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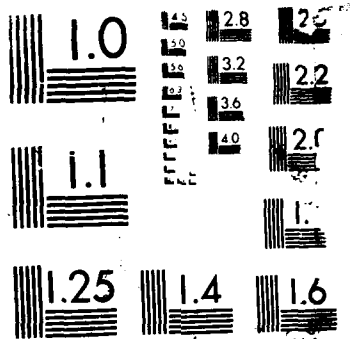
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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

→ Researchers have made substantial progress in the decomposition of large-scale nonlinear problems for accelerated convergence. A Newton method for nonsmooth equations has been developed and conditions for its convergence determined.

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# REPORT ON RESEARCH ACCOMPLISHED

Grant AFOSR-88-0090

15 January 1988-14 November 1988

Stephen M. Robinson, Principal Investigator

## 1. Summary.

This grant provided \$40,300 for a ten-month period of performance. This report outlines the areas in which work was proposed and accomplished, and lists the research papers acknowledging support from the grant. One copy of each such paper was provided to AFOSR at the time the papers were prepared, and an additional copy of each is attached to this report.

The work described here has a great deal of overlap with that taking place under the principal investigator's NSF grant: in general, the AFOSR grant has supplied most of the research assistant (graduate student) time for this work, and the NSF grant has supplied PI salary for summer, the basic computing facilities fee, and some student support. Project travel has been supplied by both grants.

## 2. Research Objectives.

The original proposal suggested work in the general area of large-scale optimization: in particular, methods for decomposition based on the so-called "bundle method," decomposition of large-scale nonlinear problems, including parameter optimization in networks of queues, and accelerating the convergence of bundle-type methods, including developing the theory base in areas such as implementable second-order models of functions to be optimized.

A particular area of work suggested as a major topic of investigation was the development of a computationally implementable and efficient Newton-type algorithm for nonsmooth problems. (good)

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### 3. Research Accomplishments.

In this section we first give a general overview of some problem areas on which much of the work sponsored by this grant was concentrated. Then we describe briefly the progress reported in each of the papers cited in §4.

A typical difficulty in optimizing a large-scale, structured and/or decentralized system occurs when one attempts to break the system into smaller pieces which can be separately optimized. In many problems of practical importance, the results of such separate optimization can indeed be "pieced together" to achieve overall optimality, but at the cost of losing smoothness in the "master" function obtained by combining the pieces. Therefore a recurring problem in the whole class of methods based on this kind of decomposition is the need to solve, at some stage, a nonsmooth optimization problem.

The most promising general line to follow in dealing with such problems seems now to be expressing the optimality conditions for the nonsmooth problem as a nonsmooth equation in  $\mathbf{R}^n$  and solving that equation. However, in solving such equations, two attributes of numerical methods become extremely important: speed of convergence, and the ability to achieve convergence from starting points that may be far from the solution.

The first is important because, in a large-scale system, each computation of a value and/or a derivative (or subgradient) of the function being optimized may actually require the solution of many subsidiary optimization problems. Accordingly, substantial computational effort may be needed for each step, so one wants a method that makes the best possible use of the function and derivative information available.

The second attribute, robustness against bad starting points, is important because in a large problem one may have very little if any information about where the optimal solution is likely to be. Also, for at least some applications reliability is important: that is, the trial-and-error method of "Start it somewhere, and if it doesn't converge, restart it somewhere else," may be unacceptable. Therefore one wants to find methods that have a high probability of convergence, even with little initial information about the location of the eventual solution.

We know from the smooth case that effective methods for solving nonlinear equations (such as Newton's method) rely heavily on knowledge of the local behavior of the function in question. Such knowledge is typically obtained from derivatives, and this derivative information is used not only to predict the function's behavior given a solution estimate, but also to generate an approximate model which is then solved to improve the current estimate of the solution.

Applying this past experience to the nonsmooth case, one could predict that a good knowledge of the local behavior of nonsmooth functions, and of related global behavior such as homeomorphism properties, would help directly in the development of better computational methods. Although no usable information appears to be possible for a general nonsmooth function, it is quite reasonable that good information might be developed for particular classes of nonsmooth equations actually occurring in applications (such as optimization). This was the reason why development of an effective nonsmooth second order (Newton) method was given a high priority in the research proposed for this grant.

To date, we have had substantial success with the class of functions of the form  $h \circ g$ , where  $g$  is a Lipschitzian function from an open subset of  $\mathbf{R}^n$  to some space  $\mathbf{R}^m$  (usually

with  $m > n$ ), and  $h$  is a function from  $\mathbf{R}^m$  to  $\mathbf{R}^n$  that is either fairly smooth (for example,  $C^1$ ) or else exhibits a certain kind of directional differentiability called Bouligand differentiability (B-differentiability), originally identified by the proposer in 1987. This class is of great importance in applications to optimization, since in particular it includes the nonsmooth equations arising from constrained nonlinear optimization, and from variational inequalities expressing problems of equilibrium.

For this class, we gave in [5] an implicit-function theorem that provides conditions under which nonsmooth equations can be solved to obtain trajectories of solutions depending on parameters, and we showed how to obtain directional derivatives of such trajectories by solving simpler (generally piecewise linear) problems. (*Note:* Numbers in brackets refer to the papers cited in §4).

The paper [6] applied the method of analysis developed in [5] to develop a workable Newton method for nonsmooth equations involving functions of the class just described. We were able to prove a Kantorovich-type existence and convergence theorem, showing conditions under which one could be sure of local R-quadratic convergence to a solution.

The analysis in [6] was local in nature, though often one wants to have also global information that could be used, for example, to support a continuation procedure yielding a path from a given starting point to a solution of the nonsmooth equation. In [7] we applied the B-differentiability concept and related results from [5] to establish an inverse-function theorem and a theorem of Hadamard type. The latter guarantees that, under a uniform invertibility condition on the (directional) derivative operator associated with the nonsmooth equation at *every* point, the function appearing in the equation will be a homeomorphism of  $\mathbf{R}^n$  onto itself. This in particular ensures that a path suitable for the continuation method exists. It also furnishes certain local information helpful in computing an approximation to that path. This approach is now being exploited by Mr. K. Park in his Ph.D. dissertation, research for which has been partially supported by this grant. Mr. Park has also obtained additional local convergence properties of the nonsmooth Newton process mentioned above.

The papers [2] and [3] exploit a somewhat different approach to the problem, not assuming B-differentiability but rather exploring a general type of "subdifferential" for nonconvex functions that shares some of the properties of the well known subdifferential for convex functions. It is smaller than the Clarke subdifferential and thus holds the potential of providing sharper necessary optimality conditions. The function itself can also be recovered from this subdifferential through an operation analogous to integration in the classical (smooth) case.

In [4], Daniel Ralph examines differentiability properties of the Euclidean distance function

$$d_C(x) = \inf \{ \|x - c\| \mid c \in C \},$$

where  $C$  is a nonempty subset of  $\mathbf{R}^n$ . His principal result is that  $d_C$  is a *primal function* in the sense of Qi's classification (that is, its Clarke generalized gradient is single-valued almost everywhere) if and only if the Lebesgue measure of the boundary of  $\text{cl } C$  is zero. He applies this result to obtain a considerable strengthening of Clarke's result on first-order optimality conditions for nonsmooth optimization.

Finally, the paper of Leung and Suri [1] studies convergence properties of a simple

algorithm for single-run optimization of a simulation. The authors investigate a simple autoregressive process involving a control parameter, in which the objective is to find a value of that parameter that will drive the expected value of the limit process as close as possible to a specified value. They show that performing a running update on the control parameter in the course of a simulation run (rather than performing many simulation runs, adjusting the control parameter after each run) produces an estimator that converges to the optimal value with probability one. This analysis provides a starting point for understanding the analytic behavior of such single-run optimization methods for more complicated systems.

#### 4. Articles Produced by the Research Program.

The following papers acknowledge support from Grant AFOSR-88-0090. One copy of each paper is attached, and copies have also been furnished previously to AFOSR as the papers were prepared.

- [1] Y.T. Leung and R. Suri, "Convergence of a single run simulation optimization algorithm," forthcoming in: *Proceedings of the 1988 American Control Conference*, held in Atlanta, GA, June 1988.
- [2] L. Qi, "The principal subdifferential," preprint, 1988.
- [3] L. Qi and J.R. Birge, "The Clarke subdifferential and operator integral," preprint, 1988.
- [4] D. Ralph, "Strict differentiability and primality of the Euclidean distance function," preprint, August 1988. This paper is submitted to *Nonlinear Analysis*; no decision has been received.
- [5] S.M. Robinson, "An implicit-function theorem for B-differentiable functions," *Working Paper WP-88-67*, International Institute for Applied Systems Analysis, Laxenburg, Austria, July 1988. This paper has been accepted by *Mathematics of Operations Research*.
- [6] S.M. Robinson, "Newton's method for a class of nonsmooth functions," preprint, August 1988. This paper is submitted to the *SIAM Journal on Numerical Analysis*; the editor has indicated the paper should be published after revision to include computational experiments.
- [7] S.M. Robinson, "Mathematical foundations of nonsmooth continuation methods," preprint, November 1988. This paper is submitted to *Mathematical Programming, Series B*; no decision has been received.

## **5. Participating Professionals and Advanced Degrees.**

a. *Advanced degrees.* Most of the money spent under this grant went toward supporting the research of the following graduate students: K. Park, D. Ralph, J. Ren, S.S. Chou, and Y.T. Leung, as part of their study toward the Ph.D. degree at the University of Wisconsin-Madison. None of these students received the Ph.D. during the reporting period; it is anticipated that the first to complete the Ph.D. will be K. Park (summer 1989).

b. *Participating professionals.* As requested in the original proposal, funds from the grant were used to assist with expenses for the visit to Madison of Dr. Claude Lemaréchal of INRIA, Le Chesnay, France. Dr. Lemaréchal is a world leader in the construction of algorithms for optimizing nonsmooth functions, and is the primary developer of the "bundle method," the most effective of the methods for general nonsmooth optimization implemented and tested to date. During the visit, Dr. Lemaréchal discussed nonsmooth optimization with persons participating in the grant's research program, and this visit provided ideas and research directions that are now being developed under the principal investigator's current AFOSR grant.

## **6. Interactions.**

The research supported by this grant has resulted in several colloquium and conference lectures, for example at the Technical University of Munich, West Germany, the Johns Hopkins University, and the Workshop on Quantitative Sensitivity in Optimization sponsored by the University of Montréal, Canada. Travel expenses for these lectures were covered from other sources. Travel funds from this grant were used to support attendance by the principal investigator (PI) at meetings of the Operations Research Society of America and The Institute of Management Sciences.

In addition, grant funds supported the PI's participation in the Technical Interchange Meeting for the Air Force's Unified Life Cycle Engineering (ULCE) Project, held at Wright-Patterson AFB in May 1988 under the sponsorship of AFHRL/LRL. The PI had previously provided technical comments on some aspects of decentralized optimization involved in the ULCE Project, at the request of the sponsors.

## **7. Inventions, Patent Disclosures, Etc.**

This research program was not planned to, nor did it, result in patentable inventions or discoveries.

## **8. Other Information.**

The general program of research in this grant is being continued under Grant AFOSR-89-0058. An interim technical report will be provided in June 1989 summarizing the progress to date under that grant.

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