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19. ABSTRACT (Continue on reverse if necessary and identify by block number) Work has continued during this period on the following projects: (1) The role of regularity in multigrid methods -- computational experiments and analytical studies on the V-cycle in an L-shaped domain; (2) Preconditioning and boundary values -- study of preconditioning of elliptic operators; and (3) Preconditioning, boundary values and mixed mode -- extension of (2) above to hyperbolic operators.					
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I. Objectives of the Program

A major thrust of this project is the study of effective means of solving large systems of linear equations with a particular emphasis on those problems which arise from the discretization of elliptic and parabolic partial differential equations. The primary topics are Multigrid and Preconditioning. The study of preconditioning leads to related studies of Condition Numbers and their behavior as h , the discretization parameter, tends to zero.

II. Personnel

A. Senior Investigator: Seymour V. Parter

B. Research Assistant: Naomi H. Decker

Ms. Decker's research was the basis for her Ph.D. thesis. The Ph.D. degree was awarded in August 1987.

III. General Activities

A. Part of the summer (1987) was spent in rewriting, revising and polishing several reports. To be specific we revised:

1. Estimates for Multigrid Methods Based on Red-Black Gauss-Seidel Smoothing (by S. V. Parter). This work was accepted for publication in *Numerische Mathematik* in early 1988.
2. On the Role of Regularity in Multigrid Methods (by N. H. Decker, Jan Mandel and S. V. Parter). This work has appeared in: *Proceedings Third Copper Mountain Multigrid Conference*, April 1987.
3. The Fourier Analysis of a Multigrid Preconditioner (by N. H. Decker). This work has appeared in: *Proceedings Third Copper Mountain Multigrid Conference*, April 1987.

B. Visit to Courant Institute

Professor Parter spent the fall semester at the Courant Institute, New York University. This visit was motivated (in part) by the desire for close relationships with researchers at this Institute. Of particular interest is the work on Domain Decomposition (Olof Widlund) and Conjugate Gradient Methods (Anne Greenbaum).

IV. Publications: The following reports and/or published papers appeared during this period.

Remarks on the "Solution of Toeplitz Systems of Equations," (by S. V. Parter) to appear: Proceedings of the 2nd International Symposium on Numerical Analysis, Prauge, August 1987, Computer Sciences Technical Report #721, University of Wisconsin-Madison.

"The k -grid Fourier Analysis of Multigrid Iterative Methods," (by N. H. Decker), Computer Sciences Technical Report #703, University of Wisconsin-Madison.

"The Analysis of Multigrid-type Iterative Methods," (by N. H. Decker) - Thesis.

"The Fourier Analysis of a Multigrid Preconditioner," (by N. H. Decker), *Multigrid Methods: Theory, Applications and Supercomputing*, edited by S. F. McCormick, pp. 117-142, Marcel Dekker, Inc., New York and Basel (1988).

"On the Role of Regularity in Multigrid Methods," (by N. H. Decker, J. Mandel and S. V. Parter), *Multigrid Methods: Theory, Applications and Supercomputing*, edited by S. F. McCormick, pp. 143-156, Marcel Dekker, Inc., New York and Basel (1988).

"Experimental Results for Multigrid and Transport Problems," (by D. Kamowitz - supported by AFOSR-82-0275), *Multigrid Methods: Theory, Applications and Supercomputing*, edited by S. F. McCormick, pp. 299-322, Marcel Dekker, Inc., New York and Basel (1988).

"On an Estimate for the Three Grid MGR Multigrid Method," (by S. V. Parter - supported by AFOSR-82-0275), *SIAM J. Numer. Anal.* 24, pp. 1032 - 1045 (1987).

V. Lectures

Professor Parter has given several lectures on this research.

Multigrid - Invited lecture as guest of the Bulgarian National Academy of Sciences, August 9-13, 1987.

On the Solution of Toeplitz Systems of Equations - Invited plenary lecture, 2nd International Symposium on Numerical Analysis, Prauge, Czechoslovakia, August 24-29, 1987.

On the Role of Regularity in Multigrid Methods - Invited participant in multigrid meeting Oberwolfach, Germany, October 25-30, 1987.

On Preconditioning and Boundary Conditions - Applied Mathematics Colloquium, Brown University, Providence, Rhode Island, November 5, 1987.

On the Role of Regularity in Multigrid Methods - Applied Mathematics Seminar, Columbia University, New York City, November 23, 1987.

On Preconditioning and Boundary Conditions - Applied Math. Colloquium, Tel Aviv University, Tel Aviv, Israel, January 8, 1988.

In addition, Professor Parter declined an invitation to participate in a "Workshop on Numerical Solutions of Convection-Diffusion Equations and Viscous Flow Computations" at the University of Maryland, November 16-17, 1987.

Further, Professor Parter received invitations to be an invited speaker at:

International Conference on Numerical Methods and Applications - Sofia, Bulgaria, August 22-17, 1988.

XIV National Summer School on Application of Mathematics in Engineering - Varna, Bulgaria, August 28 - September 4, 1988.

He declined both of these invitations.

VI. Ongoing Work

Work has been continuing on the following projects:

- A. *The Role of Regularity in Multigrid Methods* (N. H. Decker, Jan Mandel and S. V. Parter). The ongoing work on the topic consists of computational experiments and analytical studies. In particular, we have done extensive calculations on the San Diego Cray studying the V -cycle in an L -shaped domain. The theory developed earlier yields bounds on the rates of convergence which approach "one" as the number of grids increases. These calculations show that our bounds are qualitatively correct. Moreover, certain other estimates not emphasized in our earlier published reports seem to be sharp. At the same time we are working on extending our theoretical approach to describe the rates of convergence for the F -cycle in problems without H_2 regularity.
- B. *Preconditioning and Boundary Values* (T. A. Manteuffel and S. V. Parter). This project is concerned with the following question: Let $\{A_n\}$ be a family of discretizations of an elliptic operator A , and let $\{B_n\}$ be a family of discretization of an elliptic operator B . Find conditions to guarantee estimates of the form

$$\|B_n^{-1} A_n\|_{L_2} \leq k,$$

$$\|B_n^{-1} A_n\|_{H_2} \leq k,$$

$$\|A_n^{-1} B_n\|_{L_2} \leq k$$

C. *Preconditioning, Boundary Values and Mixed Mode* (David Gottlieb and S. V. Parter).

This research project is concerned with the extension of the ideas developed and being developed in (B) above to the cases where (i) A is a first order hyperbolic operation and B is an elliptic operator and (ii) A_n is a spectral discretization of A while B_n is a finite-difference discretization of B .