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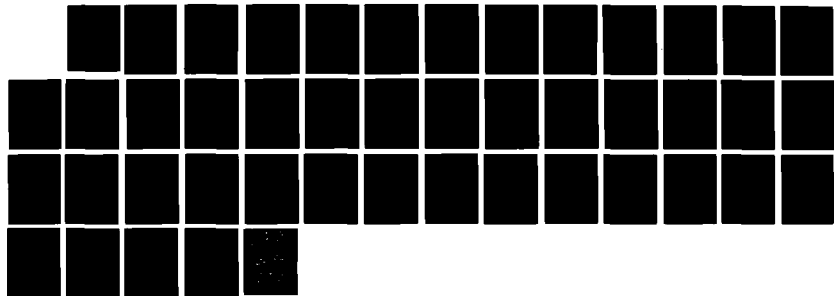
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**Nonlinear Attenuation Mechanism in Salt
at Moderate Strain Based on Salmon Data**

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SECTION 1

INTRODUCTION

In order to characterize a distant seismic pulse according to the energy which generated it, a source function¹ is needed which can be used to initiate the seismic signal at sufficient range from the event that the subsequent propagation can be described in *linear* terms according to the properties of the earth. Thus an observed seismic signal implies a source function which may be compared with known functions for discrimination and yield estimation. Therefore, the properties of such source functions must be known to use this technique. Near-field data from various events have been taken. The smallest strains observed, due to practical free field instrument placement selection, are typically no less than 10^{-5} and are often much greater. If the pulse at this extreme range undergoes no further significant nonlinear modification, its characterization can supply the needed source function. However, if any additional nonlinear changes are important, a useful source function cannot be determined. Consequently, it is important to characterize any possible nonlinear attenuation of moderate strain pulses, preferably through methods which allow generalization to all placement media of interest.

There is a long history of development of numerical methods for calculation of strongly nonlinear behavior in the near field of UGTs.^{2,3} The gross nonlinear effects of vaporization, crushing, cracking and plastic behavior induced by strains much greater than 10^{-3} must be taken into account to obtain an understanding of near field data taken at NTS. Complex equations of state or constitutive relations including effective stress and porosity are needed for a description.⁴ These methods generally apply for strains greater than 10^{-3} ; at lesser strains linear behavior is assumed. However, as indicated in the following section, it seems clear that some residual nonlinear effects continue out to strains less than 10^{-5} . Ideally one would like to have a physically based model of any nonlinear effects in order to use existing near field data to define a more distant linear source function or at least to determine what effect, if any, the nonlinearities over the moderate strain range have on the linear source function.

In order to determine this, we are attempting to develop nonlinear constitutive relations for the mildly nonlinear attenuation in the moderate strain regime (scaled ranges of 10^2 m/kT^{1/3} to 10^4 m/kT^{1/3} or strains from 10^{-3} to 10^{-6}) for explosively generated seismic pulses. The relations are to be used to determine the character of seismic pulses in the linear low-strain regime (beyond 10^4 m/kT^{1/3}) which may then

serve as source functions for regional or teleseismic propagation. A linear source function can be found by numerical propagation of an experimentally well known initial pulse through the moderate strain range to account for mild nonlinear attenuation.

Experimental data provide significant restrictions on candidate constitutive relations. Attenuation in a salt medium from Salmon,⁵⁻¹⁰ Cowboy,¹¹⁻¹³ Livermore small scale explosions,¹⁴ Rockwell damped oscillation experiments¹⁵ and New England Research ultrasonic pulse attenuation,¹⁶ with one exception, are fairly consistent internally. The first three provide a detailed indication of the change of amplitude and shape of explosively driven pulses. Proposed constitutive relations must reproduce these data. The data from the wide range of yields in salt indicate that yield^{1/3} scaling applies with a remarkable precision; if all times and distances are scaled by yield^{1/3}, the scaled amplitudes and shapes of pulses from all experiments are nearly the same.¹⁷ Thus the initial pulse on entering the moderate strain regime must scale at some small scaled range and the subsequent attenuation must result from constitutive relations which have no time or space scales which are fixed by the medium. This provides a significant reduction in the domain of allowable nonlinear behavior. For example, the relations may be a function of the strain but they may not depend on the strain rate.

In order to investigate nonlinear constitutive relations, a standard numerical time stepping method is used. The technique which we have used is that of taking the observed Salmon initial velocity pulse at small range (166 meters) as a source and comparing the resulting pulses as they are propagated through material subject to candidate constitutive relations. The results are compared with observed signals at larger ranges. For any constitutive relation the *effective Q* associated with the attenuation may be determined but it must be emphasized that nonlinear attenuation cannot be properly described by a *Q* function; still *Q* may sometimes be useful for comparison with past work. The fundamental comparison of the data with calculations is not in terms of the *Q* but in terms of reproduction of waveform including both amplitude and shape.

SECTION 2

MODERATE STRAIN ATTENUATION DATA IN SALT

The most comprehensive attenuation data, over the range of moderate strains, exists in the medium of salt. For explosive sources the laboratory results of Larson¹⁴ covers strains from 10^{-1} to 10^{-3} , the Salmon field test⁵ covers strains for 10^{-3} to 10^{-4} while the Cowboy field test¹¹ series covers strains from 10^{-4} to 10^{-5} . The Rockwell¹⁵ laboratory decaying oscillations complete the salt data by ranging from 10^{-5} to 10^{-8} . The NER¹⁶ laboratory ultrasonic pulse propagation experiments go from less than 10^{-6} to more than 10^{-5} . The essential results of these experiments will now be reviewed.

Larson's laboratory data were taken from the effects of a series of small chemical explosives in blocks of pressed salt. Velocity data were taken at several gauges such that the totality of the examples from all shots covered peak velocity to compressional velocities, which is approximately the peak strain, of 10^{-1} to 10^{-3} . Using a triplet of records from a single shot, it was estimated that over peak strains of 1.4×10^{-3} , 7.0×10^{-4} and 4.6×10^{-4} , the Q changes from 12.5 to 24.9. The corner frequency for this small scale experiment was about 5×10^4 Hertz.

The Salmon data were generated by a 5.3 kT nuclear explosion in salt. The work by McCartor and Wortman⁷ as well as McLaughlin and Gupta⁸ shows that the high quality data clearly indicate a high level of attenuation. It is estimated that for peak strains from 4×10^{-3} to 3×10^{-4} , the effective Q is to order of 10 at a corner frequency of about 6 Hertz. This Q appears nearly constant,⁷ perhaps increasing mildly with range,⁸ over the order of magnitude strain range available. However, in view of the fact that small strain data from other experiments, including the decoupled event Sterling⁸ which used the Salmon cavity, indicate a much larger Q , it seems likely that there are residual nonlinearities at the extreme range of the Salmon data so that Q must increase at larger ranges (and so smaller strains).

Tittmann¹⁵ has studied the attenuation of flexural and torsional harmonic oscillations of salt samples. It has been found that the attenuation, expressed in terms of Q^{-1} as a function of strain amplitude, tends to be nearly constant for strains from 10^{-8} to 10^{-6} , then increase for greater strains. The attenuation is a decreasing function of confining pressure with an average Q of approximately 200 at 10^{-6} strain at a frequency of 400 Hertz for confinement consistent with the explosions.

Rockwell Elastic Properties, Finite Difference

The regime covered by the COWBOY experiments overlaps that for Salmon, by beginning at strains of about 5×10^{-3} and extending out to 10^{-5} . The COWBOY experiments consisted of a series of shots over a range of yields, both coupled and uncoupled, such that the individual events generally had only a few instruments in operation. Several authors have combined these data by invoking the experimentally compelling evidence for simple scaling, which suggests that the velocity pulse data from various yields can be equated by scaling all times and distances by the cube root of the yield. This is particularly dramatic when peak velocity is plotted against scaled range giving a consistent curve over about ten orders of magnitude in yield. Larson has pointed out that the full velocity pulses, as well as their peak values, tend to be preserved through scaling.

Trulio¹⁰ has analyzed the scaling-combined Cowboy data to estimate Q as dictated by decrease in peak displacement with range by using one decade at a time in the frequency domain. He has found that attenuation decreases as range increases in a fashion inconsistent with linearity expressed as dispersive harmonic potential waves. If the coupled Cowboy data are fit assuming a Q independent of range (a possibility due only to scatter in the data), Q must be a function of frequency ranging from about 5 at low frequencies (1 Hz at Salmon scale) to nearly 100 at high frequencies (32 Hz at Salmon scale). However, simple scaling is inconsistent with this fit since scaling requires linearity coupled with a Q independent of frequency. If the decay of the reduced displacement potential is fit by the form $\exp(-\omega r/2cQ)$, it is seen that simple scaling can result so long as, for constant c , Q is a function of a variable which is unchanged by scaling – such as ωr . Trulio has shown that the a range of data from Salmon, through Cowboy and Cowboy Trails, do have the property that an effective Q can be expressed as a function of ωr for all the experiments. Obviously this indicates that at fixed frequency, the attenuation expressed in terms of Q , is dependent upon range. Since it is assumed that the medium is approximately homogeneous, the implication is clear that the attenuation must be amplitude dependent and so nonlinear.

Minster and Day¹² have used the scaled peak velocity data for COWBOY to determine if these data require an amplitude (nonlinear) or frequency dependent Q for consistency. They determined that a Q^{-1} which consists of a small constant (consistent with small strain data from Tittmann) plus a term proportional to the peak strain provides a good fit to the data. The observed attenuation effects do not firmly indicate the need for a frequency dependent Q but indicate that an amplitude dependent Q provides a more convincing fit than a constant Q . Based on the small strain Q , which is several hundred as seen in the Rockwell experiments, it is concluded that there must be nonlinear attenuation in the COWBOY strain regime.

Coyner¹⁶ of New England Research has carried out a series of laboratory experiments for which compressional and shear ultrasonic pulses consisting of about two cycles at 100-200 kHz were propagated through a sample. Attenuations were calculated using a spectral ratio technique. Variation of the attenuation with peak strain amplitude and confining pressure were determined. Experiments were carried out with several materials including Sierra White Granite, Berea Sandstone and Dome Salt. For the dome salt it was found that over a strain range of 5×10^{-7} to 3×10^{-6} and for a confining load range of 0.1 to 1 MPa, the P-wave attenuation is nearly constant and can be described by a Q of about 20. There is no particular evidence of nonlinearity in these data alone.

In summary, the salt data appear to support the hypothesis that explosively generated pulses encounter nonlinear attenuation for strains much greater than 10^{-6} . No detailed knowledge of the attenuation mechanism currently exists but there does appear to be a consistency in that the explosively generated pulses closely obey simple scaling which suggests that the mechanism must be rate independent. The *effective* Q for this process increases with increasing frequency and increases with decreasing strain. In the next section we shall attempt to exploit a feature of the Salmon near-field pulses which suggests a physical mechanism which can account for the attenuation data.

SECTION 3

MECHANISM FOR SALMON ELASTIC PRECURSOR

The Salmon data have a feature which may be useful in understanding some nonlinear effects of attenuation. Each of the pulses experimentally observed at six sensors at ranges from 166 meters to 660 meters exhibit a discontinuity in the slope upon the initial steep rise in pulse velocity, as seen in Figure 1. This appears as a toe-like behavior in the leading edge of the velocity profile which has been described by Perret⁵ as an "elastic precursor" to the main pulse. The absolute amplitude of the toe remains approximately constant with range, at a particle velocity of about 0.5 m/s, while its amplitude relative to the peak increases with range. This precursor amplitude corresponds to a compressional strain level of $\epsilon \approx 10^{-4}$. The leading edge of the pulses (i.e., that disturbance earliest in time) propagates at a speed of about 4.7 km/sec while the pulse peaks, always after the toe, propagate at a speed of about 3.7 km/sec. The elastic compressional speed of mild disturbances in this salt medium found from independent measurements is typically about 4.6 km/sec. This indicates that the precursor signal seen in the Salmon data is due to elastic behavior while the subsequent pulse suffers a lower propagation speed due to some relaxation or plastic behavior. If propagation were purely plastic the sequence of pulses seen at the Salmon sensor sites would be as shown in Figure 2.

Perret suggests an elastic-plastic material behavior might account for the data in perhaps one of two ways. First, the precursor could develop at large strain, where an elastic limit is exceeded from radii much smaller than instrumented for Salmon, and continue to propagate in front of a following plastic wave. Second, it may be that the precursor develops in the moderate strain region if dome salt has an elasto-plastic nature at such strains. In either case, the modulus of salt must be a function of the strain - that is, the medium is nonlinear.

If the precursor develops at large strain there must be an elastic limit beyond which plastic behavior provides a lower modulus. When such a medium is dynamically loaded beyond the elastic limit, a leading pulse at the elastic limiting stress is generated followed by a larger amplitude but slower plastic wave. If the elastic-plastic transition is not sharply defined, the resulting pulse could consist of a gently rising leading elastic front which smoothly merges with the main plastic pulse much as seen in the Salmon data. Perret points out that if some energy from the plastic component is fed to the elastic portion during propagation, the amplitude of the elastic piece

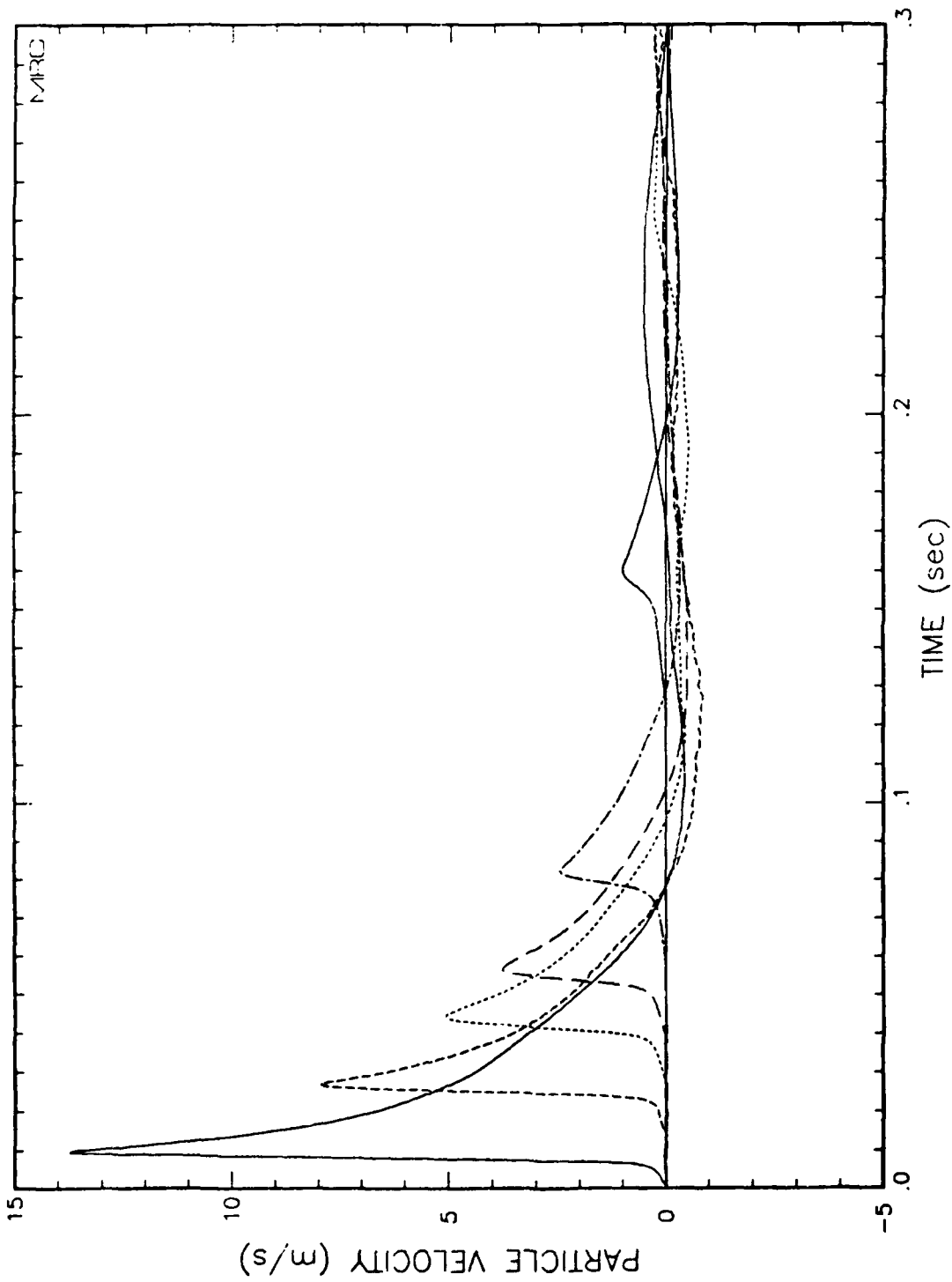


Figure 1. Salmon radial velocity pulses seen at 166, 225, 276, 318, 402 and 660 meters range (time zero on plot corresponds to initial signal arrival at 166 meters which was at 0.036 seconds after detonation).

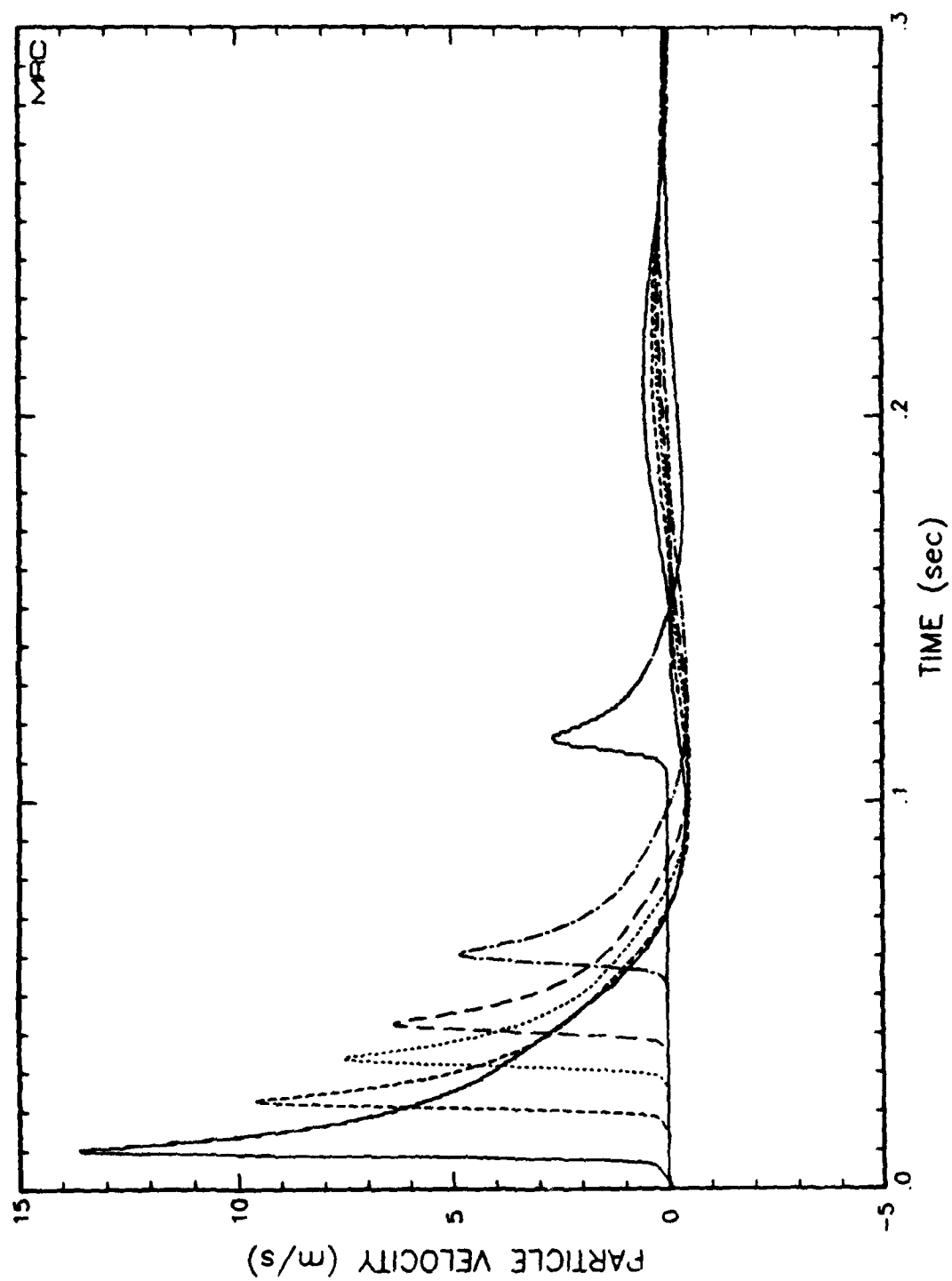


Figure 2. Calculated pulses at Salmon ranges for pure plastic medium.

could remain nearly constant, but this is quite speculative. What is known is that a variety of laboratory experiments show that strong impulsive sources can produce elastic precursors in a variety of media. For example, work of Ahrens and Duvall¹⁸ with planar pulses in quartz generated by explosives exhibits an apparent elastic stress limit of about 70 kbar corresponding to strains of about 10^{-1} . This produces a leading edge, described as the elastic shock, which propagates at a speed in excess of that of the deformational portion of the pulse which follows. The elastic shock propagates with an equivalent modulus which is greater than the static modulus at this high stress. It is speculated that the elastic wave is supported by a higher than equilibrium shear stress. After the elastic component has passed, the shear stress is apparently reduced to the static value by a plastic or fracture process. This experiment as well as others¹⁹ which show an elastic precursor consistent provide elastic limits of many or tens of kilobars in contrast with the Salmon data which gives a precursor amplitude of about 5 bars. Consequently this mechanism does not seem a likely means of accounting for the Salmon data which give a nearly constant and small precursor amplitude which seems to begin near the 166 meter sensor range rather than be well developed by this time.

The second possibility indicated by Perret is that of having the precursor develop locally in the observation region based on moderate strain plastic behavior. While there is no accepted dynamical equation of state for dome salt, Perret points out that dome salt is known to be highly plastic - under static conditions, it is nearly hydrostatic. Thus plastic deformation of salt at moderate stresses is apparently normal. In order to account for the precursor data in Salmon, the equation of state would have to provide linear behavior up to a threshold (a threshold of about 5 bars, much less than the 70 kbars found for the elastic shock discussed above) and through some deformation of the material, relax the modulus abruptly (on the Salmon time scale) to a value which provides a compressional propagation speed about 20% less than for infinitesimal strains in undisturbed material. Having a threshold at the observed and nearly constant precursor amplitude avoids having to account for the constancy of the precursor by arguments of convenience about feeding energy back from the plastic portion of the wave.

As an equation of state model which can be consistent with the Salmon data, consider a medium for which the shear modulus permanently (that is, does not recover until the pulse is past) decreases upon having a critical strain threshold exceeded; the compressional modulus before and after exceeding the strain threshold reflects the compressional speeds at the beginning and peak of the Salmon pulses, respectively. This will be referred to as a shear failure model. Depending upon the relation between the compressional and shear moduli, complete shear failure may occur, meaning that

the elastic shear modulus, μ , goes to zero. For the example used in this discussion, the compressional speed decreases to 80% of its original value. We have taken the Lamé constants λ and μ to have a ratio of 2. Thus a decrease of the compressional speed, $((\lambda + 2\mu)/\rho)^{1/2}$, where ρ is the density, of 20% corresponds to reducing μ to about 38% of its elastic value when λ is held fixed. Note that the reduction of modulus at a fixed strain is consistent with scaling since the strain is a unitless quantity and there is no rate dependence. The scaling restriction does require that the relaxation time of the modulus change be short compared with any representative time scale of the data; we take the transition to be instantaneous. Figure 3 indicates the stress-strain curve which results from this model for the modulus. In order to determine the effect on the pulse propagation we used the observed Salmon velocity at 166 meters as the source. The calculations were carried out using a standard finite difference methods whose details are provided in Appendix A.

As an initial effort, the elastic threshold was taken at a compressional strain of 10^{-4} ; the resulting pulse sequence at the ranges to observation stations for Salmon is as shown in Figure 4. Note that the character of the calculated precursor is much like that seen experimentally, in Figure 1, in that the leading feature is drawn out, the transition to the main pulse takes place at a constant amplitude and the peak now moves at a significantly lower speed. Still the amplitude of the main peak does not decrease as quickly as the data indicate.

When the modulus decreases, the elastic energy in the pulse also decreases in a manner approximately proportional to the square of the compressional wave speed. Since the modulus reduction is permanent, this energy is lost to the pulse and goes into heating the medium. For the parameter used, over a full cycle for which most of the pulse exceeds the critical strain, approximately one-third of the original elastic energy will be lost. This corresponds to an effective Q of about 13 for peak strains well in excess of 10^{-4} (for small strains less than this threshold, there will be no loss). This value of Q is far less than that expected for very small strains but it is still more than the 5 to 10 seen for Salmon attenuation. The addition of a moderate level of linear attenuation consistent with that seen for small to moderate strains in other experiments will improve the agreement (for example, the NER data suggest $Q \approx 20$). More importantly, the use of a partial shear failure mechanism will automatically terminate once the pulse weakens so that the peak strain falls below the critical strain threshold value. This will produce a sharply changing effective Q in a manner suggested by the Cowboy data as given in Figure 5. Thus this first attempt to reproduce the Salmon data is encouraging but there remain differences which may be accounted for by refining the details of the attenuation mechanism.

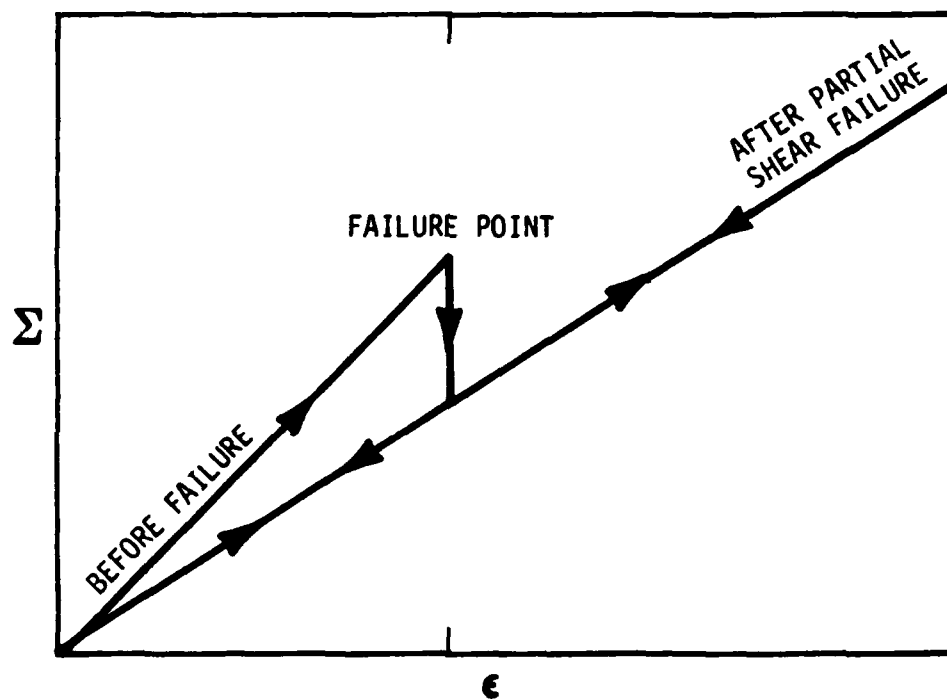


Figure 3. Stress-strain curve for partial shear failure.

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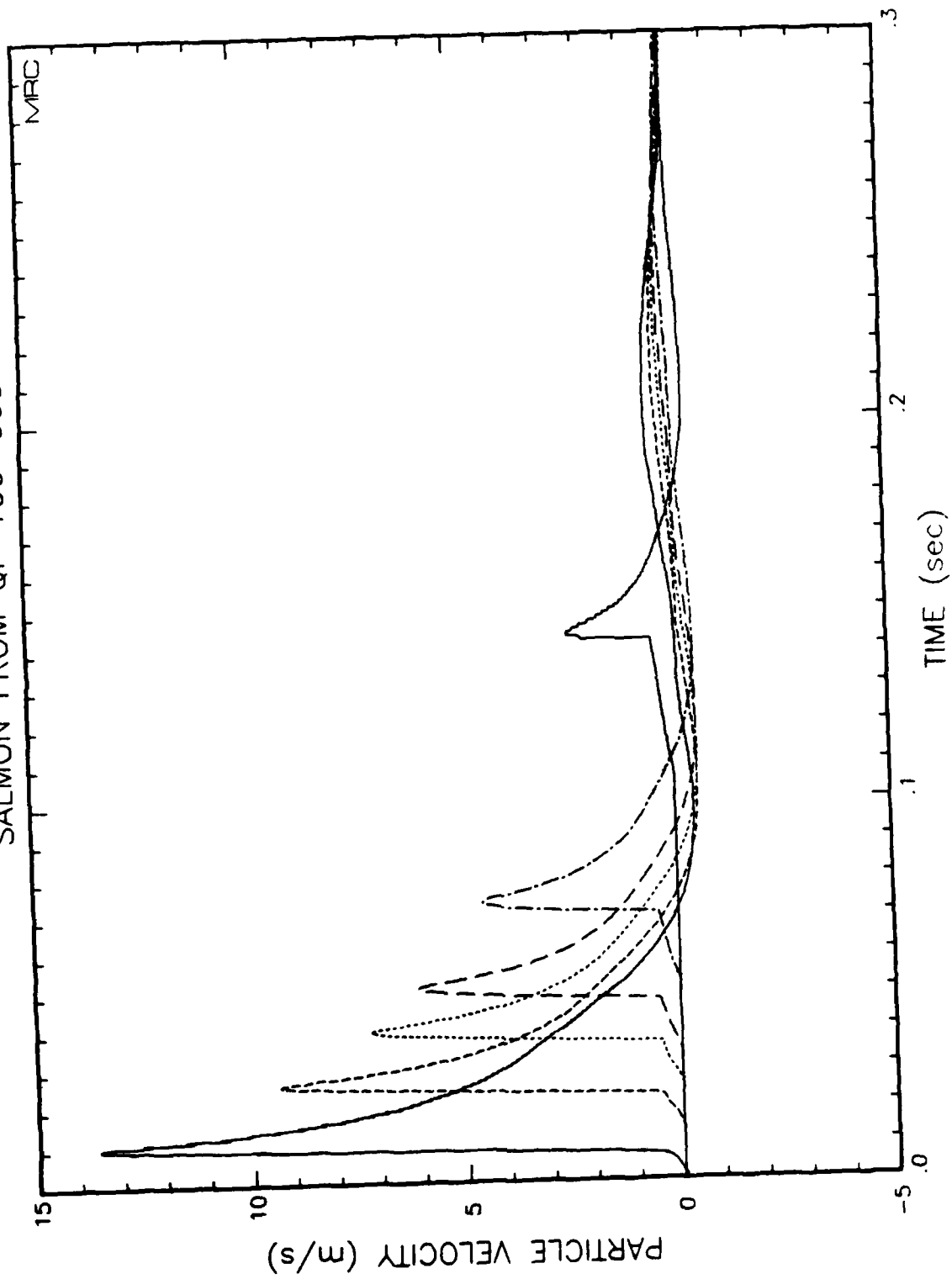


Figure 4. Calculated pulses at Salmon ranges for shear failure.

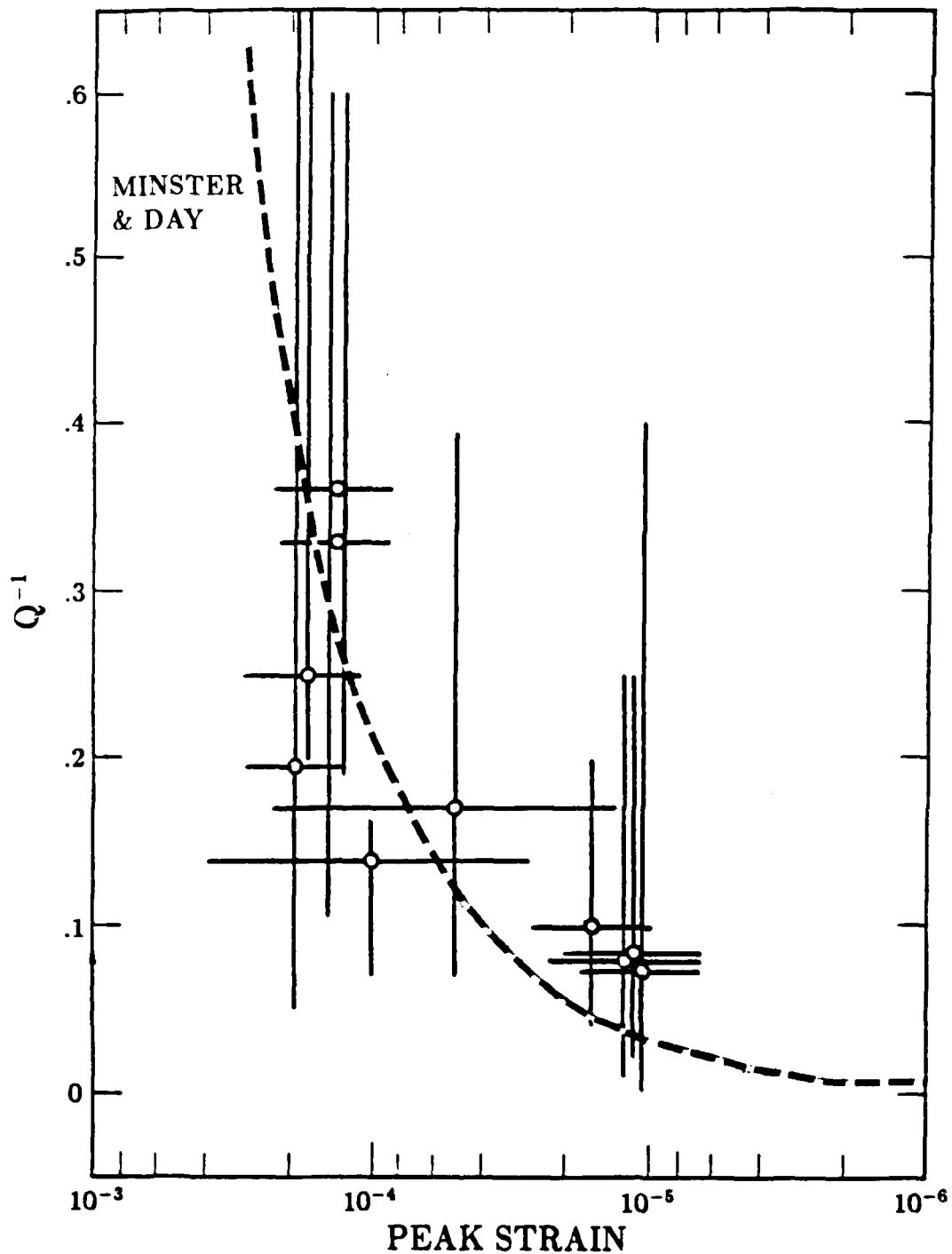


Figure 5. Cowboy Q estimates as a function of peak strain. The horizontal bars represent the two record strain values while the vertical bars indicate the spread of Q over a decade of frequency. The fit of Minster and Day is provided. (From Ref. 13).

The attenuation from partial shear failure does not produce an amplitude for the pulse at 660 meters which is as small as that seen experimentally. More attenuation can be added to attempt to match better the data by employing a linear Q of sufficient value. A method of inclusion of a linear absorption band Q in time stepping finite difference methods has been demonstrated by use of Padé approximants. Our application of this method is outlined in Appendix B. This formalism was employed using a target Q of 10 with a range of half amplitude frequencies of 1 to 100 Hertz (Q rises above 20 beyond these values). The sequence of pulses which result using both the partial shear failure and a linear Q of 10, starting with the Salmon pulse at 166 meters, is shown in Figure 6. The amplitudes for the main peaks now are in substantial agreement with the data and the length and amplitude of the precursor are also reproduced fairly well. Still there remains a very abrupt transition from precursor to main pulse which is clearly sharper than the experimental data.

One further refinement has been applied to the partial shear failure model in order to avoid the abrupt transition between precursor and pulse. Since there is certain to be a range of material properties even in the relatively homogeneous salt dome, it is reasonable to require a range of thresholds for the shear failure. The model has been altered to produce a variation of failure threshold values of compressional strain over a range of 30% with a constant probability about the 10^{-4} value. Each cell in the finite difference calculation is given its own threshold which is randomly drawn on this basis. The set of pulses at the Salmon instrument ranges then calculated is given in Figure 7. The result is a smoother transition from precursor to main pulse in a manner which is quite similar to the actual Salmon data shown in Figure 1. While it is probably possible to achieve a detailed fit to the data by further such refinements, this is not a very meaningful thing to do since the mechanisms are not understood to the required level of detail. The important point is that it is possible to reproduce the data to a substantial degree using only a few physically based parameters.

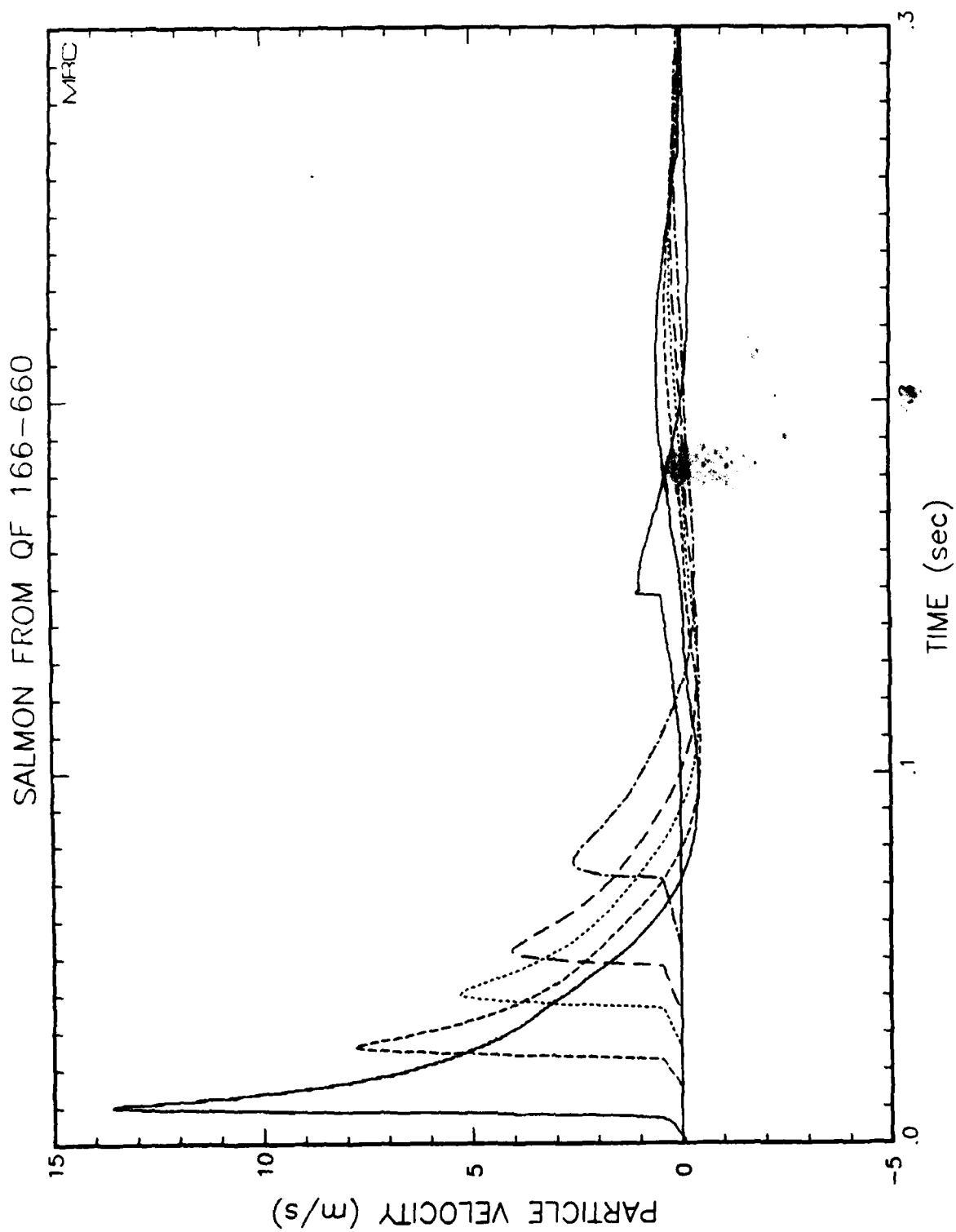


Figure 6. Calculated pulses at Salmon ranges for shear failure and $Q=10$.

SALMON FROM QF 166-660

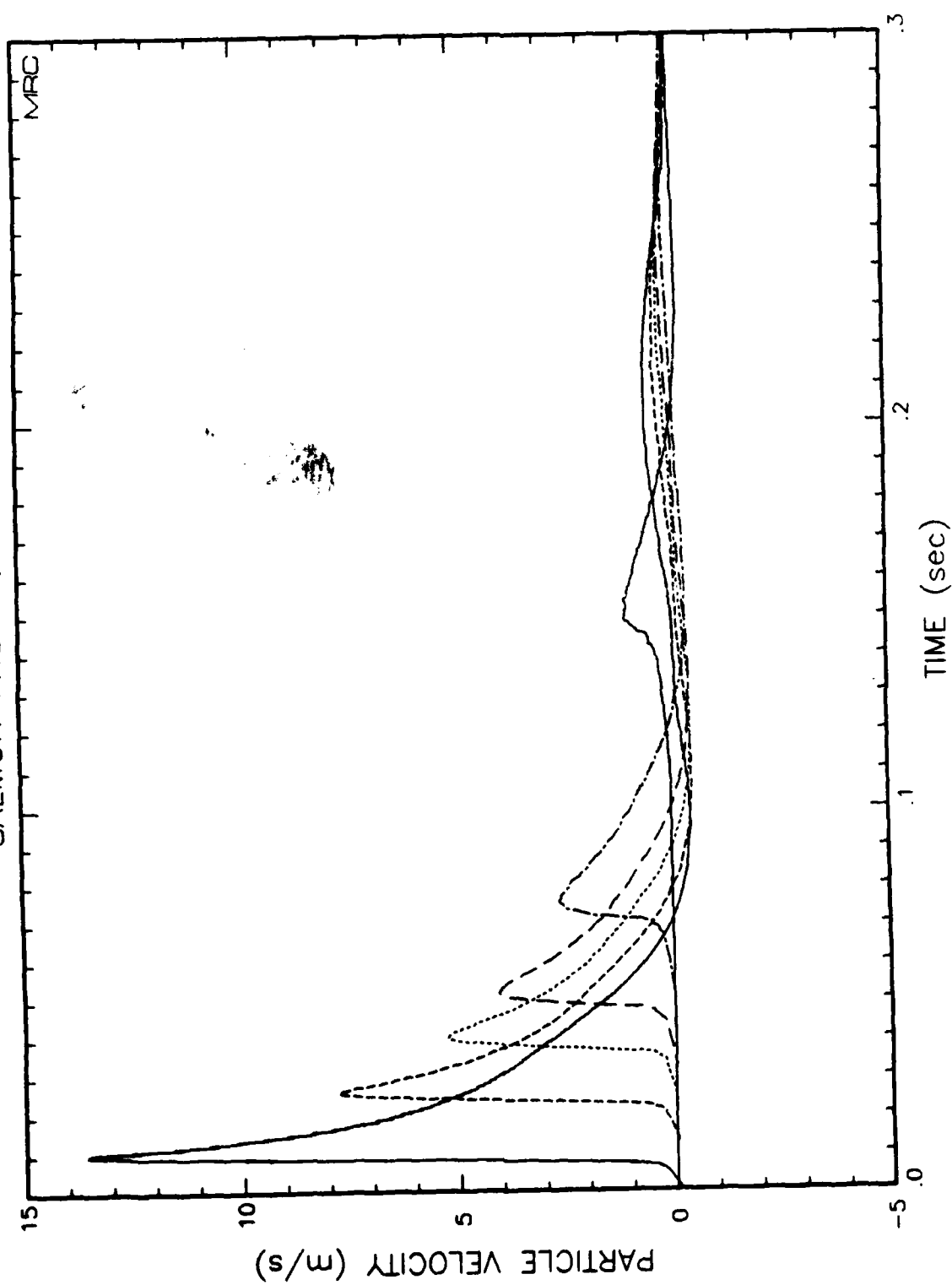


Figure 7. Calculated pulses at Salmon ranges for smoothed shear failure and $Q=10$.

SECTION 4

COMMENTS

This work indicates that the elastic precursor or leading toe seen in Salmon near-field, moderate strain, velocity data is reproduced rather well with the hypothesis of partial shear failure which is activated for the duration of the pulse when the compressional strain exceeds 10^{-4} . This also gives an attenuation mechanism which accounts for much of the energy loss seen in the decay of pulses from Salmon with range. However, the overall attenuation produced is not quite adequate to account for that seen in the data. The addition of a linear absorption band attenuation, which is active over much of the significant frequency range appropriate to Salmon and which has a Q of 10, then provides a propagation model which nearly reproduces the signals at ranges beyond 166 meters when the observed signal at this range is used as the source. Furthermore, this threshold mechanism provides a transition to more modest attenuation at small strains which is required to be consistent with Cowboy data. While there is no assurance that this mechanism applies in other than the salt medium, Perret points out that "elastic precursors" of a similar character have been seen in UGT pulses in both alluvium and dolomite.

In addition to accounting for much of the Salmon data, the shear failure mechanism also, when applied to different yield events in salt, will produce simple scaling as observed over a wide range of explosive events.

The fact that the reduction in compressional wave speed is attributed to shear failure, rather than alteration of some other material property, is largely a matter of consistency with past thinking on modes of material behavior; there is no direct experimental link to shear properties. The general agreement with data which results here could just as well have been produced by any method which reduces the compressional modulus in the required amount.

The fact that the NER experiments give an apparently linear Q of about 20 for small strains of 10^{-6} while Sterling data at comparable strains have much larger Q is a matter of some concern. However it is known that the attenuation of a material can be a strong function of the liquid content. Spencer²⁰ made a series of attenuation measurements in several types of rock in dry and water saturated states. He found that dry rocks tend to show only modest attenuation while the water saturated samples had minimum Q 's of from 4 in sandstone to about 40 in limestone. He found that the attenuation for saturated rocks is quite frequency dependent with peaks at 10 to 1000

Hz depending on the material. He further found that the attenuation spectrum is consistent with dispersion in the modulus so that the mechanism is apparently linear. This sort of result indicates that much care must be taken in comparing experimental results taken for different levels of water saturation and at different frequencies even when strain levels are quite low.

There is a second area where different experiments appear to yield different results. The work of Tittmann on decaying oscillations of salt and other bars give apparent nonlinear behavior but they also show very mild attenuation in the linear regime. As observed by Coyner, the attenuation from these multiple cycle experiments seem to be consistently much less than from propagation (single pulse) experiments at comparable strains and frequencies or from hysteresis loop experiments. For nonlinear attenuation, there does not need to be any particular relation between the effective Q from pulse and oscillatory experiments - the detailed nature of the mechanism must be known to relate these. For the example of partial shear failure, multiple cycle experiments will produce very small attenuation estimates since all the failure will take place by the time a single cycle has been completed - after that only the intrinsic attenuation will be active.

It should be emphasized that the use of a Q description for attenuation which is clearly nonlinear is probably not useful, except as a crude estimate of the degree of amplitude reduction. In fact, the use of Q in nonlinear cases can provide very deceptive results since the frequency and range dependence are generally mixed. It would be best to describe the attenuation mechanism directly in terms of an equation of state. This will allow the application to different experiments. Put another way, the result of a nonlinear attenuation for a propagation experiment can always be used to find an effective Q by, say, use of spectral ratios but the same mechanism will produce a different effective Q if the character of the initial waveform is changed. The effective Q is not a robust quantity for nonlinear attenuation.

We expect to use the finite difference propagation code, which was developed as a tool for our current work on Salmon, as a testbed for development of nonlinear constitutive relations. This avoids the use of Q and concentrates directly on the physical mechanisms which produce attenuation. Ideally one would like to express a constitutive relation in terms of parameters known to be important, such as porosity, effective stress, crack density, crack growth, etc., in order to allow a model which can be generalized to different media.

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APPENDIX A

WILKINS FINITE DIFFERENCE EQUATIONS

The difference method used to solve for the spherical propagation of a pulse is based on that given by Wilkins.¹ The essential features of this approach are as follows:

The equation of motion is

$$\frac{\rho^0 \dot{U}}{V} = \frac{\partial \Sigma_r}{\partial r} + 2 \frac{\Sigma_r - \Sigma_0}{r} \quad (\text{A} - 1)$$

where the principal stresses are

$$\Sigma_r = -(P + q) + s_1 \quad (\text{A-2})$$

$$\Sigma_\theta = -(P + q) + s_2 \quad (\text{A-3})$$

in terms of the hydrostatic pressure, P , the artificial viscous pressure, q , and the stress deviators. The reference density is ρ^0 , the relative volume is V and U is the particle velocity.

The continuity equation is

$$\frac{\dot{V}}{V} = \frac{1}{r^2} \frac{\partial(r^2 U)}{\partial r} \quad (\text{A} - 4)$$

where r is spherical radial position.

The energy equation is

$$\dot{E} - V[s_1 \dot{\epsilon}_1 + 2s_2 \dot{\epsilon}_2] + (P + q)\dot{V} = 0 \quad (\text{A} - 5)$$

where E is the internal energy and ϵ are strains:

$$\dot{\epsilon}_1 = \partial U / \partial r \quad (\text{A-6})$$

$$\dot{\epsilon}_2 = U/r \quad (\text{A-7})$$

Artificial viscous pressure is taken from Viscelli's² work as:

$$q = C_L \frac{\rho^0 \alpha}{V} \left| \frac{\partial U}{\partial r} \right| \Delta r \quad (\text{A} - 8)$$

where C_L is a constant (taken as 0.5), α is the sound speed and Δr is the cell spacing.

For the elastic case the equation of state is

$$\dot{s}_1 = 2\mu(\dot{\epsilon}_1 - \frac{1}{3}\dot{V}/V) \quad (\text{A-9})$$

$$\dot{s}_2 = 2\mu(\dot{\epsilon}_2 - \frac{1}{3}\dot{V}/V) \quad (\text{A-10})$$

$$\dot{P} = -(\lambda + \frac{2}{3}\mu)\dot{V}/V \quad (\text{A-11})$$

where λ and μ are the Lamé elastic constants ($\lambda + \frac{2}{3}\mu$ is the bulk modulus).

The finite difference version equations is taken by division of the material into N mass intervals

$$m_{j+\frac{1}{2}} = \frac{\rho^0}{3V^0} \left(\frac{(r_{j+1}^0)^3 - (r_j^0)^3}{d} \right) \quad j = 1, \dots, N \quad (\text{A} - 12)$$

The equation of motion for the velocity for the j^{th} element at the n^{th} time

$$U_j^{n+1} = U_j^{n-\frac{1}{2}} + \frac{\Delta t^n}{\phi_j^n} [(\Sigma_r)_n^{j+\frac{1}{2}} - (\Sigma_r)_n^{j-\frac{1}{2}}] + 2\Delta t^n(\beta_j^n) \quad (\text{A} - 13)$$

where

$$(\Sigma_r)_n^{j+\frac{1}{2}} = \left\{ - (P^n + q^{n-\frac{1}{2}}) + s_1^n \right\}_{j+\frac{1}{2}} \quad (\text{A-14})$$

$$(\Sigma_\theta)_n^{j+\frac{1}{2}} = \left\{ - (P^n + q^{n-\frac{1}{2}}) + s_2^n \right\}_{j+\frac{1}{2}} \quad (\text{A-15})$$

and

$$\phi_j^n = \frac{1}{2} \left[\rho_{j+\frac{1}{2}}^0 \left(\frac{r_{j+1}^n - r_j^n}{V_{j+\frac{1}{2}}^n} \right) + \rho_{j-\frac{1}{2}}^0 \left(\frac{r_j^n - r_{j-1}^n}{V_{j-\frac{1}{2}}^n} \right) \right] \quad (\text{A-16})$$

$$\beta_j^n = \frac{1}{2} \left\{ \left[\frac{(\Sigma_r)_n^{j+\frac{1}{2}} - (\Sigma_\theta)_n^{j+\frac{1}{2}}}{\frac{1}{2}(r_{j+1}^n + r_j^n)} \right] \left(\frac{V^n}{\rho^0} \right)_{j+\frac{1}{2}} + \left[\frac{(\Sigma_r)_n^{j+\frac{1}{2}} - (\Sigma_\theta)_n^{j+\frac{1}{2}}}{\frac{1}{2}(r_j^n + r_{j-1}^n)} \right] \left(\frac{V^n}{\rho^0} \right)_{j+\frac{1}{2}} \right\} \quad (\text{A-17})$$

At an outside regional boundary J

$$\phi_J^n = \frac{1}{2} \rho_{J-\frac{1}{2}}^0 \left(\frac{r_J^n - r_{J-1}^n}{V_{J-\frac{1}{2}}^n} \right) \quad (\text{A-18})$$

$$\beta_J^n = \left[\frac{(\Sigma_r)_{J-\frac{1}{2}}^n - (\Sigma_\theta)_{J-\frac{1}{2}}^n}{\frac{1}{2}(r_J^n + r_{J-1}^n)} \right] \left(\frac{V_n}{\rho_0} \right)_{J-\frac{1}{2}} \quad (\text{A-19})$$

while at an inside regional boundary

$$\phi_J^n = \frac{1}{2} \rho_{J+\frac{1}{2}}^0 \left(\frac{r_J^n - r_{J+1}^n}{V_{J+\frac{1}{2}}^n} \right) \quad (\text{A-20})$$

$$\beta_J^n = \left[\frac{(\Sigma_r)_{J+\frac{1}{2}}^n - (\Sigma_\theta)_{J+\frac{1}{2}}^n}{\frac{1}{2}(r_J^n + r_{J+1}^n)} \right] \left(\frac{V_n}{\rho_0} \right)_{J+\frac{1}{2}} \quad (\text{A-21})$$

Given the velocity, the positions are advanced by

$$r_j^{n+1} = r_j^n + U_j^{n+\frac{1}{2}} \Delta t^{n+\frac{1}{2}} \quad (\text{A-22})$$

The equation of continuity is

$$V_{j+\frac{1}{2}}^{n+1} = V_{j+\frac{1}{2}}^n + \Delta t^{n+\frac{1}{2}} \left(\frac{\rho^0}{m} \right)_{j+\frac{1}{2}} \left[U_{j+1}^{n+\frac{1}{2}} \left(r_{j+1}^{n+\frac{1}{2}} \right)^2 - U_j^{n+\frac{1}{2}} \left(r_j^{n+\frac{1}{2}} \right)^2 + \chi_{j+\frac{1}{2}}^{n+\frac{1}{2}} \right] \quad (\text{A-23})$$

with

$$\eta_{j+\frac{1}{2}}^{n+1} = \frac{1}{V_{j+\frac{1}{2}}^{n+1}} \quad (\text{A-24})$$

where

$$r_{j+1}^{n+\frac{1}{2}} = \frac{1}{2} (r_{j+1}^{n+1} + r_{j+1}^n) \quad , \text{etc.} \quad (\text{A-25})$$

defines the half time values. And, finally

$$\chi_{j+\frac{1}{2}} = \frac{(\Delta t^{j+\frac{1}{2}})^2}{12} \left[(U_{j+1}^{n+\frac{1}{2}})^3 - (U_j^{n+\frac{1}{2}})^3 \right] \quad (\text{A} - 26)$$

The strains are defined by

$$(\dot{\epsilon}_1)_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{U_{j+1}^{n+\frac{1}{2}} - U_j^{n+\frac{1}{2}}}{r_{j+1}^{n+\frac{1}{2}} - r_j^{n+\frac{1}{2}}} \quad (\text{A-27})$$

$$(\dot{\epsilon}_2)_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{U_{j+1}^{n+\frac{1}{2}} + U_j^{n+\frac{1}{2}}}{r_{j+1}^{n+\frac{1}{2}} + r_j^{n+\frac{1}{2}}} \quad (\text{A-28})$$

and the stress deviators are advanced by

$$(s_1)_{j+\frac{1}{2}}^{n+1} = (s_1)_{j+\frac{1}{2}}^n + 2\mu \left[(\dot{\epsilon}_1)_{j+\frac{1}{2}}^{n+\frac{1}{2}} \Delta t^{n+\frac{1}{2}} - \frac{1}{3} \left(\frac{V^{n+1} - V^n}{V^{n+\frac{1}{2}}} \right)_{j+\frac{1}{2}} \right] \quad (\text{A-29})$$

$$(s_2)_{j+\frac{1}{2}}^{n+1} = (s_2)_{j+\frac{1}{2}}^n + 2\mu \left[(\dot{\epsilon}_2)_{j+\frac{1}{2}}^{n+\frac{1}{2}} \Delta t^{n+\frac{1}{2}} - \frac{1}{3} \left(\frac{V^{n+1} - V^n}{V^{n+\frac{1}{2}}} \right)_{j+\frac{1}{2}} \right] \quad (\text{A-30})$$

Artificial viscous pressure is included by

$$q_{j+\frac{1}{2}}^{n+\frac{1}{2}} = C_L \alpha \rho^0 \eta_{j+\frac{1}{2}}^{n+\frac{1}{2}} \left| U_{j+1}^{n+\frac{1}{2}} - U_j^{n+\frac{1}{2}} \right| \quad ; \quad (\text{A} - 31)$$

where $C_L \approx 0.5$ (constant) and α in the compressional wave speed.

The energy equation is advanced by

$$(E_1)_{j+\frac{1}{2}}^{n+1} = (E_1)_{j+\frac{1}{2}}^n + V_{j+\frac{1}{2}}^{n+\frac{1}{2}} (s_1)_{j+\frac{1}{2}}^{n+\frac{1}{2}} (\dot{\epsilon}_1)_{j+\frac{1}{2}}^{n+\frac{1}{2}} \Delta t^{n+\frac{1}{2}} \quad (\text{A-32})$$

$$(E_2)_{j+\frac{1}{2}}^{n+1} = (E_2)_{j+\frac{1}{2}}^n + 2V_{j+\frac{1}{2}}^{n+\frac{1}{2}} (s_2)_{j+\frac{1}{2}}^{n+\frac{1}{2}} (\dot{\epsilon}_2)_{j+\frac{1}{2}}^{n+\frac{1}{2}} \Delta t^{n+\frac{1}{2}} \quad (\text{A-33})$$

$$(E_1)_{j+\frac{1}{2}}^{n+1} = \left\{ E_1^n - \left\{ \frac{1}{2} [P^{n+1} + P^n] + q^{n+\frac{1}{2}} \right\} \cdot [V^{n+1} - V^n] + \Delta E_1 + \Delta E_2 \right\}_{j+\frac{1}{2}} \quad (\text{A-34})$$

For the current application, this energy is just a diagnostic and is not fed back through any equation of state.

The hydrostatic pressure for the elastic case is

$$P_{j+\frac{1}{2}}^{n+1} = \left(\lambda + \frac{2}{3}\mu \right) \left(\eta_{j+\frac{1}{2}}^{n+1} - 1 \right) \quad (\text{A} - 35)$$

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APPENDIX B

INCLUSION OF DAY-MINSTER Q IN FINITE DIFFERENCES

Day and Minster¹ have shown how to include an arbitrary linear Q function in time stepping calculations. For an absorption band an analytic solution is available. An outline of their methods, as easily generalized to the spherical case is given here.

For a single normalized relaxation function, $m(t)$, the stresses and strains are related by

$$\Sigma_r = (\lambda + 2\mu) \int m(t - \tau) \epsilon_1(\tau) d\tau + 2\lambda \int m(t - \tau) \epsilon_2(\tau) d\tau \quad (B-1)$$

$$\Sigma_\theta = (2\lambda + 2\mu) \int m(t - \tau) \epsilon_2(\tau) d\tau + \lambda \int m(t - \tau) \epsilon_1(\tau) d\tau \quad (B-2)$$

Here the strains are

$$\epsilon_1 = \frac{\partial u}{\partial r} \quad (B-3)$$

$$\epsilon_2 = \frac{u}{r} \quad (B-4)$$

where u is displacement. Generally one could have two different relaxation functions (e.g., bulk and shear) but we shall not consider this possibility. The λ and μ are the usual Lamé constants which are now the unrelaxed or high frequency moduli of the medium. The stresses can be written in terms of Q corrected strains, e , as

$$\Sigma_r = (\lambda + 2\mu)e_1 + 2\lambda e_2 \quad (B-5)$$

$$\Sigma_\theta = (2\lambda + 2\mu)e_2 + \lambda e_1 \quad (B-6)$$

Day and Minster have shown how to express the e_i in terms of the ϵ_i

$$e_i = \int m(t - \tau) \epsilon_i d\tau \quad (B-7)$$

using a sequence of m Padé approximants to write the integral relation as a differential relation. For an absorption band attenuation with relaxation times between τ_1 and τ_2 and with a flat spectrum they show that the integral relation can be replaced by

$$e_i(t) = \epsilon_i(t) - \sum_{k=1}^m \zeta_i^k(t) \quad (B-8)$$

where

$$\frac{d}{dt} \zeta_i^k + \nu_k \zeta_i^k = \left(\frac{\tau_1^{-1} - \tau_2^{-1}}{\pi} W_k Q_0^{-1} \right) \epsilon_i(t) \quad k=1, \dots, m \quad (B-9)$$

Here Q_0 is the target Q in the absorption band, the

$$\nu_k = \frac{1}{2} [\ell_k(\tau_1^{-1} - \tau_2^{-1}) + (\tau_1^{-1} + \tau_2^{-1})] \quad (B-10)$$

where the ℓ_k are the abscissas and W_k are the weights for m -point Gauss-Legendre quadrature. The index m is that of the Padé approximant. As m increases, the solution converges to the analytic result which in the frequency domain is

$$i\omega \tilde{m}(\omega) = \tilde{m}(\omega) = 1 - \frac{2}{\pi Q_0} \ell_n \left[\frac{\tau_2}{\tau_1} \left(\frac{1 + \omega^2 \tau_1^2}{1 + \omega^2 \tau_2^2} \right)^{1/2} \right] + \frac{2i}{\pi Q_0} \tan^{-1} \left(\frac{\omega(\tau_2 - \tau_1)}{1 + \omega^2 \tau_1 \tau_2} \right) \quad (B-11)$$

The convergence of the sequence with increasing m is shown in Figure B-1. For calculations given in the main text $m=5$ was used since convergence is near. Use of significantly larger m slows down the calculations of pulse propagation since each m adds an additional differential equation which must be carried. In finite difference form the ζ are advanced for both i by

$$(\zeta^k)_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \left(\frac{2 - \Delta t^n \nu_k}{2 + \Delta t^n \nu_k} \right) (\zeta^k)_{j+\frac{1}{2}}^{n-\frac{1}{2}} + \left(\frac{\Delta t^n}{2 + \Delta t^n \nu_k} \frac{\tau_1^{-1} - \tau_2^{-1}}{\pi Q_0} W_k \right) (\epsilon_{j+\frac{1}{2}}^{n+\frac{1}{2}} + \epsilon_{j+\frac{1}{2}}^{n-\frac{1}{2}}) \quad (B-12)$$

and then the

$$\dot{\epsilon} = \dot{\epsilon} - \sum_{k=1}^m \dot{\zeta}_k \quad (B-13)$$

These are then used to advance the stresses by

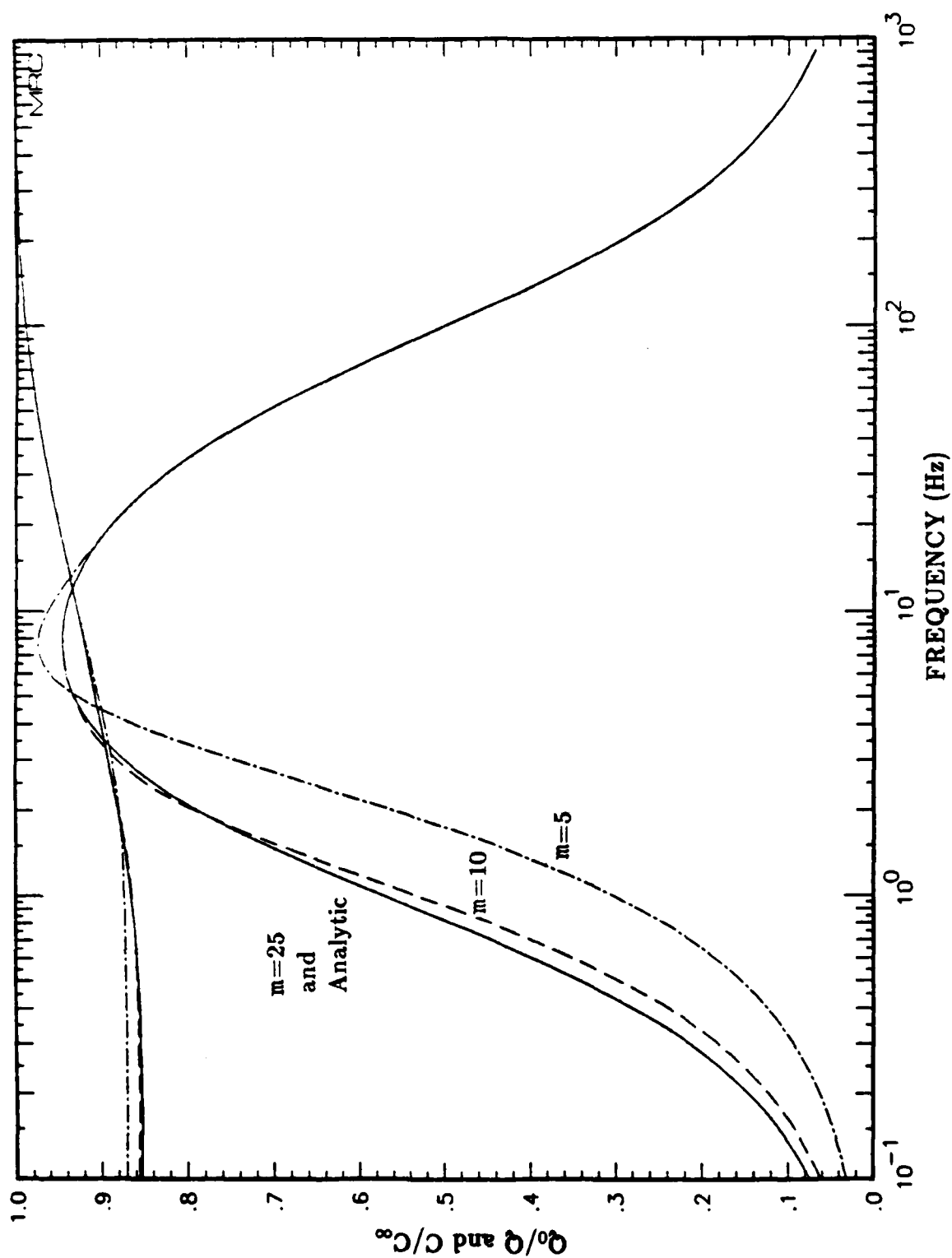


Figure B-1. Absorption band Q and propagation speed C for three values of the Padé index m for $Q_0 = 20$, $\tau_1 = 1.59 \times 10^{-3}$ and $\tau_2 = 1.59 \times 10^{-1}$.

$$\dot{s}_1 = \frac{4}{3}\mu(\dot{e}_1 - \dot{e}_2) \quad (\text{B-14})$$

$$\dot{P} = (\lambda + \frac{2}{3}\mu)(\dot{e}_1 + 2\dot{e}_2) \quad (\text{B-15})$$

so

$$(s_1)_{j+\frac{1}{2}}^{n+1} = (s_1)_{j+\frac{1}{2}}^n + \Delta t^{n+\frac{1}{2}}(\dot{s}_1)_{j+\frac{1}{2}}^n \quad (\text{B-16})$$

$$P_{j+\frac{1}{2}}^{n+1} = P_{j+\frac{1}{2}}^n + \Delta t^{n+\frac{1}{2}}\dot{P}_{j+\frac{1}{2}}^n \quad (\text{B-17})$$

$$(s_2)_{j+\frac{1}{2}}^{n+1} = \frac{1}{2}(s_1)_{j+\frac{1}{2}}^{n+1} \quad (\text{B-18})$$

are used in place of the procedure applied in Appendix A.

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