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Under this grant the following paper has been accepted for publication in the new journal edited by George Sell: Mischaikow, K., & H. Hattori, On the existence of intermediate magnetohydrodynamic shock waves. In this paper the authors have used the technique called the connection matrix to establish one of the intermediate shocks which has not been shown to exist. This technique is also applied to the above project.

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Problems in Dynamic Phase Transition

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Final Report

Reference No 87-222

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Summary of Project

There are a few approaches to the phase transition problem. One is to consider the momentum equation and another one is to consider the energy equation. The simplest in the first approach is the wave equation with nonmonotone constitutive relation, namely,

$$u_{tt} = \sigma(u_x)_x, \tag{1}$$

where u is displacement and σ is given in Fig.1. There is a difficulty for this approach. That is, when $\sigma' < 0$, the system is elliptic and may not be well posed. The "Hadamard instability" may take place. One way to overcome this difficulty is to add regularizing terms which regularize (1). Dafermos [1] and Pego [2] considered the effect of viscosity, namely,

$$u_{tt} = \sigma(u_x)_x + \nu u_{xxt}. \tag{2}$$

Slemrod [3] considered the regularization of (1) by the viscosity and the capillarity, namely,

$$u_{tt} = \sigma(u_x)_x + \nu u_{xxt} - \eta u_{xxxx}. \tag{3}$$

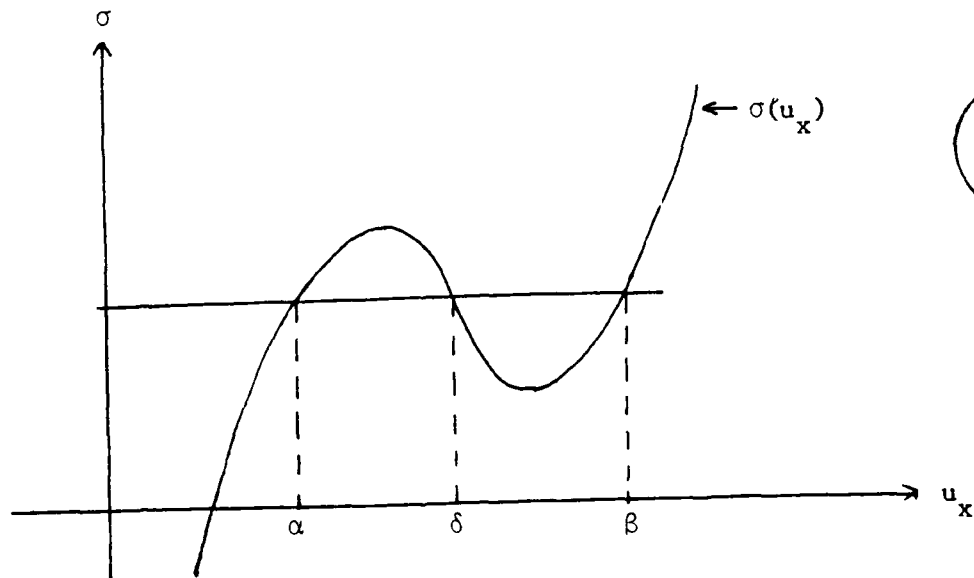


Figure 1.

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In this proposal I have considered the questions associated with the equation (3), which has the following motivation. Carr, Gurtin, and Slemrod [4] considered the variational problems associated with (3). Specifically, they considered the following problem.

(P) Minimize

$$\int_{-1}^1 [W(v) + \eta v'(x)^2/2 - Pv] dx$$

over all $v \in H^1(-1,1)$.

In (P) $W(v)$ is a primitive of $\sigma(v)$. They have shown that depending on the value of η and P there exist multiple solutions. When $\sigma_* < P < \sigma^*$, there are only constant solutions α , δ , and β if $\eta > C$ (a positive number), and among them α and β are the local minimizers of (P) and δ is not the local minimizer. When $\sigma_* < P < \sigma^*$ and $\eta < C$, there are nonconstant solutions besides the above constant solutions, but none of nonconstant solutions are the minimizers of (P).

For (P) a dynamic problem is

$$u_{tt} = \sigma(u_x)_x + \nu u_{xxt} - \eta u_{xxxx} \quad (3)$$

$$u(0,t) = 0, \quad \sigma(u_x) + \nu u_{xt} - \eta u_{xxx}|_{x=1} = P, \quad (4a)$$

$$u_{xx}(0,t) = 0, \quad u_{xx}(1,t) = 0. \quad (4b)$$

Here (4a) corresponds to the soft loading device in which the stress at $x = 1$ is kept at P , and (4b) are the natural boundary conditions to (P). Using the transform

$$p = \int_{-1}^1 u_t dx, \quad q = u_x$$

used by Pego [2] we obtain the initial boundary value problem

$$p_t = \nu p_{xx} - \eta q_{xx} + \sigma(q) - P, \quad (5)$$

$$q_t = p_{xx},$$

$$p_x(0,t) = 0, \quad p(1,t) = 0, \quad (6a)$$

$$q_x(0,t) = 0, \quad q_x(1,t) = 0. \quad (6b)$$

The advantage of this transform is that the system (5) is semilinear and the boundary conditions are linear. Therefore, we can apply the semigroup theory. The questions and results concerning the above initial boundary value problem is summarized in the following way.

Results

(i) As stated in Summary of Project, the constant solutions α and β are stable in the variational problem, but it is not clear if they are dynamically stable. I was able to show the dynamical stability of the constant solutions α and β , the instability of the constant solution δ and the nonconstant solutions.

(ii) Another interesting question is that when there are multiple steady state solutions, there may be solutions connecting them. These special solutions are called connecting orbits. I was able to show the existence of such solutions. This will provide us informations on the dynamical behavior of solutions.

These results will be submitted for publication in the near future.

Significance of The Results

The existence of connecting orbits from δ to α and β will help understand the dynamical behavior of solutions. Namely, it shows that if we set the initial data close to $(p,q) = (0,\delta)$, solution will eventually approach $(p,q) = (0,\alpha)$ or $(0,\beta)$. It also tells us the effect of the viscosity and the capillarity terms, because we do not know how solutions to (1) behave in the region where σ' is negative.

The connecting orbit problem for semi-flow has been discussed for a single parabolic equation, but not for systems. It is important to consider the connecting orbit problem for the semi-flow in systems.

This is a first step toward such a direction.

References

- [1] Dafermos, C.M., The mixed initial-boundary value problem for the equations of nonlinear one dimensional viscoelasticity, J. Diff. Eqns. 6 (1969), 71-86.
- [2] Pego, R., Phase transitions: Stability and admissibility in one dimensional nonlinear viscoelasticity, IMA Report #180 (1985).
- [3] Slemrod, M., Admissibility criteria for propagating phase boundaries in a van der Waals fluid, Arch. Rat. Mech. Anal. 5 (1982), 345-359.
- [4] Carr, J., M.E. Morton, & M. Slemrod, One dimensional structured phase transformations under prescribed loads, J. Elasticity 15 (1985), 133-142.

Publications

Under this grant the following paper has been accepted for publication in the new journal edited by George Sell:

Mischaikow, K., & H. Hattori, On the existence of intermediate magnetohydrodynamic shock waves.

In this paper the authors have used the technique called the connection matrix to establish one of the intermediate shocks which has not been shown to exist. This technique is also applied to the above project.

I contacted Dr. Mischaikow about the title of the journal but he did not remember it.

Professional Presentations

In this period the following presentation was made:

The Reimann problem for a van der Waals fluid and the entropy rate admissibility criterion, June 1988 University of Wisconsin.

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