AD-A207 629 RAPIDLY CONVERGENT ALGORITHMS FOR NONSMOOTH OPTIMIZATION(U) WASHINGTON STATE UNIV PULLMAN R MIFFLIN 15 DEC 88 AFOSR-TR-89-0592 AFOSR-83-0210 1/1

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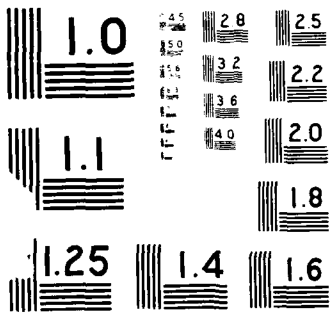
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The research supported by this grant has continued the development of efficient methods for solving optimization problems involving implicitly defined functions that are not everywhere differentiable.

Research on a rapidly convergent algorithm for the constrained single variable case where generalized derivatives are known has been completed. Significant progress has been made in extending this work to the n-variable case via the definition of "better than linear convergence." Safeguarding techniques have been developed which ensure first order convergence on problems with semismooth functions, but do not prevent better than linear convergence on piecewise second order smooth functions. For the constrained case a scale-free automatic penalty technique has been devised.

A new stable method for solving certain quadratic programming problems has been developed which includes a technique for resolving degeneracy.

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22a. NAME OF RESPONSIBLE INDIVIDUAL
Maj. James M. Crowley

22b. TELEPHONE (Include Area Code)
(202) 767-5026

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The research conducted under Grant Number AFOSR-83-0210 during the period 15 July 1983 to 14 June 1988 is partially documented in [1], [6], [10], [11], [12], [13], [14], [15], and [16].

This research is concerned with the computational and theoretical study of solution techniques for significant optimization problems involving functions that are not everywhere differentiable. In addition to being nonsmooth the functions defining such problems are often only implicitly defined. For example, the evaluation of the objective function may involve solving a subproblem that is itself an optimization problem. An example of this type of problem is the problem of minimizing the maximum eigenvalue of a symmetric matrix whose elements depend on decision variables. Other nonsmooth optimization (NSO) problems arise from applying decomposition, relaxation, duality, and/or exact penalty function techniques to large or complicated mathematical programming problems. These problems involve nonsmooth functions such as absolute value functions, maximum value functions and, more generally, sums of maximum value functions.

Having an efficient and reliable method for solving implicitly defined nonsmooth problems gives a user flexibility in modeling a problem and in writing function evaluation subroutines and the ability to exploit parallel processing in computation.

For the case of constrained minimization of a function of a single variable where generalized derivatives are known this research has produced a rapidly convergent algorithm [10] and a corresponding reliable and easy-to-use FORTRAN subroutine, called PQ1 [13, 17]. The problem functions may be nonsmooth and/or nonconvex. The method converges to stationary points of problems for which secant and/or cutting plane methods may fail and it converges rapidly on some problems for which these other methods do not exhibit superlinear convergence. The new safeguarding development that insures convergence via a sequence of bracket intervals with decreasing lengths does not detract from better that linear convergence and fits in well with numerical practice, because it keeps apart points whose difference appears in the denominator of a divided difference formula for second derivative approximation. Our new automatic penalty technique for handling a constraint is also numerically sound, because it is independent of constraint scaling and numerically well-conditioned under a mild constraint qualification. This algorithm maintains two points that define a bracket containing the problem solution. Via quadratic and polyhedral approximation each iteration determines a new point that becomes a new bracket endpoint and, hence, defines a new bracket. A recent result submitted for publication in [16] states that either the new point is superlinearly closer to the solution that both current bracket endpoints or the length of the new bracket is superlinearly shorter than that of the current bracket. This is proved under very weak convergence rate assumptions. This work was presented in October 1987 at the SIAM 35th Anniversary Meeting in Denver.

In order to show the efficacy of this method and the corresponding FORTRAN code PQ1 was used to solve both smooth [13] and nonsmooth [12] versions of a practical single resource allocation problem [5] having five bounded decision variables via a dual (min-max) technique. This application employed PQ1 in a nested manner, i.e. a single variable "outer" problem was solved where each function evaluation involved solving a five variable "inner" Lagrangian problem. The inner problem separated into five independent single variable problems that could have been solved in parallel. The original five variable primal problem is smooth. To make a nonsmooth version the five single variable functions in the objective were approximated with piecewise affine functions. Both versions were solved with about the same amount of computational effort which was about half the effort required in the best run of some popular nonlinear programming codes applied to the original primal problem [5]. These results were reported at the SIAM Conference on Numerical Optimization held at Boulder, CO in June 1984, the NATO Advanced Study Institute on Computational Mathematical Programming held at Bad Windsheim in July 1984 and at the IIASA Workshop on Nondifferentiable Optimization held at Sopron in September 1984.

A Masters degree student at WSU, Han Lim, used PQ1 in a nested manner to obtain maximum likelihood estimates of the three parameters in the Weibull probability density function [18].

In a private communication K.G. Murty has informed the principal investigator that the above algorithm will be described in a new book he is writing. Also, as did a referee of [13], Murty has reported very successful use of PQ1 by colleagues at the University of Michigan.

A modification of this subroutine was written for C. Lemarechal (INRIA, France) for performing line searches required by n-variable optimization methods. It has worked well for him and in the multivariable research described below.

A BASIC language version of the method appears in [6]. It was tested on the problem of finding a root (zero) of a function on an interval by minimizing the absolute value of the function subject to a constraint defining the interval.

Recent joint work with J.-J. Strodiot (Namur, Belgium) has produced theoretically satisfying and practically useful ideas for solving single variable minimization problems using function, but not derivative, values. The work on a safeguarded bracketing technique and on quadratic approximation in [15] will appear in *Mathematical Programming*, Vol. 43, No. 2. Also, this effort has produced, what is probably, the first instance of a function-value-only method for nonsmooth functions with proven rapid convergence. The corresponding paper [14] is under revision in order to append some figures to illustrate various cases mentioned in the introduction and considered in the proofs that are, unfortunately, rather lengthy and complicated.

In [15] a very general safeguarded bracketing technique is introduced which guarantees that the algorithm iterates are sufficiently distinct and that the iterate sequence converges to a stationary point for locally Lipschitz functions. When this technique is combined with certain sequential polynomial and/or polyhedral fitting methods it preserves certain types of rapid convergence that are properties of the fitting methods if they happen to generate distinct iterates. Each bracket has an interior point whose function value does not exceed that of those of the two bracket endpoints. The safeguarding technique consists of replacing the fitting algorithm's iterate candidate by a close point whose distance from the three bracket points exceeds a positive multiple of the square of the bracket length. Also in [15] it is shown that a given safeguarded quadratic fitting algorithm converges in a certain better than linear manner with respect to the bracket endpoints for a strongly convex twice continuously differentiable function.

In [14] an algorithm for nonsmooth problems is introduced which uses function values at five points to generate each iterate. It employs polyhedral approximations as well as quadratic ones and the above safeguard. For certain piecewise twice continuously differentiable functions it is shown that the method has the type of better than linear convergence defined earlier in work on the n-variable problem [11] discussed below.

Preliminary versions of the results in [14] and [15] were presented at the Cambridge Optimization Symposium held in England in March 1985 and at the TIMS/ORSA Joint National Meeting at Los Angeles in April 1986.

The groundwork for developing an efficient method for solving n-variable implicitly defined nonsmooth unconstrained minimization problems is established in [11]. In this paper a new concept of rapid convergence called "better that linear convergence" is defined. Also, the framework of a second order method based on piecewise quadratic approximation for obtaining this type of convergence is given. The method stores and updates several Hessian matrices that are used to generate data for quadratic programming search direction finding subproblems. It also includes a safeguarding technique to insure first order convergence on problems with semismooth [7] functions. The advantage of the form of the safeguard is that it does not prevent better that linear convergence from occurring on problems with an underlying piecewise C^2 structure. A FORTRAN code to test these ideas and guide future research has been written by a former research assistant, David Elwood. Some preliminary results of this research were presented at the May 1987 SIAM Conference on Optimization in Houston.

The above mentioned work for the unconstrained case was extended to the constrained case in the thesis [4] of the principal investigator's Ph.D. student Nilama Gupta. In [4] better than linear convergence is obtained for constrained problems having a strongly convex constraint function and a corresponding constraint set with a strict interior. This is accomplished via an n-variable generalization of the scale-free automatic penalty technique introduced in [9].

Elwood's FORTRAN code mentioned above uses the general purpose quadratic programming code QPSOL [3] to solve the search direction finding subproblems. For efficiency it might be beneficial to employ a code based on a specialized algorithm in the thesis [1] of the principal investigator's former Ph.D student Amal Al-Saket. Her work is an extension of the numerically stable constrained least squares algorithm in [8] to problems with more general quadratic objectives. It also contains a special procedure for resolving degeneracy. Two papers based upon the results in [1] are being prepared for publication submission.

Incidentally, the concept of a semismooth function mentioned above and developed under a previous grant (AFOSR-74-2695) continues to be important. For example, R.W. Chaney uses it in developing second order optimality conditions for nonsmooth optimization (See [2] and references therein).

REFERENCES:

[1] A. Al-Saket, An algorithm with degeneracy resolution for solving certain quadratic programming problems, Dissertation, Washington State University, Pullman, WA, 1985.

[2] R.W. Chaney, Second-order necessary conditions in constrained semismooth optimization, *SIAM Journal on Control and Optimization* 25 (1987) 1072-1081.

[3] P.E. Gill, W. Murray, M.A. Saunders, M.H. Wright, User's guide for SOL/QPSOL: A FORTRAN package for quadratic programming, Technical Report SOL 82-7, Department of Operations Research, Stanford University, Stanford, USA, 1982.

[4] N. Gupta, A higher than first order algorithm for nonsmooth constrained optimization, Dissertation, Washington State University, Pullman. WA, 1985.

[5] K.W. Heiner, M. Kupferschmid and J.G. Ecker, Maximizing Restitution for Erroneous Medical Payments when Auditing Samples from More than One Provider, *INTERFACES* 13 (1983) 12-17.

[6] I. Mifflin and R. Mifflin, A BASIC Program for Solving Univariate Constrained Minimization Problems, Dept. of Pure and Applied Mathematics, WSU, Pullman, WA, 1983.

[7] R. Mifflin, Semismooth and semiconvex functions in constrained optimization, *SIAM Journal on Control and Optimization* 15 (1977) 959-972.

[8] R. Mifflin, A stable method for solving certain constrained least squares problems, *Mathematical Programming* 6 (1979) 141-158.

[9] R. Mifflin, A superlinearly convergent algorithm for one-dimensional constrained minimization problems with convex functions, *Mathematics of Operations Research* 8 (1983) 185-195.

[10] R. Mifflin, Stationarity and superlinear convergence of an algorithm for univariate locally Lipschitz constrained minimization, *Mathematical Programming* 28 (1984) 50-71.

[11] R. Mifflin, Better than linear convergence and safeguarding in nonsmooth minimization, in P. Thoft-Christensen, ed., **System Modeling and Optimization** (Springer-Verlag, New York, 1984).

[12] R. Mifflin, The solution of a nested nonsmooth optimization problem, in V. Demyanov and Pallasche, eds., **Nondifferentiable Optimization: Motivation and Applications**, (Springer-Verlag, New York, 1985.)

[13] R. Mifflin, An implementation of an algorithm for univariate minimization and an application to nested optimization, *Mathematical Programming Study* 31 (1987) 155-165.

[14] R. Mifflin and J.-J. Strodiot, A rapidly convergent five point algorithm for univariate minimization, Report of the Department of Mathematics, Facultes Universitaires de Namur, Namur, 1986.

[15] R. Mifflin and J.-J. Strodiot, A bracketing technique to ensure desirable convergence in univariate minimization, to appear in *Mathematical Programming* 43.

[16] R. Mifflin, On superlinear convergence in univariate nonsmooth minimization. Department of Pure and Applied Mathematics, Washington State University, Pullman, WA, 1986, submitted for publication.

[17] K. Schittkowski, ed., **Computational Mathematical Programming** (Springer Verlag, Berlin, 1985), pp. 433-434.

[18] S.H. Zanakis, Computational experience with some nonlinear optimization algorithms in deriving maximum likelihood estimates for the three-parameter Weibull distribution, *TIMS Studies in the Management Sciences* 7 (1977) 63-77.

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