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Aerodynamics Technical Memorandum 399

**IDENTIFICATION OF AN ADEQUATE MODEL FOR
COLLECTIVE RESPONSE DYNAMICS OF A
SEA KING HELICOPTER IN HOVER**

by

R.A. Feik and R.H. Perrin

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SUMMARY

A mathematical representation of vertical acceleration response characteristics of a helicopter in hover has been developed, including blade flapping, inflow, and rotor speed dynamics. A Maximum Likelihood parameter estimation technique has been applied to assess the adequacy of the model, and to identify the relevant parameters, using flight data from a Sea King Mk 50 helicopter. A number of conclusions related to the validity of the modelling approach have resulted from comparisons between predicted and identified parameters, and further investigation of some aspects is indicated.



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NOTATION

A	System matrix
B	System matrix
C_T	Thrust coefficient
e	Flapping hinge offset distance
g	Acceleration due to gravity
I_B	Rotor blade moment of inertia about hinge
I_Z	Total moment of inertia of main rotor about the shaft
$J(\xi)$	Cost functional
K_1	Pitch - flap coupling factor
m	Helicopter mass
m_b	Mass of a main rotor blade
M_A	Total aerodynamic moment about hinge
M_B	Rotor blade moment of mass about hinge
M_{11}	Apparent additional air mass factor
n	Measurement noise vector
N	Number of blades of the main rotor
P	System matrix
Q_A	Total aerodynamic torque
Q_E	Shaft torque of engines
R	Covariance of residuals matrix
R	Radius of main rotor blade
t_i	i^{th} discrete time point
T_A	Total aerodynamic thrust
u	Control input vector
w	Vertical velocity
x	State vector
z	Measurement vector
z_{ξ}	Output vector estimate
Z	Vertical acceleration

NOTATION (cont)

β	Blade flapping angle
γ	Blade Lock number
ΔC_T	Perturbation in thrust coefficient
$\nabla \xi$	Sensitivity matrix
$\Delta \bar{u}$	Perturbation in non dimensional inflow velocity
ϵ	Ratio of flapping hinge offset to rotor radius
θ_C	Rotor blade pitch angle
u	Inflow velocity
$\bar{u}$	Non dimensional inflow velocity
$\bar{u}_{Ref}$	Reference non dimensional inflow velocity
ξ	Unknown system parameter vector
Ω	Rotor angular velocity

1. INTRODUCTION

Because of the complex nature of rotorcraft aerodynamics and flight dynamics, any global model which adequately represents dynamic flight characteristics will contain a large number of parameters based on numerous simplifying assumptions. Validation of such models against flight measurements is a difficult and time consuming exercise. One such attempt [1] with the Sea King Mk 50 helicopter, demonstrated that while good overall representation can be achieved, significant deficiencies remain in specific areas. A better understanding of these deficiencies can be gained by isolating the particular aspect of concern and focussing on the development of an adequate model structure which can be more readily verified. The simplified, special purpose models which result, are also especially useful in studies related to flying qualities, stability augmentation system design, and validation of in-flight and ground simulators [2].

Parameter estimation techniques developed over the last two decades have been used widely by the fixed wing community, and more recently are being applied in studies of rotorcraft flight dynamics [3-6]. For the latter, as well as the problems of a noisier measurement environment, particular difficulties are presented by the lack of a well defined model structure, and the presence of a large number of unknown parameters. Nevertheless, parameter estimation techniques provide a useful tool for investigating which features are more important to retain for an adequate model, and to validate theoretical estimates of the corresponding parameters. The probability of successful application of these techniques can be enhanced by making full use of any a priori information available.

Vertical axis response characteristics are important in defining flying qualities, especially in flight close to the ground involving bob-up manoeuvres [7]. A simple modelling approach as used in Reference [1] is not adequate for predicting transient response of collective inputs, including the initial peak response and subsequent periodic behavior. In particular, it has been noted [7] that blade flapping dynamics can have a significant influence on helicopter flight dynamics and handling qualities. Similarly, Reference [8] has shown that dynamic inflow can play a key role in the initial overshoot of the vertical acceleration to an abrupt collective input. Finally, the importance of engine-rotor dynamics, and hence rotor speed changes, on vertical axis handling qualities has been emphasized in Reference [9]. This suggests the desirability of including rotor speed dynamics in any model of vertical axis response to collective inputs.

This paper develops a model for vertical response in hover incorporating each of the above features, and by use of time domain parameter estimation techniques, in conjunction with available flight data for the Sea King Mk 50 helicopter, establishes the need to retain them in an adequate model. Secondly, parameter values extracted from measured responses to collective step and pulse inputs are compared with theoretical estimates. Practical difficulties due to limited data are discussed, and some suggestions made for further model refinement and validation.

2. MATHEMATICAL REPRESENTATION

Equations for vertical motion, blade flapping, inflow and rotor speed dynamics were derived using the following assumptions:

- (i) Rigid rotor blade with linear twist and uniform section
- (ii) Pure vertical motion, i.e. all angular rates, fore/aft and sideways velocities are zero
- (iii) Uniform inflow
- (iv) Cyclic flapping coefficients are zero
- (v) Incompressible, attached flow over blades with tip loss factor of 1
- (vi) Changes in fuselage download neglected
- (vii) Flapping and inflow angles small
- (viii) Only first order terms in hinge offset retained

2.1 Vertical Motion

The equation for vertical velocity, w (positive down), can be written as follows:

$$m\dot{w} - mg - \frac{Nm_b R}{2}(1-\epsilon)\dot{\beta} = -T_A \quad (1)$$

where m is the helicopter mass, m_b is the blade mass and R the rotor radius, N is the total number of main rotor blades, ϵ is the ratio of the hinge offset to rotor radius, e/R , β is the flapping angle (positive up), and T_A is the total aerodynamic thrust.

2.2 Blade Flapping

The equation for blade flapping dynamics is:

$$I_B \ddot{\beta} + \Omega^2(I_B + eM_B)\beta - M_B \dot{w} = M_A \quad (2)$$

with I_B and M_B the blade moment of inertia and moment of mass, respectively, about the flapping hinge, Ω is the rotor angular velocity and M_A is the aerodynamic moment about the flapping hinge.

2.3 Inflow

The inflow dynamics equation follows the derivation of Chen and Hindson [8]

$$\frac{M_{11}}{\Omega} \frac{d\bar{u}}{dt} + 2 \left(\bar{u} - \frac{w}{\Omega R} + \frac{2}{3} \frac{\dot{\beta}}{\Omega} \right) \bar{u} = C_T \quad (3)$$

where $\bar{u}$ is non dimensional inflow velocity, C_T is the thrust coefficient and M_{11} is related to the apparent additional air mass introduced by Carpenter and Fridovich [10].

Values of M_{11} vary from 0.849 in Reference [10] to a 'corrected' value of $128/75\pi$ (0.543) in the dynamic inflow theory of Pitt and Peters [11]. The latter also imply a simpler, linear alternative to Equation (3)

$$\frac{M_{11}}{\Omega} \frac{d\Delta\bar{u}}{dt} + 2 \bar{u}_{Ref} \Delta\bar{u} = \Delta C_T \quad (3a)$$

where $\Delta\bar{u}$ and ΔC_T are perturbations in inflow and thrust coefficient.

2.4 Rotor Speed

The equation for the rotor speed variations can be written

$$I_Z \dot{\Omega} - 2N(I_B + eM_B)\beta\dot{\beta}\Omega = Q_E + Q_A \quad (4)$$

where Q_E is the shaft torque provided by the engines, Q_A is the total aerodynamic torque about the shaft, I_Z is the total rotor moment of inertia about the shaft.

The force and moment terms on the right hand side of the above equations were derived using a simple strip theory detailed in Reference [12]. In order to facilitate comparisons between model and flight data via parameter estimation, the equations (1) to (4) are linearized about a reference state of steady hover with zero vertical velocity. The linearized small perturbation model has the form

$$\begin{bmatrix} 1 & Z_{\beta} & 0 & 0 & 0 \\ \beta_w & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & v_{\Omega} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{w} \\ \ddot{\beta} \\ \dot{\beta} \\ \dot{v} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} Z_w & Z_{\beta} & Z_{\beta} & Z_v & Z_{\Omega} \\ \beta_w & \beta_{\beta} & \beta_{\beta} & \beta_v & \beta_{\Omega} \\ 0 & 1 & 0 & 0 & 0 \\ v_w & v_{\beta} & v_{\beta} & v_v & v_{\Omega} \\ \Omega_w & \Omega_{\beta} & \Omega_{\beta} & \Omega_v & \Omega_{\Omega} \end{bmatrix} \begin{bmatrix} w \\ \beta \\ \beta \\ v \\ \Omega \end{bmatrix} + \begin{bmatrix} Z_{\theta} & 0 \\ \beta_{\theta} & 0 \\ 0 & 0 \\ v_{\theta} & 0 \\ \Omega_{\theta} & \Omega_Q \end{bmatrix} \begin{bmatrix} \theta_C \\ Q_E \end{bmatrix} \quad (5)$$

The input vector in (5) includes changes in collective pitch, θ_C , and engine torque. A priori values for each of the matrix elements can be obtained from the expressions summarized in Appendix A. Relevant physical data used for the estimating a priori values for the Sea King Mk 50 helicopter is given in Table 1

Table 1 - Sea King Physical Characteristics

Rotor radius (ft), R	31
Blade chord (ft), c	1.52
Hinge offset (ft), e	1.05
Blade twist (deg), θ_t	-8
Pitch - flap coupling, K_1	0.08
Blade mass (lb), m_b	181
Angular velocity (rad/s), Ω	21.89
Lock number, γ	11.51
Ratio of rotor/blade inertia, $\frac{I_Z}{I_B}$	6

3. FLIGHT DATA

A flight test program using the Sea King Mk 50 helicopter has been described in detail in Reference [13], and has provided an extensive data base for both performance and flight dynamics characteristics. Data was recorded in 12 bit form at a sampling rate of 60Hz and subject to a range of post-processing procedures to reduce random noise levels, correct various error sources such as 'drop-outs' and time delays, and to ensure kinematic consistency. The data of particular relevance here are the vertical acceleration responses to collective step and pulse inputs in hover. The cases selected for the current investigation are summarized in Table 2, including information on input type and all up weight (AUW), which affects directly or indirectly several of the parameters listed in Appendix A. Unfortunately other input shapes, such as doublets, 3211, or sweeps were not available. Nevertheless, the table shows that there were nine cases, each between five and eight seconds duration, included in the analysis. The first six cases being at a higher AUW of around 18500 lb and the remainder at a lower AUW of about 16500 lb. In all cases the altitude was close to sea level. For each case, measurement records were available for collective input and vertical acceleration response, as well as engine torque and rotor speed, but no information was available for the inflow or blade flapping angle variations.

Table 2 - Flight Cases

Case	Collective Input Type	AUW (lb)
1	Step up	18925
2	Step up	18835
3	Step down	18719
4	Step up	18487
5	Step up	18449
6	Step down	18335
7	Step down	16535
8	Step up	16590
9	Pulse	16500

4. PARAMETER ESTIMATION METHOD

The linear system given by Equation (5) has the general form

$$P \dot{x} = A x + B u \quad (6)$$

where the vectors x and u are the state and input respectively, while matrices P , A , and B contain the system parameters. The output at time t_i is modeled as

$$z(t_i) = C x(t_i) + D u(t_i) + n(t_i) \quad (7)$$

where $n(t_i)$ is the measurement noise vector and C and D are also system matrices.

In the present formulation, the state equation (6) is assumed free of process noise while the measurement noise is assumed to be zero mean white Gaussian noise. The maximum likelihood (ML) of the unknown system parameters, ξ , is that value of ξ that maximizes a likelihood function, defined as the probability of obtaining the outputs, z , given the parameters, ξ . Details of the method can be found in standard references [14,15], and amounts to the minimization of a cost functional

$$J(\xi) = \frac{1}{2} \sum_{i=1}^N [z(t_i) - z_{\xi}(t_i)]^T R^{-1} [z(t_i) - z_{\xi}(t_i)] \quad (8)$$

with an estimate of R being given by

$$R = \frac{1}{N} \sum_{i=1}^N [z(t_i) - z_{\xi}(t_i)] [z(t_i) - z_{\xi}(t_i)]^T \quad (9)$$

where $z_{\xi}(t_i)$ is the estimate of the output at time t_i . A modified Newton-Raphson algorithm is used to achieve the minimization iteratively. A measure of the accuracy of the estimates is given by the Cramer-Rao bound [14]:

$$\text{Covariance}(\xi) = \left[\sum_{i=1}^N [\nabla_{\xi}(z_{\xi}(t_i))]^T R^{-1} \nabla_{\xi}(z_{\xi}(t_i)) \right]^{-1} \quad (10)$$

where the gradient, ∇ , term is the sensitivity matrix obtained from the system equations. The Cramer-Rao bound is a lower bound, and Reference [15] points out that it is significantly smaller than the scatter of the results. Nevertheless, it is a reasonable indicator of relative accuracy of various parameters from different cases. Correlations between parameters can also be derived from the covariance matrix (10)

A particular advantage of the output error method is that not all the outputs need to be measured for parameter estimates to be obtained. For the present, only the vertical acceleration and rotor speed are available for matching, while the total number of parameters indicated in Equation (5) is 28. With the limited information available and the non-optimal input shapes, it is not possible to obtain reliable estimates of all 28 parameters, many of which are highly correlated, even if convergence of the algorithm were achieved. In order to reduce the total number of unknown parameters, the a priori information has been used in two ways. Firstly, an order of magnitude analysis was undertaken to establish which parameters were likely to be the most significant. As a result, several of the less important parameters were fixed at their a priori values. Secondly, many of the remaining parameters were closely related, differing from one another only by a constant multiple. By maintaining these constraints during the parameter estimation process, the total number of independent variables can be reduced considerably. In Appendix A, with fixed and constrained parameters indicated in the right hand column, the remaining number of independent parameters has been reduced to 14, of which five relate to the torque equation, and three relate to each of the other equations. A priori values were used as a guide to initial estimates for the parameters during the identification process. Even so, convergence was marginal when all 14 parameters were simultaneously identified, and it was necessary to vary the starting values of several of the parameters, particularly in the rotor speed equation, in order to achieve convergence.

Because of the convergence difficulties, an alternative, more robust identification strategy was developed by decoupling the rotor speed equation from the rest of the system equations, as follows:

1. Set to zero all parameters in the rotor speed equation except for Ω_Ω , Ω_θ , and Ω_Q . Match the rotor speed and vertical acceleration records for initial estimates of Ω , z , β , and v parameters. Initial fits are quite good.
2. With the z , β , and v parameters fixed at the values obtained at step 1, obtain an improved Ω match with additional parameters Ω_β , Ω_w , and all constrained parameters, included.
3. Use the Ω record from step 2 as an input to the system, to obtain an improved vertical acceleration match and revised z , β , and v parameters.

Steps 2 and 3 above could be iterated further if necessary but in practice this does not produce any significant improvement, either in terms of fit errors or parameter accuracy as measured by the Cramer - Rao bound. The three step procedure was successful in all cases tried, and the results were in good agreement with those obtained with all 14 parameters simultaneously identified, when convergence was achieved for the latter.

5. RESULTS

5.1 Time Histories

The effect of including flapping and/or inflow dynamics in the vertical response model is illustrated in Figure 1, which shows the best match for vertical acceleration achievable by each model against the flight data of Case 1. In Figure 1(a) the model includes only inflow dynamics while Figure 1(b) has flapping but not inflow dynamics represented. Figure 1(c) has both flapping and inflow dynamics, but in none of the cases has any account been taken of rotor speed variations.

While Figure 1(a) illustrates that inflow dynamics cannot reproduce the initial acceleration peak, it is clear from Figure 1(b) that flapping dynamics on its own is also inadequate. However, with both effects included an excellent match with the initial acceleration response to a step input can be achieved, as seen in Figure 1(c). Nevertheless, the longer term periodic variations cannot be reproduced without taking into account rotor speed dynamics. When the complete model, as described in Section 2, is used, an excellent match of both vertical acceleration and rotor speed records can be achieved. This is illustrated in Figures 2 to 4 for three different cases. Figure 2 (Case 4) is the response to a collective step up (note that positive acceleration is downwards), Figure 3 (Case 7) is for a collective step down input, while Figure 4 (Case 9) illustrates the response to a collective 'pulse' type of input. For the step response cases it is clearly possible to distinguish the initial, high frequency, response from the longer term dynamics, but the time history for the pulse input is a combination of both high and low frequency dynamics. While band pass filtering could be used to separate these, this has not been found necessary in this investigation, since the uncoupling procedure outlined in the previous section largely achieves the same end. The clear separation between short and long term effects is probably the reason why that procedure has proved successful here.

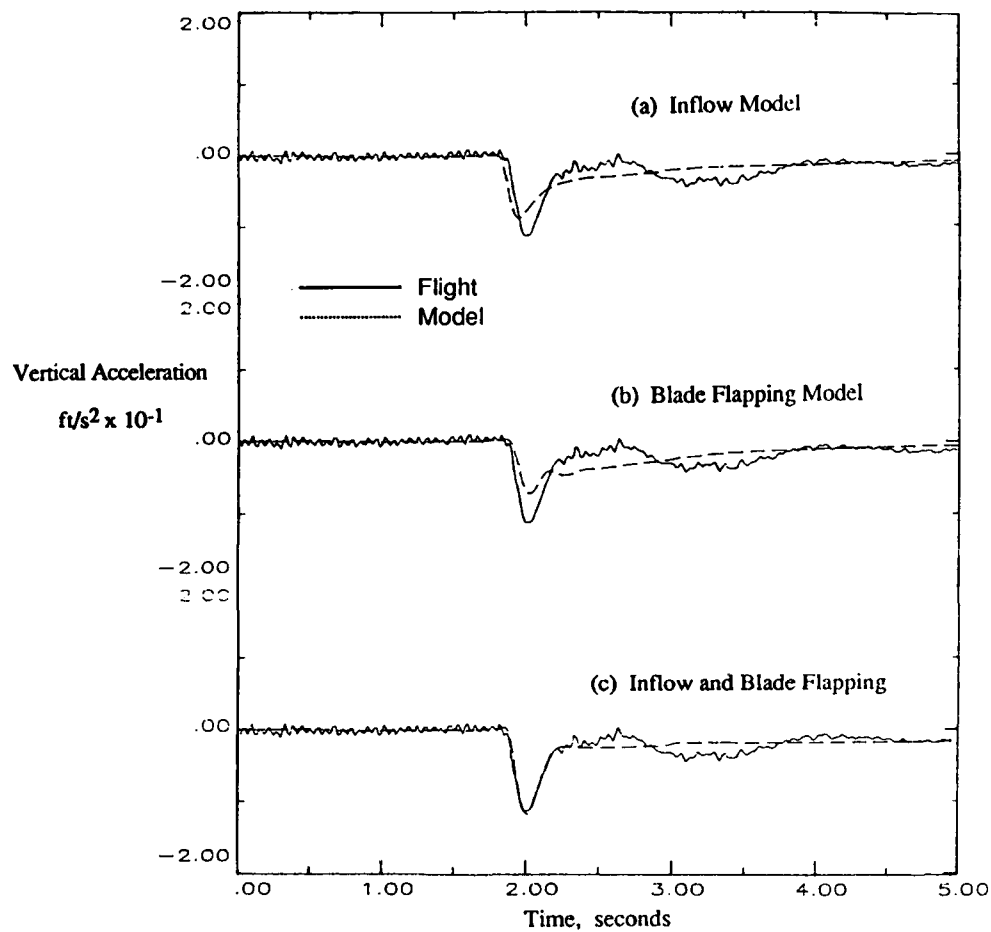


Figure 1 Effects of Inflow and Blade Flapping on Vertical Acceleration

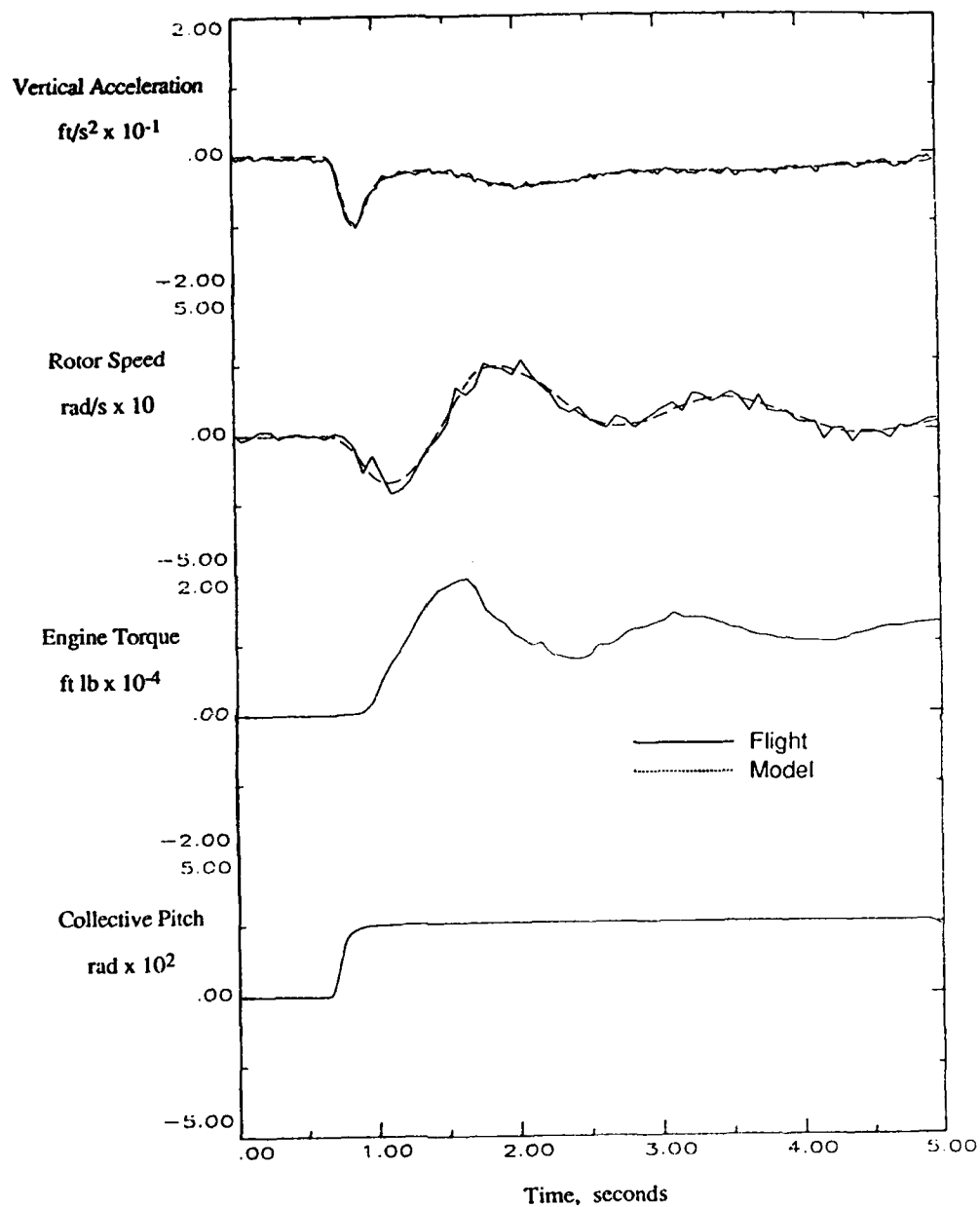


Figure 2 Response to Collective Step Up (Case 4)

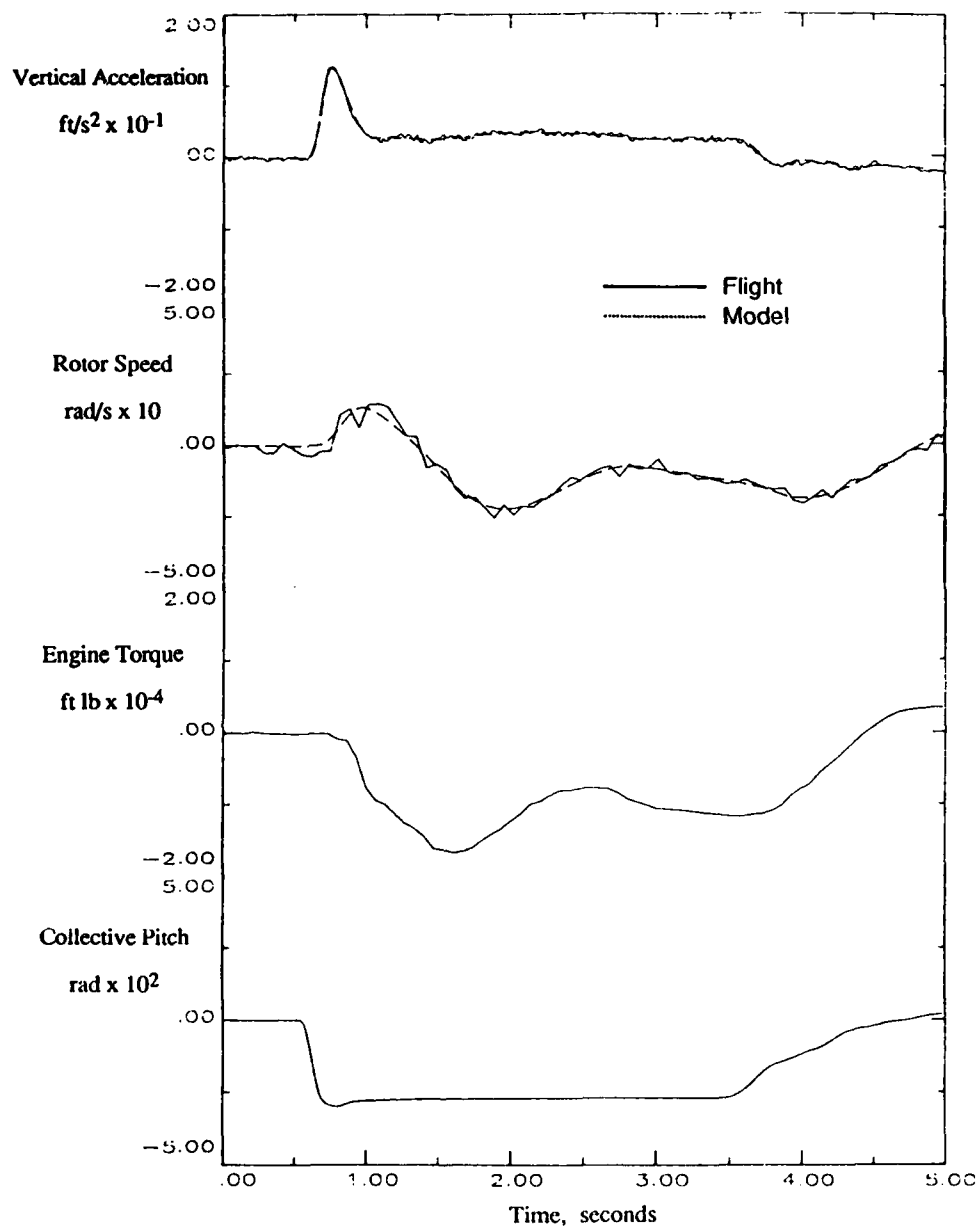


Figure 3 Response to Collective Step Down (Case 7)

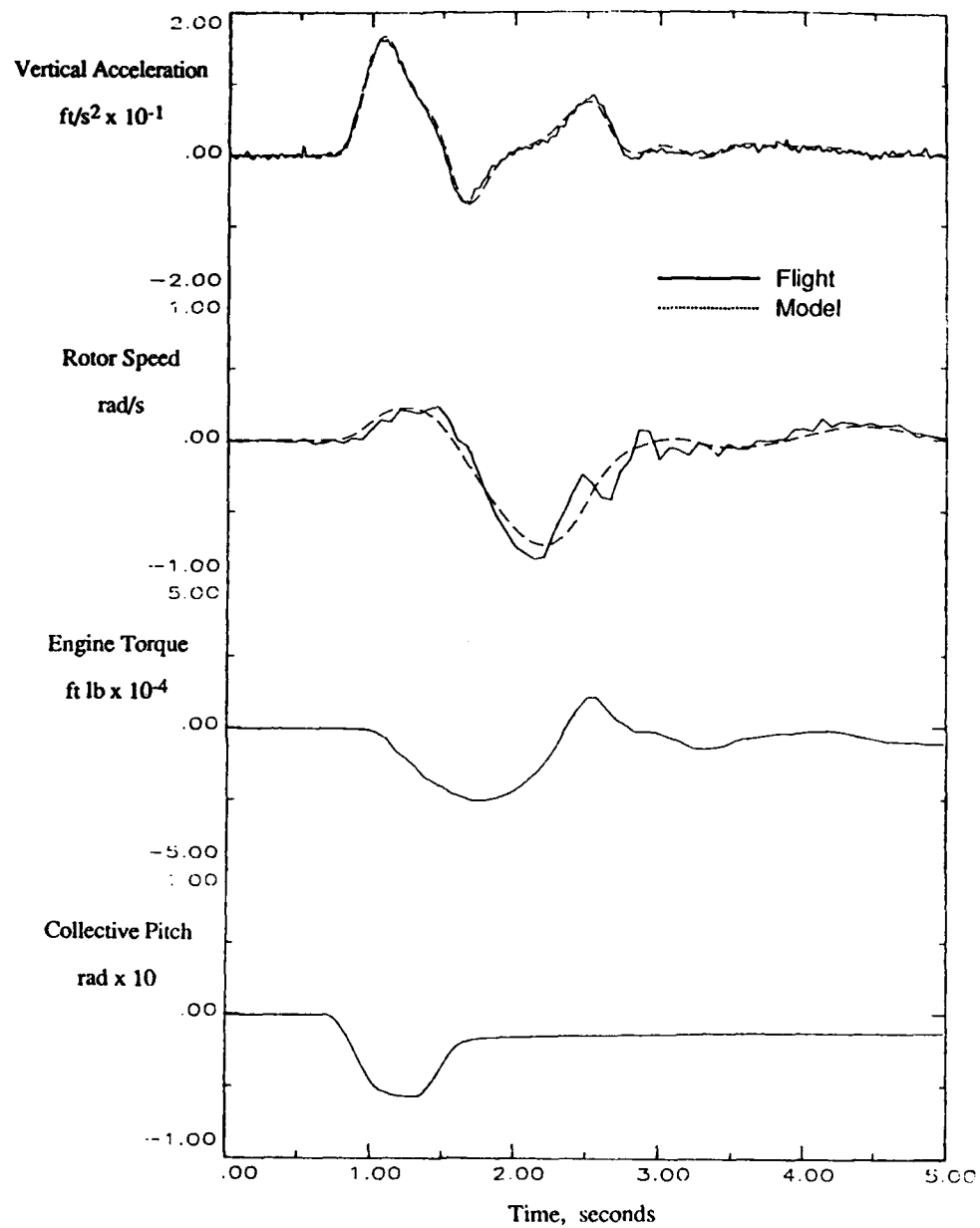


Figure 4 Response to Collective Pulse (Case 9)

5.2 Identified Parameters (except rotor speed)

The identified values of all parameters not associated with rotor speed are shown for all cases analyzed in Figures 5 to 10, while the resulting mean values and standard deviations are summarized in Table 3. In the figures the identified values as well as Cramer - Rao bounds are shown, both normalized by the respective a priori values. The figures illustrate that although there is significant scatter in some cases, the results are mostly clustered about well identified means, i.e. the standard deviations are small enough to provide meaningful confidence intervals. The a priori estimates are also shown in the table, with a distinction made between high and low AUW estimates where appropriate. Note that the predicted values are derived using a blade lift curve slope of 6.2 and an 'uncorrected' M_{11} value of 0.85

A comparison of predicted estimates with values identified using the current model structure reveal a number of interesting features. The vertical damping derivative, $-Z_w$, is significantly higher than the predicted value. This may be attributed to the simple rotor aerodynamics representation and/or may reflect the neglect of any fuselage aerodynamic effects. For the Z_θ derivative the identified and predicted values are within one standard deviation of one another for high AUW, but the low AUW mean is approximately 1.3 standard deviations less, in absolute value, than the prediction. Nevertheless, this represents a discrepancy of only 11%. Note also that there is some scope for varying the predicted parameter values since some of the physical data, e.g. blade inertia, are subject to some degree of uncertainty. For both Z_w and Z_q the trends with AUW appear to be consistent with that expected, but statistically meaningful conclusions are not possible due to the relatively large standard deviations.

The identified values of the flapping derivatives, β_β and β_θ , are generally within one standard deviation of their a priori estimates. The large scatter for the low AUW cases is due to high values for the two derivatives obtained from Case 7, as well as to the small number of samples.

The mean value of the inflow derivative, v_θ is in good agreement with the predicted estimate, assuming M_{11} equal to 0.85. From Appendix A, v_θ is seen to be inversely proportional to M_{11} , so that a 'corrected' value for M_{11} of 0.54 would imply a v_θ of over 2200, well above the identified value, even accounting for the scatter. Further, the identified value of v_w is most closely approximated by the formula derived from Equation 3 with M_{11} equal to 0.85. Thus, within the constraints of the present model, the dynamic inflow model of Carpenter and Fridovich [10] appears to provide estimates of v_θ and v_w consistent with the flight data.

With regard to the scatter of the identified parameters, it is of interest to compare the computed standard deviations with the Cramer - Rao lower bounds. While the latter vary somewhat from case to case, the average Cramer - Rao bounds for the present results are typically 3 to 5 times less than the standard deviation which characterizes the scatter. This is roughly in line with the experience of the fixed wing community where it is common practice to multiply the Cramer - Rao bounds by a 'fudge factor' of 5 to 10 (Ref. 16). The present results confirm the usefulness of the Cramer - Rao bound as an estimate of the accuracy of identified helicopter parameters, provided a suitable multiplying factor is used. Alternatively, it provides some indication of relative accuracy of a given parameter from case to case. For example, the relatively large Cramer - Rao bounds for v_θ in Cases 1, and 3 suggest a lower confidence in these points, whereas, for Z_w , the relatively low Cramer - Rao bound for Case 9 adds credibility to the somewhat lower value obtained in that run, the only example of a pulse input.

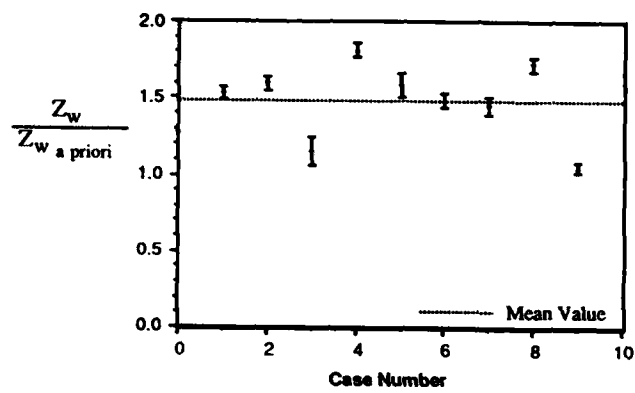


Figure 5 Identified Results for Z_w

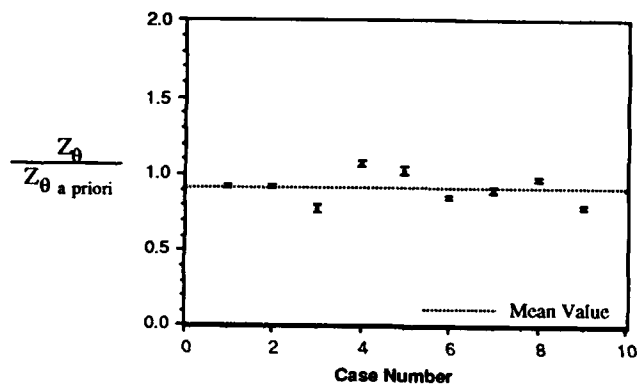


Figure 6 Identified Results for Z_θ

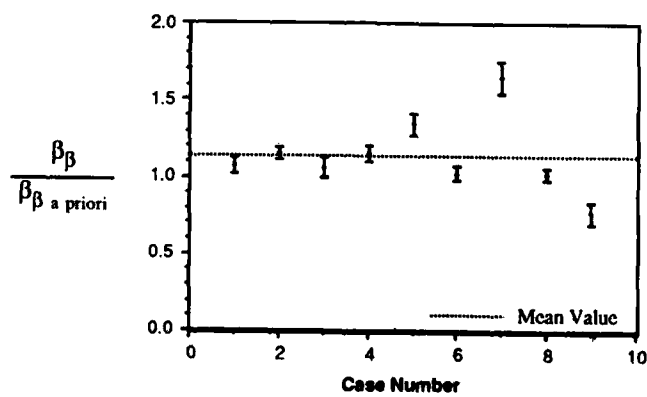


Figure 7 Identified Results for β_β

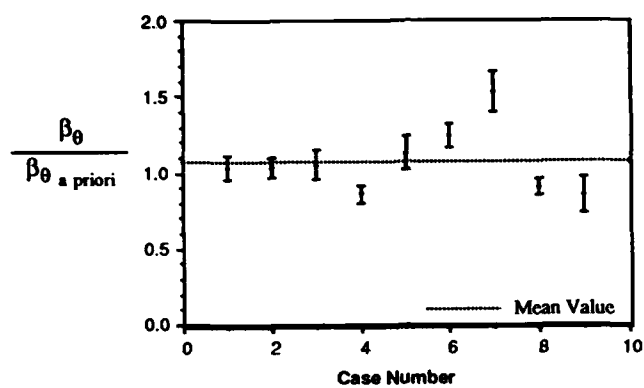


Figure 8 Identified Results for β_0

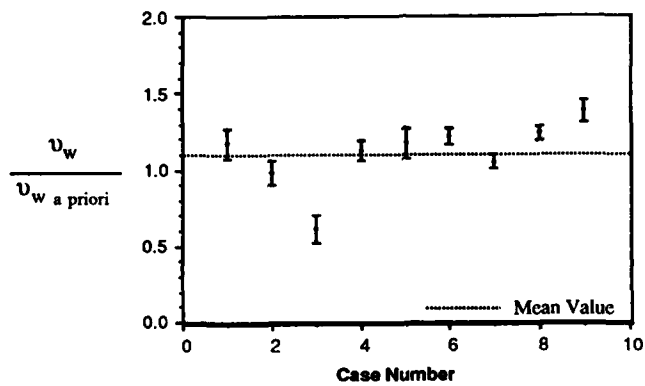


Figure 9 Identified Results for v_w

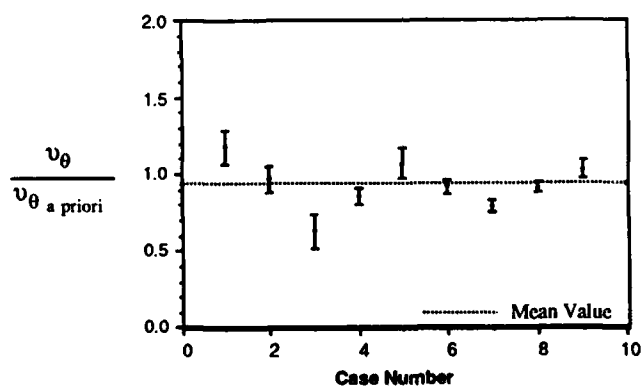


Figure 10 Identified Results for v_θ

Table 3 Means and Standard Deviations of Identified Parameters

Parameter	A Priori (High AUW)	A Priori (Low AUW)	Identified (High AUW)	Identified (Low AUW)	Identified (All Cases)
Z_w	-1.03	-1.16	-1.57±.22	-1.63±.39	-1.59±.26
Z_θ	-465	-521	-431±48	-462±45	-441±47
β_β	-557	-557	-631±64	-639±251	-633±135
β_θ	658	658	695±83	719±248	703±141
v_w	5.8	5.7	6.1±1.3	7.0±.92	6.4±1.2
v_θ	1409	1409	1323±268	1284±166	1310±229
Z_Ω	-4.6	-4.7	14.6±5	18.0±6	15.7±5
β_Ω	2.1	2.0	22.3±23	38.4±22	27.7±23
v_Ω	14	13	-108±42	-123±76	-113±51
Ω_Ω	-0.49	-0.43	-2.2±0.4	-1.5±0.5	-1.9±0.5
Ω_θ	-41	-38	-32±5	-25±.2	-29±5
Ω_Q	.000093	.000093	.000097 ±.000013	.000085 ±.000012	.000093 ±.000013

5.3 Rotor Speed Parameters

Table 3 lists values of several parameters associated with the rotor speed. While the current model has successfully accounted for the effects of rotor speed variations on vertical acceleration response, and has achieved a good fit to the rotor speed record, it is clear from the identified parameters that the model is not physically correct. Apart from Ω_Q (the inverse of the rotor inertia), and possibly Ω_θ , the other rotor equation parameters differ considerably from their predicted values. Similarly, the identified values of Z_w , β_Ω , and v_Ω , while subject to considerable scatter are generally much greater than the predicted values, i.e. the predicted values underestimate the coupling between rotor speed and vertical response. Examination of the rotor speed record reveals a period of around 1.5 to 2 seconds. The estimated undamped natural frequency of blade lagging motion is about 0.9 Hz, which, for a damping ratio of 0.7, translates to a damped natural frequency of 0.64 Hz, or a period of 1.6 seconds. The suggested implication is that the blade lagging motion may be an important factor in coupling the rotor speed variations to vertical response. This needs further investigation, but it is clear that the extra parameters introduced by modelling the blade lagging dynamics imposes an additional burden on the identification algorithm.

While the coupling between rotor speed and vertical response needs further investigation, it is not expected that the parameters not associated with the rotor speed will be much affected, due to the separation in frequency of the initial response, which largely determines the values of those parameters, and the subsequent periodic response.

Possible exceptions are Z_w and v_w which require time for vertical velocity to develop. This has been confirmed by matching a simplified model, without any rotor speed effects, with the vertical acceleration response. The match shown in Figure 1(c) is typical of the results. Significantly, the pulse response record, Case 9, fails to converge with the simplified model. This is quite likely due to the difficulty in separating initial and medium term responses with this input. Neglecting this case, the mean parameter values for the remaining cases are similar to those given in Table 3, mostly lying within one standard deviation of them. The main exception is $-Z_w$ which at 1.93 is somewhat greater, probably reflecting the poorer overall match of the vertical acceleration, and hence vertical velocity. Significantly u_q at 1425, is very close to the previous value although v_w , at 7.6, is slightly higher, most likely for the same reason as for Z_w . As expected, the Cramer - Rao bounds for the simpler model are higher, by about 50%, than those for the model including rotor speed effects.

6. CONCLUSIONS

This paper has illustrated the use of time domain Maximum Likelihood parameter estimation techniques in developing a model of rotorcraft vertical acceleration response in hover, capable of reproducing all the features observed in the flight data of a Sea King Mk 50 helicopter. A linearized vertical response model has been derived from first principles to include both blade flapping and inflow dynamics, as well as rotor speed dynamics. It has been demonstrated that an excellent match of the flight data can be obtained with a model including all these features. A total of nine flight cases has been analyzed to identify model parameters for comparison with the a priori predictions. By making use of information from an order of magnitude analysis and by imposing constraints suggested by the a priori model, the total number of unknown parameters has been reduced to manageable proportions. In addition, a procedure to uncouple the rotor speed dynamics from the remaining system was developed to take advantage of the difference in frequency content.

The scatter in the results and the problems with convergence of the algorithm were due in some part to the large number of system parameters, the limited number of flight response characteristics recorded, noise in the recorded data, and the non-optimal input shapes used. Improvements in many of these areas would be desirable in future trials, especially in the measurement of additional outputs, such as blade flapping, and possibly blade lagging angle. This would make possible the relaxing of some of the constraints found necessary in the current investigation.

Despite the experimental scatter, a number of meaningful conclusions can be drawn from a comparison of the identified parameters with predictions. In particular the inflow parameters are consistent with the Carpenter - Fridovich dynamic inflow model with an apparent additional air mass parameter, M_{11} , of 0.85. The vertical damping parameter is consistently higher than predicted possibly due to simplifications in the aerodynamic model used. Other identified parameters, apart from those associated with the rotor speed, are in reasonable agreement with the predictions. The Cramer - Rao bound appears to be a good indicator of relative accuracy, and greater weighting could be given to those identified parameters with lower Cramer - Rao bounds, all else being equal. Poor agreement for the rotor speed parameters indicates a shortcoming in the underlying physical model. The results suggest that blade lagging dynamics may need to be included to explain the interaction between the rotor speed and vertical acceleration response. Further investigation is required in this area.

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Appendix A - A Priori Expressions for Model Parameters

Parameter	A Priori Information	Constraint
Z_w	$-\frac{NI_B}{4mR^2} \gamma \Omega$	Fixed $-\frac{1}{\Omega} \left(1 - \frac{3\epsilon}{2}\right) \times Z_\theta$ $-K_1 \times Z_\theta$ $-Z_w$
Z_β	$-\frac{Nm_B}{2m} (1-2\epsilon)$	
Z_β	$\frac{NI_B}{6mR} \gamma \Omega \left(1 - \frac{3\epsilon}{2}\right)$	
Z_β	$\frac{NI_B}{6mR} \gamma \Omega^2 K_1$	
Z_v	$\frac{NI_B}{4mR^2} \gamma \Omega$	
Z_Ω	$-\frac{NI_B}{4mR} \gamma \Omega \bar{v}_{Ref} - \frac{2g}{\Omega}$	
Z_θ	$-\frac{NI_B}{6mR} \gamma \Omega^2$	
β_w	$-\frac{M_B}{I_B}$	Fixed $\frac{4}{3R\Omega} \frac{\left(1 - \frac{3\epsilon}{2}\right)}{\left(1 - \frac{4\epsilon}{3}\right)} \times \beta_\theta$ $-\frac{1}{\Omega} \frac{\left(1 - \frac{8\epsilon}{3}\right)}{\left(1 - \frac{4\epsilon}{3}\right)} \times \beta_\theta$ $-\frac{4}{3R\Omega} \frac{\left(1 - \frac{3\epsilon}{2}\right)}{\left(1 - \frac{4\epsilon}{3}\right)} \times \beta_\theta$
β_w	$\frac{\gamma \Omega}{6R} \left(1 - \frac{3\epsilon}{2}\right)$	
β_β	$-\frac{\gamma \Omega}{8} \left(1 - \frac{8\epsilon}{3}\right)$	
β_β	$-\Omega^2 \left[\left(1 + \frac{eM_B}{I_B}\right) + \frac{\gamma K_1}{8} \left(1 - \frac{4\epsilon}{3}\right) \right]$	
β_v	$-\frac{\gamma \Omega}{6R} \left(1 - \frac{3\epsilon}{2}\right)$	
β_Ω	$\frac{\gamma \Omega}{6} \left(1 - \frac{3\epsilon}{2}\right) \bar{v}_{Ref}$	
β_θ	$\frac{\gamma \Omega^2}{8} \left(1 - \frac{4\epsilon}{3}\right)$	

$$I_B = \frac{m_B R^2}{3}$$

$$M_B = \frac{m_B R}{2}$$

Appendix A - A Priori Expressions for Model Parameters (cont)

Parameter	A Priori Information	Constraint
v_w^1	$\frac{2\Omega}{M_{11}} \left(\bar{v}_{Ref} + \frac{\sigma a}{8} \right)$ or $\frac{\sigma a \Omega}{4M_{11}}$	
v_β^1	$-\frac{4\Omega R}{3M_{11}} \left(\bar{v}_{Ref} + \frac{\sigma a}{8} \left(1 - \frac{3\varepsilon}{2} \right) \right)$ or $-\frac{\sigma a \Omega R}{6M_{11}} \left(1 - \frac{3\varepsilon}{2} \right)$	$-\frac{2R}{3} \times v_w$
v_β	$-\frac{\sigma a R \Omega^2}{6M_{11}} K_1$	$-K_1 \times v_\theta$
v_v	$-\frac{4\Omega}{M_{11}} \left(\bar{v}_{Ref} + \frac{\sigma a}{16} \right)$	$\frac{3}{2\Omega R} \left(1 + \frac{16 \bar{v}_{Ref}}{\sigma a} \right) \times v_\theta$
v_{Ω^1}	$-R \bar{v}_{Ref}$ or 0	Fixed
v_{Ω^1}	$\frac{4\Omega R}{M_{11}} \left(\bar{v}_{Ref} + \frac{\sigma a}{16} \right) \bar{v}_{Ref}$ or $\frac{\sigma a \Omega}{4M_{11}} \bar{v}_{Ref}$	
v_θ	$\frac{\sigma a R \Omega^2}{6M_{11}}$	
Ω_w	$\frac{N\gamma_z \Omega}{6R} \left[(\theta - K_1 \beta)_{Ref} + \frac{3}{4} \theta_t - 3 \bar{v}_{Ref} \right]$	
Ω_β	$-\frac{N\gamma_z \Omega}{8} \left[(\theta - K_1 \beta)_{Ref} \left(1 - \frac{4\varepsilon}{3} \right) + \frac{4}{5} \left(1 - \frac{5\varepsilon}{4} \right) \theta_t \right.$ $\left. - \frac{8}{3} \left(1 - \frac{3\varepsilon}{2} \right) \bar{v}_{Ref} - \frac{16(l_B + cM_B)\beta_{Ref}}{\gamma_z l_z} \right]$	
Ω_β	$\frac{N\gamma_z \Omega^2}{6} K_1 \bar{v}_{Ref}$	$-K_1 \times \Omega_\theta$
Ω_v	$-\frac{N\gamma_z \Omega}{6R} \left[(\theta - K_1 \beta)_{Ref} + \frac{3}{4} \theta_t - 3 \bar{v}_{Ref} \right]$	$-1.0 \times \Omega_w$
Ω_Ω	$-\frac{N\gamma_z \Omega}{6} \left[(\theta - K_1 \beta)_{Ref} + \frac{3}{4} \theta_t - 3 \bar{v}_{Ref} \right] \bar{v}_{Ref}$ $-\frac{2Q_{F,Ref}}{\Omega l_z}$	
Ω_θ	$-\frac{N\gamma_z \Omega^2}{6} \bar{v}_{Ref}$	
Ω_Q	$\frac{1}{l_z}$	

$$\gamma_z = \frac{l_B}{l_z} \gamma$$

$$\sigma = \frac{Nc}{\pi R}$$

¹ Alternative expressions from Equations (3) or (3a) respectively

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