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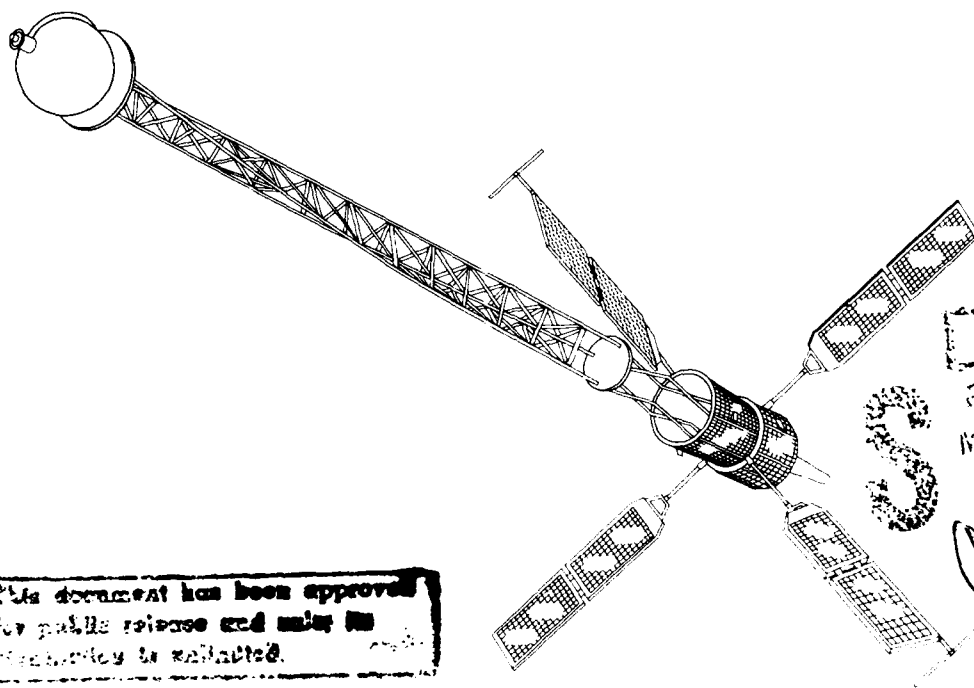
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Oct. 1, 1987 - Sept. 30, 1988



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Nonlinear Dynamics and Control of Flexible Structures

CORNELL UNIVERSITY

Upson Hall, Ithaca, N.Y. 14853

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October 1, 1987 - September 30, 1988
University Research Initiative

NONLINEAR DYNAMICS AND CONTROL OF FLEXIBLE STRUCTURES

Contract No. F49620-87-C-0011

Principal Investigator: Francis C. Moon
Mechanical and Aerospace Engineering

Co-Principal Investigator: Peter Gergely
Civil and Environmental Engineering

Co-Principal Investigator: James S. Thorp
Electrical Engineering

Acting Principal Investigator: John F. Abel
(August '88 - January '89) Civil and Environmental Engineering



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University Research Initiative

NONLINEAR DYNAMICS AND CONTROL OF FLEXIBLE STRUCTURES

Annual Report 1988

Cornell University

Francis C. Moon - Principal Investigator

Peter Gergeley - Co-Principal Investigator

James S. Thorp - Co-Principal Investigator

John F. Abel - Acting Principal Investigator (August '88-January '89)

EXECUTIVE SUMMARY

The second years' research has seen the transition from theoretical analysis to the building of laboratory models for testing new ideas in nonlinear dynamics and control of flexible structures. Among the goals for this interdisciplinary project is to develop analytic and computational tools to predict, design, and control the dynamics of large space structures taking into account nonlinearities such as large deformations, buckling, friction, nonlinear joints, nonlinear materials behavior, and nonlinear control. In the past year several planar and three-dimensional truss structures have been built and tested.

This project is a joint effort of faculty and students in civil, mechanical, and electrical engineering, and theoretical mechanics. Summaries of the different projects and subprojects are presented below, and copies of more extended reports and papers may be obtained by writing to the faculty member associated with that subproject (see the list of Faculty Participants below). A brief overview of accomplishments is described in this executive summary.

Experimental-Theoretical Studies

- Chaotic vibrations have been shown to exist in a 3-meter long space truss with pin connections. The use of cable induced compressive prestress in the truss led to a reduction of the region of chaotic behavior. Analytical models for play in pin-connected truss structures are under study (**Moon; Li, Feeny**: page 6). Study of the effect of friction in joints on nonlinear dynamics of structures has been underway with both experimental and theoretical analysis (**Moon; Feeny**).
- Active control of large amplitude vibrations in a planar truss have been achieved using a servo-hydraulic actuator, and a 10-meter three-dimensional experiment is under design. (**Gergely, Ingrassia, Abel; Larsen**: page 15). This work is based on theoretical studies of optimal control of nonlinear structures using Differential Dynamic Programming (**Shoemaker, Abel Thorp; Liao, Aubert**: page 21).
- An analysis of joint-to-joint self-equilibrating actuators in truss structures has shown the importance of actuator location, as well as the saturation of active control damping. A 4-meter long space truss has been built with electro-mechanical actuators to test these ideas (**Moon; Chen, Davies**). The use of nonlinear rubber elasticity for support of structures under dynamic testing to simulate free space vibrations has been successfully implemented (**Moon; Davies**). Theoretical analysis of optimal actuator location for space trusses has been undertaken (**Thorp, Chiang; Lu**: page 36).
- The accuracy of finite element codes with respect to prediction of natural frequencies in 3-D space trusses has been investigated using experimental data from a 23-bay structure. The use of frame-type finite elements has shown an accuracy of from 2 to 12% in the 5 lowest modes. The use of truss-type codes, however, can

lead to large errors of up to 50% in the 5th mode (**Abel, Moon**; Aubert, Li: page 40).

- Nonlinear vibrations of a truss with a buckled link have been studied in a 3 meter space truss. The dynamics appear to be chaotic. Theoretical studies are underway to include the effect of elasto-plastic buckling on dynamics of structures (**Mukherjee, Moon**; Pratap: page 43).

Theoretical Studies

- Finite element codes and associated computer graphics to predict the dynamics of large deformations of truss and frame type structures have been applied to studies of collocated velocity feedback control (**Abel, Moon**; Aubert, Chen: page 46) and of the numerical effects of mixing linear and geometrically nonlinear modelling (**Abel, Thorp, Shoemaker**; Aubert, Liao: page 49). This code has been integrated with an optimal nonlinear control algorithm, Differential Dynamic Programming (DDP), to achieve simulation of active damping in structures undergoing large deformations. This research has shown a 28% improvement in damping in the use DDP over linear closed-loop schemes (**Shoemaker, Abel, Thorp**; Liao, Aubert: page 21).
- More efficient algorithms for the use of Differential Dynamic Programming (DDP) have been developed for application to control of flexible structures. (**Shoemaker**; Liao: page 56).
- An analysis of the effect of structural nonlinearities on the stability of linear feedback control has shown that such nonlinearities can severely shrink stability domains and degrade overall system performance (**Chiang, Thorp**; Lu, Fekih-Ahmed: page 59) and (**Thorp**; Naqavi: page 60).

- New theoretical methods have been proposed for reducing the dimension of eigenstructure assignment problems in the modal control of large space structures (Thorp, Chiang; Lu: page 66).
- The use of group theoretic methods to improve the efficiency of calculating eigenvalues of large truss structures with symmetries has been accomplished. An example of an antenna or dish-type structure is given as a test case (Healey; Treacy: page 70).
- The effect of finite precision in feedback control systems for flexible, lightly damped structures has been studied (Delchamps: page 74).

The focus of research in the coming year will involve two experimental-theoretical projects. In the first, a 4-meter space truss structure will be tested using active damping and self-equilibrating electro-mechanical actuators. The MOOG Corp. of Buffalo, New York will participate in this effort in the development of light-weight actuators. A key question involves the optimum placement of actuators and spillover problems. A second project involves construction of a 10-meter, three-dimensional multibay truss which will test ideas involving optimal control schemes for flexible structures.

LIST OF FACULTY PARTICIPANTS

Professor Francis C. Moon
Mechanical and Aerospace Engineering
105 Upson Hall
(607) 255-7146

Professor James S. Thorp
Electrical Engineering
304 Phillips Hall
(607) 255-3347

Professor Hsiao-Dong Chiang
Electrical Engineering
302 Phillips Hall
(607) 255-5270

Professor Mircea Grigoriu
Civil and Environmental Engineering
367 Hollister Hall
(607) 255-3334

Professor Anthony R. Ingrafea
Civil and Environmental Engineering
317 Hollister Hall
(607) 255-3336

Professor Christine A. Shoemaker
Civil and Environmental Engineering
210 Hollister Hall
(607) 255-9233

Professor Peter Gergely
Civil & Environmental Engineering
324 Hollister Hall
(607) 255-4217

Professor John F. Abel
Civil & Environmental Engineering
323 Hollister Hall
(607) 255-7582

Professor David F. Delchamps
Electrical Engineering
315 Phillips Hall
(607) 255-6447

Professor Timothy J. Healey
Theoretical and Applied Mechanics
233 Thurston Hall
(607) 255-3738

Professor Subrata Mukherjee
Theoretical and Applied Mechanics
233 Thurston Hall
(607) 255-7143

NOTE: All addresses are at Cornell University, Ithaca, NY 14853

PROJECT SUMMARY

Project Title:

Chaotic Vibration in Pin-Connected Truss Structures

Faculty Leader:

Professor Francis C. Moon

Mechanical and Aerospace Engineering

Graduate Research Assistants:

Brian Feeny

Other Participants:

Dr. G.X. Li, Postdoctoral Research Associate

Executive Summary:

The dynamics of a pin-jointed space truss has been studied. Experimental results show that the response of the truss, under sinusoidal excitation, exhibited broadband chaotic-like vibrations. When a tension cable was added to place the structure under compressive load, the level of chaos was reduced. An analytical model has been developed by including the small free-plays in the truss joints, and was subsequently numerically implemented.

In addition, a simple oscillator of one-degree-of-freedom with dry friction is under study to understand the effects of dry friction in the truss joints.

Description of Project, Progress and Results

The structure under study is a 3.5 meter, 16 bay 3-D truss built with aluminum rods. The truss members were connected at joints with pins. While no attempt was made to introduce gaps in the pin joints, the truss showed a large amount of accumulated free-play.

In the idealized case, where no gaps exist in the joints, the truss structure has model shapes as shown in Figure 1. The first two natural frequencies of the bending are 65.5 and 167.7 Hz, respectively, and the first natural frequency of the twisting is 98.8 Hz. In the real case, however, the model shapes, tested out in our laboratory, were greatly distorted due to the loose joints. The first two such mode shapes of the bending are shown in Figure 2. In the dynamical testings, a sinusoidal force was applied to the truss, which was hung to the ceiling by two soft rubber bands. Although the input was periodic, the response of the truss,

measured by a small accelerometer, was chaotic. In Figure 3, the input signal contains one main peak (driving frequency) and several superharmonic components in the frequency domain. However the output signal contains infinitely many frequency components, a characteristic feature of chaotic motions. To control chaos a tension cable was added to the truss along the longer direction in hopes of killing some of the loose joints and bringing the truss close to its linear regime. As expected, the chaos level, measured in terms of the magnitude of the frequency response, was substantially reduced, and in some cases weak chaotic motions became periodic ones when the cable tension was increased to a certain level.

In order to understand the underlying mechanism of the chaos in the truss, an analytical vibration equation has been formulated for the truss. Each member in the truss was modeled as a trilinear stiffness oscillator, as shown in Figure 4. Using the standard technique of assembling in finite element analysis, these finitely many elemental trilinear oscillators could be put together yielding an analytical model of the whole truss. Model reduction technique was used to solve for solutions for the first few lower modes. Figure 5 shows the response of the first bending mode of the truss, which clearly does not look like periodic. Its frequency representation, seen in Figure 6, once again appears to be broadband indicating the chaotic nature.

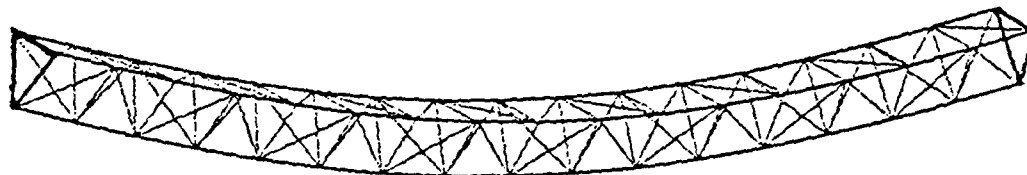
An experiment on an oscillator with dry friction (Figure 7) has revealed some nonperiodic motions, such as that shown in the delay map of Figure 8. We would like to support these results with some theory or numerics, but first we must answer some questions about the dynamic nature of the friction law. Can we use Coulomb's law? Is there hysteresis in the friction law? Does the dynamic friction law change greatly if we use different materials? We are trying to answer these questions by calculating the frictional force exerted during oscillation. This can be obtained from measured displacement, velocity, acceleration, and input. Given the friction vs. velocity and friction vs. displacement relations, we can perform some analysis. Experiments with other materials, such as teflon, will give insight to the role of the material in the motion.

Published Papers and Reports:

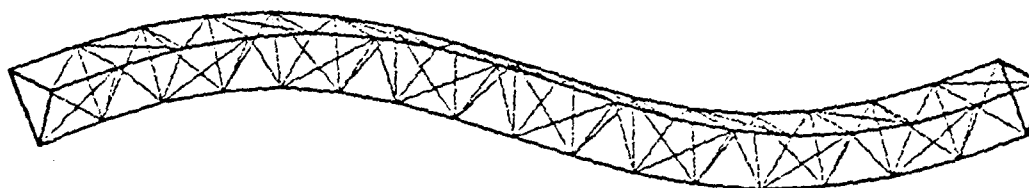
Moon, F.C., and G.X. Li (1988). *Experimental Study of Chaotic Vibrations in a Pin-Jointed Space Truss Structure*, submitted to the AIAA Journal

Li, G.X., and F.C. Moon (1988). *Numerical Simulations of the Dynamics of a Space Truss with Complex Joints* (in preparation)

(a) First bending mode, $f \approx 65.5$ Hz



(b) Second bending mode, $f \approx 167.7$ Hz



(c) First twisting mode, $f \approx 98.8$ Hz

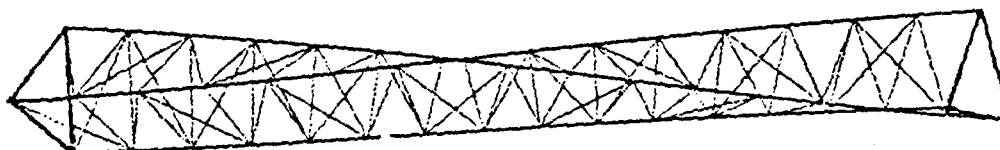


Figure 1

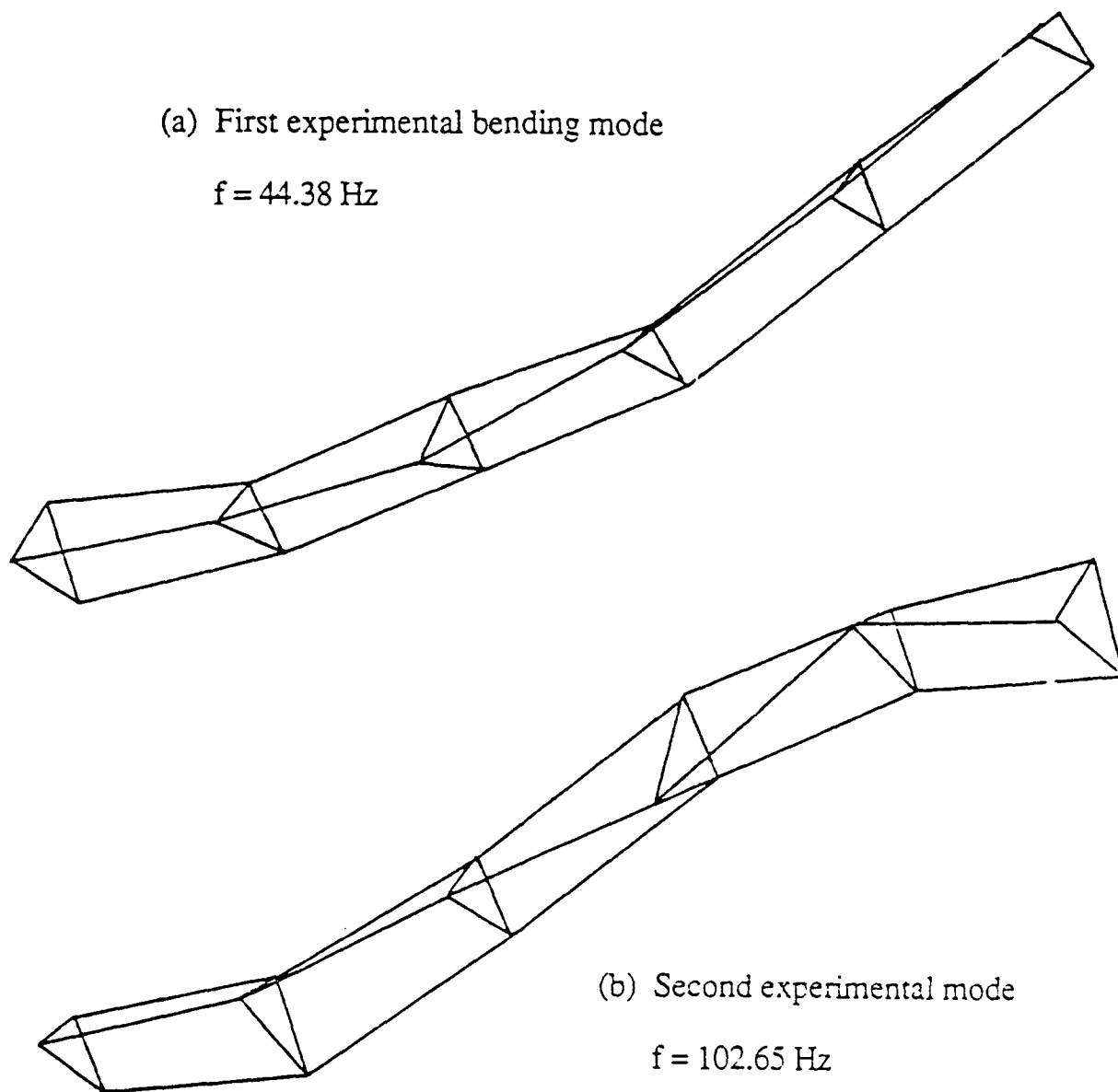


Figure 2

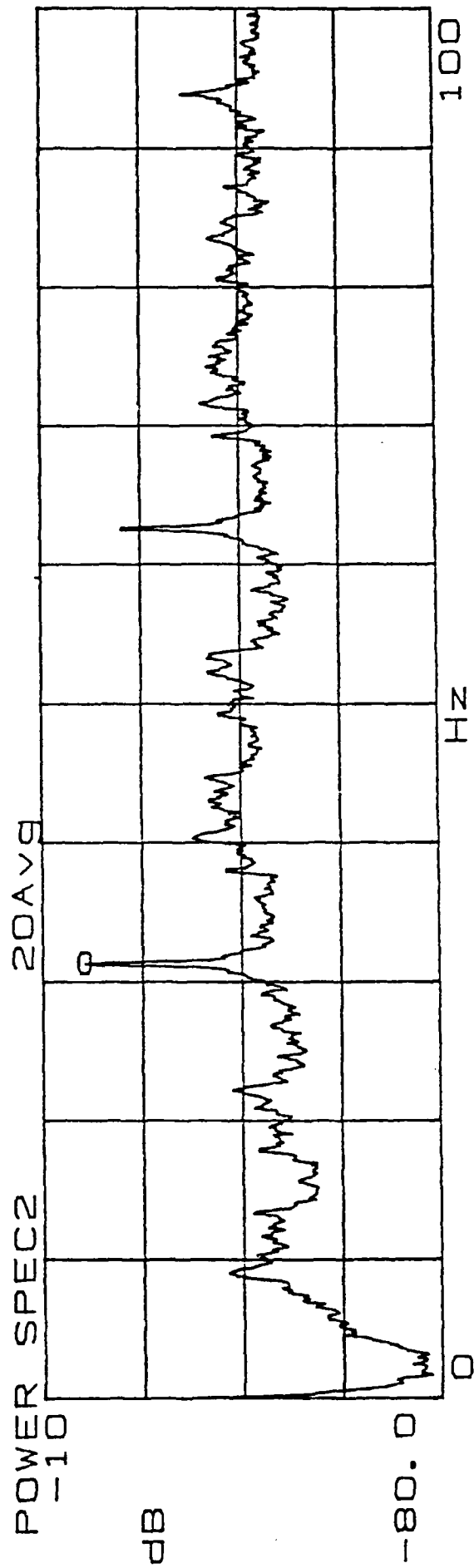
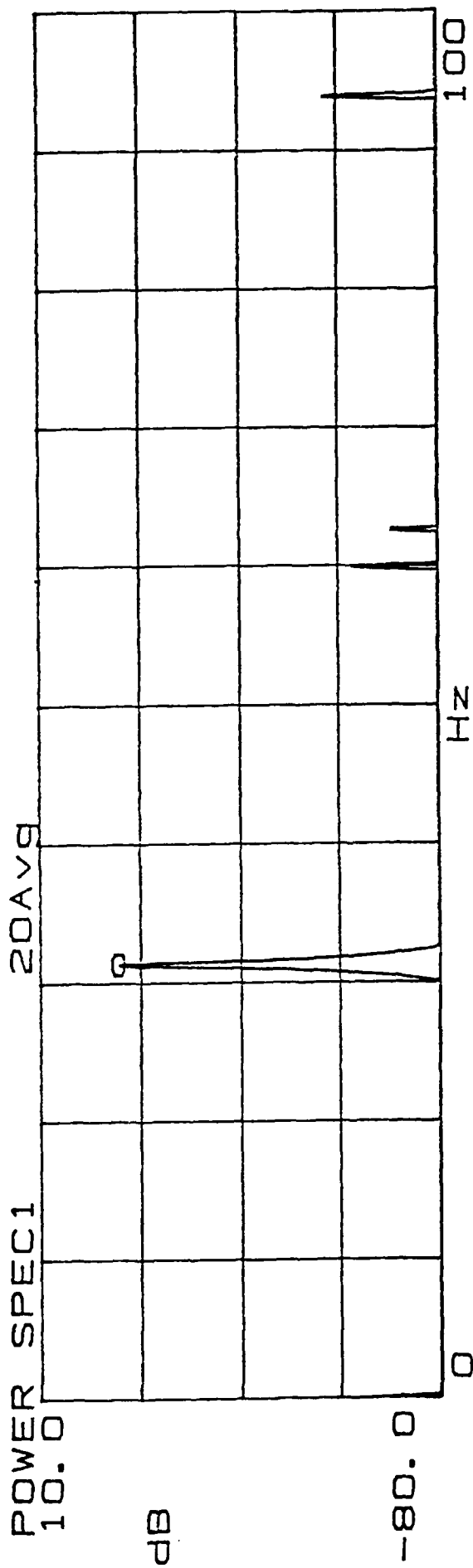


Figure 3

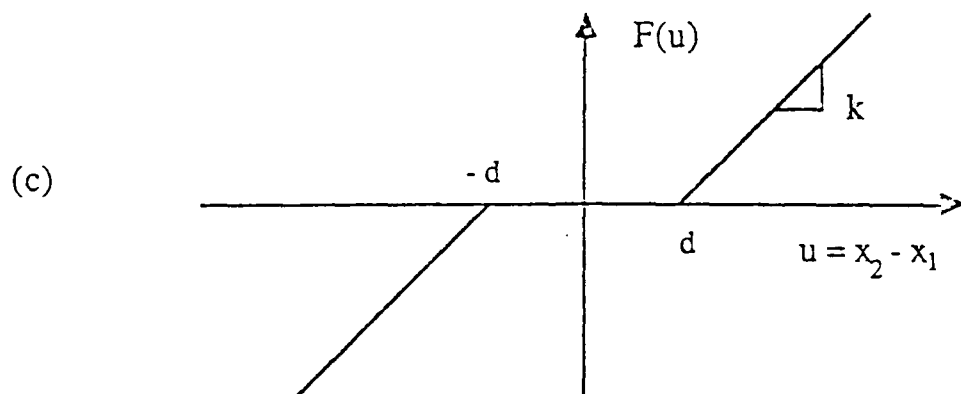
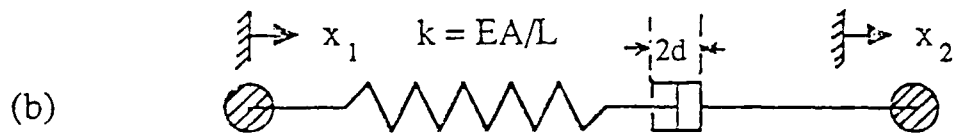
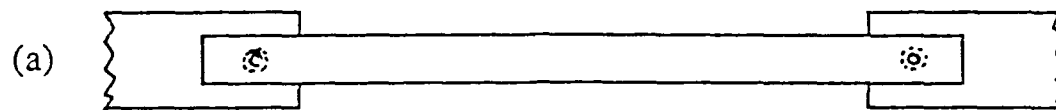


Figure 4

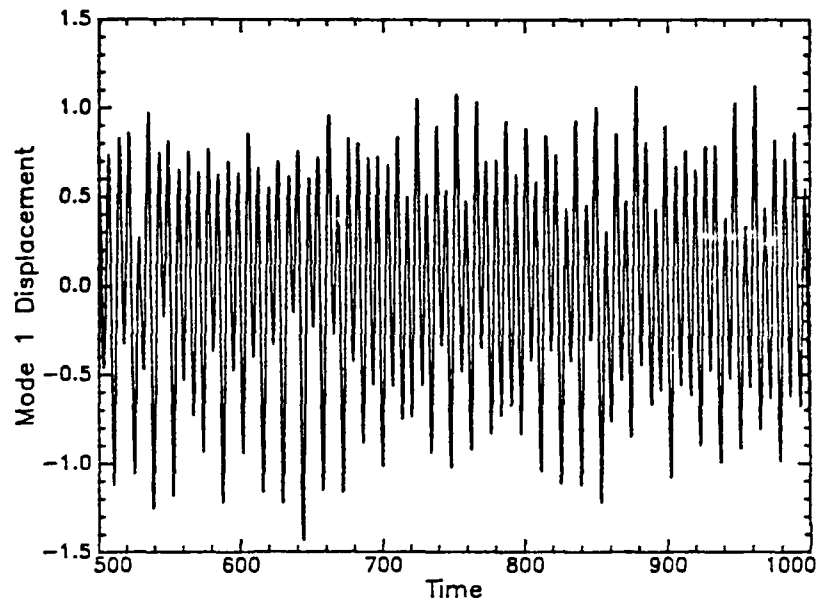


Figure 5

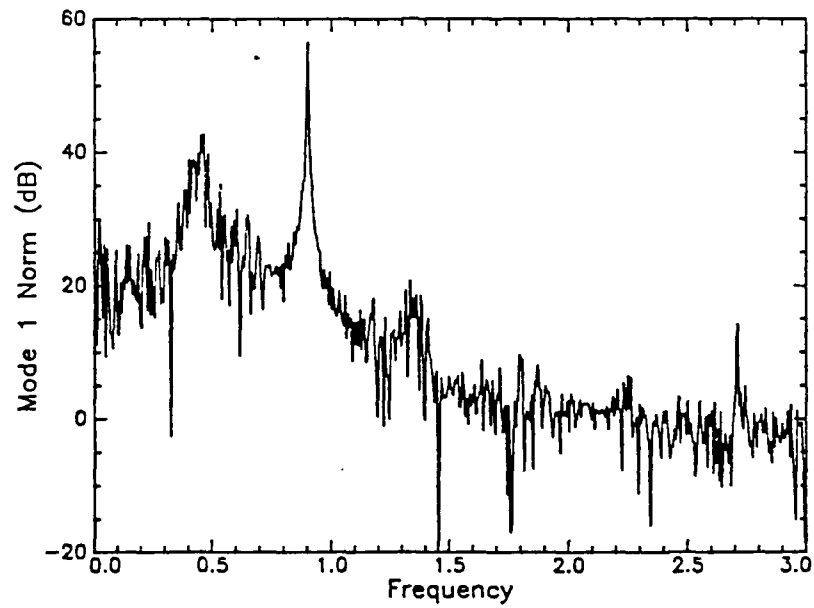


Figure 6

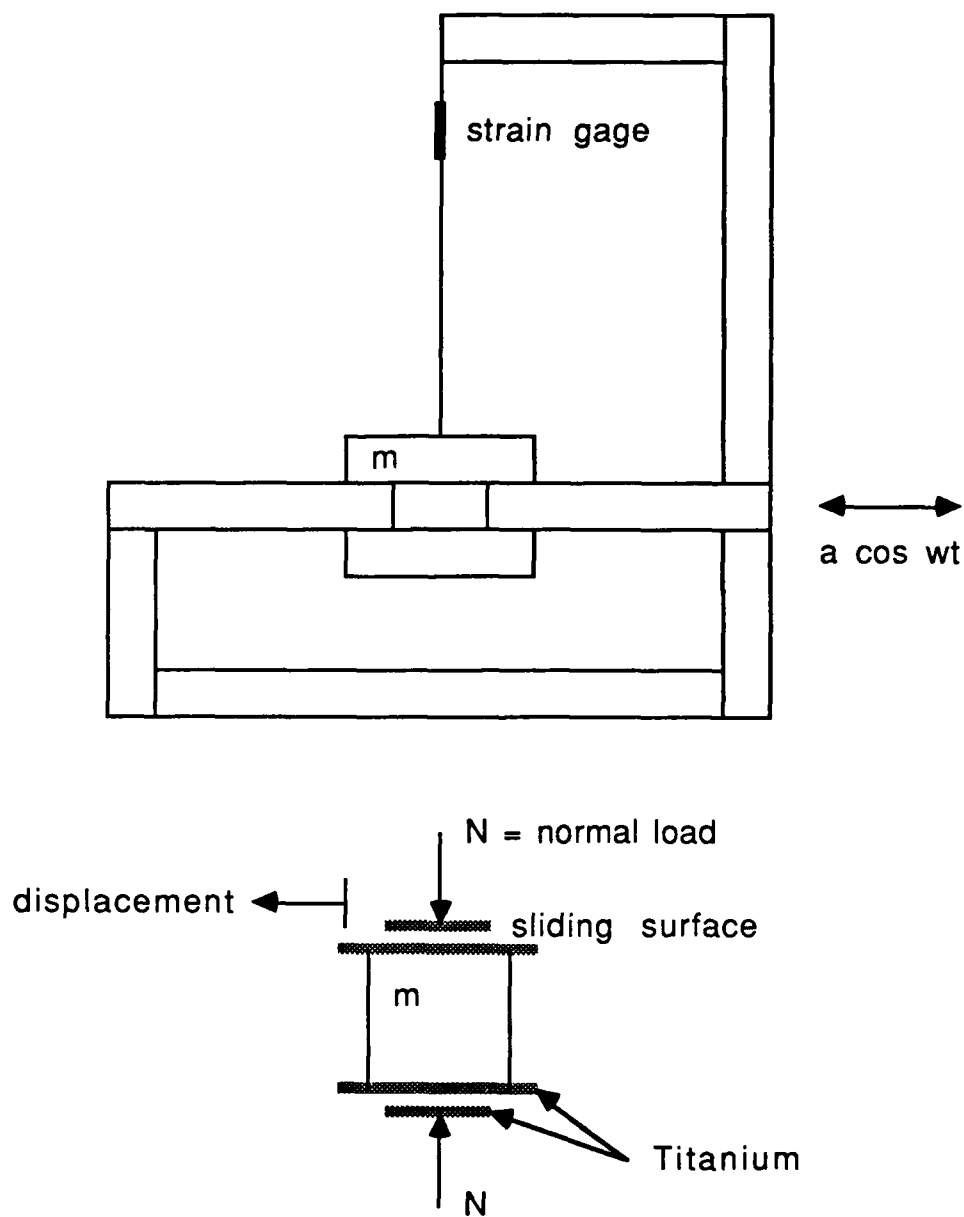
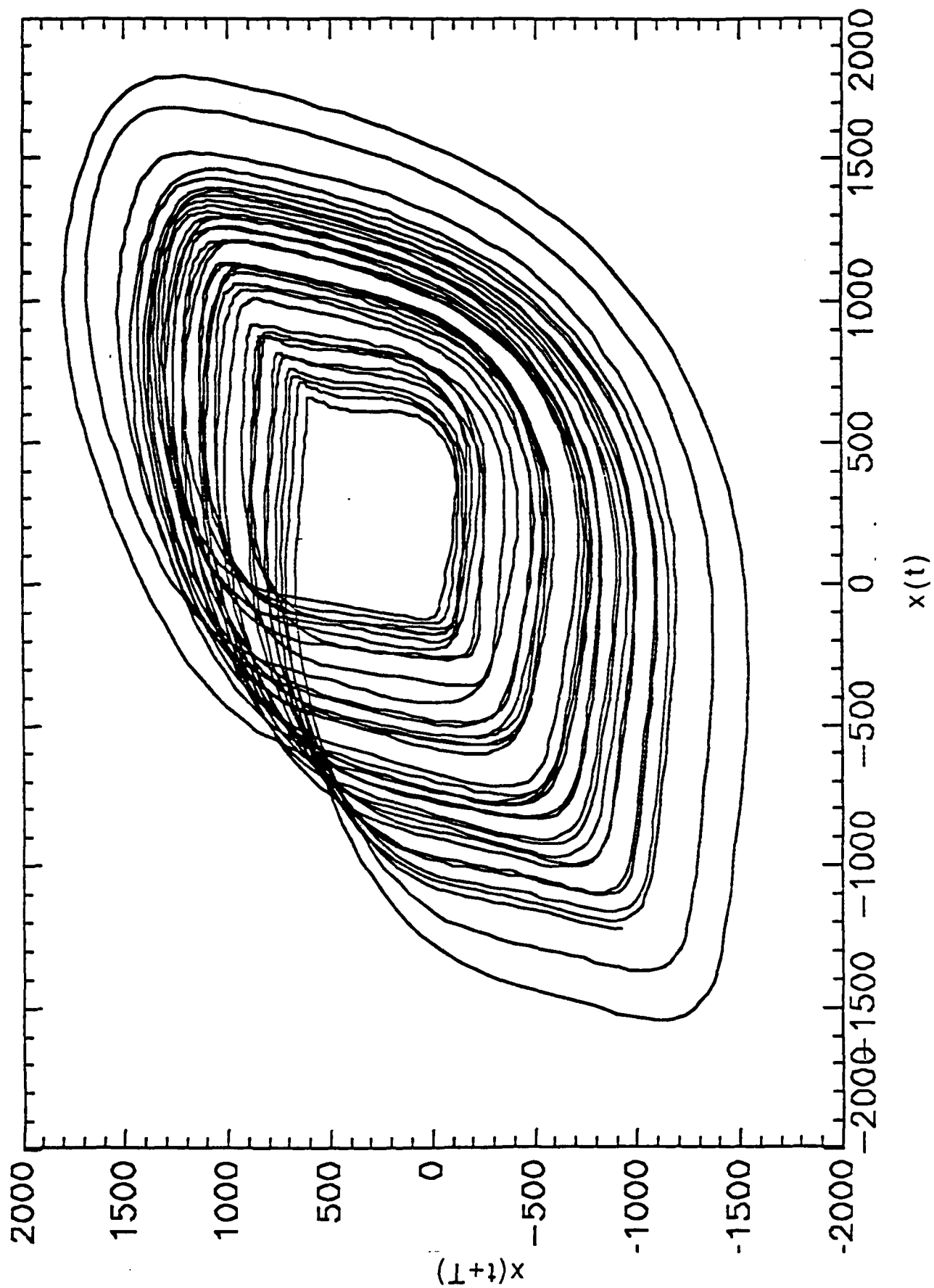


Figure 7. Experimental set up

Figure 8. Delay map with delay $T=0.04s$, and a 4.45hz driving frequency. Here, the natural frequency is 3.6hz .



PROJECT SUMMARY

Project Title:

Experimental Trusses : Open-Loop Active Control

Faculty Leader:

Professor Peter Gergely
Civil and Environmental Engineering

Graduate Research Assistant:

Lauran Larson

Other Participants:

Professor Anthony Ingraffea
Professor John Abel
Brian Aubert
Li-zhi Liao
Michael Minter

Executive Summary:

Dynamic testing of a small truss subject to active control is in progress. A typical relationship between uncontrolled and controlled experimental responses is shown in figure 1. The primary purpose of the small truss is to complete the correlation of finite element and experimental dynamic response results, the verification of the precomputed time varying linear feedback control waveforms, and the development of experimentation and data acquisition techniques. These tasks are considered preliminary to the construction and testing of a 10 meter truss. Working drawings and design calculations for the large test truss are now in a stage of refinement.

Project Description:

The small preliminary test truss consists of four 38cm x 38cm bays supported in a horizontal plane 90cm above the laboratory floor by lightweight aluminum tubes which pivot freely at each node. (See figure 2.) This support results in minimal restraint and damping of lateral free vibration of the specimen cantilevered from the laboratory wall. To bring the dynamic response of an inherently stiff system into a frequency range compatible with that of the servo-hydraulic control link, several members have been replaced by "flexible" links and each nodal point has been weighted with a lumped mass of 9kg.

The active control link, based upon force feedback gains, is mounted in parallel with a flexible link and delivers both tension and compression to one chord. The force control waveform is based upon the application of a nonlinear differential dynamic optimal control program to this preliminary linear problem. Excitation consists of a 1.9cm initial lateral displacement of the cantilevered end. Although

the first mode dominates the response, the servo-hydraulic control has been effective in delivering to the system predetermined force waveforms of frequency content in the range of the second and third modes.

The experimental and finite element model uncontrolled responses agree to within 2% on the first and third modes, 8% on the second mode. However, we've encountered some difficulty in correlating the finite element model controlled response with that of the experimental response. Precomputed time varying linear feedback control waveforms which perform well in the model do not produce similar damping when applied to the experimental truss. More specifically, the finite element model has, as yet, been unable to predict the pronounced period shortening evident in the first period of controlled motion. (See figure 1.) Efforts to achieve a finite element characterization which models this early period shortening have included variation of mass at the actuator location, variation of the control link stiffness proportional to actuator velocity, and variation of control link stiffness proportional to actuator force level. Apparently, the dynamics of the servo-hydraulic actuator itself is significantly altering the total system response in the early periods of motion. For this reason, finite element model / experimental correlation efforts are now focused on the actuator dynamics.

Configuration of the proposed 10 meter truss is based upon an effort to provide elastic deformations of sufficient magnitude to introduce geometric nonlinearities to the control problem. (See figures 3 & 4.) Thus, the high aspect ratio in the primary direction of motion. The vertically hung cantilever installation provides nearly undamped free vibration and results in slight dead load pretension to offset axial buckling of the chord members adjacent to the support. Diagonals are absent in the "weak" direction for maximization of deflections within that plane of motion, diagonals are present in the "strong" direction to minimize out-of-plane motions.

Details as yet undecided in the large specimen configuration are:

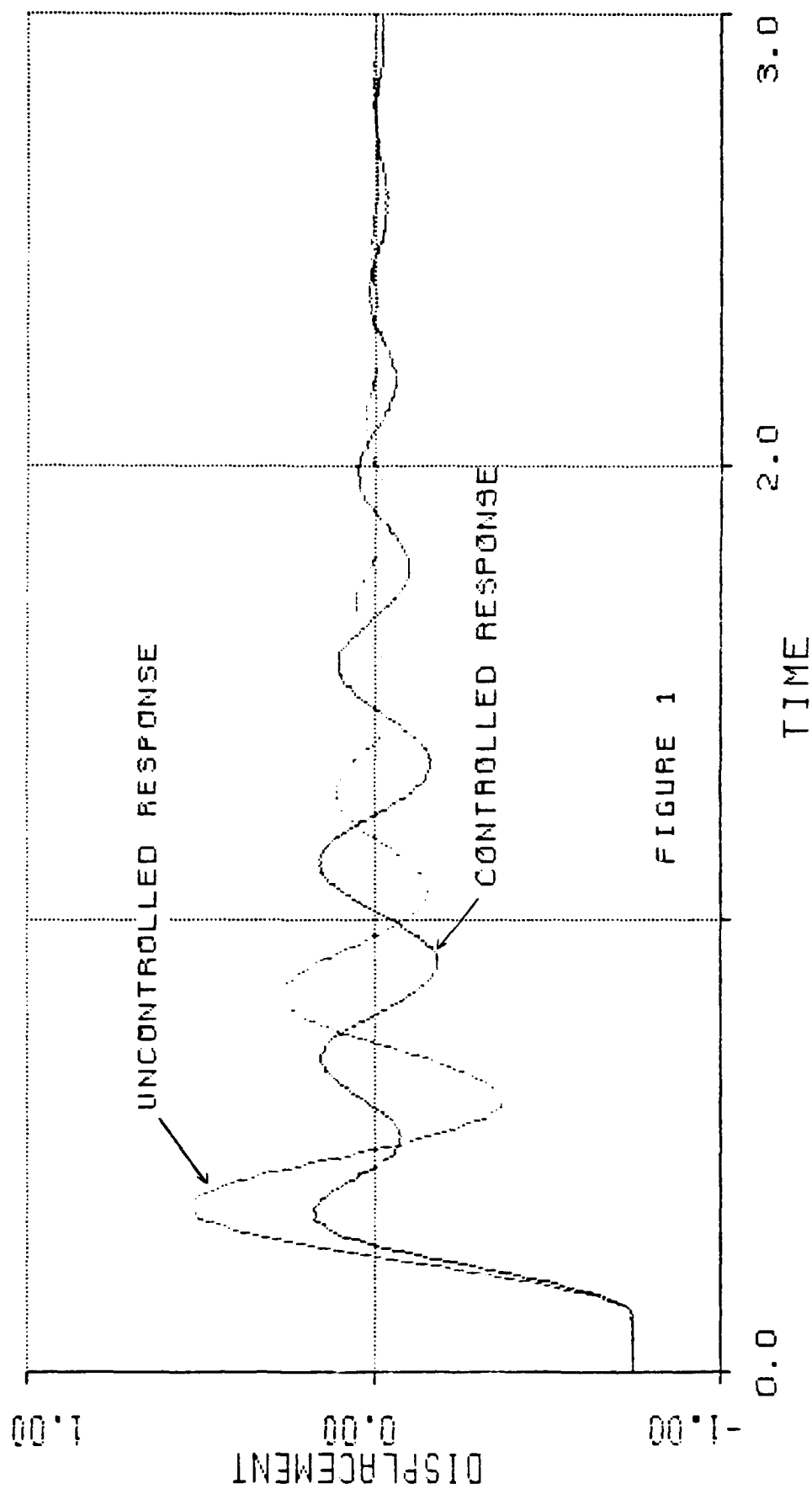
Member materials; aluminum or wound reinforced composites.

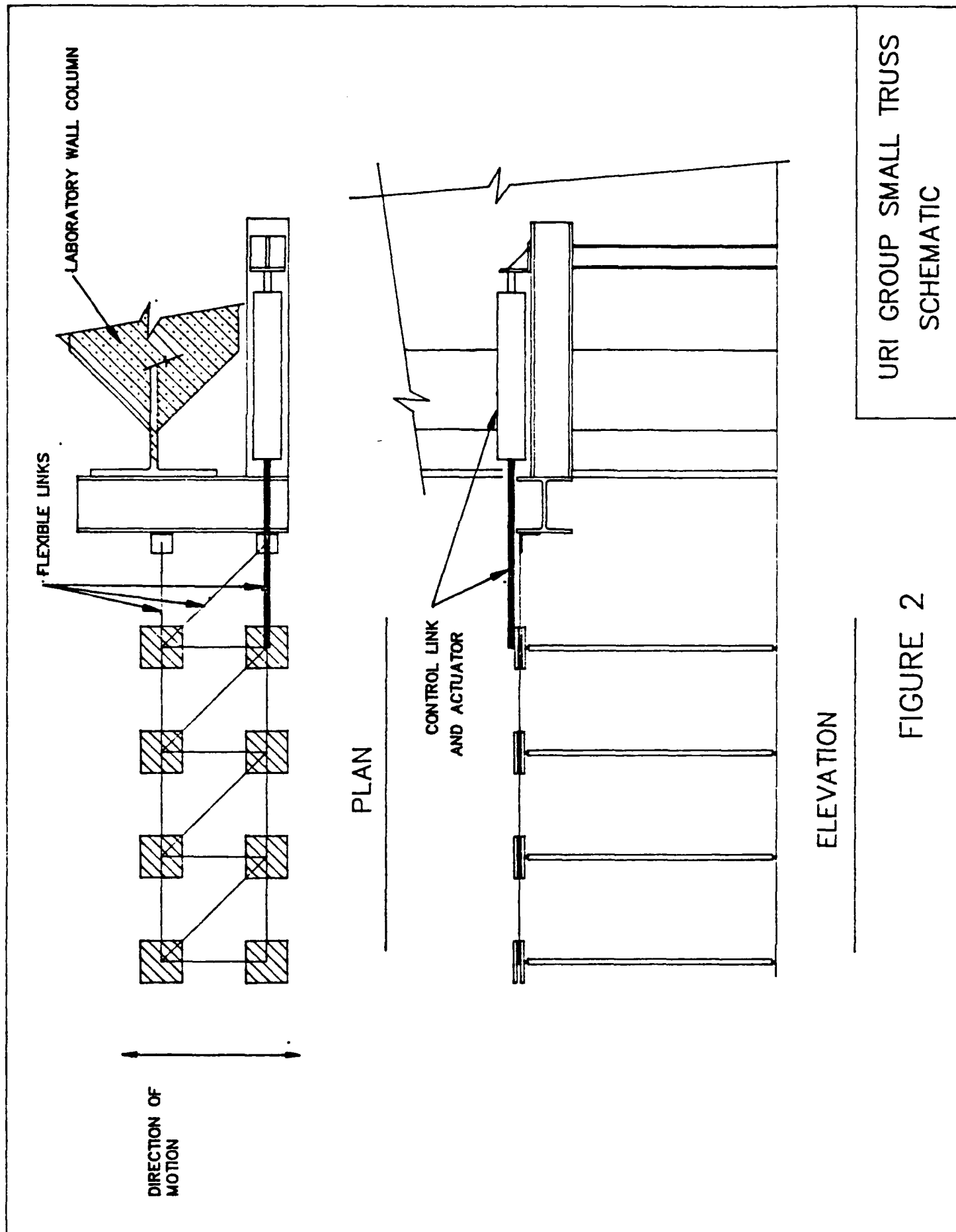
Control linkage; tension and compression parallel links or tuned tension only tendons.

The first phase of testing of the large specimen will involve open-loop force feedback with an initial displacement. Subsequent to this first phase, the large specimen will serve as a test bed for a variety of studies within the global project objective. These include:

- Passive control
- Control placement
- Closed-loop active control
- Chaotic dynamics

FIGURE 1
SMALL TEST TRUSS
TIME HISTORY RESPONSE



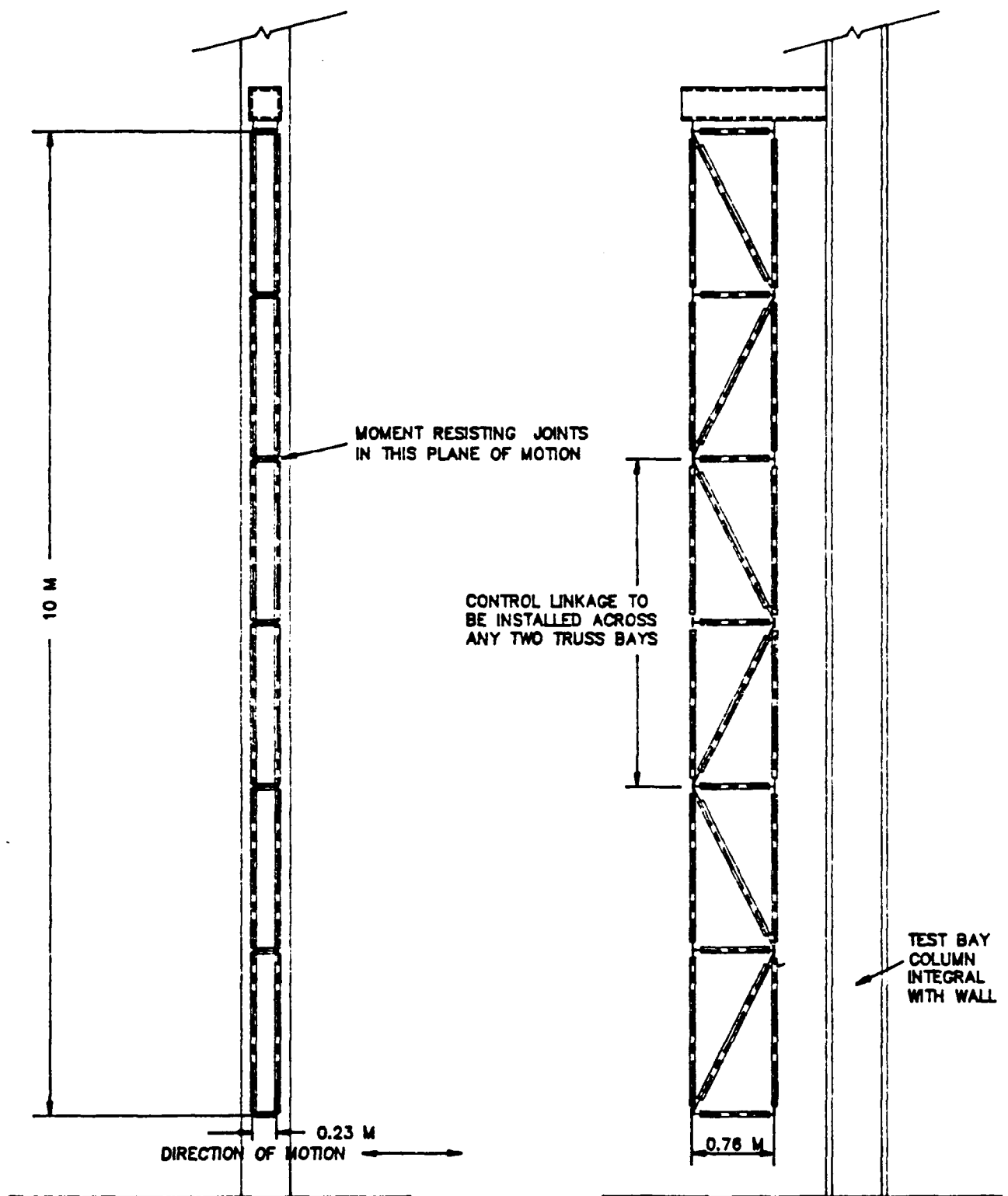


URI GROUP SMALL TRUSS
SCHEMATIC

FIGURE 2

ELEVATION

PLAN



ELEVATION

SECTION

URI GROUP LARGE TRUSS
SCHEMATIC

FIGURE 3

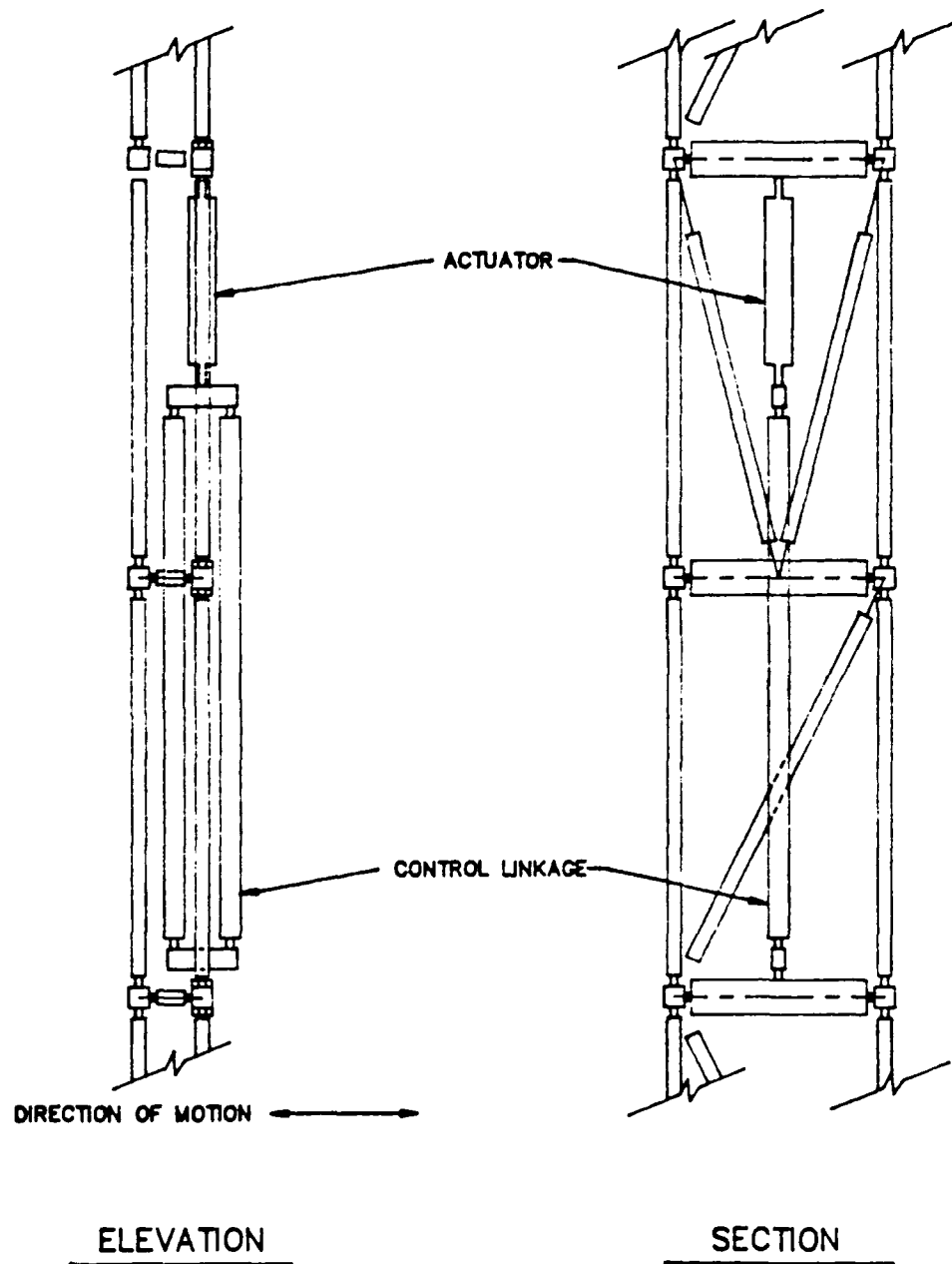


FIGURE 4

URI GROUP LARGE TRUSS
CONTROL LINKAGE DETAIL

PROJECT SUMMARY

Project Title:

Computing Optimal Nonlinear Control of Nonlinear Structures by Coupling Differential Dynamic Programming and Finite Element Algorithms

Faculty Leaders:

Christine A. Shoemaker and John F. Abel, School of Civil and Environmental Engineering

Graduate Research Assistants:

Li-zhi Liao and Brian Aubert

Other Participants:

Prof. James S. Thorp, Department of Electrical Engineering

Executive Summary:

To compute the optimal control of nonlinear structures we have coupled the optimal control algorithm Differential Dynamic Programming (DDP) to finite element models of geometrically nonlinear structures. We have computed optimal control of structures with two and with four bays with as many as 20,000 time steps in the finite element model. Our results indicate that, for a nonlinear system, the optimal control computed by DDP is significantly better than that obtained by application of a linear feedback rule.

Project Description:

We have imbedded the structural finite element model into the differential dynamic programming algorithm by using the finite element explicit time-marching equations as the transition function in the DDP algorithm. Since the DDP algorithm utilizes partial derivatives with respect to displacement, velocity, and control, it is necessary to rescale parameters in the transition equations to insure numerical stability.

The DDP-finite element analysis has been completed for a number of test problems including: a) a four bay example based on the physical characteristics of the truss that has been built in the laboratory by Professor Gergely, b) a four-bay example with most members being very flexible to illustrate the impact of strong nonlinearities, and c) several two-bay examples that were discussed in last year's report.

Table 1
Computational Results for 4-bay Problem (12 Soft Members)
Nonlinear K and Linear B System

	Energies (objective function components)			
	Strain	Kinetic	Potential	Total
No Control	17298.5	16162.7	0.0	33761.2
Linear System linear control	1546.40	990.621	3077.18	5614.20
Linear Feedback closed-loop	1231.22	797.113	2801.17	4829.51
1 Iter DDP shift=400	1203.79	766.477	2701.10	4671.36
5 Iter DDP shift=300	1151.59	711.039	2335.27	4200.90
10 Iter DDP shift=50	1176.59	707.757	1786.66	3671.01
15 Iter DDP no shift	1250.62	719.767	1538.76	3509.15
17 Iter DDP no shift	1264.75	723.820	1508.20	3196.77

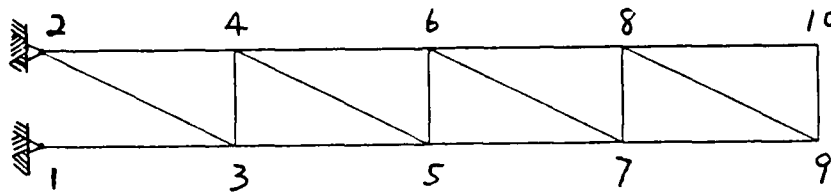


FIGURE 1

The four-bay, two-dimensional truss with 12 soft members has , Figure 1, a typical bay length of 15 inches and a bay height of 7.5 inches. The chord and diagonal members are modelled as having a stiffness value of $AE=6.4$ kips, and the vertical members a value of $AE=5440$ kips,. The mass of the members was lumped at the nodes, and the net effect is a typical nodal mass of approximately 22 lbs. The objective function used is the sum of the strain, kinetic, and potential energies.

The results for this problem (Table 1) show that the DDP's solution is significantly better than the closed-loop linear feedback's solution. The result obtained after 17 iterations of DDP, which is very close to the optimal solution, has 28% improvement in the value of the objective function over the closed-loop, linear-feedback result. Figure 2 through 6 compare the displacements, velocities, and controls between the DDP and the closed-loop linear feedback.

FIGURE 2

Control Comparison

Nonlinear K and Linear B

closed-loop lin feed vs DDP

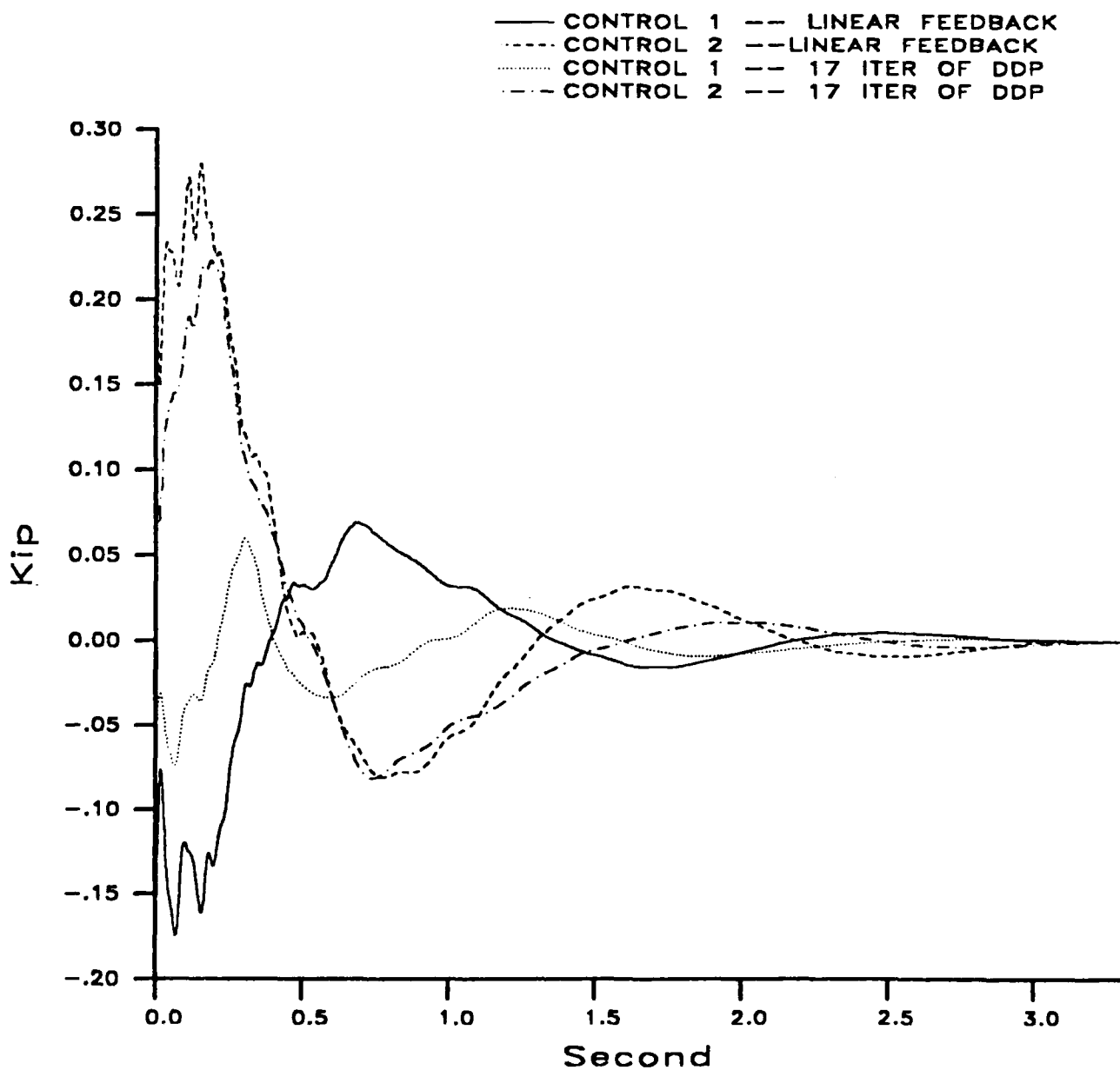


FIGURE 3

Displacement Comparison

Nonlinear K and Linear B

Horizontal Direction Node 10

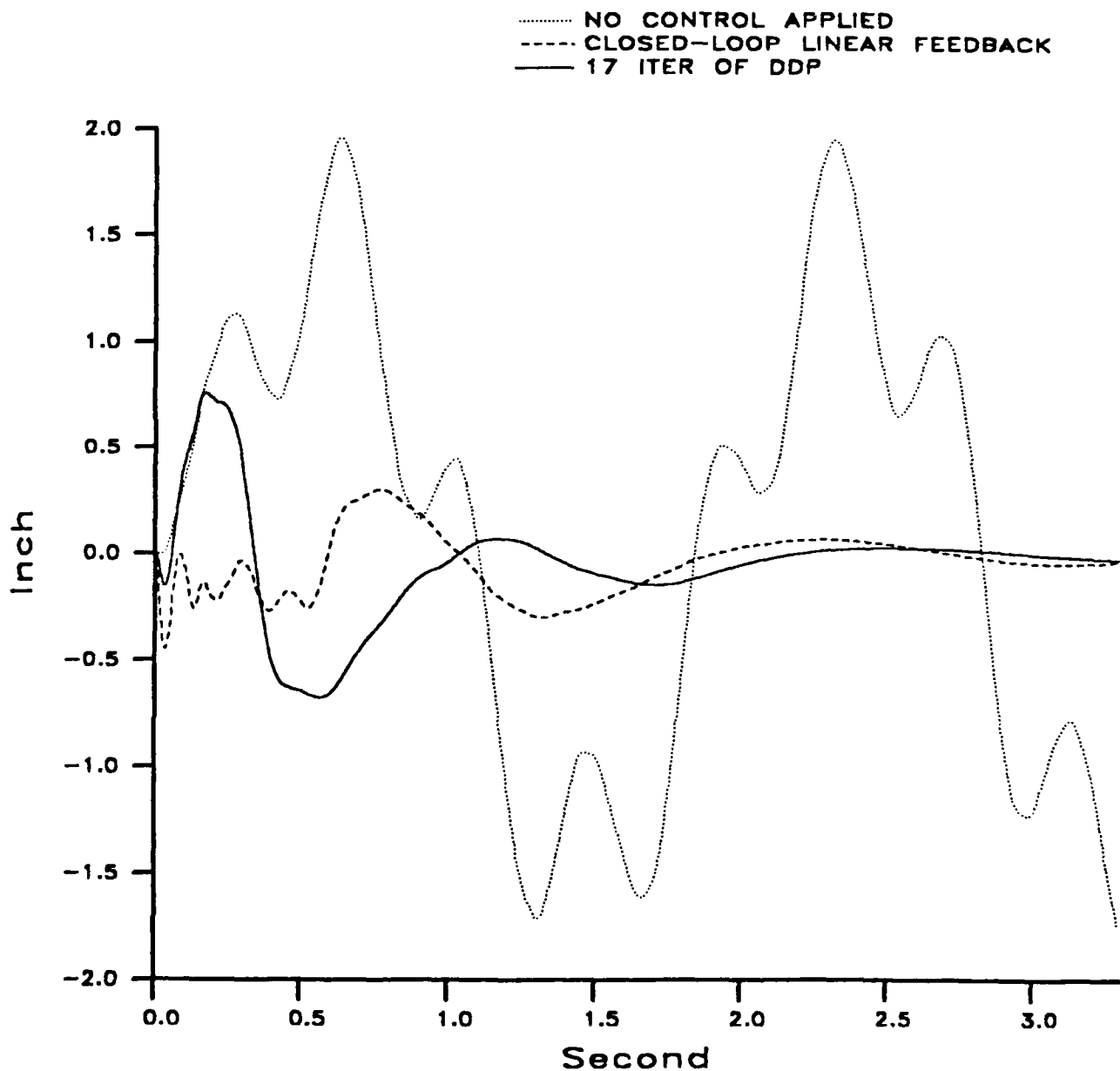


FIGURE 4

Displacement Comparison Nonlinear K and Linear B Vertical Direction Node 10

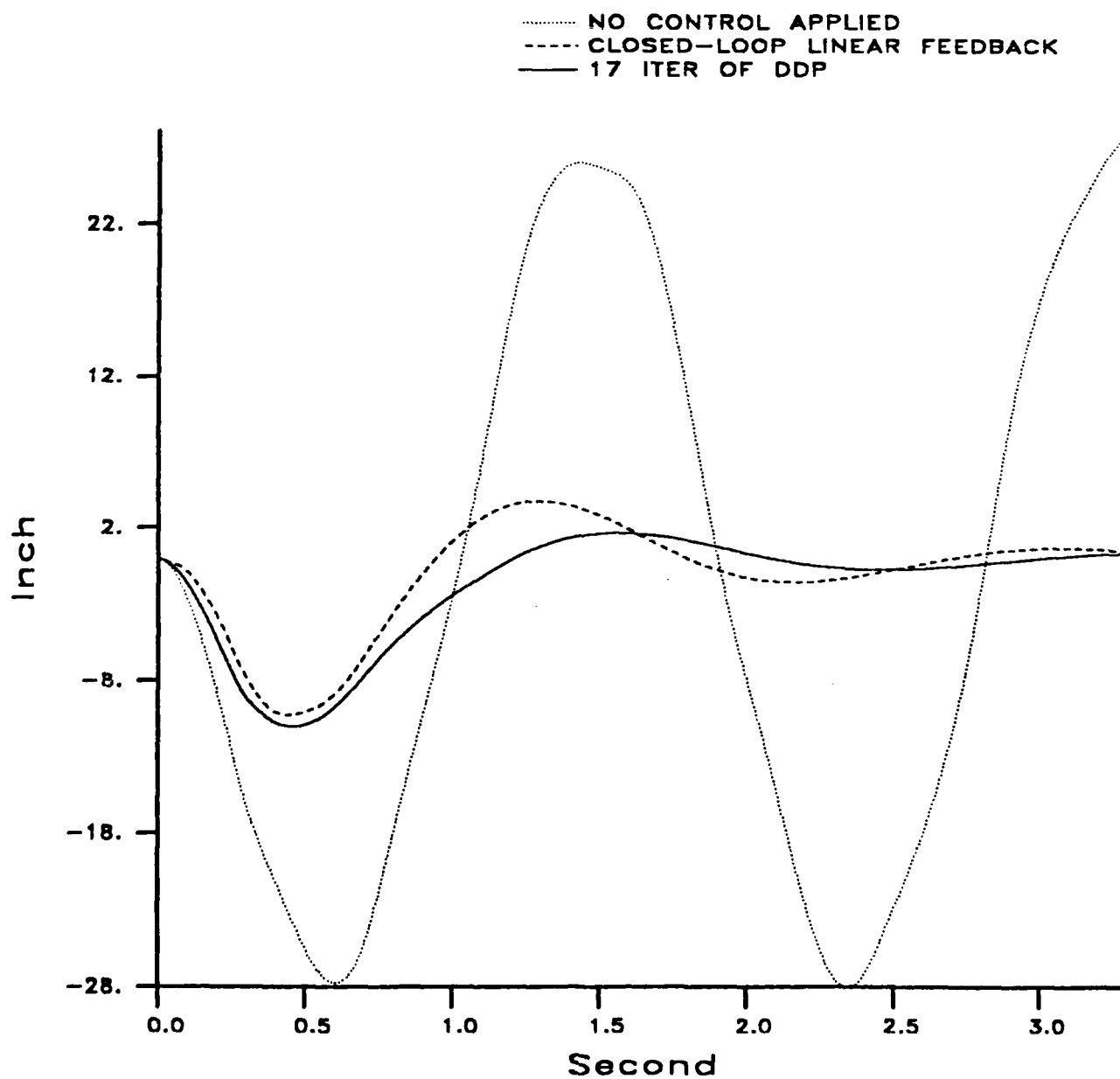


FIGURE 5

Velocity Comparison Nonlinear K and Linear B Horizontal Direction Node 10

..... NO CONTROL APPLIED
----- CLOSED-LOOP LINEAR FEEDBACK
—— 17 ITER OF DDP

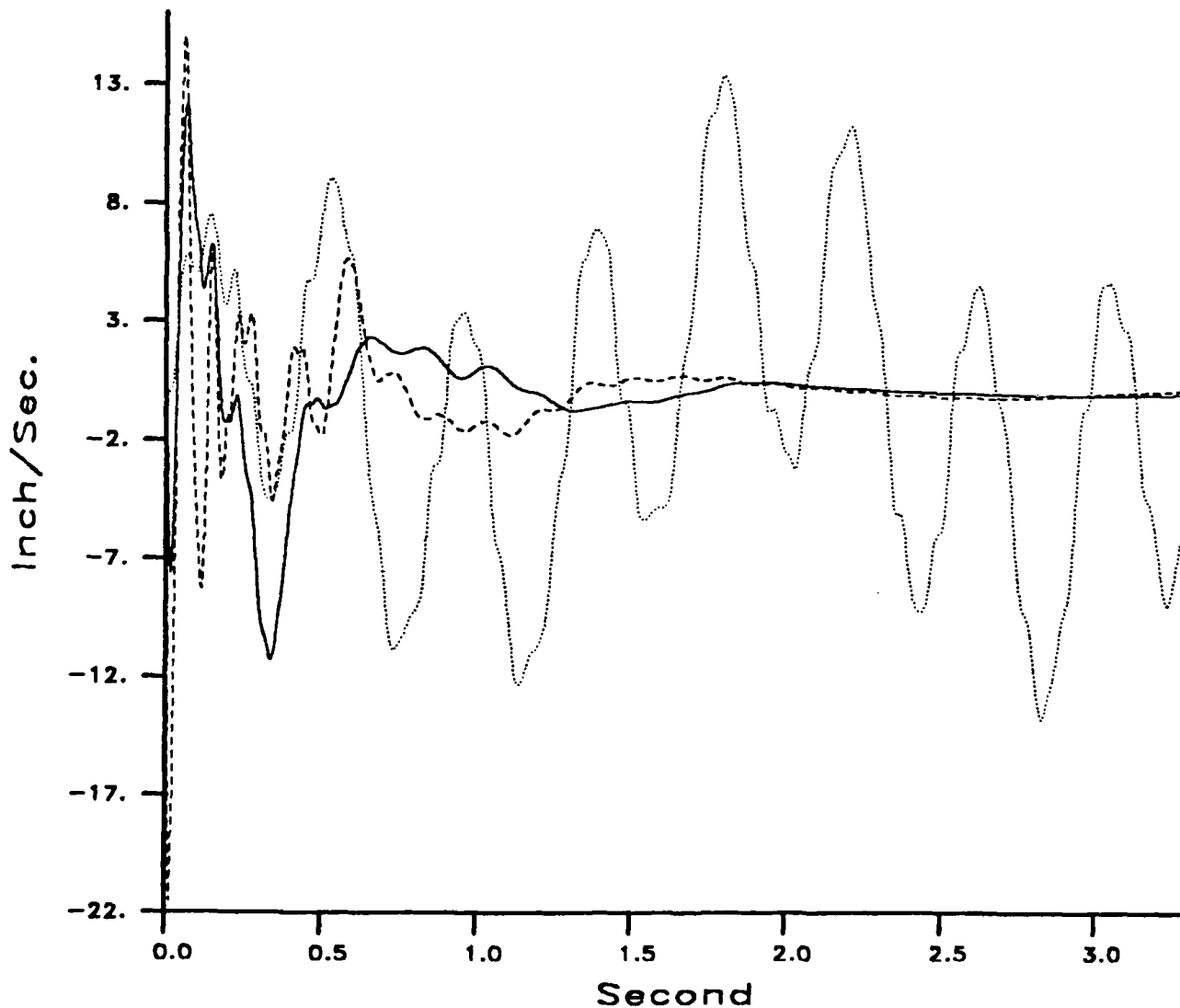
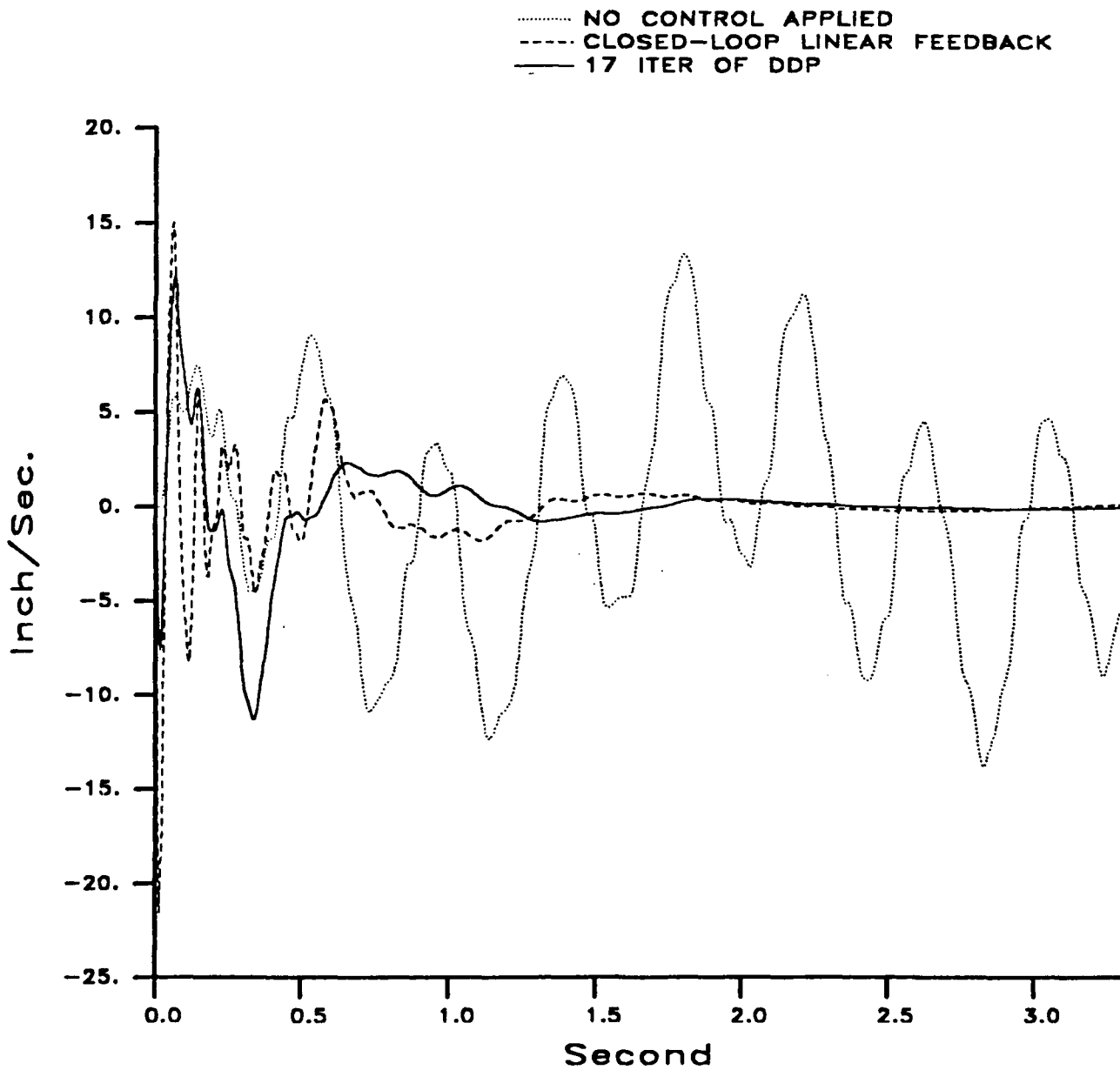


FIGURE 6

Velocity Comparison Nonlinear K and Linear B Vertical Direction Node 10



PROJECT SUMMARY

Project Title:

Collocated Feedback Control for the Vibration Suppression in Truss-Type Structures

Faculty Leader:

Professor Francis C. Moon
Mechanical and Aerospace Engineering

Graduate Research Assistants:

Pei-Yen Chen
Matt Davies

Other Participants:

None

Executive Summary:

Numerical simulation shows that the vibrations of truss-type structures with rigid joints can be damped out efficiently by the multiple-input / multiple-output collocated feedback control. A simplified algorithm is developed to draw the root loci of the closed loop system using this control strategy. The experiment to implement this idea is under study. Also, in order to simulate structures in the free space, we investigate the suspension system which must be very soft, compared to the truss itself.

Project Description:

A 6.5-meter long experimental space truss with rigid joints, shown in Figure 1, is designed to implement the idea of multiple-input / multiple-output (MIMO) collocated feedback control. The natural frequencies and vibration modes are shown in Figure 2 & Figure 3. In order to make the control system (actuators & feedback circuits) practically realizable, the fundamental natural frequency is lowered to around 10 Hz. Also, this linear truss is designed to prevent from buckling when the vibration amplitudes of its both ends are less than 10MM.

In principle, the fundamental-mode type disturbance can be damped out efficiently by choosing an appropriate feedback gain. Some results of numerical simulation (finite element method) in figure 4 indicate that the damping ratio is about 43%. In some other cases, the closed loop system even has overdamped behavior, but the damping capability for higher modes is worse. Furthermore, we intend to improve the performance of feedback control by incorporating the concept of optimal control.

A simplified algorithm is developed to draw the root loci of the closed loop system using the MIMO collocated feedback control. Therefore, it can be used to determine an appropriate feedback gain. Now, the experiment under study is designed to demonstrate the capability of vibration suppression.

Another prime concern in the earth-based testing of space structures is the design of an adequate suspension system. It is necessary that such a suspension system, when coupled with the truss, have a natural frequency much lower than the fundamental natural frequency of the truss itself. This allows the data from the truss vibration tests to be easily recognized and separated from rigid body motion introduced by oscillation in the suspension system. At this point, it seems that suspending the truss with strips of pure gum rubber is an effective way of lowering the truss-suspension system natural frequency. Figure 5 shows some of the data which has been collocated for pure gum rubber supporting different masses. It has been found that, for most of our experimental trusses, this natural frequency can be made as low as 0.3 Hz simply by adjusting the length and width of the rubber strips used to suspend them.

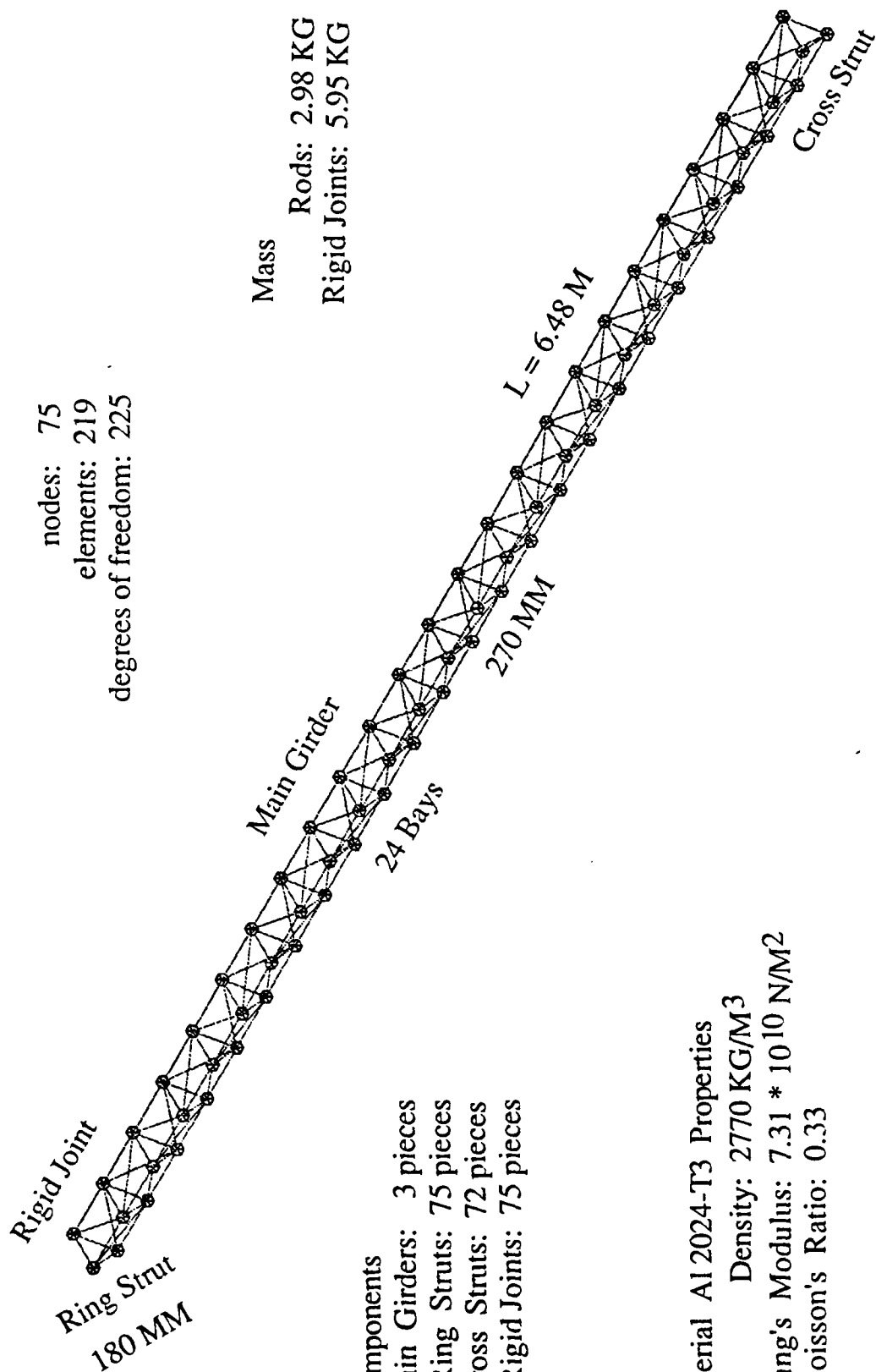
Published Papers and Reports:

Chen, P.Y., and Moon, F.C. (1988). Multiple-Input / Multiple-Output Collocated Feedback Control for the Vibration Suppression of a Large Linear Structure System (in preparation).

rod diameter: 5 MM (Al 2024-T3)
 rigid joint cube size: 31 MM (Al 6061-T6)

nodes: 75
 elements: 219
 degrees of freedom: 225

Mass
 Rods: 2.98 KG
 Rigid Joints: 5.95 KG

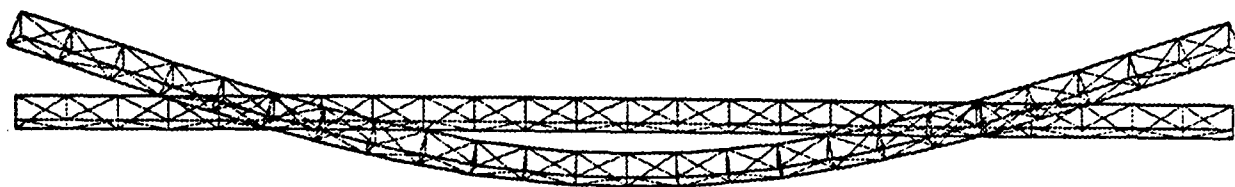


Components
 Main Girders: 3 pieces
 Ring Struts: 75 pieces
 Cross Struts: 72 pieces
 Rigid Joints: 75 pieces

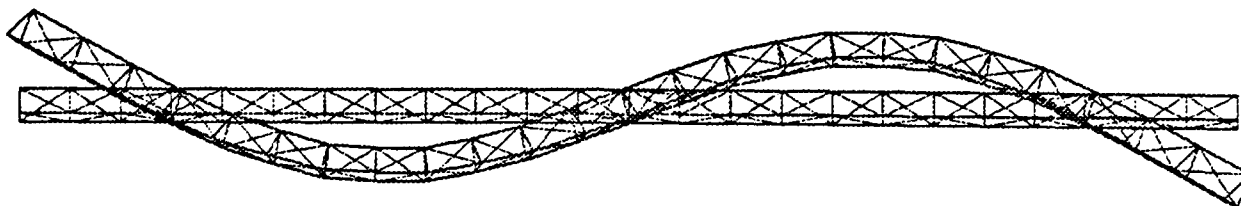
Material Al 2024-T3 Properties
 Density: 2770 KG/M³
 Young's Modulus: 7.31 * 10¹⁰ N/M²
 Poisson's Ratio: 0.33

Experimental Space Truss

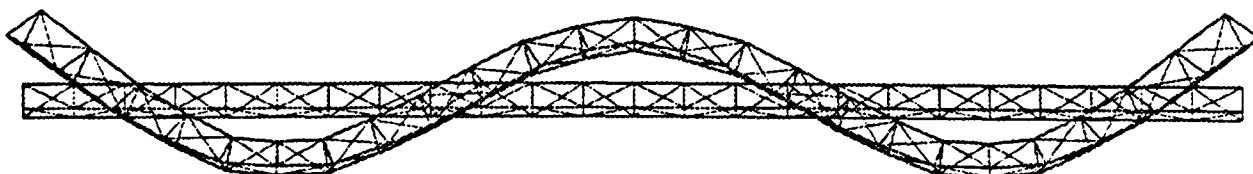
Figure 1.



First Bending Mode $f_{b1} = 10.1 \text{ Hz}$



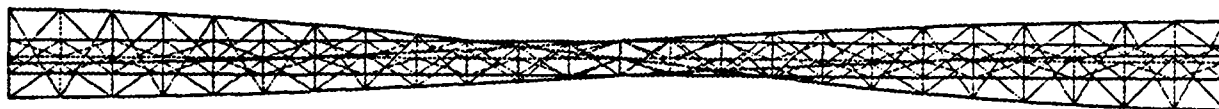
Second Bending Mode $f_{b2} = 27.5 \text{ Hz}$



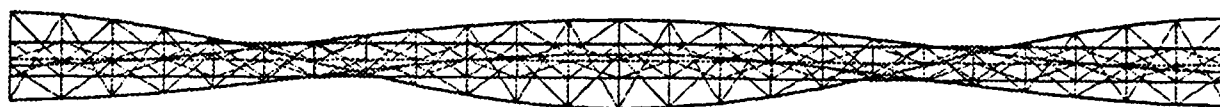
Third Bending Mode $f_{b3} = 52.6 \text{ Hz}$

Bending Vibration Modes of Experimental Space Truss
(no control element)

Figure 2.



First Torsional Mode $f_{t1} = 27.8$ Hz



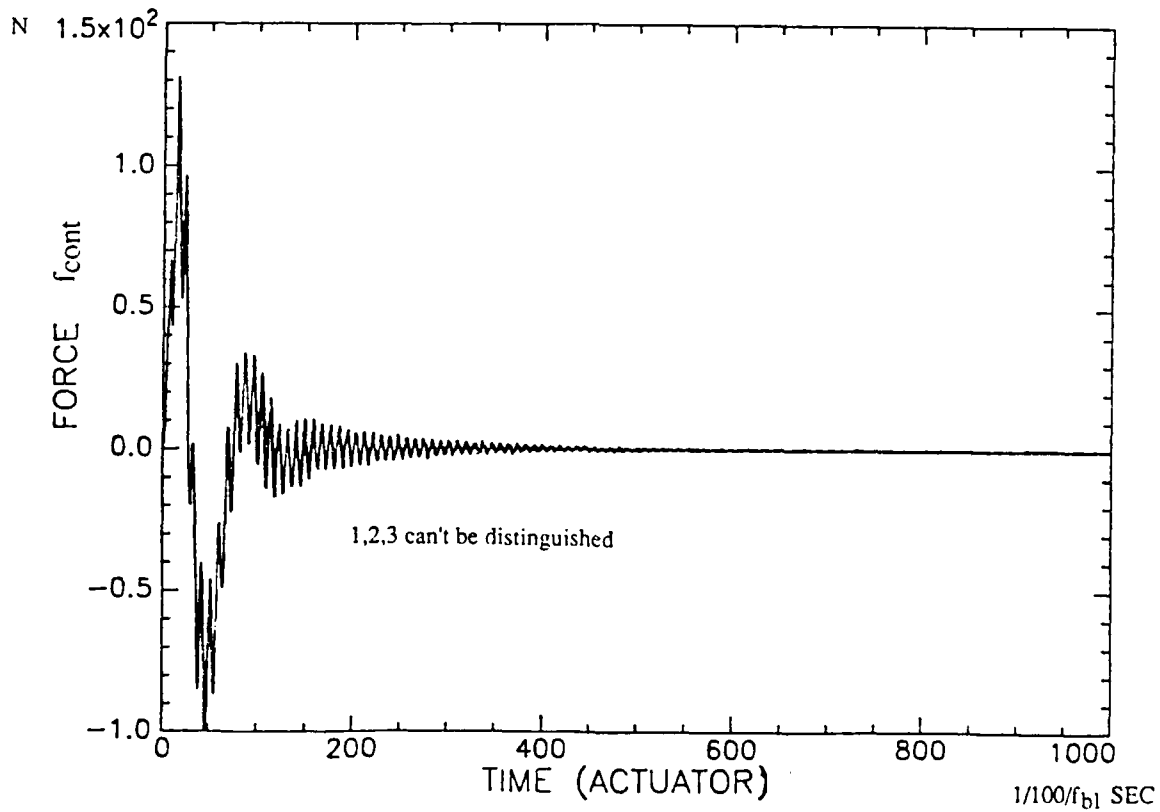
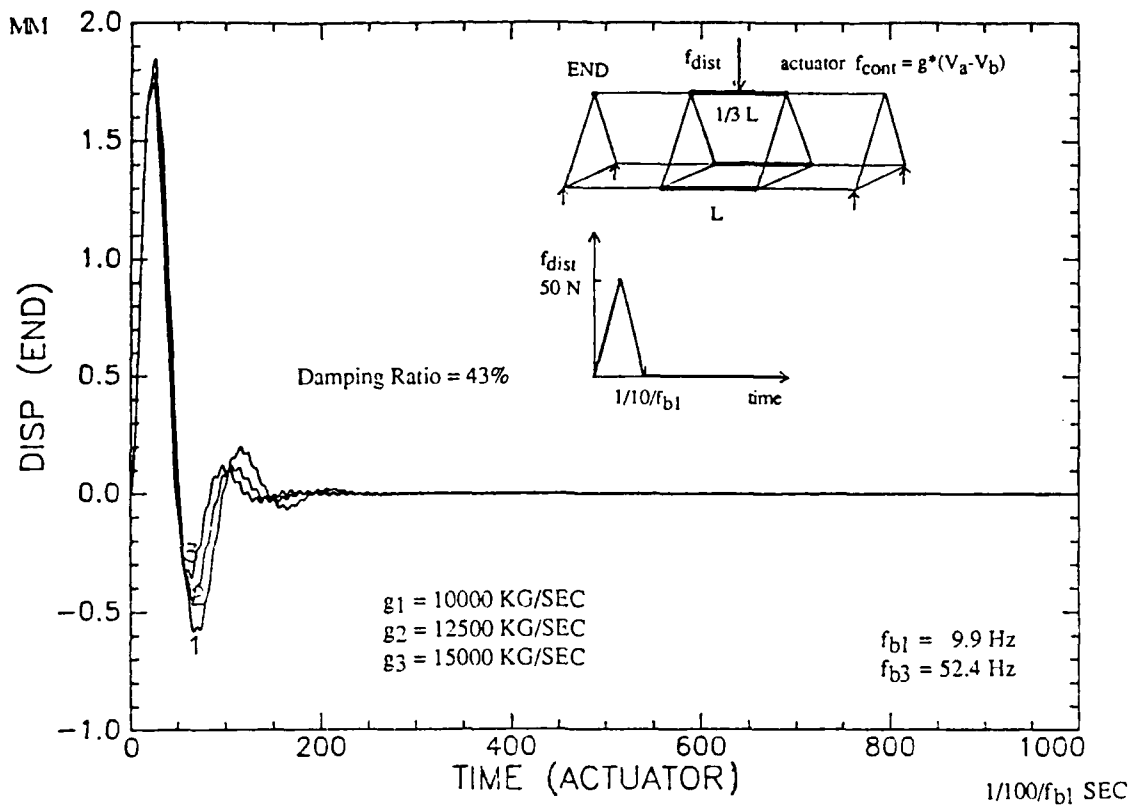
Second Torsional Mode $f_{t2} = 55.5$ Hz



Third Torsional Mode $f_{t3} = 82.9$ Hz

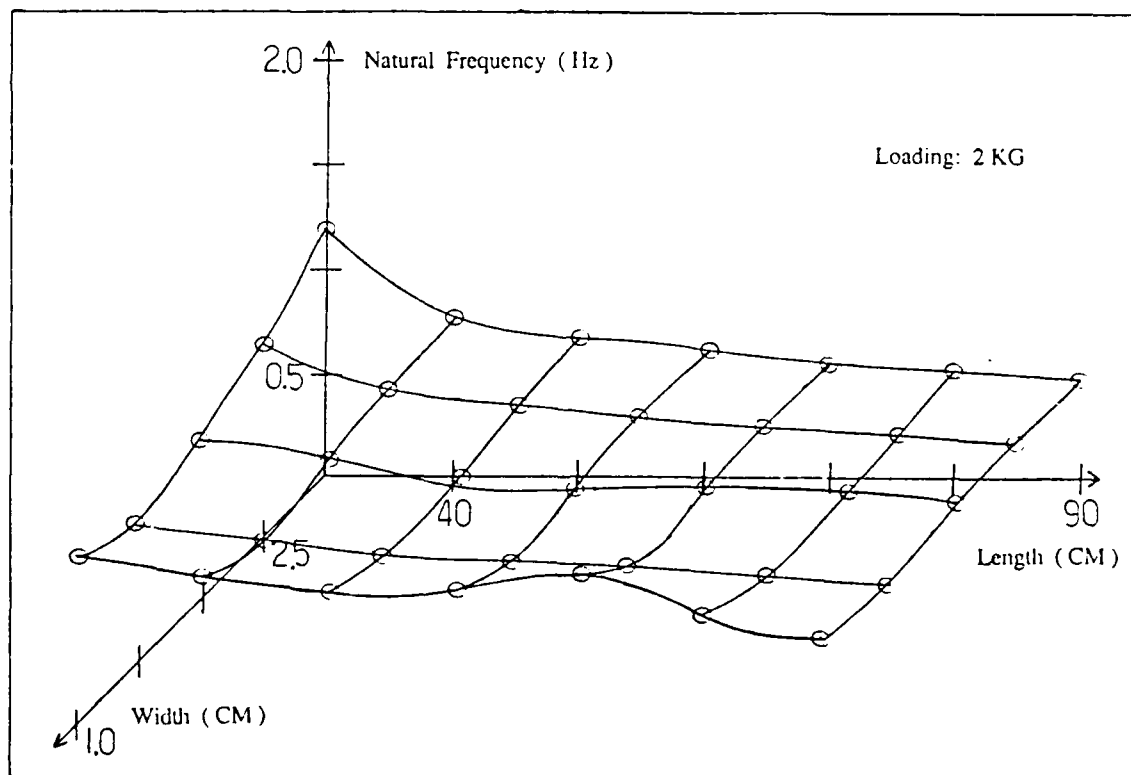
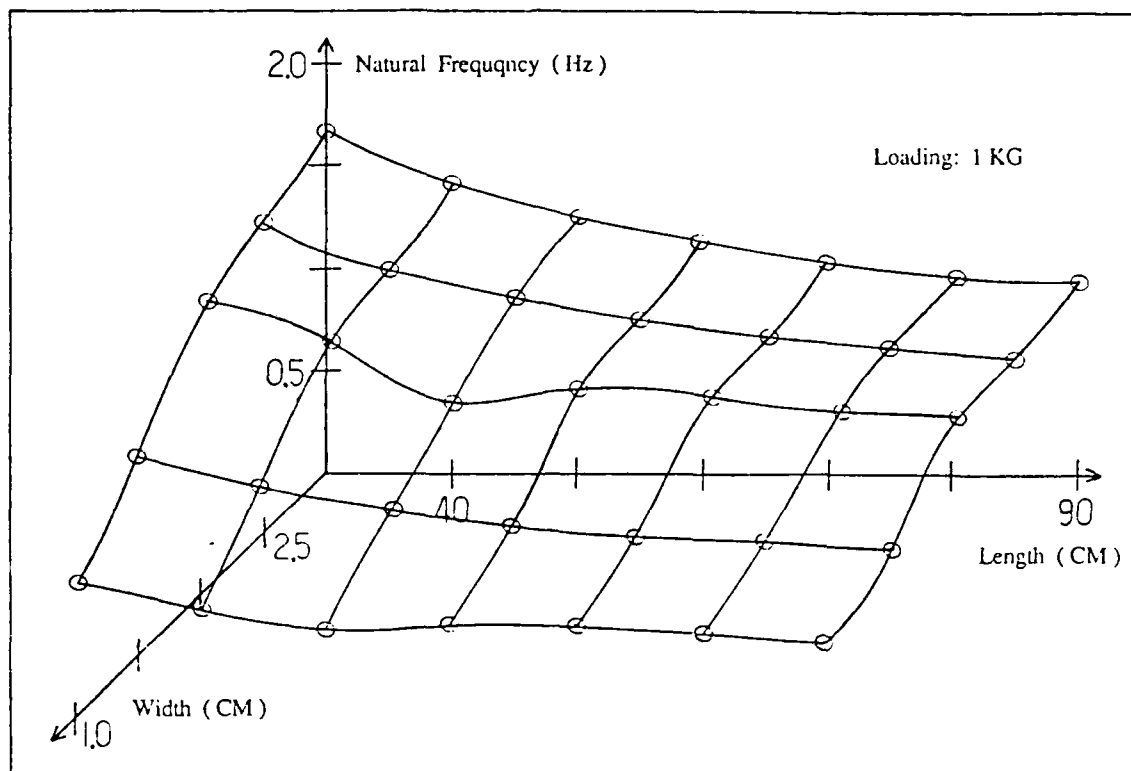
Torsional Vibration Modes of Experimental Space Truss
(no control element)

Figure 3.



Numerical Simulation for the Closed Loop System

Figure 4.



Plot of Natural Frequency vs. Length vs. Width

Figure 5.

PROJECT SUMMARY

Project Title

Actuator Locations for the Modal Control of Large Space Structures

Faculty Leader

Professor James S. Thorp
Electrical Engineering

Graduate Research Assistant

Jin Lu

Other Participants

Professor H. D. Chiang - Electrical Engineering

Executive Summary

The problem of actuator locations in dynamic systems has received considerable attention recently. Methods of finding optimal actuator locations under certain criteria have been presented. However, they are far from enough. A method to find good actuator locations for the modal control for large space structure is presented in this work. This method is based on the interpretation of the connection between an objective function and the actuator locations. The objective function is minimized subject to constraints on the closed-loop mode assignment to obtain a group of good actuator locations. This method avoids choosing good actuator locations combinatorially as some existing techniques do and numerically efficient.

Project Description

Large space structures (LSS) tend to have extremely low-frequency, lightly damped modes which are closely spaced in the frequency domain, making it vulnerable to system vibration. Structural damping can be enhanced actively by using automatic control systems. If the locations of controllers (actuators) are given, then control scheme can be determined by existing control techniques, such as optimal control^[1]. The issue is where to place the controllers appropriately. This problem is especially important from the point of view of reliability, economy and effectiveness. Since the choices of the locations of controllers for a LSS are numerous, this problem is by no means a trivial one.

Efforts have been made to find optimal control locations under some performance criteria^[2,3]. In [2] the optimal locations are chosen combinatorially such that an energy objective function is minimized for modal control. Such a scheme works well for the choice among a limited number of control locations, but is not numerically practical for the

choice among a large number of control locations. In [3], the control matrix B is specified for a minimal energy control problem such that the minimal control energy as the function of B achieves the minimum at the specified B . The question of where the best control locations are, however, is unanswered with this method.

In this paper, multi-input state feedback control is considered. The control goal is to move the poles of the uncontrolled systems into a specified area of the pole plane so as to achieve required dynamical properties. Given the controller number, say k , we wish to find the k control locations such that the feedback gains of the k controllers are as small as possible.

We may think of such control locations as inexpensive control locations.

First, we interpret the connection between the Frobenius norm of the feedback gain and the locations of controllers. The determination of the inexpensive control locations is then expressed as a constrained optimization problem with the Frobenius norm of the feedback gain being the objective function. As pointed out in [4], the specification of closed loop eigenvalues does not define a unique multi-input state feedback control system. Such freedom makes it possible to optimize some functions of the feedback gain subject to constraints.

The way to solve the optimization in this paper is motivated by Roppeneckers' method[5,6] and differs from it as follows. In the case where open loop and closed loop systems have common eigenvalues, we let the open loop and closed loop systems have same eigenvectors associated with the common eigenvalues. The reason of doing this is that (i) if an eigenvalue of the open loop system is not to be changed, then there is usually no need to change the associated eigenmode ($=\exp(\lambda t)v$, where λ is the eigenvalue and v is the associated eigenvector, see[6] for the definition); (ii) if some eigenvectors of the closed loop system are fixed, then the number of variables of the optimization is reduced and less computer processing time for the optimization algorithm is needed.

The technique discussed is applied to a four-bay truss system shown in Fig.1. The interesting control locations are shown in Fig. 2. Suppose that two controllers are used. To modify the lowest frequency mode, the inexpensive control locations are control location 21, 23. To modify the two lowest frequency modes, the inexpensive control locations are control location 12, 23. The results agree with the optimal two control locations (in terms of feedback gain) obtained by exhausting all the interesting groups of two control locations.

References

- [1] David F. Miller, Vipperla B. Venkayya, Victoria A. Tischler, "Integration of Structures and Controls-Some Computational Issues," Proc. of 24th Conf. on Decision and Control.
- [2] Min I. J. Chang, T. T. Soong, "Optimal Controller Placement in Modal Control of

Complex Systems", J. of Mathematical Analysis and Applications 75, 340-358 (1980).

- [3] Ami Arbel, "Controllability measures and actuator placement in oscillatory systems," Int. J. Control, 1980, Vol. 33, No. 3, 565-574.
- [4] B. C. Moore, "On the Flexibility Offered by State Feedback in Multivariable Systems Beyond Closed Loop Eigenvalue Assignment," IEEE Trans. Automat. Contr., Vol. AC-21, No. 5, PP689-692, Oct. 1976.
- [5] G. Roppennecker, "Minimum norm output feedback design under specific eigenvalue areas," Systems & Control Letters 3 (1983) 101-103, North-Holland.
- [6] G. Roppennecker, "On parametric state feedback design," Int. J. Control, 1986, Vol. 43, No. 3, 793-804.

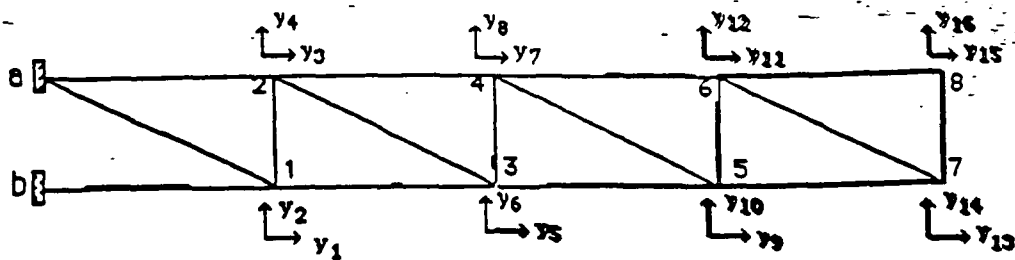


Fig. 1: A four-bay truss

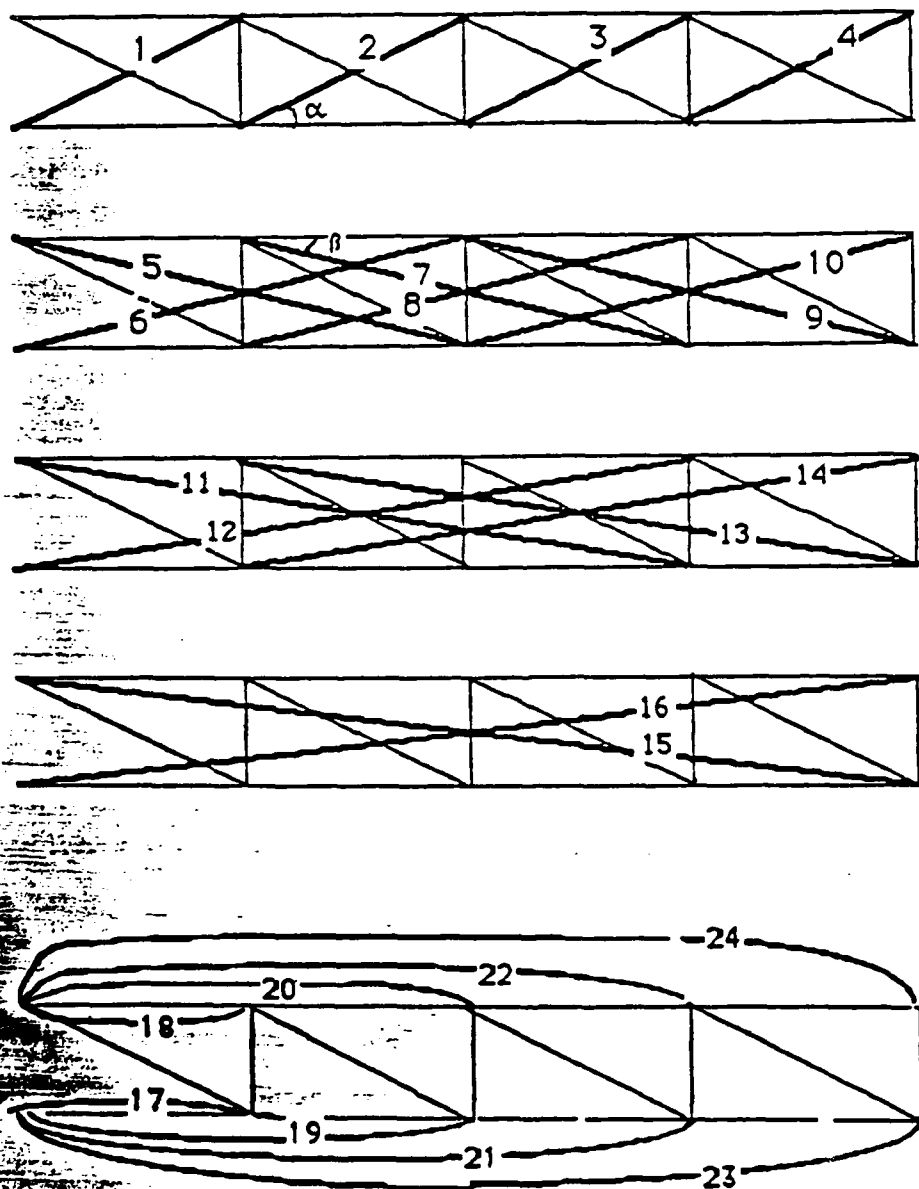


Fig. 2: Twenty four actuator locations of interest

PROJECT SUMMARY

Project Title:

Comparison of Theoretical and Experimental Frequencies of a Frame Structure

Faculty Leader:

John F. Abel, Civil and Environmental Engineering and Program of Computer Graphics

Graduate Research Assistant:

Brian H. Aubert

Other Participants:

**Professor Francis C. Moon, Mechanical and Aerospace Engineering
Dr. G.-X. Li, Postdoctoral Research Assistant, Theoretical and Applied Mechanics**

Executive Summary:

A triangular aluminum structure with welded connections was fabricated for experiments involving active control and chaos. This truss was excited to determine its low frequency modes of vibration. A set of finite element analyses using beam-column and truss elements were run to evaluate the effectiveness of computerized structural simulators. The comparison of the finite element results with the observed frequencies is good.

Project Description:

A series of eigenvalue analyses was run to determine the theoretical vibration frequencies of a three dimensional, twenty-three bay truss. The results of the analyses were compared with the observed frequencies from a structure fabricated by the Department of Theoretical and Applied Mechanics.

The triangular structure, shown in Figure 1, is fabricated from aluminum. The main longitudinal members are tube stock with an outer diameter of 6.35 mm and an inner diameter of 3.91 mm. The vertical and diagonal members spanning between the longitudinal tubes are bar stock with a diameter of 3.18 mm. The average bay length is 156.8 mm. The longitudinal members are 160 mm apart. The modulus of elasticity was

Table 1 - Frequency (Hz.) / % of Experimental

MODE	TYPE	EXPTL.	Numerical Analyses			
			#1	#2	#3	#4
1-6	RB	-	-	-	-	-
7	B	56.7 -	60.9 (106%)	60.1 (106%)	58.0 (102%)	58.0 (102%)
8	B	56.7 -	60.9 (107%)	60.1 (106%)	58.1 (102%)	58.0 (102%)
9	T	77.5 -	88.4 (114%)	84.4 (109%)	85.5 (110%)	82.4 (106%)
10	B	128.5 -	155.6 (121%)	153.2 (119%)	148.5 (115%)	148.3 (115%)
11	B	128.5 -	155.6 (121%)	153.2 (119%)	148.7 (116%)	148.4 (115%)
12	T	147.8 -	176.0 (119%)	166.4 (112%)	170.3 (115%)	162.1 (110%)
13	B	181.2 -	276.4 (152%)	216.2 (119%)	264.6 (146%)	202.7 (112%)
14	B	181.2 -	276.4 (152%)	216.2 (119%)	264.6 (146%)	202.7 (112%)

RB, B and T indicate rigid body, bending and torsion respectively.

- 1 = Ideal truss model, density 0.0027.
- 2 = Ideal frame model, density 0.0027.
- 3 = Ideal truss model, density 0.0026, accurate dimensions and weight.
- 4 = Ideal frame model, density 0.0026, accurate dimensions and weight.

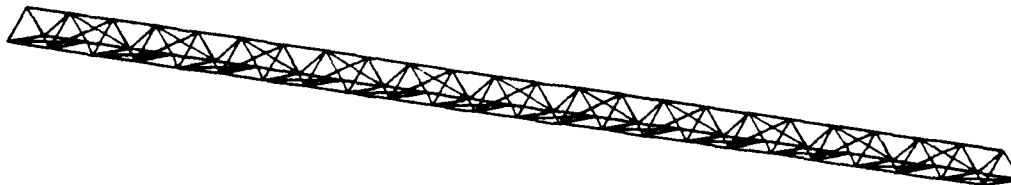


Figure 1 - Structure used in finite element analyses

experimentally determined to be 68.95 Mpa for the bar stock and 66.05 Mpa for the tube stock. The density was experimentally found to be 0.0026 grams per cubic millimeter for both the tube and bar stock. The total structural mass was found to be 1194 g. The joints of the structure are fabricated by welding the vertical and diagonal members to the exterior of the longitudinal members, thus the members are not concentrically connected.

The results of four eigenvalue analyses are presented in Table 1. The results were obtained on a VAX 8300 computer at the Program of Computer Graphics. EISPAC subroutines were used to obtain the solution. The structure was analyzed both as a pin-jointed truss with uniaxial truss elements and as a frame structure with beam-column elements. In all the analyses the joints were assumed to be concentric and the masses were lumped at the nodes. The first two sets of results used dimensions and masses which were later found to be inaccurate. The dimensions used were, average bay length 156 mm, outer tube diameter 6.40 mm, inner tube diameter 4.00 mm, and a bar diameter of 3.2 mm. A tabulated value for the modulus of elasticity of 68.95 Mpa was used for both the tube and bar stock. A mass value of 1021 g was used in the first two analyses. The third and fourth analyses used the more accurate values of the dimensions, moduli of elasticity, and mass.

The experimental results were obtained using a Zonic 6088 multichannel processor. The response of the structure was decomposed in the frequency domain and the structural frequencies were identified. A frequency range of 200 hertz was used. The Zonic has 512 spectral lines so an accuracy of ± 0.2 hz is the best that can be obtained. Since the numerical analyses showed that the bending modes occur in pairs which are virtually indistinguishable, it was assumed that the small difference in the frequencies of the bending modes was not discernable experimentally.

The accuracy of the best finite element calculations, model 4, is good. The frame models, which more closely model the welded joint behavior, perform better than the truss models. If only the lowest mode is needed, either model provides accurate answers. If more frequencies need to be found, clearly a frame model provides better estimates of the frequencies.

The results of the finite element calculations could be improved by taking into account the imperfections in the experimental truss. The eccentric joints, the small variations from the average dimensions in each bay, and the variation in the amount of weld material at each node could be incorporated into the finite element analysis to provide an even more accurate set of frequencies.

PROJECT SUMMARY

Project Title:

Snap Through Analysis of Shallow Arches under Periodic Axial Excitation.

Faculty Leader:

Professor Subrata Mukherjee
Theoretical and Applied Mechanics

Graduate Research Assistant:

Rudra Pratap

Other Participants:

Professor Francis C. Moon
Mechanical and Aerospace Engineering

Executive Summary:

If a straight elastic bar is subjected to some pulsating axial force, lateral vibrations are produced. During these vibrations, at certain frequencies, the external pulsating force may produce positive work resulting in increasing amplitude of the vibrations, a situation known as the condition of instability. It is possible to solve this kind of problem analytically within the framework of linear theories. The corresponding problem of vibration of arches, however, becomes highly complicated due to the introduction of the geometric non-linearity right at the outset. The objective of this study is to formulate and solve the arch vibration problem for both the elastic and elasto-plastic cases allowing for large deformations. The main emphasis of this dynamic analysis is on the snap through phenomenon, symmetric and antisymmetric modes of vibration, a possible interplay of the two modes and the investigation of the range of frequencies for possible chaotic vibrations.

Project Description:

The motivation for this study comes from the vibration of space trusses. In fig.1, AB is part of a space truss which is free to undergo large vibrations. A typical beam member like CD can deform as shown by the dotted line under such motion. This member is likely to experience pulsating excitation from the pin joints at C and D. For analysis, this member can be modelled as a shallow arch subjected to a periodic axial force as shown in fig.2. For simplicity one end of the arch has been hinged to a fixed support. It is proposed to try also a roller support at that end later.

Under the given conditions the beam is most likely to buckle with a possible snap through. The analysis undertaken will include both elastic and plastic buckling. A very preliminary and crude experiment with an aluminium arch has demonstrated the possibilities of symmetric as well as unsymmetric modes of vibration and an interplay between these modes as shown in fig.3.

For the numerical formulation by the finite element method a Galerkin spatial discretization of the equations of motion as referred to the inertial frame is proposed. This has been shown to be very efficient for large overall motions by Simo and Vu-Quoc recently. It is also proposed to do a boundary element formulation for the problem and compare the results for the suitability of the methods. The possibility of chaotic motion of the arch is also under investigation. This project has started very recently and hence the investigators are presently in the stage of literature survey.

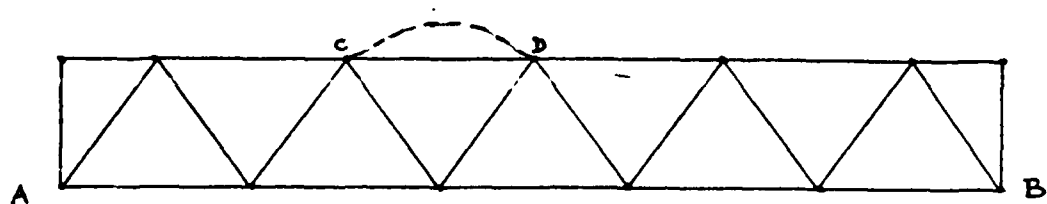


FIGURE 1

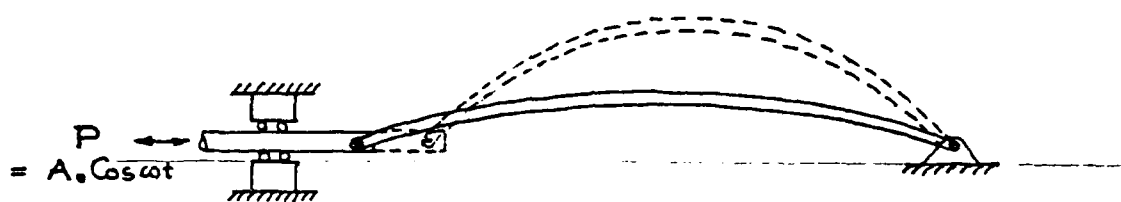


FIGURE 2

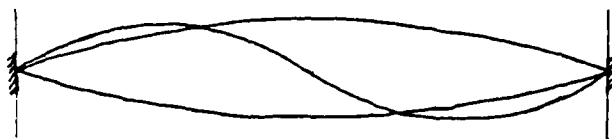


FIGURE 3

PROJECT SUMMARY

Project Title:

Simulation of Collocated Velocity Feedback Control on an Actively Damped Structure

Faculty Leader:

John F. Abel, Civil and Environmental Engineering and Program of Computer Graphics

Graduate Research Assistant:

Brian H. Aubert

Other Participants:

**Professor Francis C. Moon, Mechanical and Aerospace Engineering
P.-Y. Chen, Theoretical and Applied Mechanics**

Executive Summary:

Controllers based on collocated velocity feedback were used in a simulation to provide active damping for a planar truss. Collocated velocity feedback provides a control strategy which requires limited state variable information. The numerical results show that collocated velocity feedback provides relatively low percentages of critical damping. The results indicate that a maximum value of critical damping exists for a given structural configuration. Attempts to increase the amount of damping above the maximum value causes less damping to be present in the system.

Project Description:

Dynamic analyses were performed for a hypothetical two-dimensional, pin-jointed truss. The control method chosen to provide active damping in the structure was collocated velocity feedback. Collocated velocity feedback is simple to implement because the method requires only a small number of state observations. The method uses the relative velocity between two nodes to calculate an opposing control force.

The results of three of the analysis sets are presented. The structure used in the computer simulations, shown in Figure 1, is a sixteen-bay truss. Each bay was modelled as six inches square. The members used were 1/8 inch diameter aluminum rods with a cross sectional

area of 0.0123 square inches and a modulus of elasticity of 10,000 ksi. The mass of the members was lumped at the nodes. The controls were assumed to be applied by actuators capable of both tension and compression. The mass of the actuators was lumped at the exterior control nodes. The excitation of the truss consisted of self-equilibrating loads applied at the center and end nodes. The load history attached to the center node starts with an initial zero force, increases linearly to a maximum value of 100 pounds at 0.0005 seconds and then decreases to zero at 0.001 seconds. The load histories attached to the end nodes each have half the magnitude and opposing directions as the center load history.

The analyses were carried out on a VAX 8700 at the Program of Computer Graphics. A conditionally stable explicit time integration method was used. The time step was chosen to meet the stability limit and to avoid control saturation. The first step in the analysis process was to determine the maximum uncontrolled relative velocity between the desired control nodes. The proportionality constants between the control force and the collocated velocity were based on the the maximum uncontrolled relative velocity between the control nodes. The maximum control forces, varied from 0.01 k to 1.0 k. The analyses assumed linear structural behavior.

The first two sets of analyses each used four controllers located as shown in Figure 1 and Figure 2. The third set of analyses used two controllers as shown in Figure 3. The results of the analyses are presented in Figure 4. In Figure 4 the percentage of critical damping obtained in the fundamental mode is plotted versus the maximum control force.

The location of the controllers has a significant effect on the efficiency of the active control. The percentage of critical damping obtained is relatively low for even the best location. Addition of control effort above a certain force level actually provides a less effective control. The drawbacks of the method must be balanced against the advantages. Since collocated velocity control requires only a very limited observation of the state vector of the system, it is more feasible to implement than methods which require full state feedback. Control based on collocated velocity feedback will not cause any destabilization of a linear system even though limited state observations are made.



Figure 1 - Control locations for the first analysis set

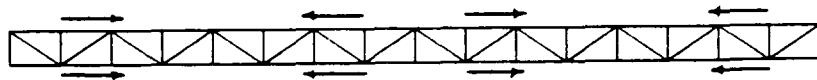


Figure 2 - Control locations for the second analysis set

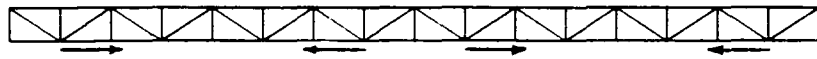


Figure 3 - Control locations for the third analysis set

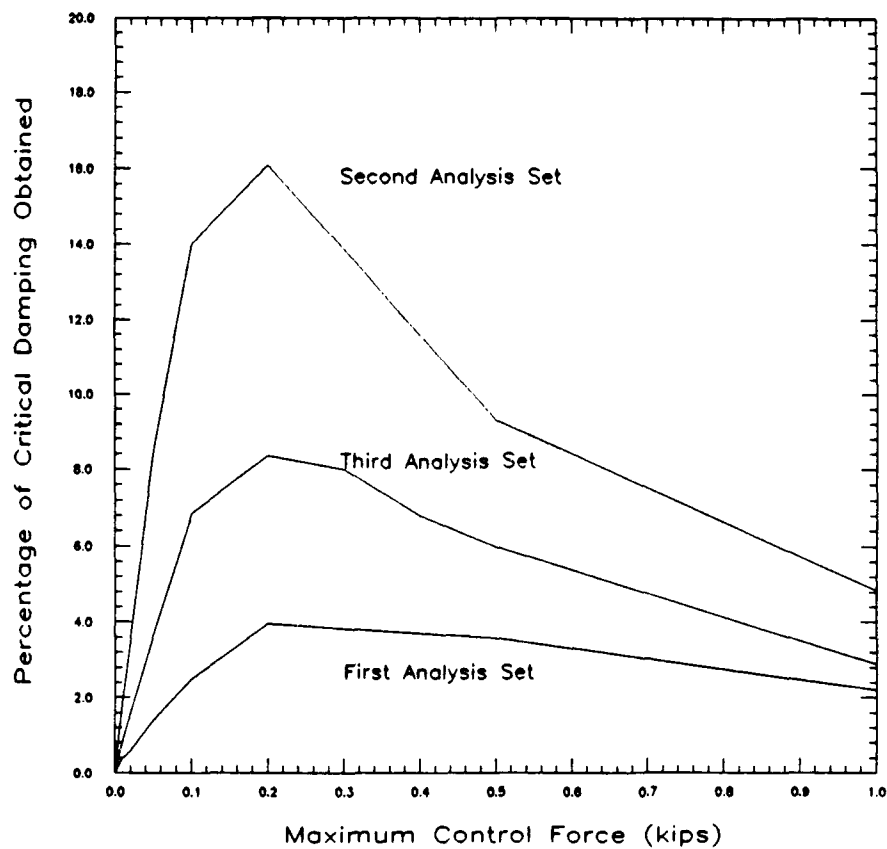


Figure 4 - Results of control based on collocated velocity feedback

PROJECT SUMMARY

Project Title:

On the Numerical Effects of Mixed Linear and Geometrically Nonlinear Structural Modelling

Faculty Leader:

John F. Abel, Civil and Environmental Engineering and Program of Computer Graphics

Graduate Research Assistant:

Brian H. Aubert

Other Participants:

**Professor James Thorp, Electrical Engineering
Professor Christine Shoemaker, Civil and Environmental Engineering
L.-Z. Liao, Operations Research and Industrial Engineering**

Executive Summary:

A simulation of a four-bay truss was conducted to determine the effects of using linear initial conditions for the starting point of a nonlinear analysis. It was found that the mixture of linear initial conditions with the geometrically nonlinear analysis, and vice versa, results in excitation of a broader spectrum of vibration modes than are observed in the consistent simulations. Additionally it was discovered that use of a linear stiffness matrix to calculate the cumulative strain energy of a nonlinear structure results in large errors despite the fact that the linear and nonlinear displacements are only slightly different.

Project Description:

This study consisted of two parts: (1) the effects of employing dissimilarly modelled initial conditions and transient dynamic analyses and (2) the effects of mixed modelling on strain energy evaluations.

Initial Condition Study

The four-bay truss, shown in Figure 1, was analyzed to determine the effects of mixing initial conditions from a linear analysis with a

geometrically nonlinear analysis and vice versa. The truss was modelled as having an typical bay length of 15 inches and a typical height of 7.5 inches. The members were modelled as rectangular steel sections with $AE = 5440$ kips. The 3 members in the bay adjacent to the supports were modelled as having a reduced stiffness, $AE = 6.4$ kips, to allow a more flexible response.

The analyses were run on a VAX 8700 computer at the Program of Computer Graphics. A conditionally stable time integration method was selected for the analysis. The masses were lumped at the nodes. The results are based on dynamic simulations which had initial displacements and velocities and no externally applied loads. Two sets of initial conditions were used. The first set of initial conditions was obtained from a linear analysis of the structure subjected to a suddenly applied vertical force at the lower tip node. The force had a constant magnitude of 0.05 kips and a duration of 0.25 seconds. The first set of initial conditions were obtained from the free vibration response of the linear structure 0.75 seconds after the external load was removed. The second set of initial conditions were obtained for the same instant from a geometrically nonlinear analysis of the same structure subjected to the same load history. In all cases the structure was undamped.

The results of four analyses are examined. In the first case, the initial conditions from the linear analysis were used as a starting point for another linear analysis. The result for the vertical tip velocity of the upper right node is the smooth line in Figure 2. The second analysis used the initial conditions from the linear structure as the starting point for a geometrically nonlinear analysis. The result for vertical tip velocity is the jagged line in Figure 2. The third analysis used the initial conditions from the geometrically nonlinear analysis as the starting point for a geometrically nonlinear analysis. The result for the vertical tip velocity is the smooth line in Figure 3. The final analysis used the nonlinear initial conditions as a starting point for a linear analysis, and the result for the vertical tip velocity is shown as the jagged line in Figure 3. The velocities for the cases where linear and nonlinear conditions are not mixed are compared in Figure 4. The displacements agree even more closely than the velocities. It is to be noted that although there is some difference in the amplitudes of the linear and nonlinear velocities, there is virtually no difference in the periodicity of the response.

Results from mixed simulations with small percentages ($\approx 1\%$) of critical damping exhibit high-frequency responses similar to those in Figures 2 and 3 which do not persist in the long-term response.

One significance of the results of this study is that perceptively small perturbations in the initial conditions can disproportionately excite higher modes of lightly damped systems. Since any control force history may be viewed as a time series of superimposed initial conditions, imprecise controls may have the potential of

disproportionate excitation.

Strain Energy Study

The second aspect of mixing linear and nonlinear assumptions deals with the calculation of strain energy. The calculation of a cumulative relative strain energy may be used in optimal control schemes as one factor in an objective function. The objective function is used to evaluate the performance of the control by comparing uncontrolled and controlled responses of the structure.

The results from four loaded trusses show an enormous discrepancy between strain energies calculated by consistent and inconsistent methods. The first truss is the truss used in the initial conditions study. The second truss has the same dimensions as the first but includes the more flexible members in all the horizontal and diagonal positions. The third truss is a three-dimensional truss supported at both ends. The fourth truss is a two-dimensional truss which is unsupported and subjected to self-equilibrating loads. The third and fourth trusses were simulated with a constant AE value for all the members. The results of three types of strain energy calculations are shown in Table 1. The first case uses the linear stiffness matrix and the linear displacements to calculate the cumulative strain energy. The second case uses the geometrically nonlinear stiffness matrix and the geometrically nonlinear displacements to calculate the cumulative strain energy. The third, inconsistent case uses the linear stiffness matrix and the geometrically nonlinear displacements. A comparison of the linear and nonlinear tip displacements for the first truss is shown in Figure 5. There is a small phase shift and a small change in amplitude between the two curves.

Table 1 - Relative Cumulative Strain Energy

Case	<u>Consistent</u>		<u>Inconsistent</u>
	$\{u\}_L^T [K]_L \{u\}_L$	$\{u\}_{NL}^T [K]_{NL} \{u\}_{NL}$	$\{u\}_{NL}^T [K]_L \{u\}_{NL}$
1	6,720	6,480	4,130,000
2	4,320	4,210	967,400
3	1,920,000	2,000,000	3,180,000
4	1,580	1,580	2,240

[K] = stiffness, {u} = displacement, L = linear, NL = nonlinear

In the calculation of the cumulative strain energy of a structure which is geometrically nonlinear by a small degree it is tempting to use the linear stiffness matrix to economize on computations. The results in Table 1, which have been independently verified, indicate that this is a poor choice. The strain energy used in an objective function should be calculated from a consistent set of linear or nonlinear quantities.

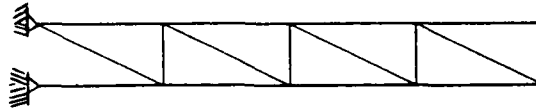


Figure 1 - Structure used in the analyses

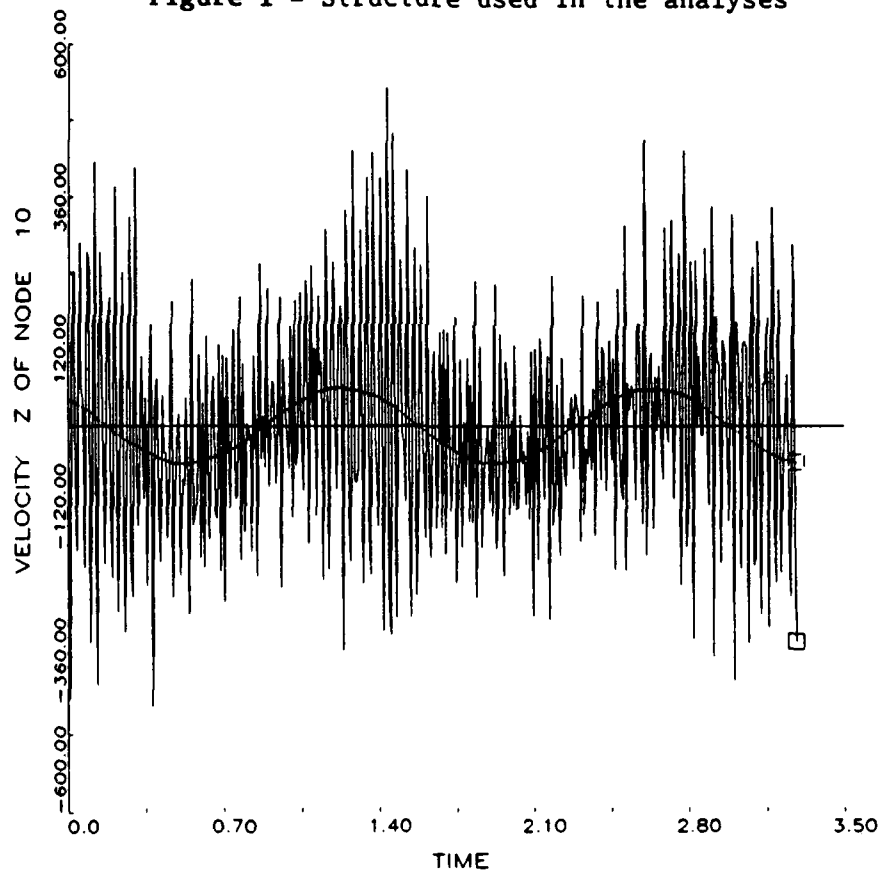


Figure 2 - Linear versus nonlinear velocities, linear initial conditions

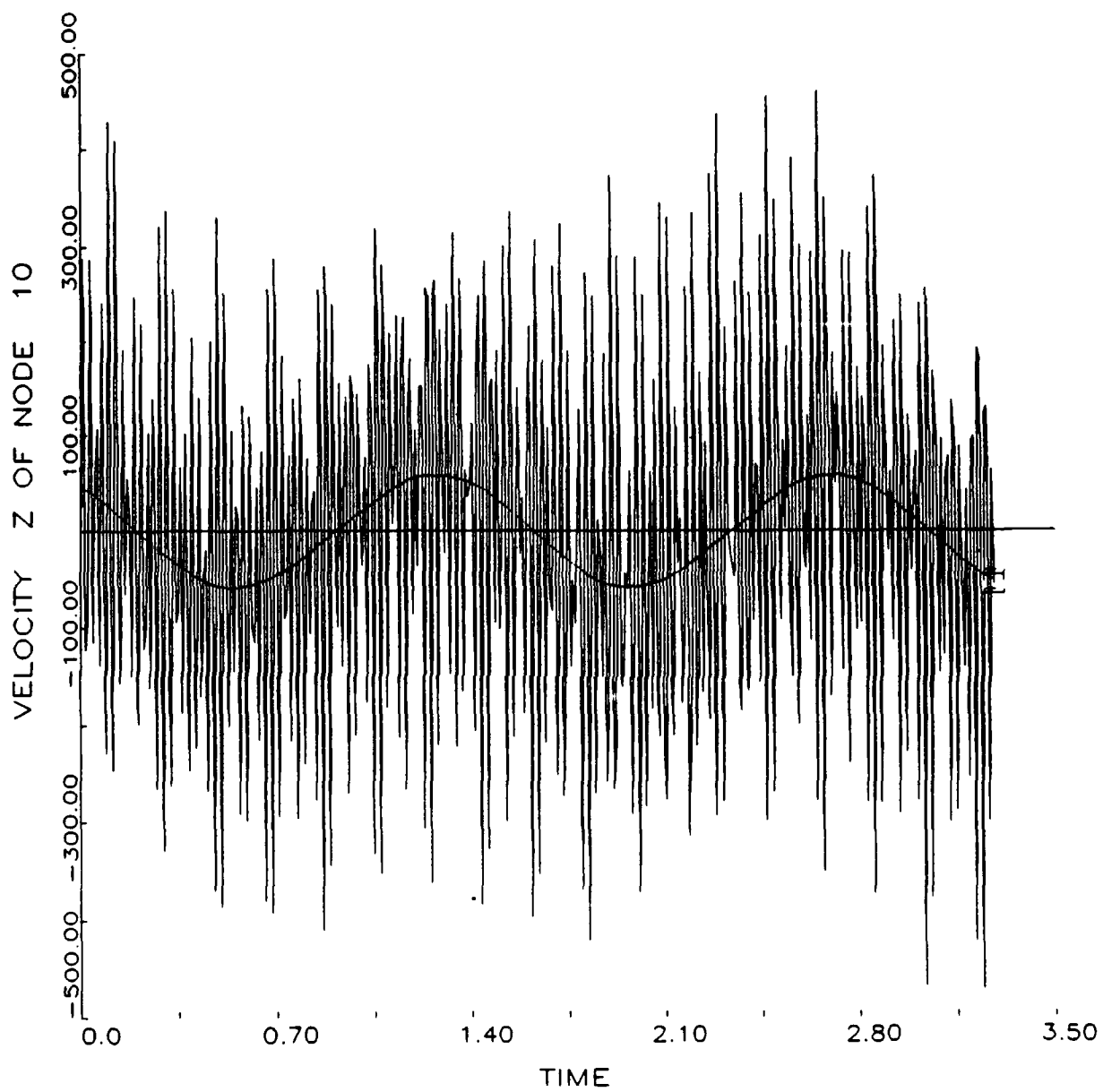


Figure 3 - Linear versus nonlinear, nonlinear initial conditions

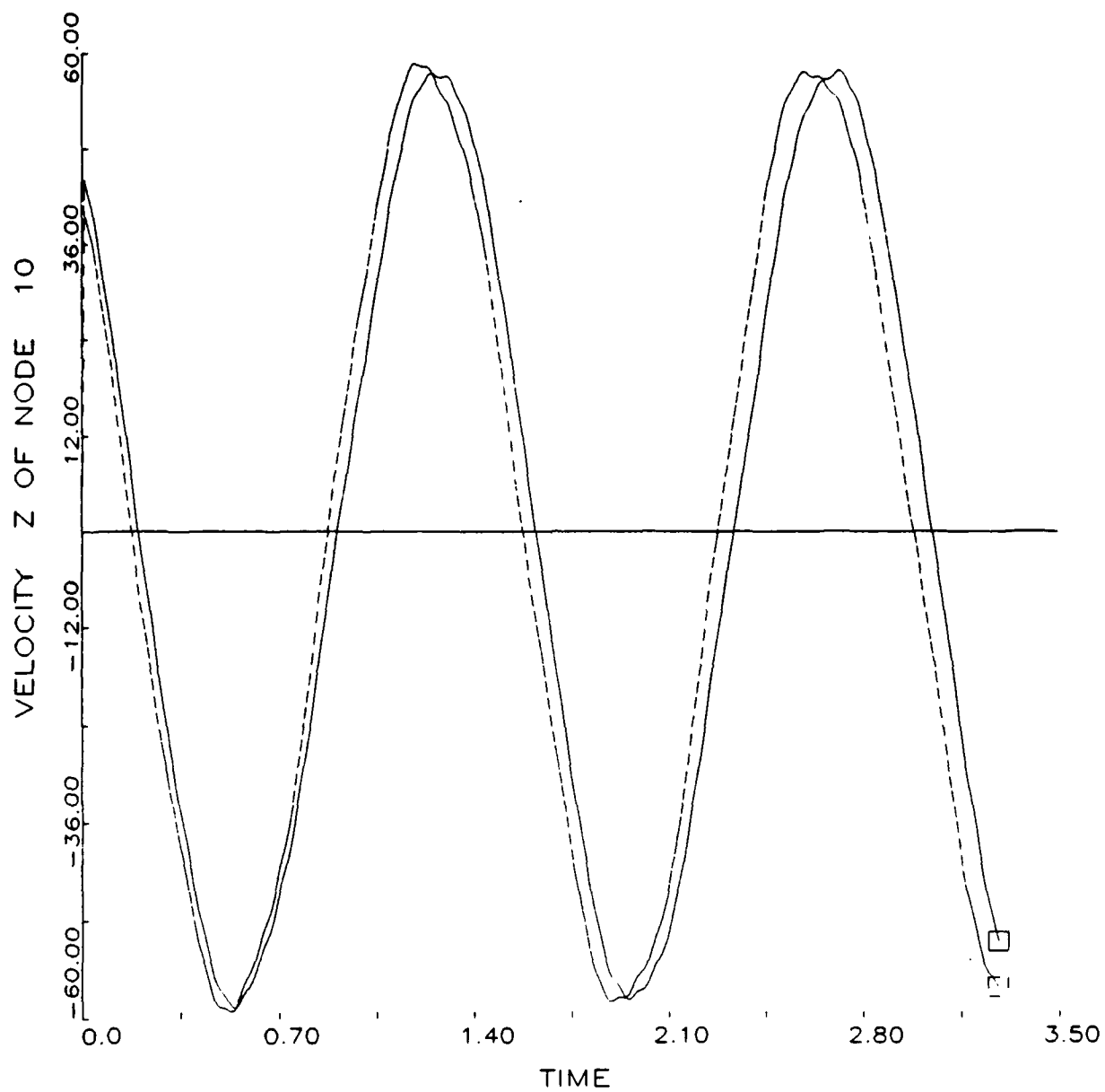


Figure 4 - Linear versus nonlinear, consistent initial conditions

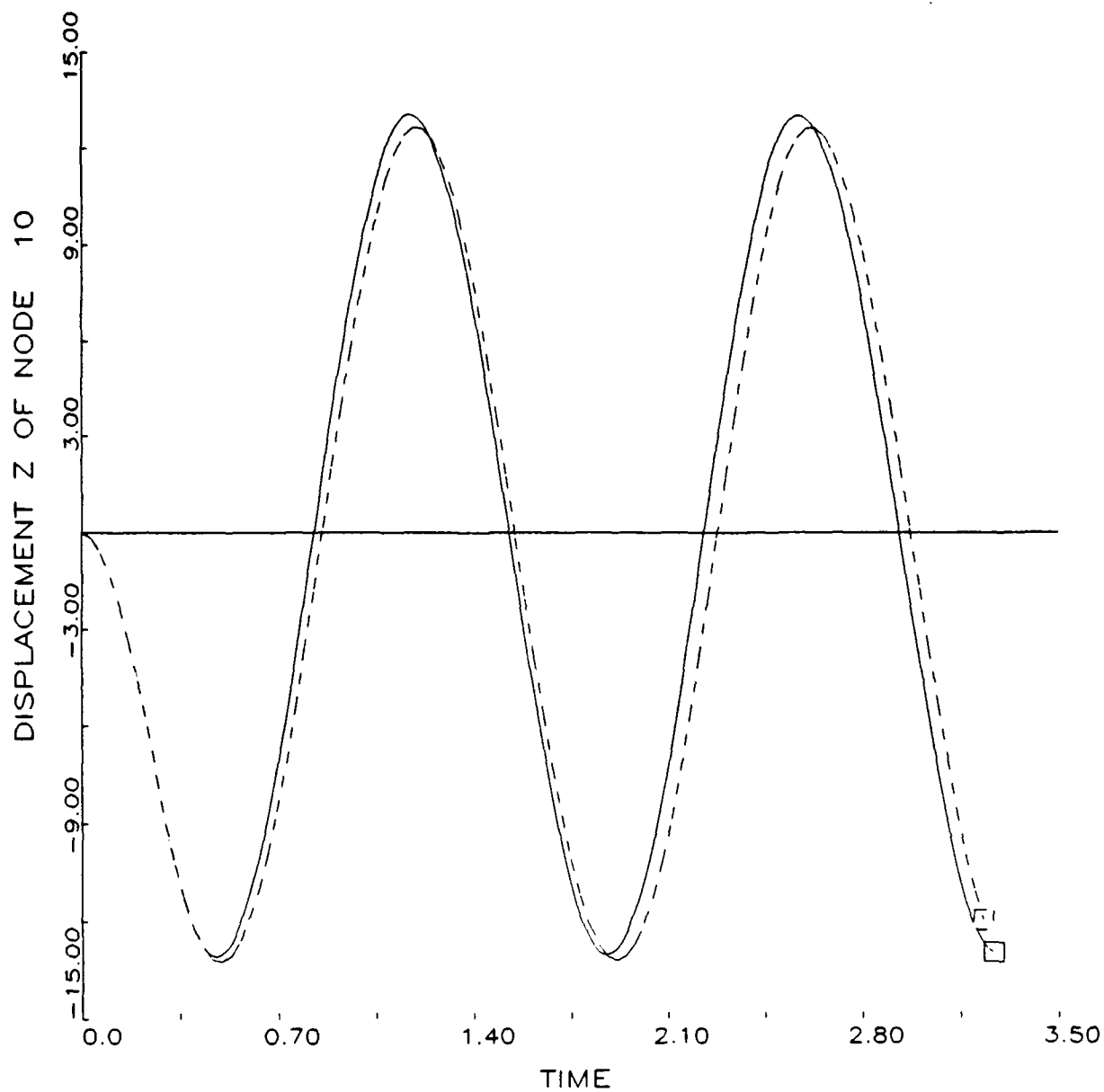


Figure 5 - Linear versus nonlinear tip displacements

PROJECT SUMMARY

Project Title:

Improving the Convergence and Computational Efficiency of Differential Dynamic Programming for Large-Scale Nonlinear Dynamical Systems

Faculty Leader:

Christine A. Shoemaker, School of Civil and Environmental Engineering

Graduate Research Assistant:

Li-zhi Liao, School of Operations Research and Industrial Engineering

Other Participants:

None

Executive Summary:

We are using the nonlinear control algorithm Differential Dynamic Programming (DDP) to compute the optimal control of nonlinear flexible structures. The DDP algorithm computes the optimal control iteratively, but the method is computationally feasible for large scale systems only if it converges in a relatively few iterations. Our theoretical investigations have proven that DDP converges quadratically if the C_i matrices computed in the DDP algorithm are positive definite. We have shown that the C_i will always be positive definite if the objective functions are all strictly convex and the transition functions are all linear. However, if the transition functions are nonlinear (which is the case for a nonlinear structure), we show that the C_i cannot be guaranteed to be positive definite even if both the objective function and transition functions are convex. To overcome these problems, we have developed an "active shift" method to improve the convergence properties for situations with C_i that are not positive definite. Numerical results are presented that illustrate the advantage of the active shift method over the no shift or constant shift methods that have been used previously.

Description of Project:

Because we have been able to prove that for a wide range of nonlinear dynamical systems, the DDP algorithm will not converge, we have mathematically identified when positive definiteness may disappear and have developed a numerical active shift method to overcome the problem. The active shift method converges linearly until it becomes close enough to the optimum to shift into a different mode, at which time the convergence begins to be quadratic.

The use of the active shift method is very important for applications to large-scale problems including those arising in the control of nonlinear flexible spacecraft. In our

experience we have found that for some nonlinear problems, the DDP algorithm will not converge at all. In other cases, the number of iterations will be much larger without the use of the active shift method. Given the size of the problems arising in control of nonlinear flexible spacecraft, the computational cost per iteration of DDP is sufficiently high that it is necessary to reduce the number of iterations as much as possible. Hence the identification of the mathematical characteristics of problems where the DDP without shift will not work and the development of the active shift method is important in computing optimal control of nonlinear flexible structures. We have found the active shift method to be most robust if it is preceded by several iterations of DDP with the constant shift.

Table 1 illustrates the advantage of the active shift method over the no shift or constant shift options. The example problem solved is:

$$\begin{aligned} \text{Min } & \sum_{t=1}^{20} (x_t - \sin u_t)^2 \\ & x_{t+1} = x_t^2 \sin u_t - x_t \cos u_t \quad t = 1, \dots, 19 \\ & x_t \text{ and } u_t \text{ are all scalars.} \\ & x_1 \equiv 1 \end{aligned}$$

In which x_t is the state variable and u_t is the control variable.

The optimal solution to this example has an objective function value of zero. The fourth column of Table 1 indicates that using the active shift after a few iterations of the constant shift method results in a much lower value of the objective function in 9 iterations than the constant shift from a starting point of $u_t = 0$. From a starting point of $u_t = 1$, the DDP algorithm stops long before reaching an optimal value. Hence the DDP with an active shift both speeds convergence and prevents the cessation of the algorithm due to lack of positive definiteness in the C_t matrices.

Table 1. Computational Results for the Example

	Objective Value			
	No Shift	Constant Shift $\varepsilon^c=2$	Constant Shift $\varepsilon^c=2$ for 5 iter. then Active Shift $\delta=1.0$	No Shift
No. Iter.	Initial $\bar{u}_t=0.0$	Initial $\bar{u}_t=1.0$	Initial $\bar{u}_t=1.0$	Initial $\bar{u}_t=1.0$
0	0.2000×10^2	0.1316×10^2	0.1316×10^2	0.1316×10^2
1	0.2518×10^{-1}	0.7691×10^1	0.7691×10^1	0.1145×10^2
2	0.4892×10^{-2}	0.3834×10^1	0.3834×10^1	0.1088×10^2
3	0.1266×10^{-2}	0.1755×10^1	0.1755×10^1	$(0.1030 \times 10^2)^*$
4	0.4040×10^{-3}	0.7967×10^0	0.7967×10^0	
5	0.1380×10^{-3}	0.3580×10^0	0.3580×10^0	
6	0.5583×10^{-4}	0.1610×10^0	0.9174×10^{-1}	
7	0.2543×10^{-4}	0.7216×10^{-1}	0.4995×10^{-7}	
8	0.1291×10^{-4}	0.3231×10^{-1}	0.4243×10^{-7}	
9	0.7156×10^{-5}	0.1447×10^{-1}	0.4243×10^{-7}	
10	0.4271×10^{-5}	0.6489×10^{-2}	optimal	
11	0.2681×10^{-5}	0.2917×10^{-2}		
12	0.1775×10^{-5}	0.1316×10^{-2}		
13	0.1203×10^{-5}	0.5965×10^{-3}		
14	0.8566×10^{-6}	0.2726×10^{-3}		
15	0.6123×10^{-6}	0.1259×10^{-3}		
20	0.1755×10^{-6}	0.3694×10^{-5}		
	stopped by user	stopped by user		

*: Automatically stopped because the Hessians are not positive definite in this process.

PROJECT SUMMARY

Project Title:

Nonlinear Analysis of Large Flexible Space Structure with Linear Controllers

Faculty Leader:

Dr. Hsiao-Dong Chiang - Assistant Professor
Electrical Engineering

Graduate Research Assistant:

Jin Lu and Lazhar Fekih-Ahmed

Other Participants:

Professor James S. Thorp - Electrical Engineering

Executive Summary:

The linear controllers designed on the basis of the linearized system of the LFSS may de-stabilize the original system which is a nonlinear system. This is due to the fact that the stability regions of nonlinear systems are changed with the introduction of controllers. Qualitative analysis of the change of the stability regions associated with the LFSS using linear controllers and co-location linear controllers are undertaken in order to assess the effectiveness of different linear controllers from the state-space point of view.

Project Description:

Nonlinear analysis of the LFSS with and without linear controllers are performed. These include: (a) qualitative analysis of system trajectory, (b) some topological and dynamical properties and (c) characterization of stability region.

The relationship between the stability region of the LFSS without controllers and the stability region of the LFSS with co-location linear controllers is under development. Simulation study is also underway to examine the theoretical results.

An attempt is also made to establish the relationship between the stability region of LFSS without controllers and that with linear controllers.

Published Papers:

- [1] H. D. Chiang, J. S. Thorp and J. Lu, "Nonlinear Analysis of Large Flexible Space Structure with Linear Controllers", in preparation.
- [2] H. D. Chiang, "Global Analysis of Nonlinear Dynamical Systems by a V-Function", to be submitted to Automatica for publication.

PROJECT SUMMARY

Project Title:

Use of Stability Domain Distortions to Determine Effectiveness of Linear Feedback Control.

Faculty Leader:

Professor James S. Thorp
Department of Electrical Engineering.

Graduate Research Assistant:

Syed Akbar Hasan Naqavi.

Other Participants:

None.

Executive Summary:

For purposes of modal control, Large Flexible Space Structures, on account of their high dimensionality, are modelled as linear systems. Design methods available in the literature on issues dealing with linear time-invariant systems can then be made full use of to design linear state-feedback controllers employing velocity feedback for these space structures. Controllers designed using this approach work well for as long as the linear model is valid, and are expected to be, at worst, less efficient otherwise. However, computer simulations and other analytic methods applied to the true model of at least one space structure have shown that its inherent nonlinearities cause the linear feedback controller to actually severely degrade the overall system performance. Based on suspicions that the introduction of the controller was causing the stability domain of the uncontrolled system to shrink, and that the extent of shrinkage was also related to the controller gain, it was decided to study a lower-order system (described below) with similar characteristics to confirm or deny their validity. These studies have shown that the phenomenon of stability domain shrinkage of large space structures due to linear feedback is indeed possible.

Description of Project:

The low-order system studied is shown in Figure 1. It is a simple structure consisting of two members that obey the same nonlinear force law that is obeyed by

the members of the actual space structure. It has two degrees of freedom, and two normal mode frequencies of oscillation. The lower frequency is related to transverse oscillations (or oscillations in the x -direction) of the structure. It is desired to suppress these oscillations by means of a controller that will damp out the low-frequency modes. Although the system is described by two coupled nonlinear second-order differential equations, the controller is designed on the basis of the linearized model. This controller is then applied to the true nonlinear system and its effects on system performance are studied with the help of computer simulations. These effects are studied in terms of changes in the stability domain of the system in the x - x' phase plane due to the application, and changes in the gain of the feedback controller. The results for two controller designs, velocity-feedback and colocated, are shown in figures 2 and 3 respectively. The black regions in both figures represent the stability domain of the desired one of the system's two stable equilibria, while the grey region represents that of the other. The white region in Figure 2 shows the set of initial conditions for which the system performance is severely degraded by the controller. Although not shown in the figure, this region was found from other simulations to be very striated, with a very dense set of alternate black and grey lines running down the length of the band. The cross-hatched region represents initial conditions for which the fixed duration simulations did not yield conclusive results regarding which stable equilibrium the system tended to converge to, but in all probability it coincides with the white region. Notice how the reduction of the controller gain, shown as percentages of γ , the gain required to achieve critical damping of the low-frequency modes, causes the stability domain to get bigger and the white band to get pushed out of it. Notice also how a comparison of the stability domains for both types of controllers can be used to compare their relative effectiveness.

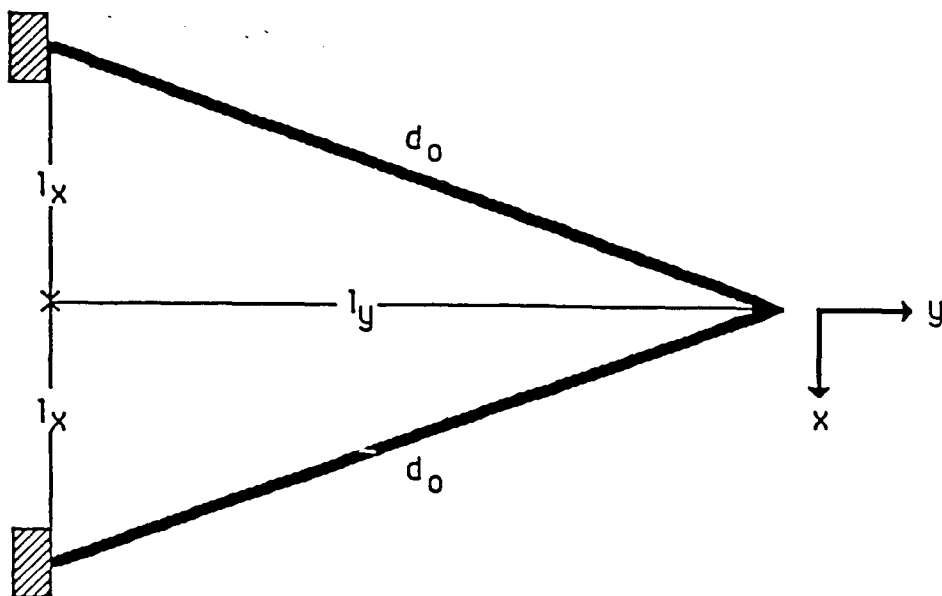


FIGURE 1: The Elementary Flexible-Joint Structure

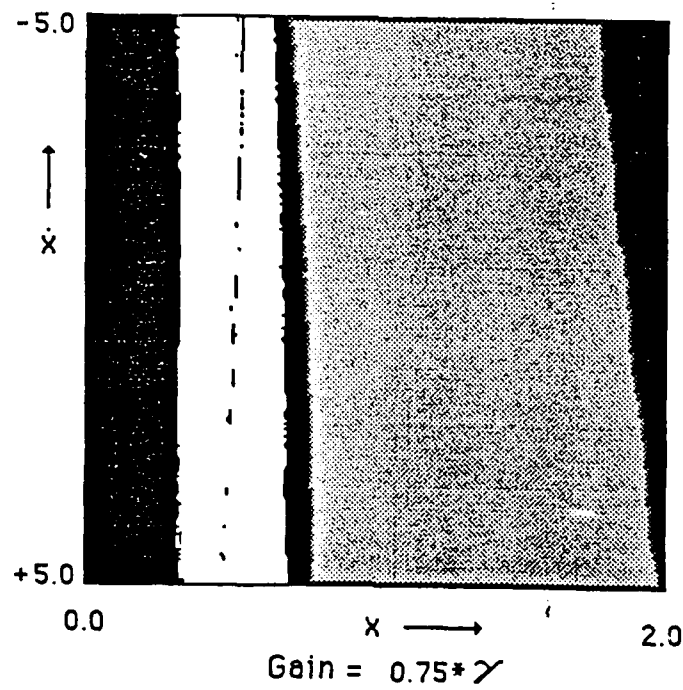
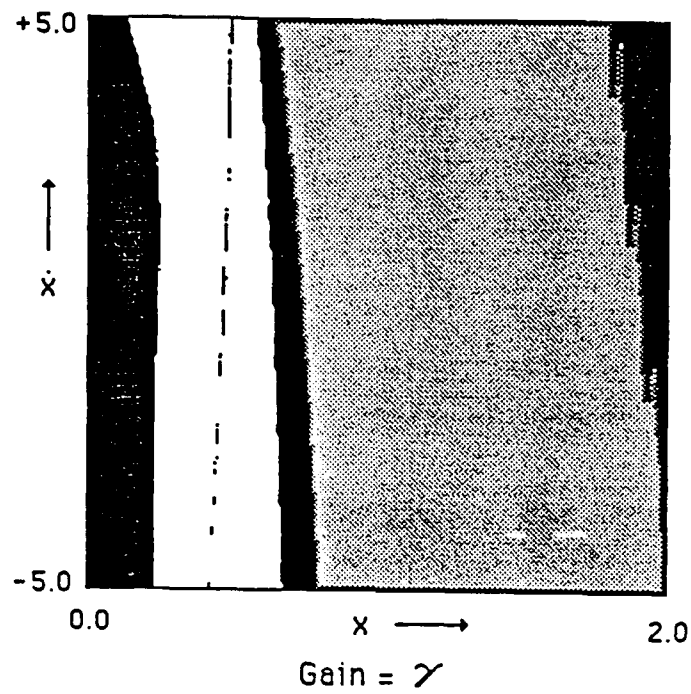
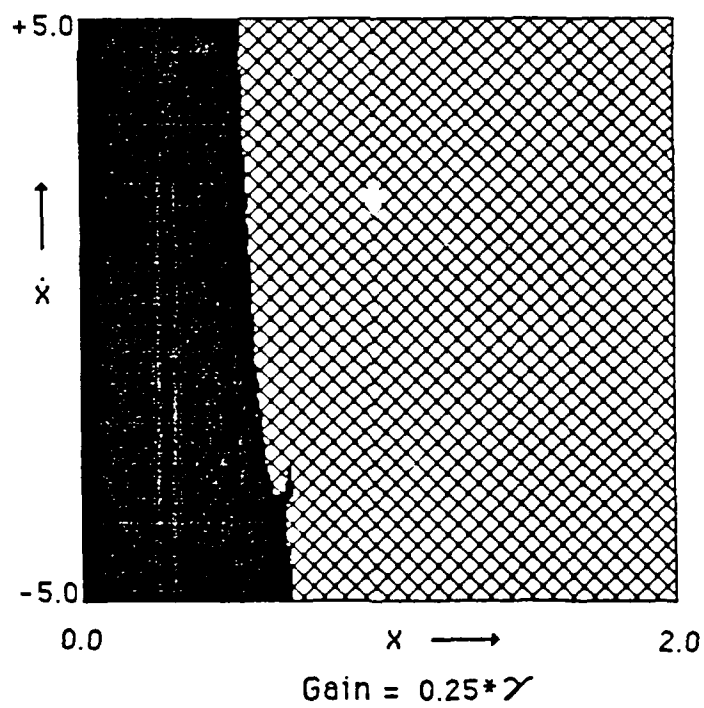
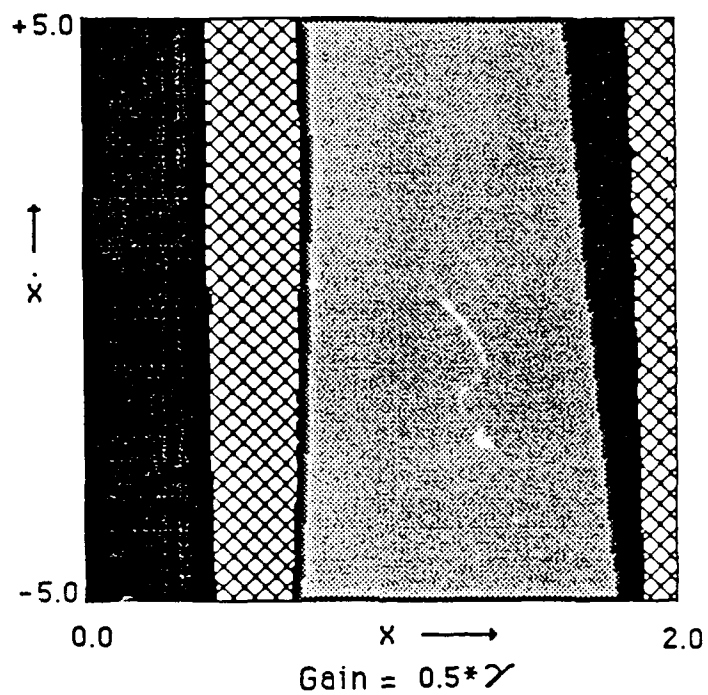


FIGURE 2: Effect of reduction in controller gain on stability domain

FIGURE 2 (Continued)



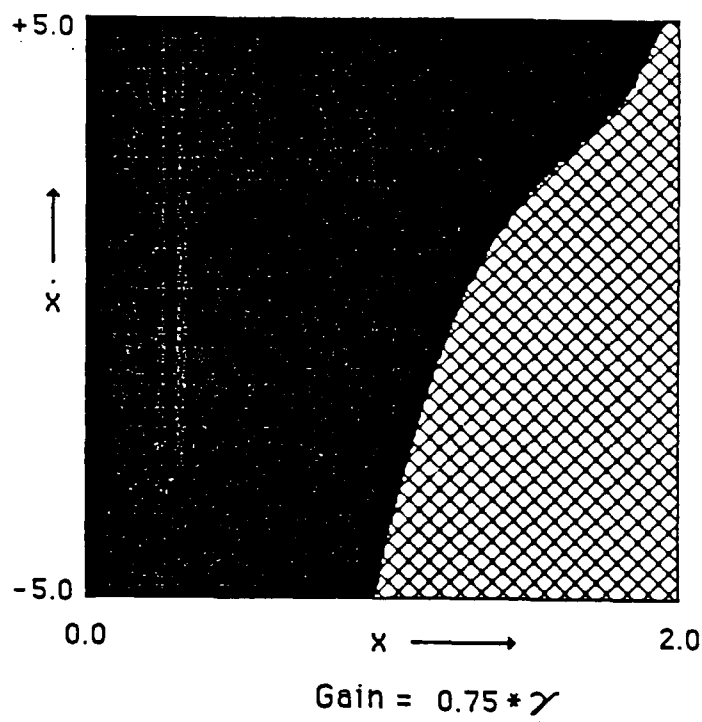
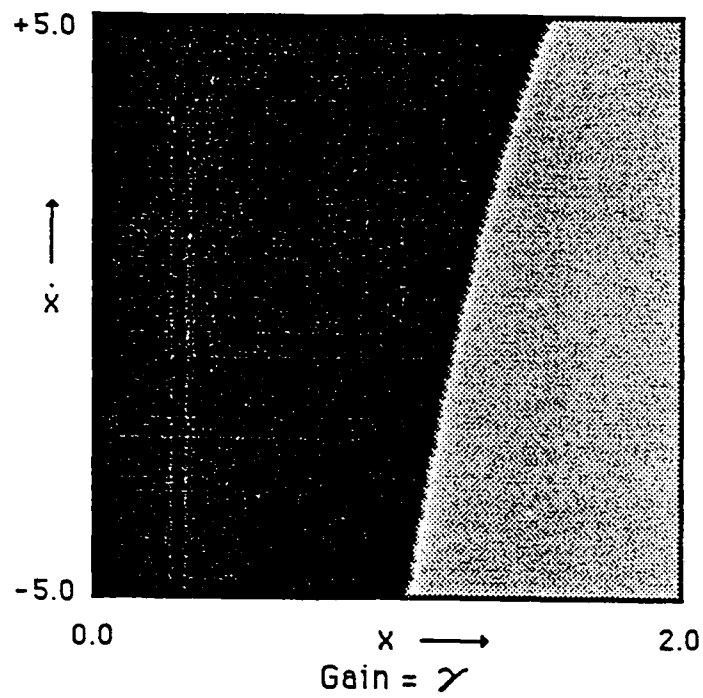


FIGURE 3 : Effect of colocated control on stability domain

PROJECT SUMMARY

Project Title

Partial Eigenstructure Assignment and its Application to the Modal Control of Large Space Structures

Faculty Leader

Professor James S. Thorp
Electrical Engineering

Graduate Research Assistants

Jin Lu

Other Participants

Professor H. D. Chiang-Electrical Engineering

Executive Summary

The modal control of Large space structures is featured by (1) the systems are so large that existing modal control techniques may confront numerical difficulties; (2) only part of the modes (some lowest frequency modes) are modified and the rest of the modes are left unchanged. Recently, methods have been presented to deal with this problem. The idea is to take the advantage of the feature (2) by projecting the original large system onto a subspace associated with the modified mode. The new projected system has the same dimension as the number of the modified eigenvalues. Feedback gain for the modal control of large space structure can be determined by considering this new system. Our contribution is to incorporate the closed-loop eigenvector assignment for large space structures in the modal control with these methods. This is important because the "shape" of the transient response to the disturbance depends on the closed-loop eigenvectors.

Description of Project, Progress and Results

The problem of eigenstructure assignment (simultaneous assignment of eigenvalues and eigenvectors) via linear state feedback control in a linear multivariable system (with n states and m control inputs) is of great importance in control theory and application. The control of the system behaviour is achieved by assigning a certain set of eigenvalues and an associated set of eigenvectors and generalized eigenvectors to the closed-loop system. In general terms, the speed of response of the closed-loop system is determined by the assigned eigenvalues whereas the "shape" of the transient response depends on the associated eigenvectors and generalized eigenvectors. This problem has received considerable attention recently[1]-[10]. The approaches taken in the existing eigenstructure assignment techniques can be summarized

briefly as follows. For a set of n desired self-conjugate closed-loop eigenvalues, n linearly independent vectors (which turn out to be the closed-loop eigenvectors and generalized eigenvectors) are selected from the n admissible subspaces associated with the n closed-loop eigenvalues, with no two belonging to the same subspace, and a feedback gain matrix is formed from the set of n closed-loop eigenvalues and the n linearly independent vectors, which gives the desired eigenvalue-eigenvector sets. The existence of the above-mentioned n linearly independent vectors is discussed in [1],[2].

In many realistic situation, what is desired is to modify part of the open-loop eigenvalues and leave the rest of the open-loop eigenvalues unchanged. This is called partial eigenvalue assignment[15]. This arises, for example, in large space structure control problems[16]. In doing this, in order to control the shape of the part of the transient response corresponding to the unchanged eigenvalues, we wish to preassign the closed-loop eigenvectors associated with the unchanged eigenvalues before assigning the modified eigenvalues (and, if we like, the associated eigenvectors). We refer to this as partial eigenstructure assignment. To apply the existing eigenstructure assignment techniques to the partial eigenstructure assignment, a problem needs to be clarified. It is stated in [1], [2] that under some conditions, it is possible to select n linearly independent vectors from the n admissible subspaces, with no two belonging to the same subspace. We would like to show that under the similar conditions, given k ($< n$) linearly independent vectors from k of the n admissible subspaces, with no two belonging to the same subspace, it is possible to select $n-k$ vectors from the other $n-k$ admissible subspaces, with no two belonging to the same subspace, such that all the n vectors are linearly independent. It will be clear later that what we would like to show implies the statement in [1], [2], but not vice verse. A parametric expression of the feedback gain matrix for partial eigenstructure assignment is given based on this result.

Applying existing eigenstructure assignment techniques to partial eigenstructure assignment for large scale systems may cause numerical problems due to the large dimension matrix computation. Motivated by this, an algorithm for partial eigenstructure assignment is presented in this paper. With this algorithm, the system is projected onto a subspace of lower dimension, and the feedback gain matrix is determined by computing matrices of smaller dimension. Methods of this kind are especially important in the optimization of feedback gain matrix under certain performance index since feedback gain matrix is calculated many times during the optimization.

Finally in this paper, the partial eigenstructure assignment is applied to the modal control of large space structures to achieve the optimal control in certain sense.

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PROJECT SUMMARY

Project Title:

Dynamics of Lattice Structures with Symmetry

Faculty Leader:

Professor T.J. Healey
Theoretical and Applied Mechanics

Graduate Research Assistants:

J. Treacy

Executive Summary:

We are developing and implementing several techniques for the efficient analysis of large flexible space structures. Our initial efforts were directed at group-theoretic techniques for linear eigenvalue problems associated with symmetric lattice structures. More recently we have been studying the problem of relative equilibria of rotating structures and their stability and bifurcation.

Project Description:

I. Linear eigenvalue problems of skeletal structures with symmetry. A group-theoretical approach.

Most man-made structures have some degree of symmetry and repetivity, especially large lattice structures appropriate for deep-space applications. In this project we have implemented some techniques based on a group representation theory in a novel computational setting. Once the symmetry of the structure has been identified, our methodology a-priori identifies invariant subspaces (dependent only on the particular symmetry type) along which the full eigenvalue problem *decouples* into several smaller eigenvalue problems.

For example, we have analyzed the antenna structure, depicted in Figure 1, and the lattice beam shown in Figure 2. The antenna has 57 degrees of freedom (dof). We computed all frequencies and mode shapes of the structure by solving 6 smaller problems: a 3 dof, a 4 dof, a 5 dof, a 7 dof, a 9 dof, and a 10 dof problem. The beam has 117 dof. Its frequencies and mode shapes were computed by solving a 19 dof, a 20 dof, and a 39 dof problem. In both cases a significant reduction in solution time was achieved. For the antenna the solution time utilizing the symmetry was 6.9 sec. compared to 16.1 sec. from a general eigenvalue solver. For the beam the solution time was reduced from 110.6 sec. to 36.0 sec.

II. Relative equilibria of rotating structures. Stability and bifurcation.

In this project we are studying large rotating states of lattice structures, for which we employ geometrically exact, nonlinearly elastic models. We believe that such a formulation is crucial to understanding the dynamics of such *flexible* systems.

The computation of steady rotating states is facilitated by a description of the motion relative to a rotating frame. This delivers a nonlinear "static" problem parameterized by angular velocity, which is a standard one-parameter bifurcation problem. Such problems require the solution of linearized eigenvalue problems. Hence, there is a connection between projects I and II.

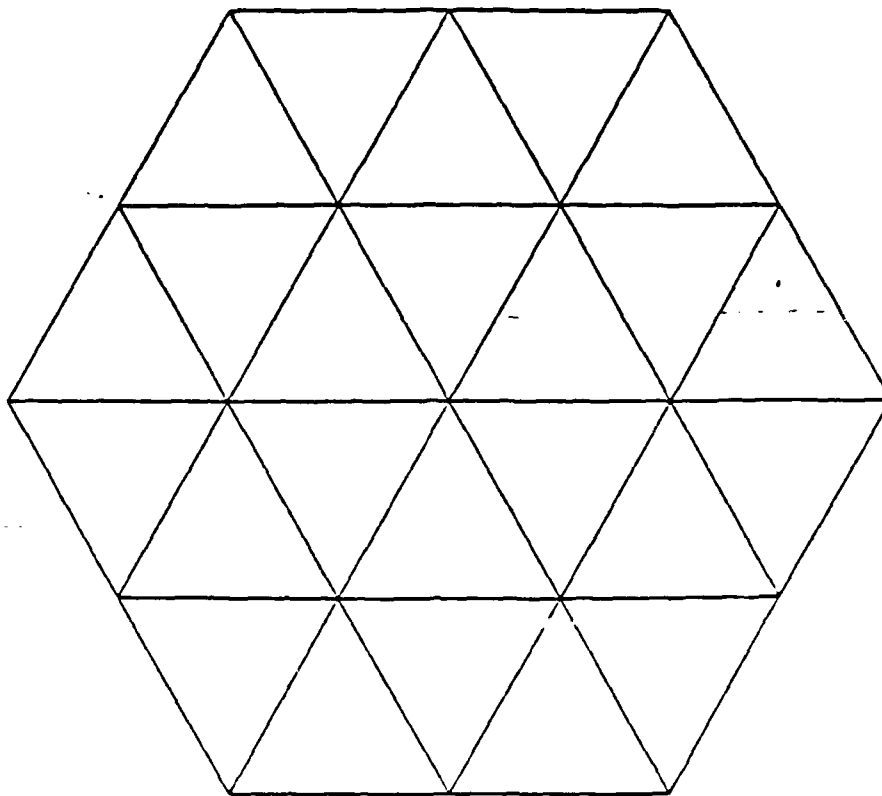
Of course the determination of relative equilibria is not enough--it is crucial to establish their stability. For the statics of conservative systems, it is sufficient to seek relative minima of the potential energy. The analogous procedure for relative equilibria is not so obvious. However, recent work on a simple prototype problem [2] has shed light on this question. In the absence of forces (other than central forces), the total angular momentum of the structure is conserved. Hence, it is reasonable to seek minimizers of the total energy with the side condition that the angular momentum is prescribed and constant. This energy-momentum approach enables a rigorous stability analysis in both analytical and large-scale computational settings.

Figures:

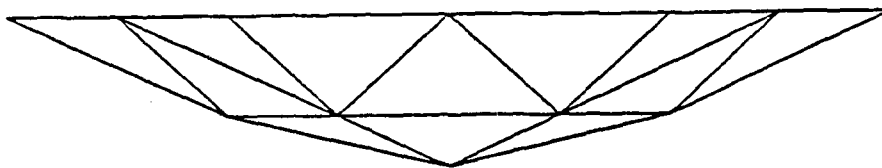
Attached

Papers:

- [1] T.J. Healey and J. Treacy, Linear eigenvalue analysis of skeletal structures with symmetry. A group-theoretic approach (working manuscript to be submitted to Int. J. Num. Meth. Engrg. by October 1, 1988.)
- [2] T.J. Healey and J. Papadopoulos, A class of shape independent motions of strings (submitted to J. Appl. Mech.).



Elevation



Plan

FIGURE 1

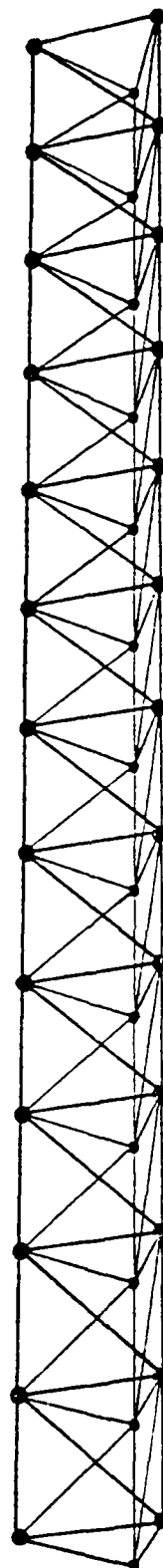


FIGURE 2

PROJECT SUMMARY

Project Title:

Finite-Precision Effects in Feedback Control Systems for Flexible Space Structures

Faculty Leader:

Assistant Professor David F. Delchamps
Electrical Engineering

Graduate Research Assistants:

None

Other Participants:

None

Executive Summary:

Analytical models for lightly damped systems are close to being unstable over a range of operating conditions. If finite-precision measurements are employed by a feedback controller designed to stabilize or place the closed loop poles of the linearization of such a system, then complicated dynamical behavior often results. We have discovered that the design of stabilizing controllers for such systems must pay close attention to finite-precision constraints on measurements and on arithmetic in potential digital implementations. Moreover, we have discovered ways in which long records of finite-precision measurements may be employed in designing feedback controllers for lightly damped systems which are more effective than standard schemes based on instantaneous feedback laws.

Project Description:

We have subjected to a careful analysis the dynamical behavior that arises in control systems which employ feedback based on quantized measurements of real-valued time functions. The models we have considered are all discrete-time, but have important continuous-time analogues. Of particular interest to us have been two general questions. First, how does the finite-precision constraint on measurements affect one's ability to stabilize unstable systems or place the poles of lightly damped systems by means of feedback? Second, how can one make intelligent use of long records of finite-precision measurements in more general control problems involving trajectory following or trajectory optimization? Many established techniques from the ergodic theory of dynamical systems and from information theory have proven extremely useful to us in our research.

With regard to the stabilization problem, we have established under some mild assumptions the existence of a bounded invariant region in the state space of a closed loop system whose controller is based on the instantaneous feedback of a finite-precision measurement of the system's state which would stabilize the system if perfect measurements were available. In this invariant region, almost all trajectories are chaotic, and under additional assumptions there

exists on the region an invariant measure which is absolutely continuous with respect to Lebesgue measure and with respect to which the closed-loop dynamics are ergodic. The ergodicity would seem to make such systems amenable to computer simulation. The invariant measure differs significantly from the measure which would be obtained if the quantization errors were modeled as uniform white noise. For details, see [2].

Furthermore, we have discovered ways of using feedback to make a long record of finite-precision measurements of a system's state reveal asymptotically perfect knowledge of the current state if the system is stable or just barely unstable. If the system is too unstable for such strategies to work, there exist other schemes which give a current state estimate which is asymptotically quite a bit better than that which may be calculated from instantaneous finite-precision measurements. We have obtained an upper bound on the amount of information about the state which may be rendered available by such strategies; this bound might be construed as an upper bound on the useful arithmetic precision in a digital controller for the system. See [1] for a complete discussion.

Papers

[1] Delchamps, David F., "New Techniques for Analyzing the Effects of Output Quantization in Feedback Systems," in *Proceedings of the 1988 Conference on Information Sciences and Systems*, Princeton, NJ, March, 1988.

[2] Delchamps, David F., "The 'Stabilization' of Linear Systems With Quantized Feedback," to appear in *Proceedings of the 27th IEEE Conference on Decision and Control*, Austin, TX, December, 1988.