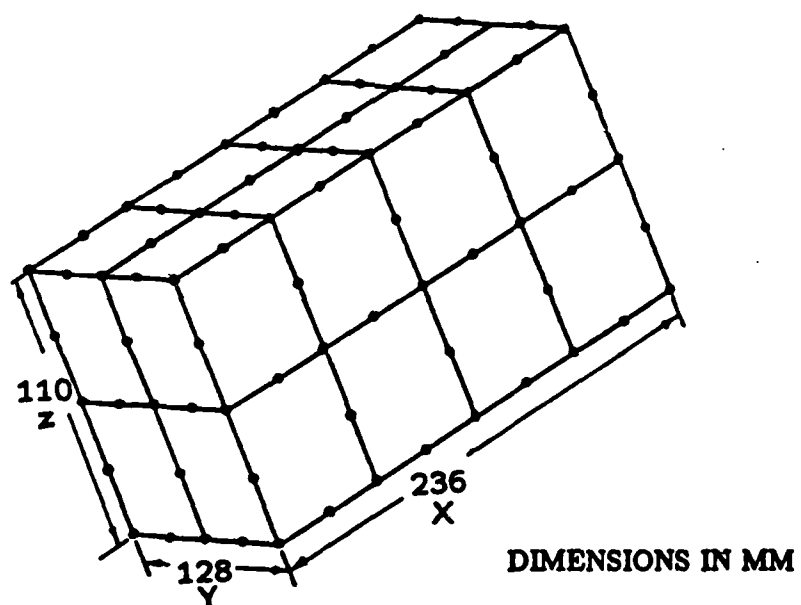
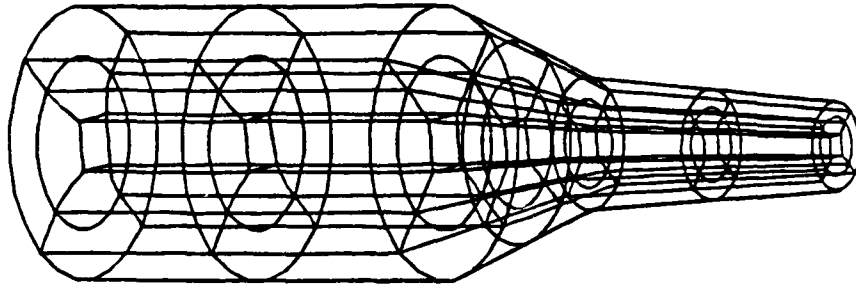
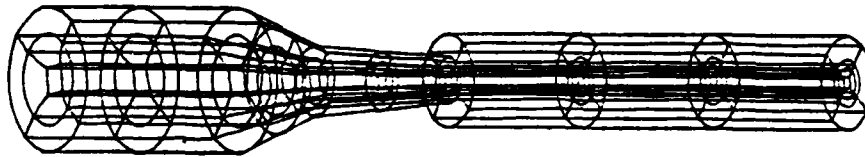


Figure 2: Geometry and finite element model of the rectangular cavity.

MODEL 1



MODEL 2



MODEL 3

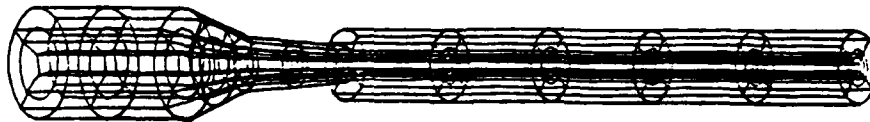
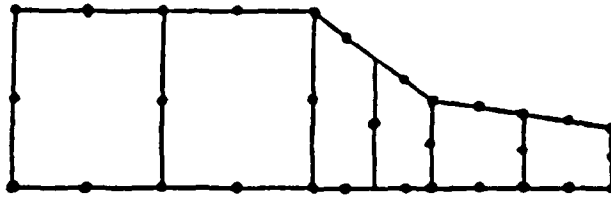
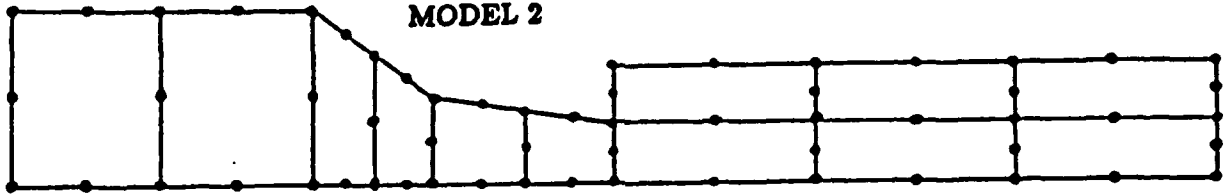


Figure 3: Three brick element models of the bottle and end-piece.

MODEL 1



MODEL 2



MODEL 3

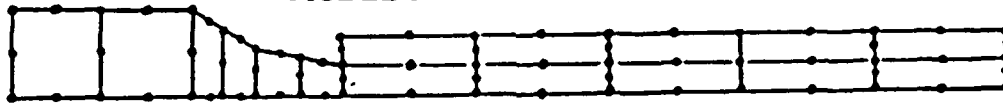


Figure 4: Axisymmetric models of the bottle and end-piece equivalent to the solid models.

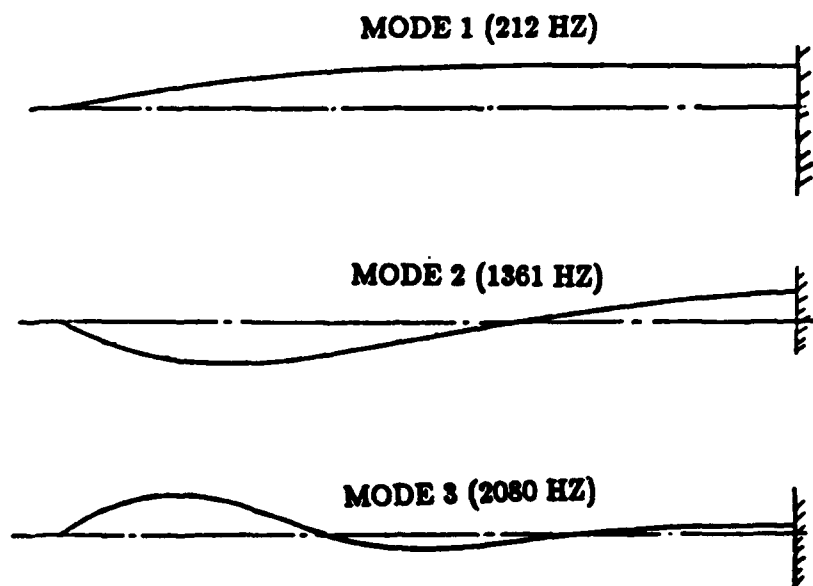


Figure 5: Axial distribution of pressure for the first three modes as determined with axisymmetric Model 1.

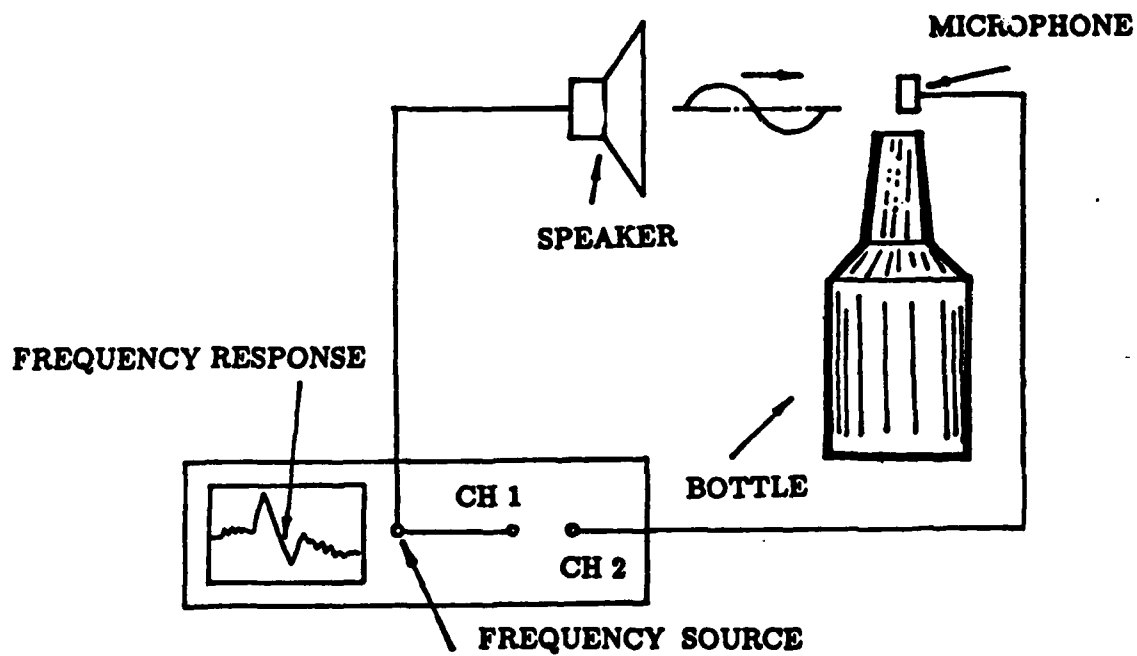


Figure 6: Experimental set-up for determination of the higher cavity modes of the bottle.

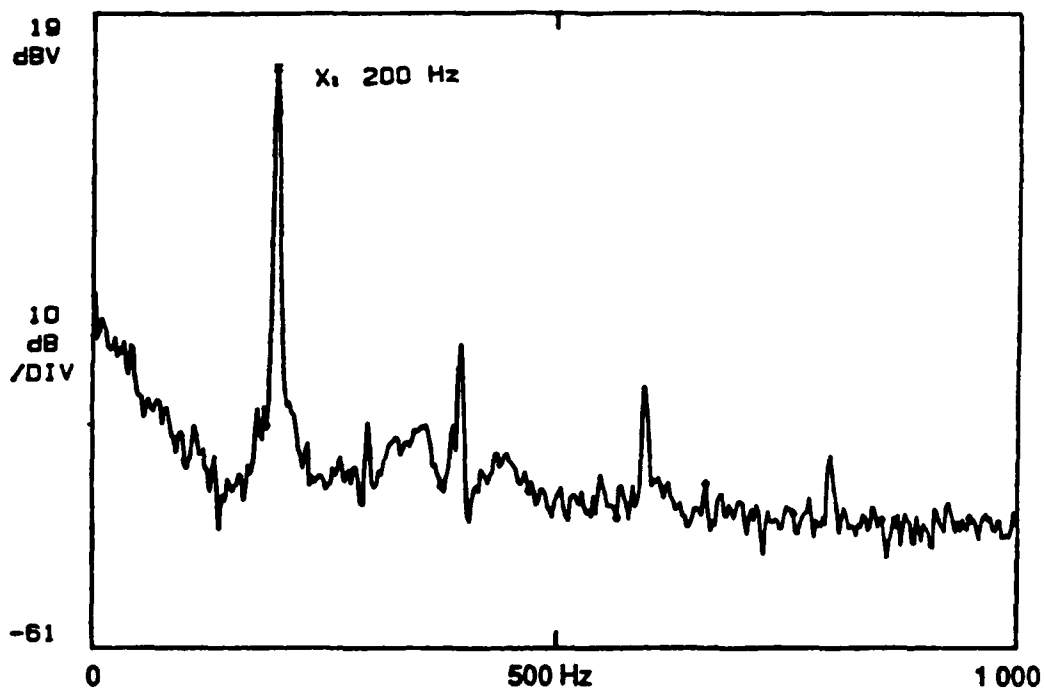


Figure 7: Power spectrum obtained from blowing over the bottle lip.

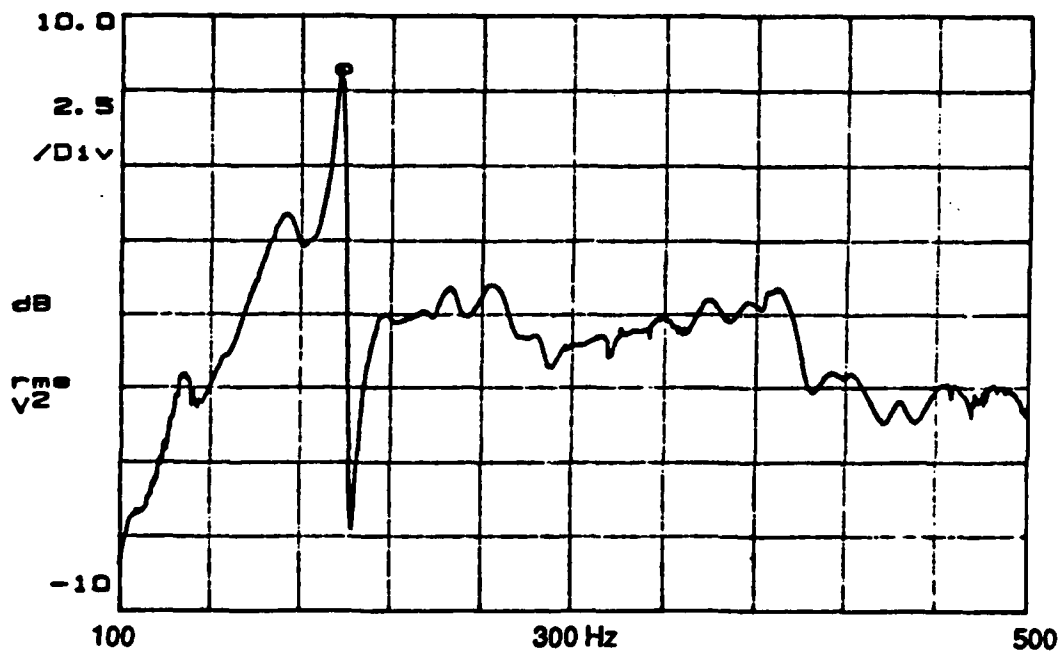


Figure 8: Frequency response curve obtained from the loudspeaker/bottle system.

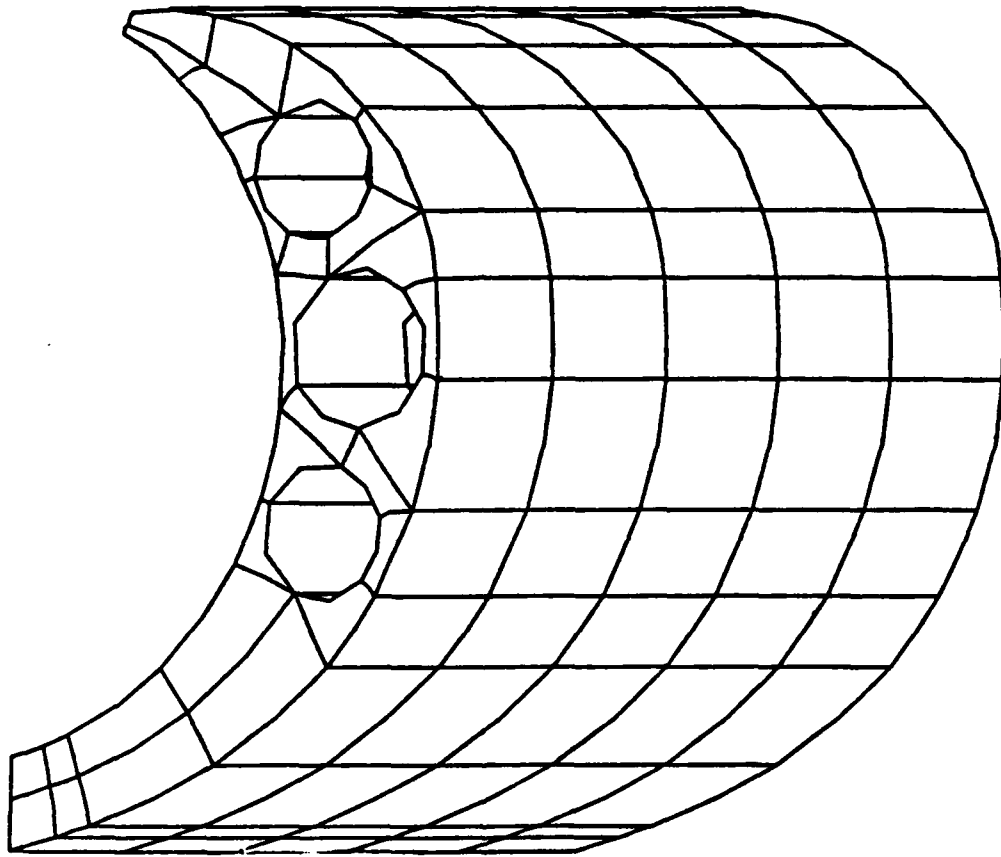


Figure 9: Tank modelled with 20-noded brick elements.

Appendix A - Program ACOUSTIC

Following the standard VAST format, the program ACOUSTIC requires the following input files:

1. PREFIX.USE
2. PREFIX.GOM

The format of these files can be found in Reference 1. The .USE file must contain a master control line, the IELEMA header and control line, and the IBANRD header and control line, in this order. Since it is likely that a natural frequency analysis will be carried out using the same .USE file, that file ends up looking essentially the same as would be required for a complete VAST run. Note that substructuring of the model is not currently permitted.

For the ACOUSTIC program, the master control line switches IELEMS, IBANRD, LASSEM, and ISTIFM must be set. The program will restart at either bandwidth minimization or matrix assembly. For assembly (LASSEM = 1), the master control variable ISTIFM is used to control the form of the assembled stiffness matrix. If ISTIFM = 1, ACOUSTIC produces a PREFIX.T46 file, anticipating that boundary conditions are to be imposed. If ISTIFM = 0, ACOUSTIC produces the PREFIX.T48 file directly, which will then require shifting to decompose. If boundary conditions are required, they are put under the ISTIFM section in the .USE file (or boundary condition file) and VAST is used to create the PREFIX.T48 file.

There is one small difference in the .GOM file used by the ACOUSTIC program; the element group sound speed, in length units/second, replaces the material modulus specification. The Poisson's ratio and material density are not required. As is the case for material property variations, elements with different sound speeds must be in different groups. The element type codes are the same as for the VAST structural elements; IEC = 2 and IEC = 15 for the 20-noded and axisymmetric elements respectively.

The output files created by ACOUSTIC are as follows:

1. PREFIX.LPA - formatted output data file
2. PREFIX.T31 - element mass matrices
3. PREFIX.T32 - element stiffness matrices
4. PREFIX.T41 - geometry control file for VAST compatibility
5. PREFIX.T45 - bandwidth reduction mapping file

6. PREFX.T46 - assembled stiffness matrix before boundary conditions are imposed
7. PREFX.T48 - assembled stiffness matrix
8. PREFX.T49 - assembled mass matrix

All files are binary with the exception of the .LPA file, which resembles the .LPT file produced by VAST. The print control parameters in the geometry and bandwidth control lines in the .USE file are active. The .T41 file produced by ACOUSTIC does not contain the geometric data as is the case for files created by VAST. Unfortunately, this data is required for the eigenvector plotting routines of the VASTG⁹ postprocessor. In general, plotting of the pressure eigenvectors is not particularly useful, since they are interpreted as x-axis displacements in VASTG. If plotting is desired, the element formulation section of VAST must be executed to obtain a correct .T41 file.

The only input required by ACOUSTIC is the 5 character file prefix variable PREFX and the reference sound speed. The program offers a list of defaults for the latter; a choice can be made depending on the units of the geometry. For models which include fluids of different density, a median value of reference sound speed is recommended. The value is echoed in the .LPA file, for future reference in factoring the calculated eigenvalues. The program can also be run in batch mode.

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→ A finite element method for acoustic cavity modal analysis is presented. The approach utilizes the VAST finite element program eigen-solution routines operating on equivalent acoustic mass and stiffness matrices generated from the program ACOUSTIC. Both closed and open cavities can be treated, within the assumption of rigid boundaries. Two elements have been implemented in ACOUSTIC: a 20-noded isoparametric brick element and an 8-noded isoparametric axisymmetric element. The performance of the analysis system has been evaluated via comparison to both analytical and experimental results, and good agreement has been found.

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Finite elements
Frequency analysis
Vibration
Cavities
Modal analysis
Acoustics
Numerical methods
Eigenvalues
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