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THESIS

ACTIVE DAMPING OF VIBRATION IN LARGE
SPACE
STRUCTURES USING A KARHUNEN–LOEVE RE-
DUCED ORDER MODEL

by

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The Karhunen-Loeve basis functions or mode shapes must first be found to identify the reduced order model. Previous results have shown that in the limit as the structural damping approaches zero the Karhunen-Loeve mode shapes and natural mode shapes converge. Numerical techniques are applied to evaluate the structural damping required for convergence. Once the Karhunen-Loeve mode shapes are determined, the reduced order control model is applied to the full order system. The performance of various Karhunen-Loeve models is compared by measuring the modal energies in the controlled and uncontrolled modes.

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Active Damping of Vibration in Large Space Structures Using a Karhunen–Loeve Reduced Order Model

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ABSTRACT

Large space structures are difficult to control because of the high order of their mathematical models. The high order mathematical model makes the use of a reduced order model to control the structure desirable. The Karhunen-Loeve expansion along with Galerkin's method is used to generate a reduced order model. A control algorithm is achieved by applying linear quadratic regulator theory to the reduced order model.

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THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

A. LARGE SPACE STRUCTURES

The lightweight, flexible materials used to construct large space structures (LSS), like a space station, are lightly damped and when disturbed will vibrate for a considerable amount of time. This prolonged vibration could jeopardize the structural integrity of the structure or disturb experiments on the LSS. The purpose of this thesis is to study the effects of controlling this structural vibration with a reduced order model, specifically a Karhunen-Loeve reduced order model.

B. PROBLEM APPROACH

The solution of the vibration control problem requires the use of a mathematical model that describes the behavior of the system in time. The space structure is modeled as a combination of small plates of unit mass connected to form the complete structure. The vibrational motion of these plates can be modeled as a set of coupled damped harmonic oscillators. Using modal analysis the mathematical model of the structure can be decoupled to yield a set of uncoupled simultaneous second order differential equations.

For LSS this model is of very high order, making control design difficult. It is therefore desirous to use a reduced order model (ROM) to control the structure. A control system is designed for the LSS using the Karhunen-Loeve (reduced order) model. The control system is then applied to the LSS after it has been disturbed by an impulse. The performance of this system and a control system based on a modal model, truncated by frequency, are compared.

The LSS used in this thesis is a dual keel space station, see Figure 1 on page 2. A mathematical model of this space station is provided courtesy of McDonnel Douglas Astronautics. A computer simulation of this space station is used to determine the effectiveness of the control system. Karhunen-Loeve (reduced order) models of increasing size are simulated and the modal energies are calculated and used to determine the relative effectiveness of the control models. These results will then be compared to results obtained for the modal model truncated by frequency.

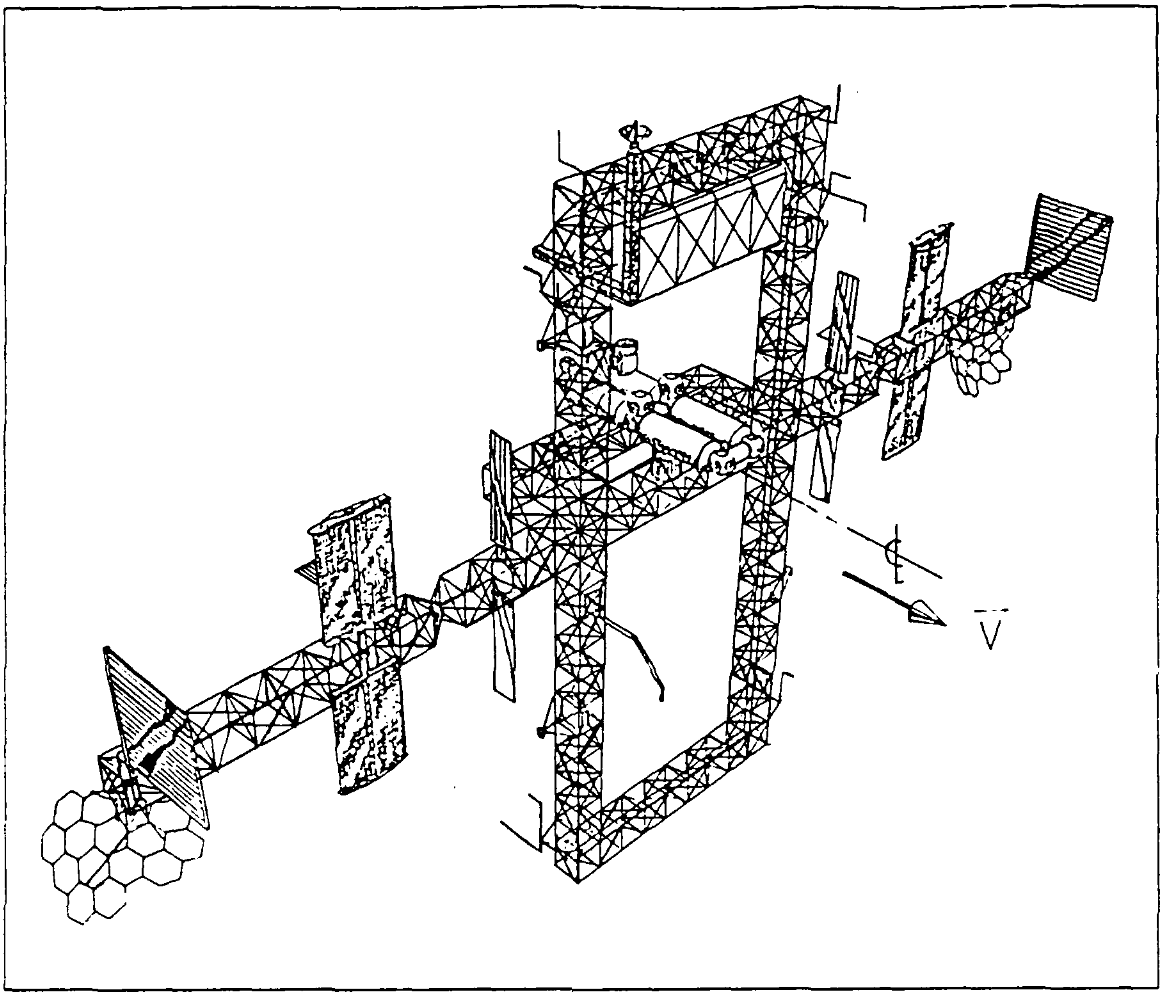

Figure 1. Representation of a Dual Keel Space Station from [Ref. 1: p. 1]

C. ORGANIZATION

- Chapter II is a statement of the mathematical model of the space station.
- Chapter III discusses the Karhunen–Loeve expansion, the relationship between the Karhunen–Loeve mode shapes and the natural mode shapes and how a reduced order model can be synthesized using the Karhunen–Loeve expansion and Galerkin's method.
- Chapter IV discusses the determination of the Karhunen–Loeve modes.
- Chapter V describes the control solution and presents the control simulation results.
- Chapter VI discusses the conclusions based on the simulation results and a recommended design procedure.

II. THE MATHEMATICAL MODEL

A. INTRODUCTION

Large space structures are flexible, lightly damped structures that can vibrate for a considerable amount of time when disturbed by an external force. To effectively control such a system, a mathematical model describing the evolution of that system in time is required.

B. THE MODAL MODEL

The space station is modeled as a combination of small plates of unit mass connected to form the complete structure. This model can be visualized as a system of masses connected by springs and dashpots (the springs representing the stiffness factor and the dashpots representing the damping factor). The displacement of the masses can be described by a second order matrix differential equation of motion:

$$M\ddot{x}(t) + \frac{d}{\omega_f} K\dot{x}(t) + Kx(t) = F(t) \tag{1}$$

where

- x is the generalized coordinate vector
- M is the diagonal system mass matrix
- $\frac{d}{\omega_f} K$ is the structural damping term
- d is the damping constant
- ω_f is the frequency of oscillation of the system
- K is the symmetric system stiffness matrix
- F(t) is the system forcing function

This equation represents a system of simultaneous, second order differential equations that are coupled by the K matrix.[Ref. 1: p. 3]

This equation can be uncoupled and the system represented by a set of independent second order differential equations. This is done through the process of modal analysis which is outlined in [Refs. 2, 3]. The resulting modal model consists of a set of independent second order differential equations:

$$\left[\ddot{\eta} + d\,\Omega\,\dot{\eta} + \Omega^2\,\eta = X^T F\right] \tag{2}$$

where

- η is the coordinate vector or modal amplitude vector
- $\Omega^2 = \mathrm{diag}[\omega_{o1}, \omega_{o2}, \ldots, \omega_n]$
- $X^T = [x_1\ x_2\ \ldots\ x_n]$ the transpose of the modal matrix or mode shape vector
- F is the torquing force applied at a point

Next a discrete-time equation describing the motion of the space station in terms of its natural modes of vibration is developed. The discrete-time state equation for the ith equation of motion is:

$$Z_i(kT+1) = \Phi_i(T)\, Z_i(kT) + \Gamma_i(T)\, X_i^T\, [F(kT) + W(kT)] \tag{3}$$

where

- Z_i is a vector of the ith modal amplitude and the ith modal velocity
- Φ_i is the ith state transition matrix
- Γ_i is the ith input vector
- X_i^T is the transpose of the ith mode shape vector
- F is the control torque force vector applied at a point
- T is the sampling time
- k is the time index
- W is the disturbance input

This equation is used for computer simulation of the space station and control solution.[Ref. 1: p. 4]

III. APPLICATION OF THE KARHUNEN-LOEVE EXPANSION TO THE REDUCED ORDER CONTROL OF LARGE SPACE STRUCTURES

A. INTRODUCTION

Large flexible structures, such as a space station, as a class of distributed parameter systems (DPS) require a finite dimensional model for control design. This model may be achieved by approximating the state of the LSS using the Karhunen-Loeve (KL) expansion. The expansion is truncated to provide the finite dimensional approximation of the state for control design. The KL model that results describes the evolution of the approximated state of the structure.

The natural mode shapes are normally used for modeling and control of flexible structures. The relationship between the natural mode shapes and the KL mode shapes is described in this chapter as well as the use of Galerkin's method. Galerkin's method is used to generate a reduced order model (ROM), using both the natural mode shapes and the KL expansion.

B. THE KARHUNEN-LOEVE EXPANSION

The purpose of the KL expansion is stated by Stark and Woods [Ref. 4: p. 322]. "The idea [of the KL expansion] is to decompose a general second-order random process into an orthonormal expansion whose coefficients are uncorrelated random variables." The state of a large space structure (LSS), y(x), can be modeled as random process since it depends on random excitations, i.e., noise from onboard machinery and actuators. The second order moments of the LSS are proportional to the physical energy and therefore exist. The LSS can therefore be approximated using the KL expansion [Ref. 5: p. 12]. This is done by projecting the random process onto an orthonormal basis and truncating to N terms. The value chosen for N is a matter of "engineering judgment". [Ref. 5: p. 13]

The selection of the orthonormal basis is made by solving the KL eigenequation:

$$< R_{yy}(x,z), \phi_i(z) > = \lambda_i \phi_i(x) \tag{4}$$

where

- $\phi_i(x)$ is referred to as an eigenfunction (or the KL mode shapes)
- λ_i is the eigenvalue and is a measure of the excitation of the ith basis function

- $R_{yy}(x,z)$ is the correlation function of y(x): $R_{yy}(x,z) = E[y(x)\,y^T(x)]$
- $< \bullet , \bullet >$ is an inner product: $< a(z), b(z) > = \int_\Omega a^T(z)\, M(z)\, b(z)\, dz$
- M(z) is the mass density of the structure
- Ω is the spatial extent of the structure

The state of the LSS is approximated by:

$$y_a = \sum_{l=1}^{N} \zeta_i \phi_i(x) \tag{5}$$

where

- y_a is an approximation to the state of the space structure
- ζ_i is a set of coordinates found by $\zeta_i = < \phi_i(x), y(x) >$
- $\phi_i(x)$ is the ith basis function or KL mode shapes

The expansion is truncated by keeping the eigenfunctions (KL mode shapes) associated with the N largest eigenvalues. [Ref. 5: p. 13]

The KL expansion yields the best approximation to the random process, i.e., minimizes the expected value of the norm of the error, of any orthogonal expansion. The approximation error is defined as:

$$e(x) = y(x) - y_a(x) = \sum_{l=N+1}^{\infty} \zeta_i \phi_i(x) \tag{6}$$

For a proof of the optimality of the KL expansion see [Ref. 6: p. 11] or [Ref. 5: p. 15].

C. RELATIONSHIP BETWEEN THE KARHUNEN-LOEVE MODE SHAPES AND THE NATURAL MODE SHAPES

The relationship between the KL mode shapes or KL basis functions and the natural mode shapes is taken from Burl [Ref. 5: p. 13]. The KL mode shapes of a structure are related to the natural mode shapes of that structure by a linear transformation representing a change of basis which can be written:

$$\phi_i(x) = \sum_{j=1}^{\infty} c_i^{2j-1} \begin{bmatrix} \eta_j(x) \\ 0 \end{bmatrix} + c_i^{2j} \begin{bmatrix} 0 \\ \eta_i(x) \end{bmatrix} = \eta^T(x) c_i \tag{7}$$

where

$$\eta^T(x) = \begin{bmatrix} \eta_1(x) & 0 & \eta_2(x) & 0 & \eta_3(x) & \cdots \\ 0 & \eta_1(x) & 0 & \eta_2(x) & 0 & \cdots \end{bmatrix} \tag{8}$$

$$c_i^T = \begin{bmatrix} c_i^1 & c_i^2 & c_i^3 & \dots \end{bmatrix} \tag{9}$$

and $\{\eta_i(x)\}$ is the set of natural mode shapes. The state of a structure consists of a generalized position and velocity which can be expanded in terms of the natural mode shapes:

$$y(x,t) = \sum_{j=1}^{\infty} \alpha_j(t) \begin{bmatrix} \eta_j(t) \\ 0 \end{bmatrix} + \dot{\alpha}_j(t) \begin{bmatrix} 0 \\ \eta_j(x) \end{bmatrix} = \eta^T(x)\alpha(t) \tag{10}$$

where

$$\alpha^T(t) = [\alpha_1(t) \quad \dot{\alpha}_1(t) \quad \alpha_2(t) \quad \dot{\alpha}_2(t) \quad \alpha_3(t) \quad \dots] \tag{11}$$

are the coordinates and velocities of the natural mode shapes. The vectors, c_i, can be found by solving the equation:

$$E\left[\alpha(t)\,\alpha^T(t)\right] \mathrm{Q}\, c_i = \lambda_i\, c_i \tag{12}$$

where

$$\mathrm{Q} = \mathrm{diag}\left[\begin{bmatrix} \omega_7^2 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} \omega_8^2 & 0 \\ 0 & 1 \end{bmatrix}, \dots, \begin{bmatrix} \omega_{56}^2 & 0 \\ 0 & 1 \end{bmatrix}\right] \tag{13}$$

Equations 10 and 11 give the KL mode shapes in terms of the natural mode shapes. Equation 12 is an infinite dimensional eigenvalue problem that can be solved practically by truncating it to the most significant terms.

D. REDUCED ORDER MODELING USING THE KARHUNEN-LOEVE MODES

Galerkin's method is used to produce reduced order state equations. This can be done with either the KL modes or the natural modes.

The discrete-time state equation of a distributed parameter system is:

$$y(x,k) = F\,y(x,k-1) + G\,f(k) \tag{14}$$

where

- y(x,k) is the state
- f(k) is the input
- k is the time index
- F is an operator on the state space
- G is an operator from $\mathscr{R}^m$ to the state space
- For each k, y(x,k) $\in$ the state space, f(k) $\in \mathscr{R}^m$

A finite dimensional approximation to this equation can be obtained using Galerkin's method

$$P_n\,y(x,k) = (P_n\,F\,P_n)\,y(x,k-1) + P_n\,G\,u(k) \tag{15}$$

where P_n can be written in terms of a basis $[\eta_i \mid i = 1,2,\ldots,n]$

$$P_n(\bullet) = \sum_{i=1}^{n} \eta_i(x) < \eta_i(x),\ \bullet > \tag{16}$$

The KL mode shapes (basis functions) can be used in the above equations and the KL model results. The natural mode shapes can be used in the above equations, which is equivalent to truncating the modal equations, producing the modal model.[Ref. 5 : p. 13]

E. SUMMARY

The KL expansion can be used to approximate the state of a LSS. This is done by projection of the state, which is modeled as a random process, onto an orthonormal basis function. The basis function can be found by solving the KL eigenequation. These KL basis functions will yield the best approximation to the state of the LSS. Then using Galerkin's method a reduced order model of the LSS is produced. This reduced order model is used to generate a control which is applied to the entire system.

IV. DETERMINATION OF THE KARHUNEN-LOEVE BASIS FUNCTIONS

A. INTRODUCTION

The KL basis functions (or mode shapes) can be determined by solving the eigenequation 12. This is a tedious and laborious processes. It can be shown that for lightly damped structures the KL mode shapes can be determined from the open loop response by ordering the natural mode shapes in order of decreasing modal energies.

The KL mode shapes were calculated using the KL mode program in **Appendix B**. These calculations were compared to the mode shapes selected by observing the open loop response to verify that this is a valid method of determining the KL mode shapes.

The value of the structure's damping factor used in the program was increased until the KL mode shapes no longer were the same as those determined from the open loop response. This was done to determine how large the structure's damping factor could be and still have the KL mode shapes converge to the natural mode shapes.

B. NUMERICAL DETERMINATION OF THE KARHUNEN-LOEVE MODE SHAPES

The KL mode shapes are found by determining the eigenvalues and eigenvectors (equation 12) of the covariance matrix $E[\alpha_i(t)\,\alpha_j(t)]$. This is an infinite dimensional matrix which is truncated to a finite dimensional square matrix. There are three terms in $E[\alpha(t)\,\alpha^T(t)]$, where $\alpha(t)$ is defined by equation 11. The first of these terms is computed, for a white noise input

$$E[\alpha_i(t)\,\alpha_j(t)] = \int_0^{\infty} h_i(\tau)\, h_j(\tau)\partial\tau \tag{17}$$

where $h_i(t)$ is the impulse response for $\alpha_i(t)$ [Ref. 6: p. 66]. The impulse response is given by:

$$h_i(t) = \frac{b_{2i}}{\omega_{oi}\sqrt{1-\gamma^2}}\; e^{-\gamma\omega_{oi}t}\; \sin(\omega_{oi}\sqrt{1-\gamma^2}\,t) \tag{18}$$

where

- b_{2i} is the modal slope in the x direction
- γ is the damping coefficient; $\gamma = \frac{d}{2}$ is a constant
- ω_{oi} is the natural frequency

Performing the integration in equation 17 produces the result:

$$E[\alpha_i(t)\,\alpha_j(t)] = K_i\,K_j\left[\frac{a}{2[a^2+b^2]} - \frac{a}{2[a^2+c^2]}\right] \tag{19}$$

where

- $K_{i,j} = \dfrac{b_{2i,j}}{\omega_{oi,j}\sqrt{1-\gamma^2}}$
- $a = \gamma\,\omega_{oi} + \gamma\,\omega_{oj}$
- $b = \omega_{oi}\sqrt{1-\gamma^2} - \omega_{oj}\sqrt{1-\gamma^2}$
- $c = \omega_{oi}\sqrt{1-\gamma^2} + \omega_{oj}\sqrt{1-\gamma^2}$

The other terms in $E[\alpha(t)\,\alpha^T(t)]$ can be found in a similar way. They are:

$$\begin{aligned} E[\dot{\alpha}_i(t)\,\alpha_j(t)] = {} & K_i\,K_j\,L_i\left[\frac{b}{2[a^2+b^2]} + \frac{c}{2[a^2+c^2]}\right] \\ & - K_i\,K_j\,M_i\left[\frac{a}{2[a^2+b^2]} - \frac{a}{2[a^2+c^2]}\right] \end{aligned} \tag{20}$$

and

$$\begin{aligned} E[\dot{\alpha}_i(t)\,\dot{\alpha}_j(t)] = K_i\,K_j \Bigg[& \frac{L_iL_ja - L_iM_jb - L_iM_ib + M_iM_ja}{2[a^2+b^2]} \\ & + \frac{L_iL_ja - L_iM_jc - M_iM_ja}{2[a^2+c^2]} \Bigg] \end{aligned} \tag{21}$$

where

- $L_{i,j} = \omega_{oi,j}\sqrt{1-\gamma^2}$
- $M_{i,j} = \gamma\,\omega_{oi,j}$

Equations 19, 20, and 21 specify the eigenvalue problem equation 12.

The eigensolution specifies the transformation from the natural mode shapes to the KL mode shapes. This finite dimensional eigenvalue problem is solved and the KL mode shapes are approximated as a linear combination of the first fifty (flexible) natural mode shapes. The program in **Appendix B** computes the KL mode shapes by solving the eigenvalue problem equation 12.

C. EMPIRICAL DETERMINATION OF THE KARHUNEN-LOEVE MODE SHAPES

The numerical computation of the KL mode shapes is difficult. Solving the eigenvalue problem requires the determination of the impulse response and solving for all the terms in the covariance matrix $E[\alpha_i(t)\,\alpha_j(t)]$. It turns out that, in the limit as the damping factor of the structure approaches zero, the KL mode shapes converge to the natural mode shapes [Ref. 7].

If the modal energies for the open loop response are computed and the mode shapes ordered by decreasing modal energy, these mode shapes should be the same as those computed numerically. The output of the system, y, is defined:

$$\begin{bmatrix} y_7 \\ \vdots \\ y_{56} \end{bmatrix} = \begin{bmatrix} \omega_7 & 0 & & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \omega_{56} & 0 \\ 0 & 0 & & 0 & 1 \end{bmatrix} \begin{bmatrix} x_7 \\ \vdots \\ x_{56} \end{bmatrix} \tag{22}$$

where $y_i \in \mathscr{R}^2$ and

$$E[\text{ith Modal Energy}] = E[\,\|y_i\|^2]. \tag{23}$$

The energy, given in equation 23 can be written [Ref. 8]:

$$E[\,\|y_i\|^2] = \int_0^\infty \|h_{y_i}(t)\|^2 \partial t \tag{24}$$

where h_{y_i} is the response of y_i due to an impulse applied at the disturbance input (node 69 or 55). Equation 24 is evaluated using computer simulation. The mode shapes found are compared to those determined numerically for noise input at node 69 or node 55, see Table 1 on page 12.

Table 1. DETERMINATION OF THE FIRST FIFTY FLEXIBLE KARHUNEN-LOEVE MODE SHAPES

Karhunen-Loeve mode shapes determined from the open loop response.		Karhunen-Loeve mode shapes numerically calculated.	
Noise input Location:			
Node 55	Node 69	Node 55	Node 69
40	54	40	54
43	51	43	51
7	31	7	31
17	52	17	52
28	36	28	36
15	7	15	7
44	30	44	30
35	28	35	28
39	48	39	48
33	26	33	26
41	35	41	35
31	50	31	50
42	15	42	15
45	55	45	55
53	41	53	41
48	56	48	56
11	44	11	44
36	53	36	53
26	33	26	33
21	34	21	34
8	25	8	25
16	37	16	37
38	38	38	38
29	46	29	46
50	27	50	27
30	43	30	43
23	40	23	40
19	23	19	23
32	21	32	21
25	16	25	16
51	8	51	8
27	45	27	45
22	42	22	42
37	11	37	11
20	47	20	47
10	49	10	49
52	18	52	18
55	20	55	20
9	29	46	29
46	10	49	10
49	24	9	24
34	17	34	17
56	39	56	39
24	22	24	22
54	32	54	32
13	9	13	9
18	13	18	13
47	14	47	14
14	19	14	19
12	12	12	12

Table 1 shows that the mode shapes are the same. Therefore, the KL mode shapes can be determined from the open loop response when the structural damping factor is sufficiently small. In the case of the space station simulated in this thesis a value of 0.001 was used. The next question is, how big can the damping factor of the structure be before the KL mode shapes fail to converge to the natural mode shapes?

1. Required Magnitude of the Damping Factor for Convergence

When the structural damping factor is small enough, the eigenvectors solved for in equation 12 approximate the natural basis

$$e_1^T = [1\ 0\ 0\ \dots\ ] \tag{25}$$

or

$$e_2^T = [0\ 1\ 0\ 0\ \dots\ ] \tag{26}$$

etc.; one element is unity and the other elements are zero. When these vectors are substituted into equation 7 it is easy to see that the KL mode shapes are the natural mode shapes. If the damping factor is increased, at some value the eigenvectors have more than one non-zero element and the KL mode shapes become a linear combination of natural mode shapes.

The damping factor was increased successively by a factor of two from a starting value of 0.0005. The norm of the error (or error norm) is used as a means of measuring how closely the eigenvectors obtained from the KL mode program approach the ideal of equation 25. The error norm is defined as follows:

$$\| \varepsilon \|_k = \underset{j}{\text{minimum}} \| c_k - e_j \| = \sqrt{(1 - x_j)^2 + \sum_{\substack{i \neq j \\ i=2}}^{50} x_i^2} \tag{27}$$

where

$$c_k^T = [x_1\ x_2\ x_3\ \dots\ x_j\ \dots\ x_{50}] \tag{28}$$

and for simplicity x_i is assumed to be positive and the largest component of c_i. Equation 27 can be simplified to

$$\| \varepsilon \|_k = \sqrt{1 - 2x_j + \sum_{i=1}^{50} x_i^2} \tag{29}$$

Since the eigenvectors are normalized,

$$\sum_{i=1}^{50} x_i^2 = 1 \tag{30}$$

and substituting equation 30 into equation 29 yields

$$\| \varepsilon \|_k = \sqrt{2(1 - x_j)} \tag{31}$$

a. Results

The norm of the error, ε, and the natural mode shape sequence obtained from the KL mode program were used to determine how large the damping factor could be and still have the KL mode shapes and natural mode shapes converge. The error norm is presented graphically for each KL mode; all cases are for disturbance torques due to actuator noise. The cases presented are:

- Figure 2 on page 16: for a damping factor of 0.0005 all but two KL modes have an error norm below 0.2. The natural mode sequence is as in Table 1 on page 12.
- Figure 3 on page 17; for a damping factor of 0.001 nearly 74 percent of the KL modes have an error norm of 0.2 or below, the natural mode sequence is presented in Table 1 on page 12. The natural mode shapes are a good approximation of the KL mode shapes.
- Figure 4 on page 18; for a damping factor of 0.002 only 46 percent of the KL modes have an error norm at or below 0.2, and the natural mode sequence no longer conforms to that in Table 1 on page 12, i.e., the KL modes and natural modes are not converging.
- Figure 5 on page 19; for a damping factor of 0.005 only 28 percent of the KL modes have an error norm of 0.2 or less and the KL modes and the natural modes are more divergent.

b. Conclusion

If the damping factor is greater than 0.001 the natural mode shapes are not a good approximation of the KL mode shapes. The convergence criteria is that the norm of the error, for 50 percent of the KL modes or greater, is more than 0.2. The value for the damping factor used in the space station simulations as noted in previous chapters was 0.001. The assumption that the KL mode shapes and natural mode shapes converge is valid for this damping factor and determining the KL modes from the modal energies is a good approximation.

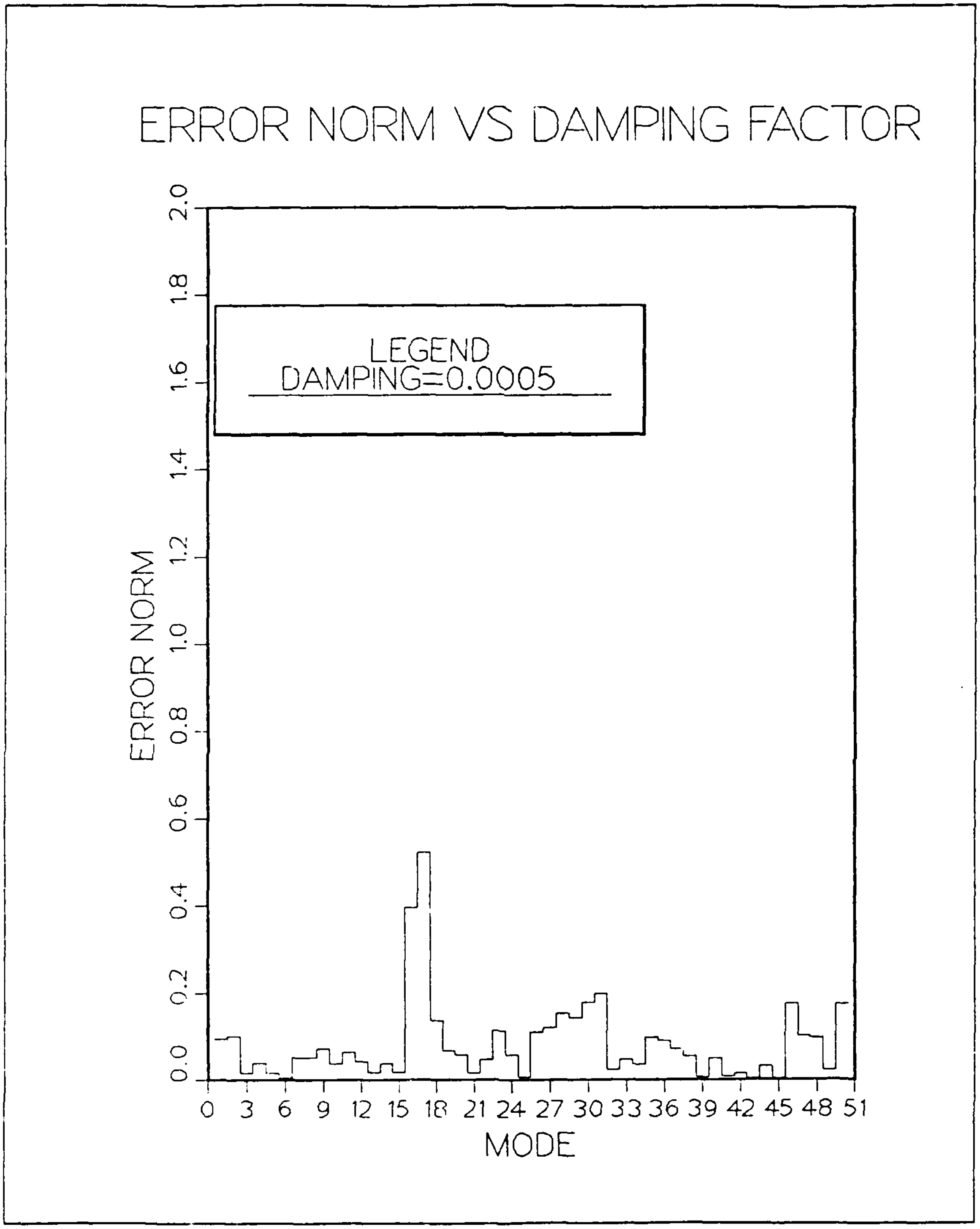

Figure 2. Norm of the error values for the KL modes, excitation at node 69 , damping factor = 0.0005

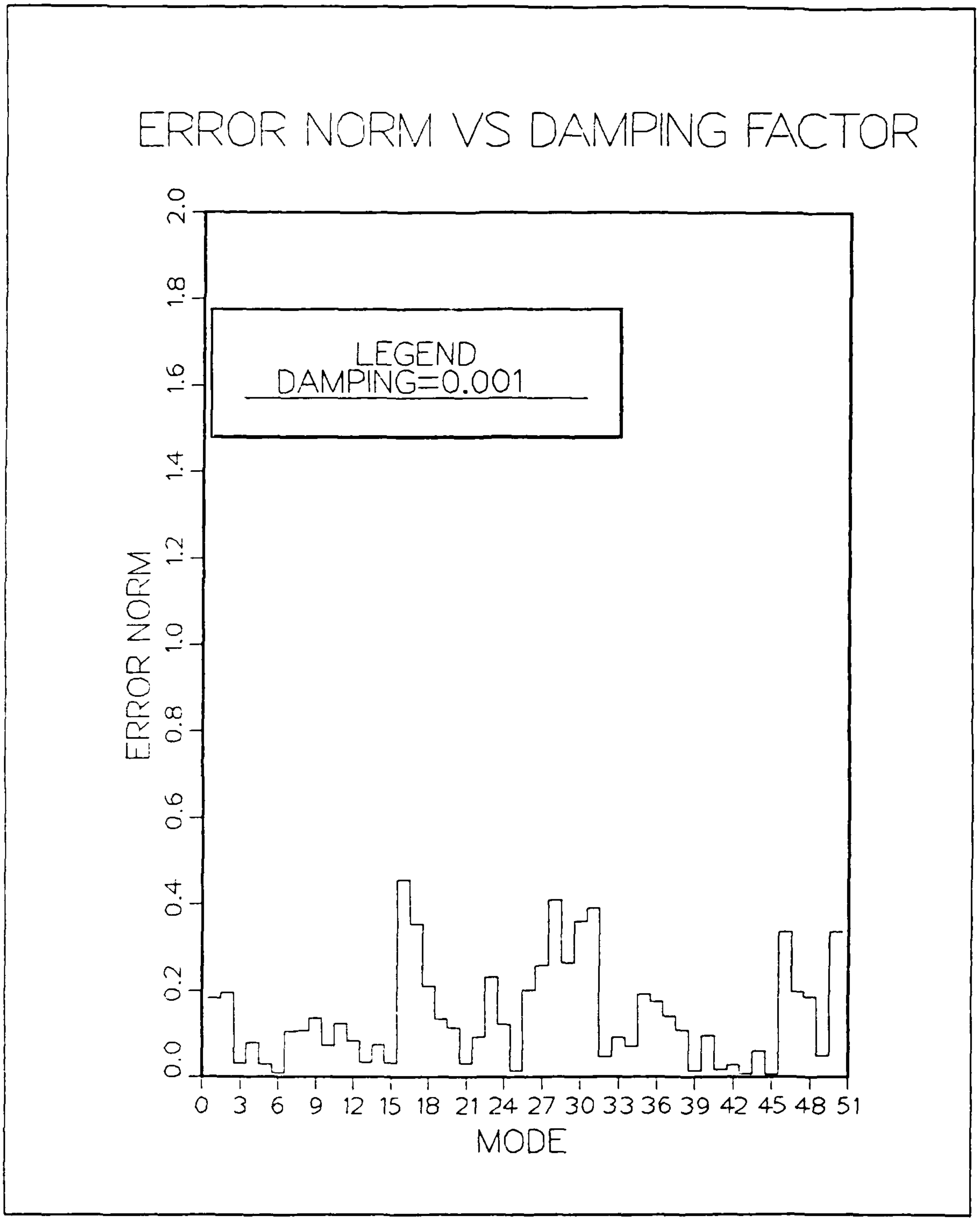

Figure 3. Norm of the error values for the KL modes, excitation at node 69 , damping factor = 0.001

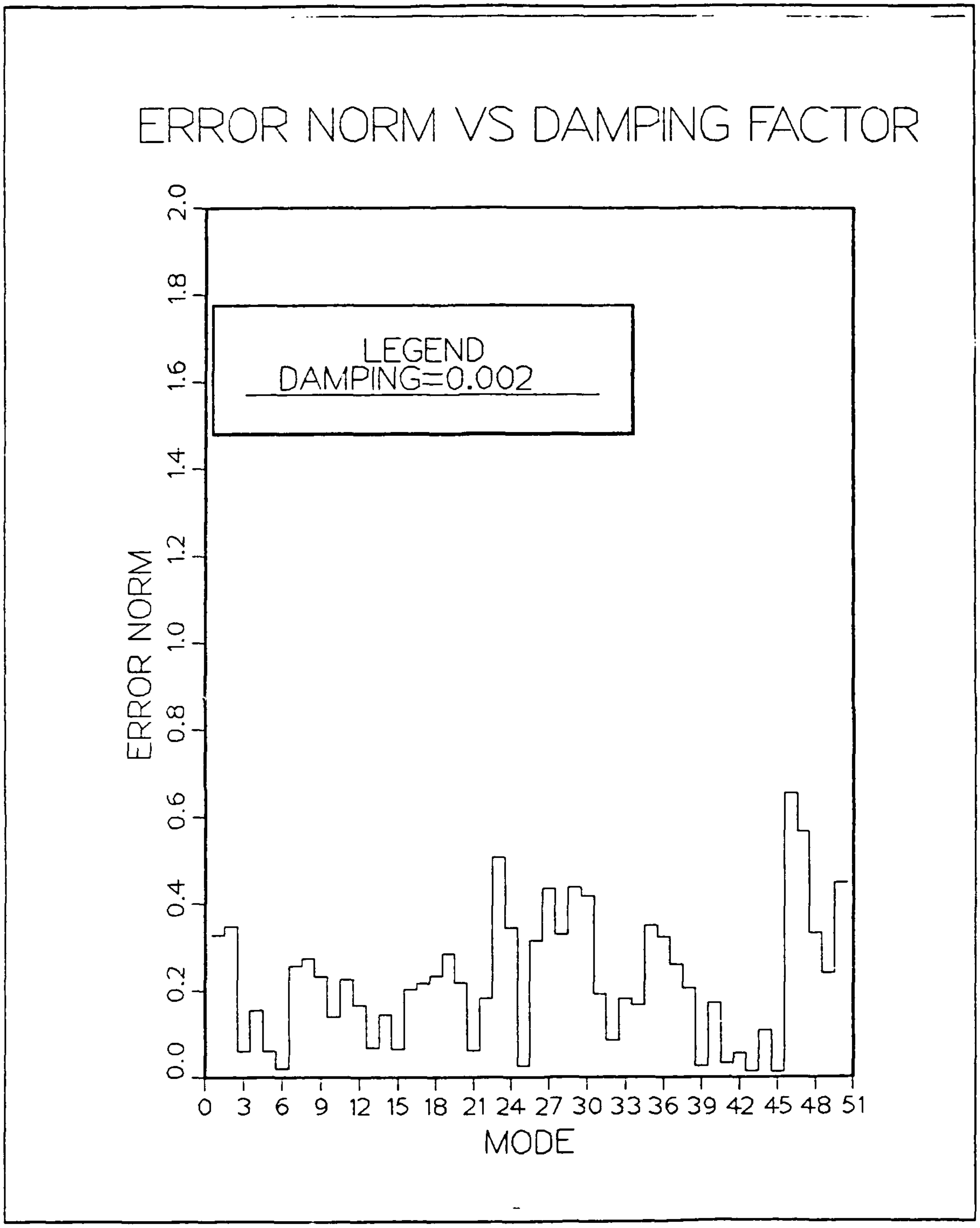

Figure 4. Norm of the error values for the KL modes, excitation at node 69 , damping factor = 0.002

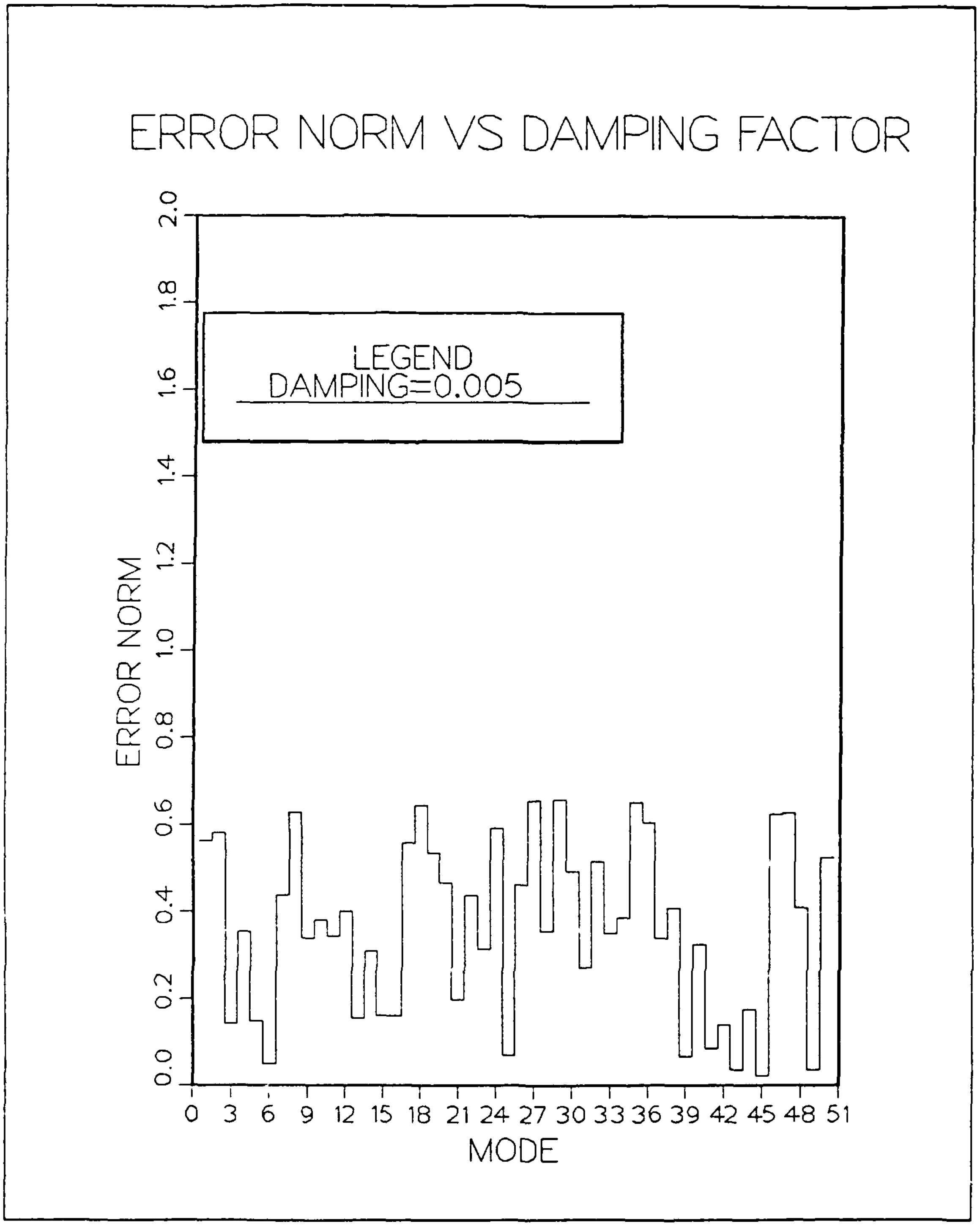

Figure 5. Norm of the error values for the KL modes, excitation at node 69 , damping factor = 0.005

V. CONTROL SOLUTION

A. INTRODUCTION

The response of the space station to disturbance and control inputs is simulated using the modal model discussed in Chapter II. The Fortran program that simulates this model was written by Preston [Ref. 1: p. 47], and is used in this thesis with minor modification, (see Appendix A). The objective of the simulation is to determine the system response to disturbances applied at modes 69 and 55 (see Figure 6) on the structure, using increasing sizes of KL models (i.e., 5, 10, 20 modes). The response of the system is depicted graphically by displaying the energy in each mode in english units of inch–pounds (in–lbs).

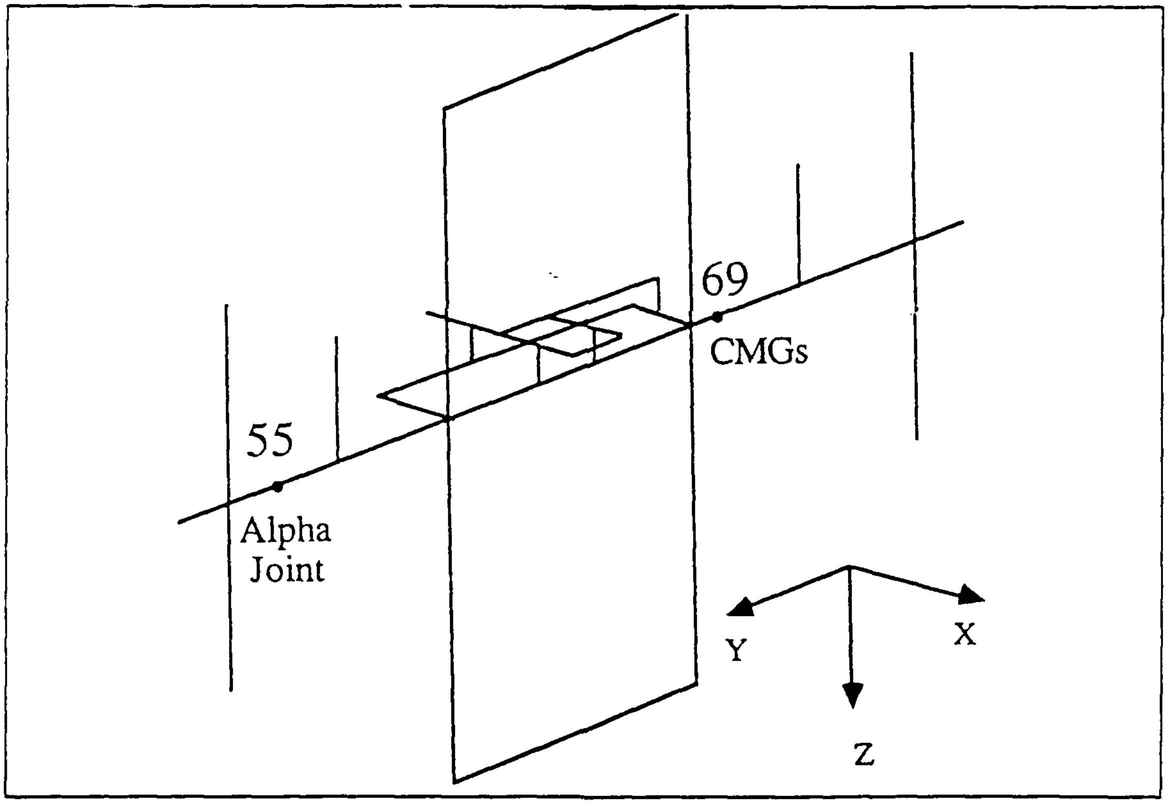

Figure 6. Disturbance Locations

B. FULL ORDER SPACE STATION MODEL

The plant being controlled in this thesis, as stated in Chapter I, is a preliminary version of the NASA dual keel space station. NASTRAN was used to generate the first fifty flexible modes (starting with mode seven). The modal model that forms the full order model is composed of these flexible modes:

$$
\begin{bmatrix} \dot{x}_7 \\ \vdots \\ \dot{x}_{56} \end{bmatrix} = \begin{bmatrix} 0 & 1 & & 0 & 0 \\ -\omega_7^2 & -d\omega_7 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & & 0 & 1 \\ 0 & 0 & & -\omega_{56}^2 & -d\omega_{56} \end{bmatrix} \begin{bmatrix} x_7 \\ \vdots \\ x_{56} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \phi_7(p_c) & \phi_7(p_n) \\ \vdots & \vdots \\ 0 & 0 \\ \phi_{56}(p_c) & \phi_{56}(p_n) \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix} \tag{32}
$$

where

$$
x_i = \begin{bmatrix} \text{modal amplitude} \\ \text{modal velocity} \end{bmatrix} \in \mathscr{R}^2 \tag{33}
$$

and

- ω_i is the natural frequency of the ith mode
- d = 0.001 is the damping coefficient
- $u \in \mathscr{R}^3$ is the control input torques from three orthogonally oriented control moment gyros
- $w \in \mathscr{R}^3$ is a random, white noise disturbance input (three orthogonal disturbance torques)
- $\phi_i(p) \in \mathscr{R}^3$ is the modal slopes at p
- p_c is the location of the control moment gyros (node 69 on Figure 6 on page 20)
- p_n is the location of the disturbance input (either the control moment gyro location, node 69, or the alpha joint, node 55, as seen in Figure 6 on page 20).

This model is used as the full order model in the simulation of the space station. However, it is itself a reduced order model since the "infinite" number of vibrational modes of the structure are truncated to fifty.

It is assumed that perfect modal amplitude and modal velocity information is available. This assumption of perfect sensor information is not realistic, but isolates the effect of modal truncation on control algorithm synthesis.

C. THE REDUCED ORDER MODEL

The reduced order model of the space station is obtained by truncating modes. The criteria for truncating modes is either the modal frequency or the Karhunen-Loeve ordering (the KL ordering found in the previous chapter to be equivalent to an ordering based on the energy measured during open loop excitation).

D. PERFORMANCE FUNCTION AND OPTIMAL CONTROL

The full-order model and a reduced order model have been established. Next, a mathematical expression of system performance is required. The performance function, J, stated here is developed in detail in [Ref. 1: p. 10]. J consists of the total energy in the modeled modes plus a control energy term:

$$J = \begin{bmatrix} x_7^T & \cdots & x_{imax}^T \end{bmatrix} \begin{bmatrix} \omega_7^2 & 0 & & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & & \omega_{imax}^2 & 0 \\ 0 & 0 & & 0 & 1 \end{bmatrix} \begin{bmatrix} x_7 \\ \vdots \\ x_{imax} \end{bmatrix} + ru^T u \tag{34}$$

where

- imax is the number of modes in the reduced order model
- $r = 10^{-12}$ is a weighting coefficient on the control energy term selected to yield time constants on controlled modes of approximately 30 seconds.

Vibration damping is achieved by application of steady state, linear quadratic regulator theory [Ref. 9] to the KL (reduced order) model. The control torque vector, u(k), is the product of an optimal gain matrix, L, and the time varying state matrix, Z (equation 3). The L matrix is found by solution of the Ricatti equations to minimize the performance function, J, [Ref. 1: p. 12].

E. SIMULATION RESULTS

The space station was simulated and the modal energies calculated for disturbance inputs applied at nodes 69 and 55 as shown in Figure 6 on page 20. The control system consists of KL (reduced order) models of five, ten or twenty modes. The data is presented graphically as energy in each of the modes. The open loop response is included for comparison except where the difference in scale precludes it. The closed loop response for a reduced order model generated by modal truncation is included for comparison with the KL model. The cases presented are:

- Open loop (no control) response which identifies the reduced order model to be used. See Figure 7 and Figure 8.
- Closed loop response for a reduced order model generated by modal truncation with the modes ordered by natural frequency. See Figure 9 to Figure 14.
- Closed loop response for a Karhunen-Loeve (reduced order) model. The KL modes were determined from the open loop responses. These results are shown in Figure 15 through Figure 19.
- Closed loop response, with a KL model based on the open loop response, for the system excited by the control actuators. These results are shown in Figure 20 through Figure 22.

These results are discussed in detail in Chapter VI.

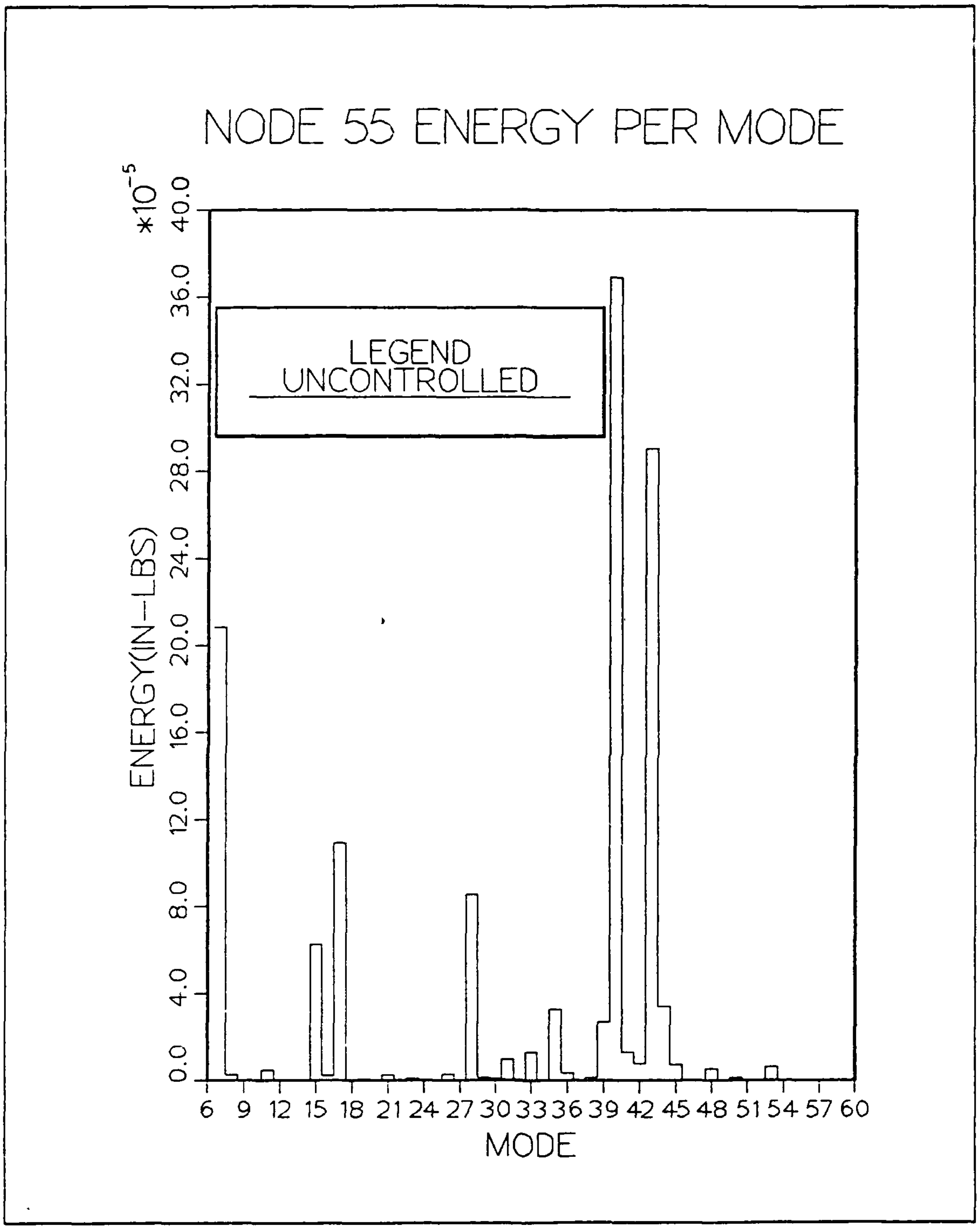

Figure 7. Open Loop Response, excitation at node 55.

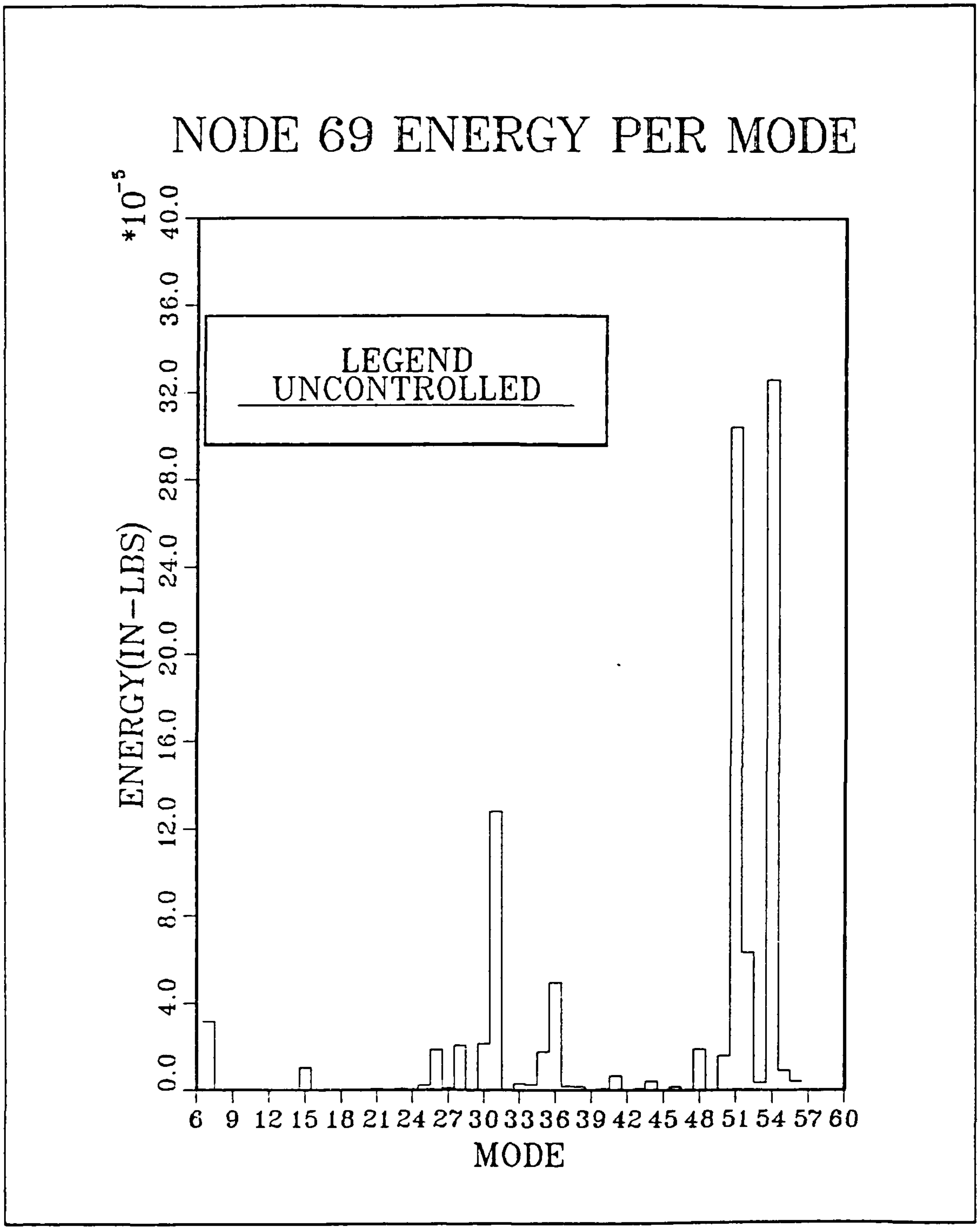

Figure 8. Open Loop Response,excitation at node 69

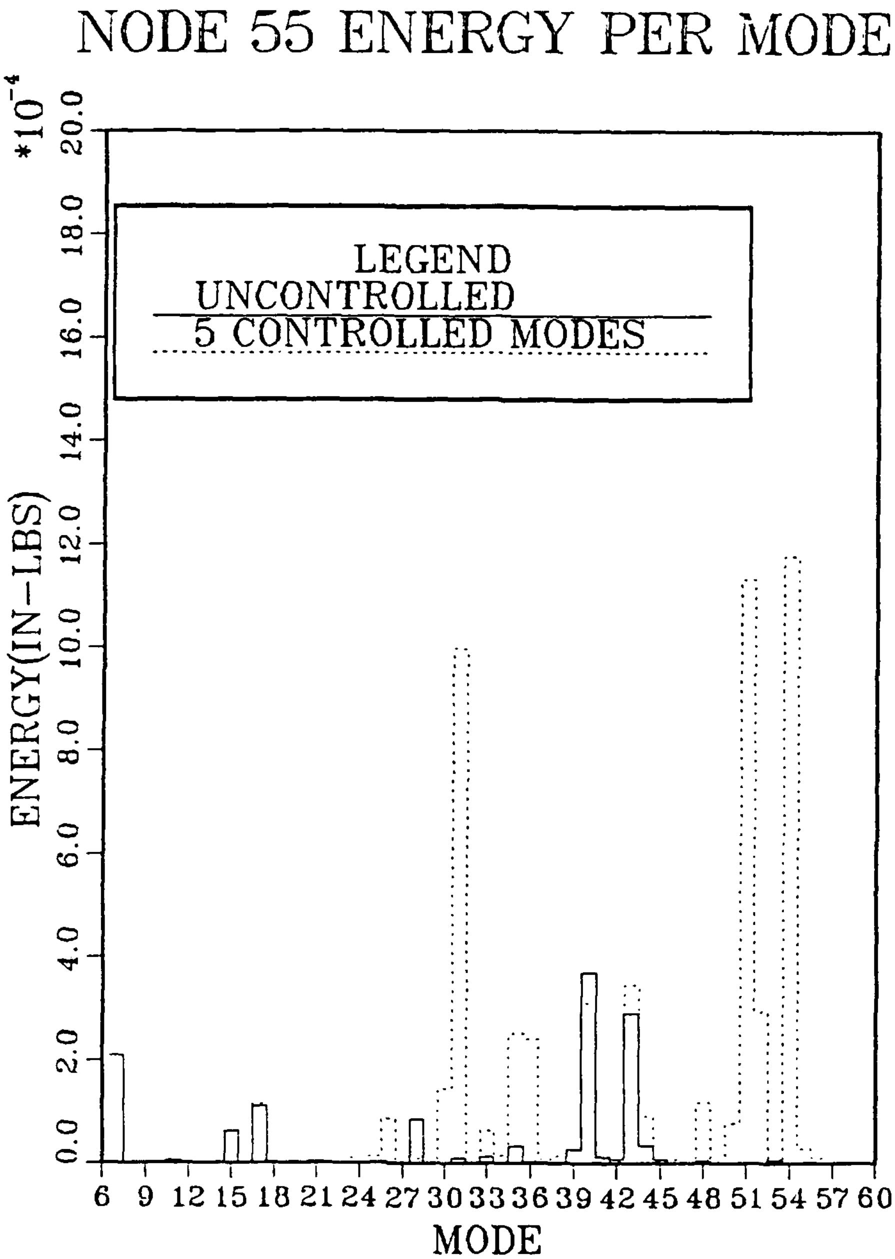

Figure 9. Five Controlled modes selected by frequency, excitation at node 55.

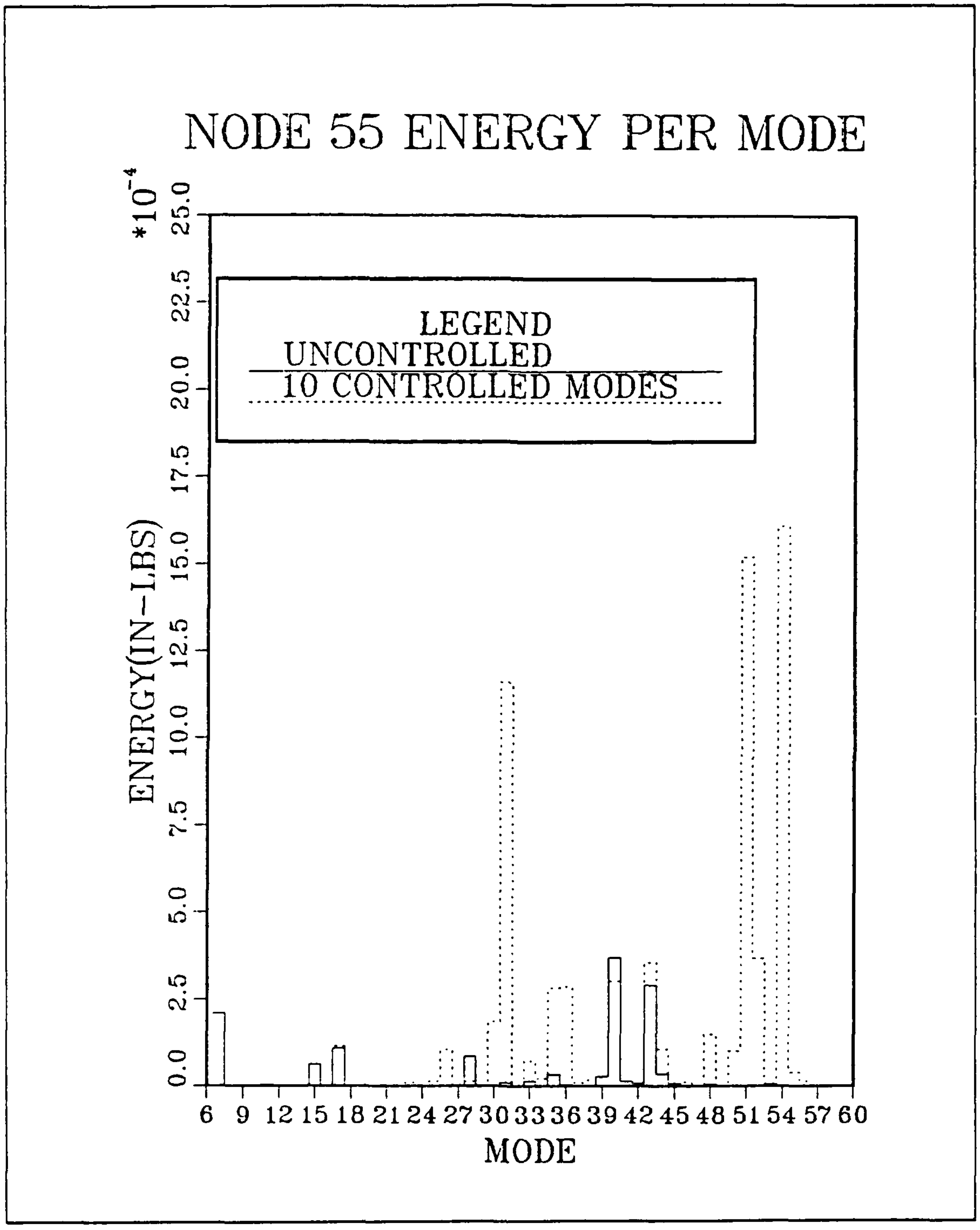

Figure 10. Ten controlled modes selected by frequency, excitation at node 55.

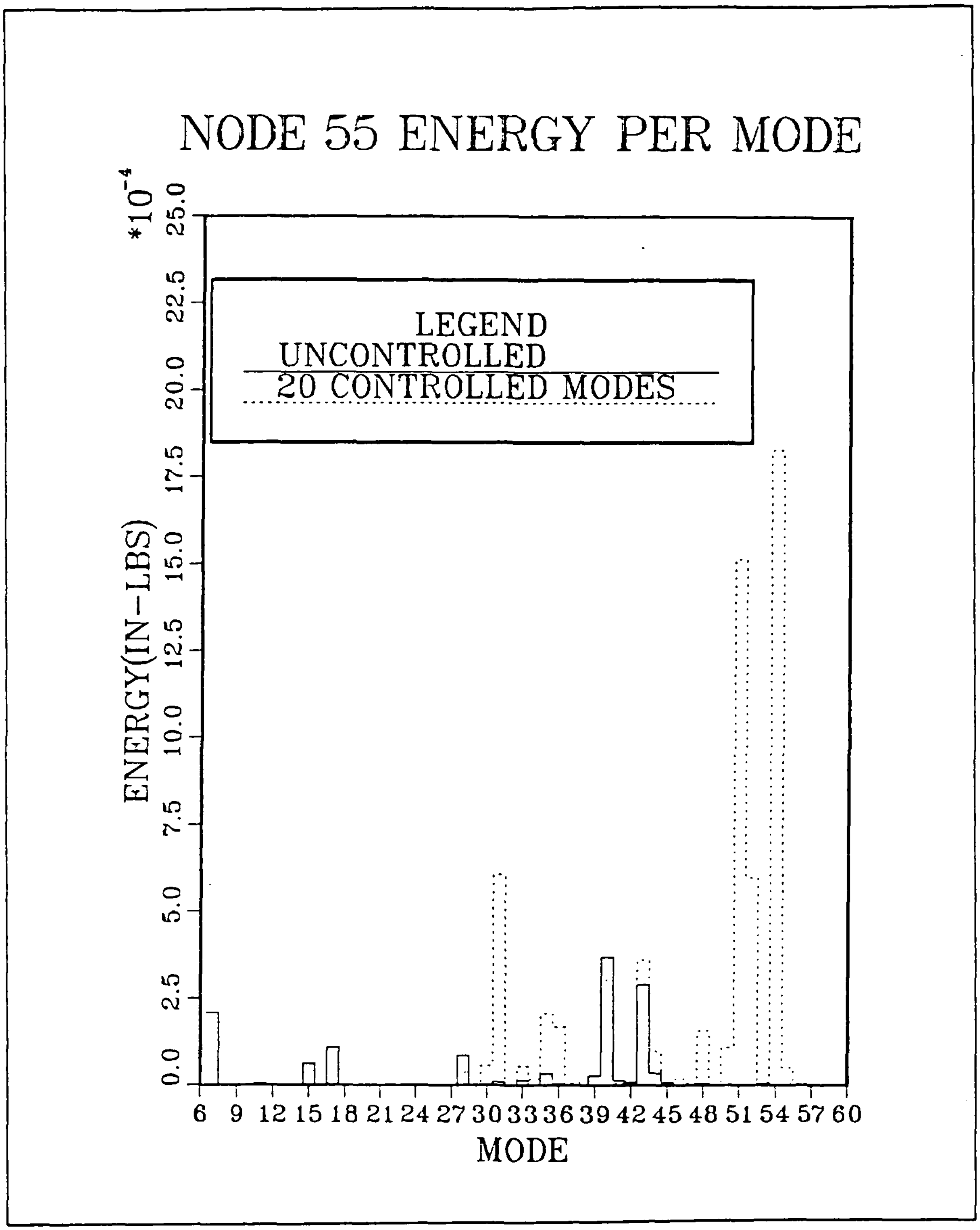

Figure 11. Twenty controlled modes selected by frequency,excitation at node 55.

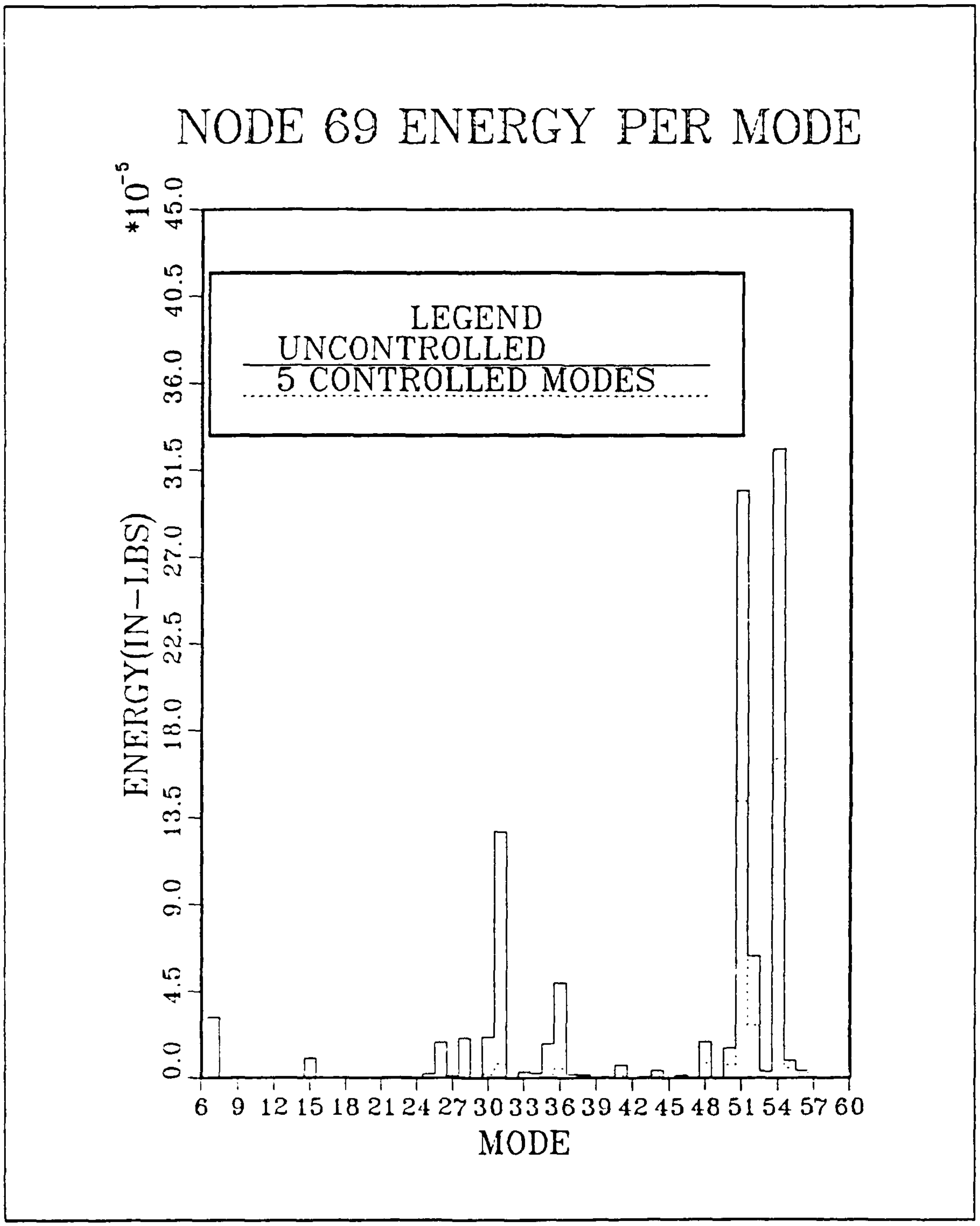

Figure 12. Five controlled modes selected by frequency, excitation at node 69.

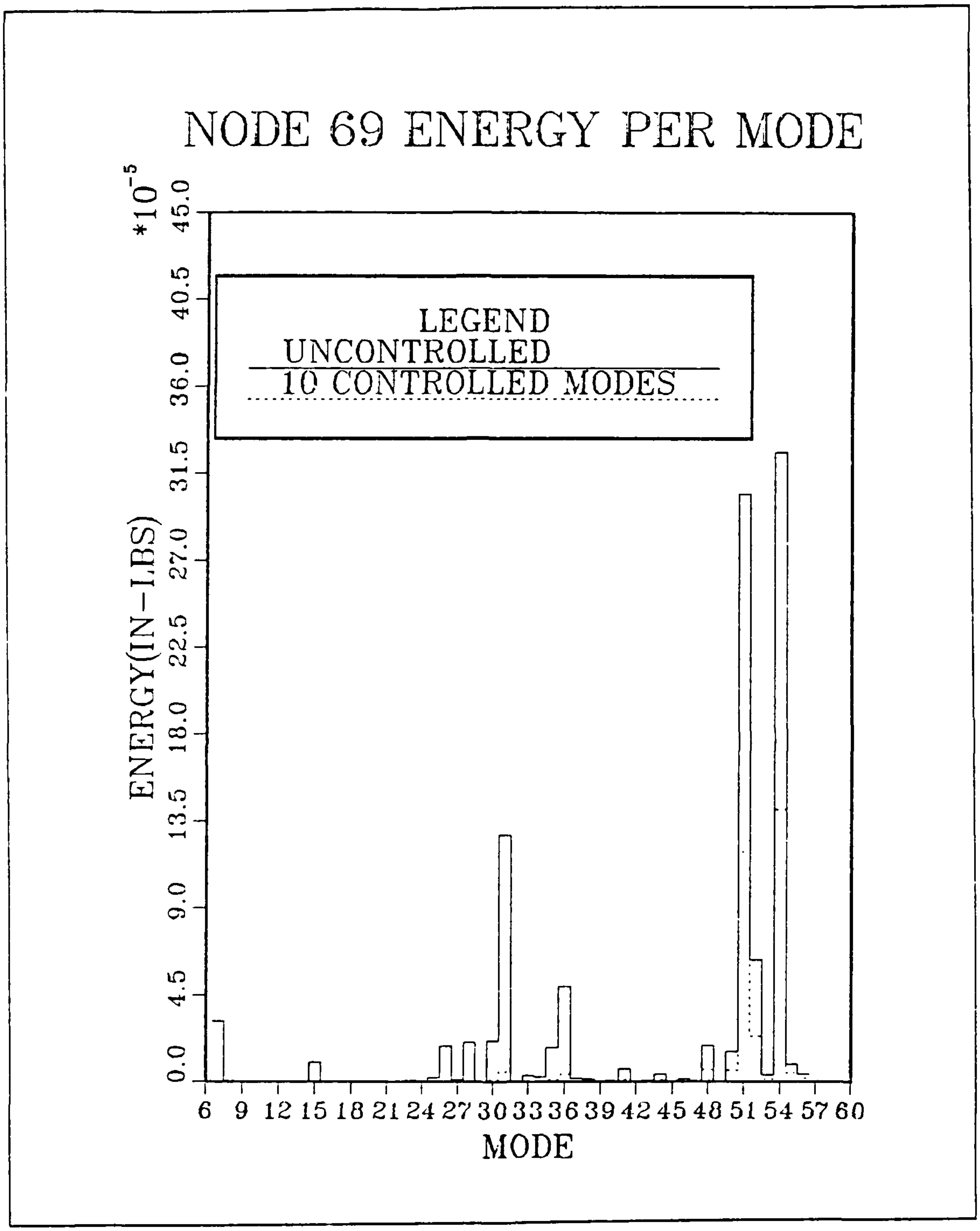

Figure 13. Ten controlled modes selected by frequency, excitation at node 69.

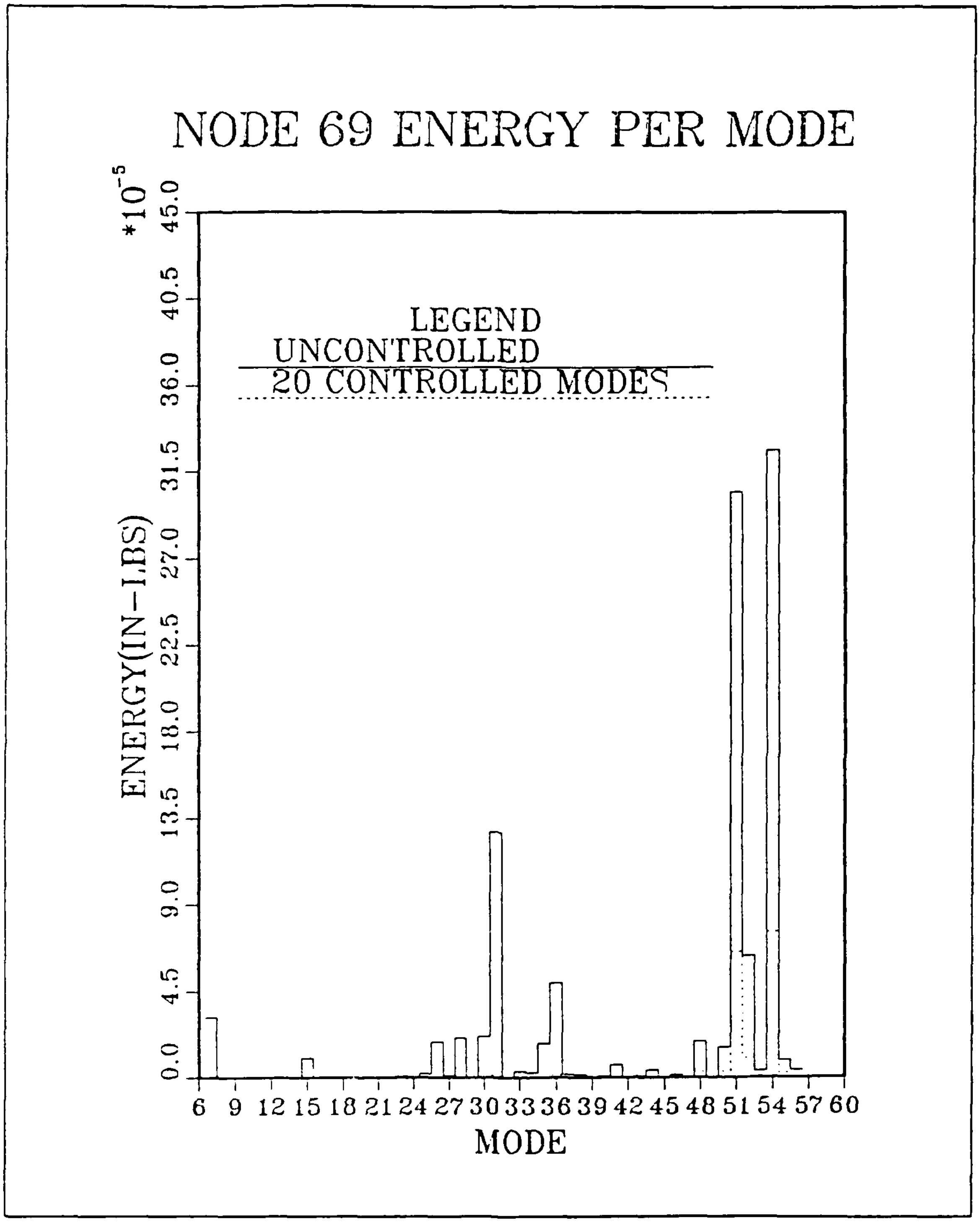

Figure 14. Twenty controlled modes selected by frequency, excitation at node 69.

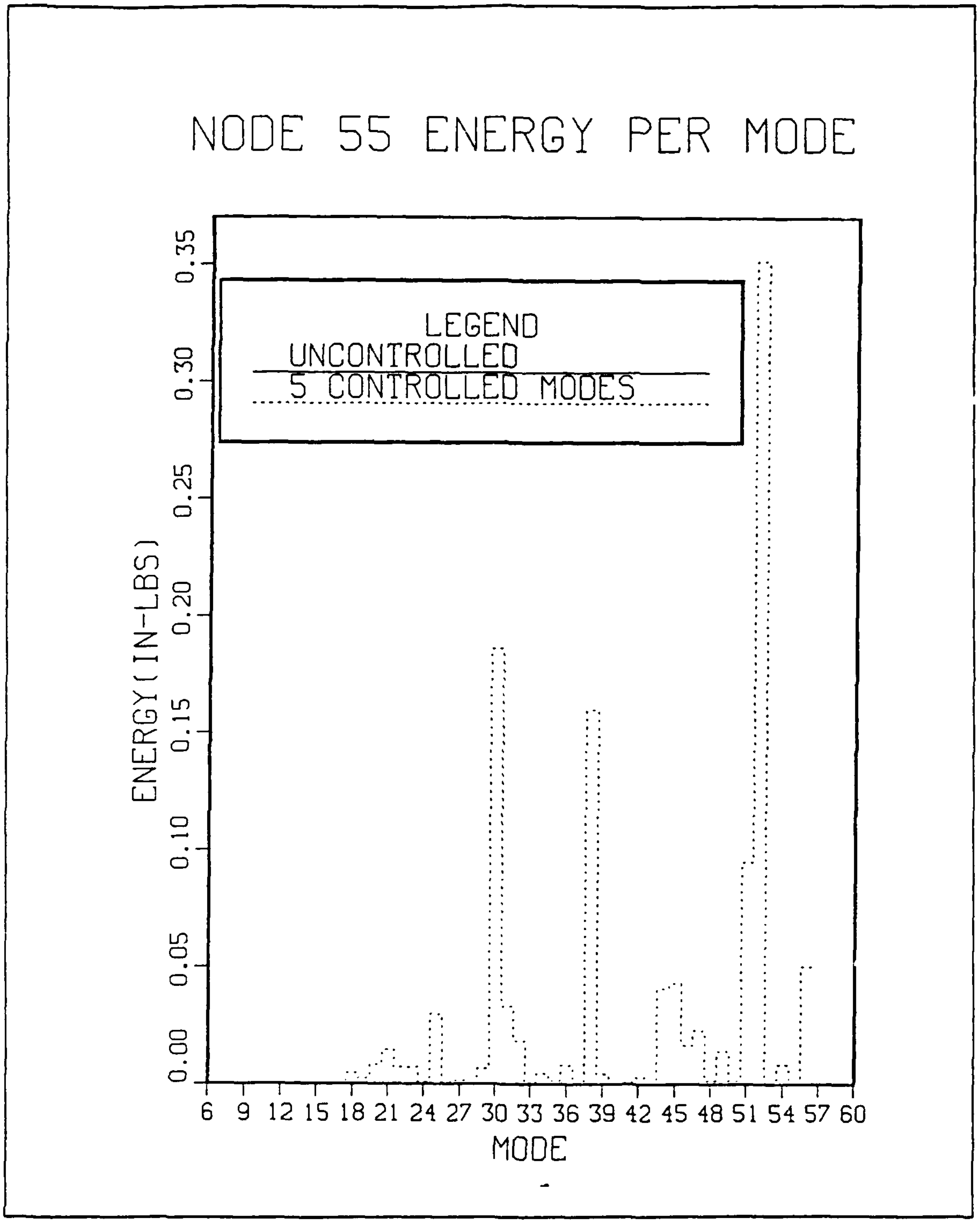

Figure 15. Five controlled modes selected by modal energy, excitation at node 55.

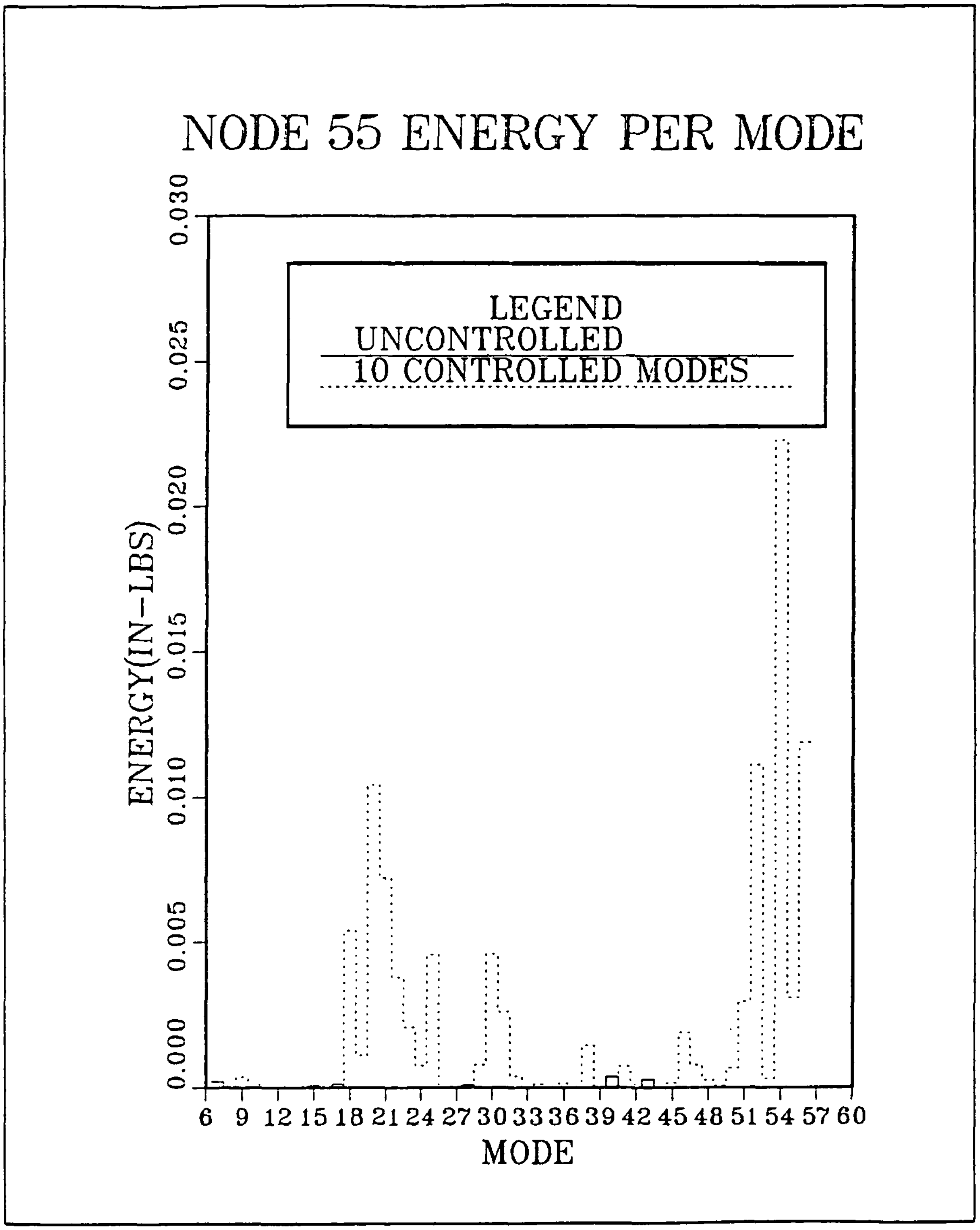

Figure 16. Ten controlled modes selected by modal energy, excitation at node 55.

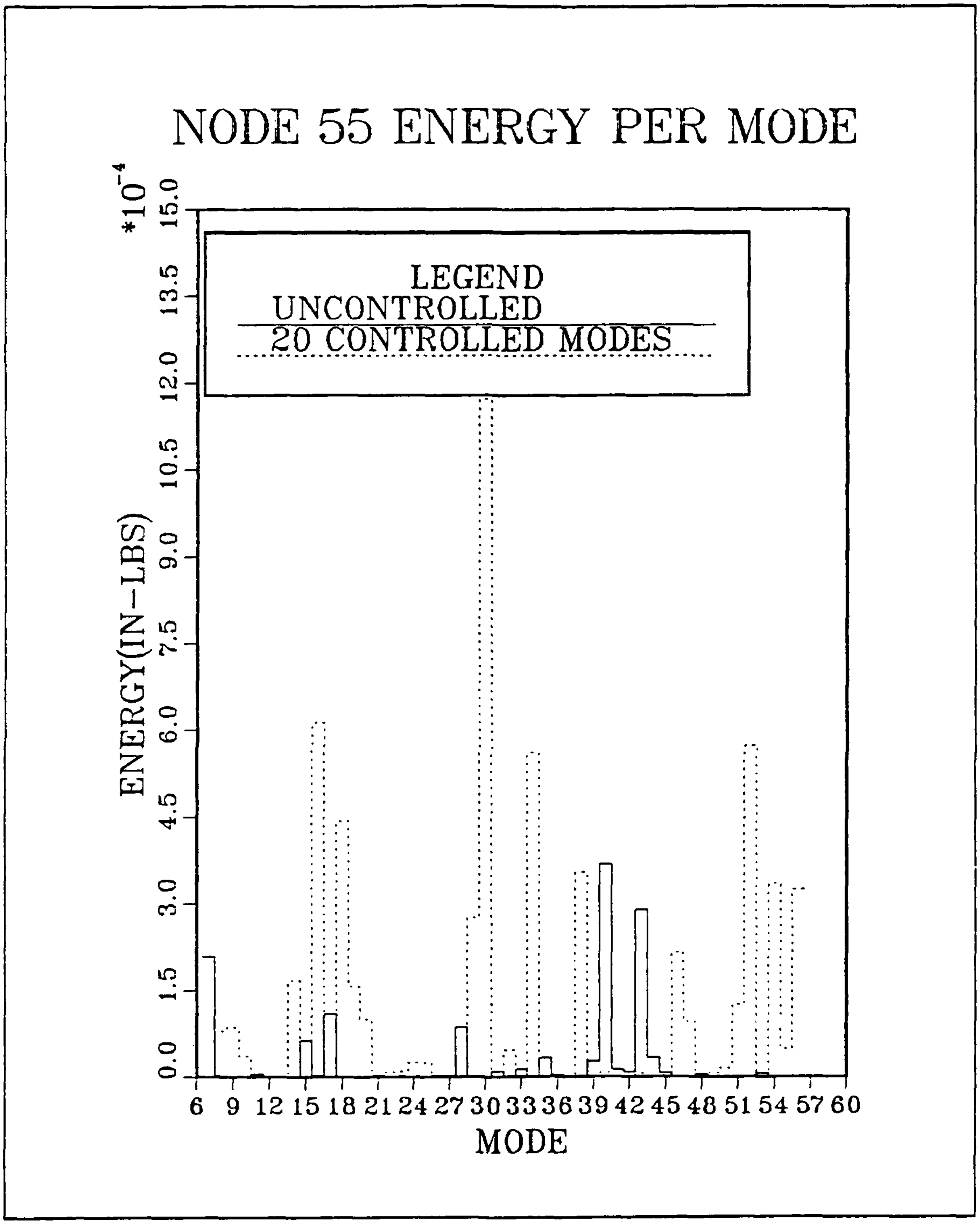

Figure 17. Twenty controlled modes selected by modal energy, excitation at node 55.

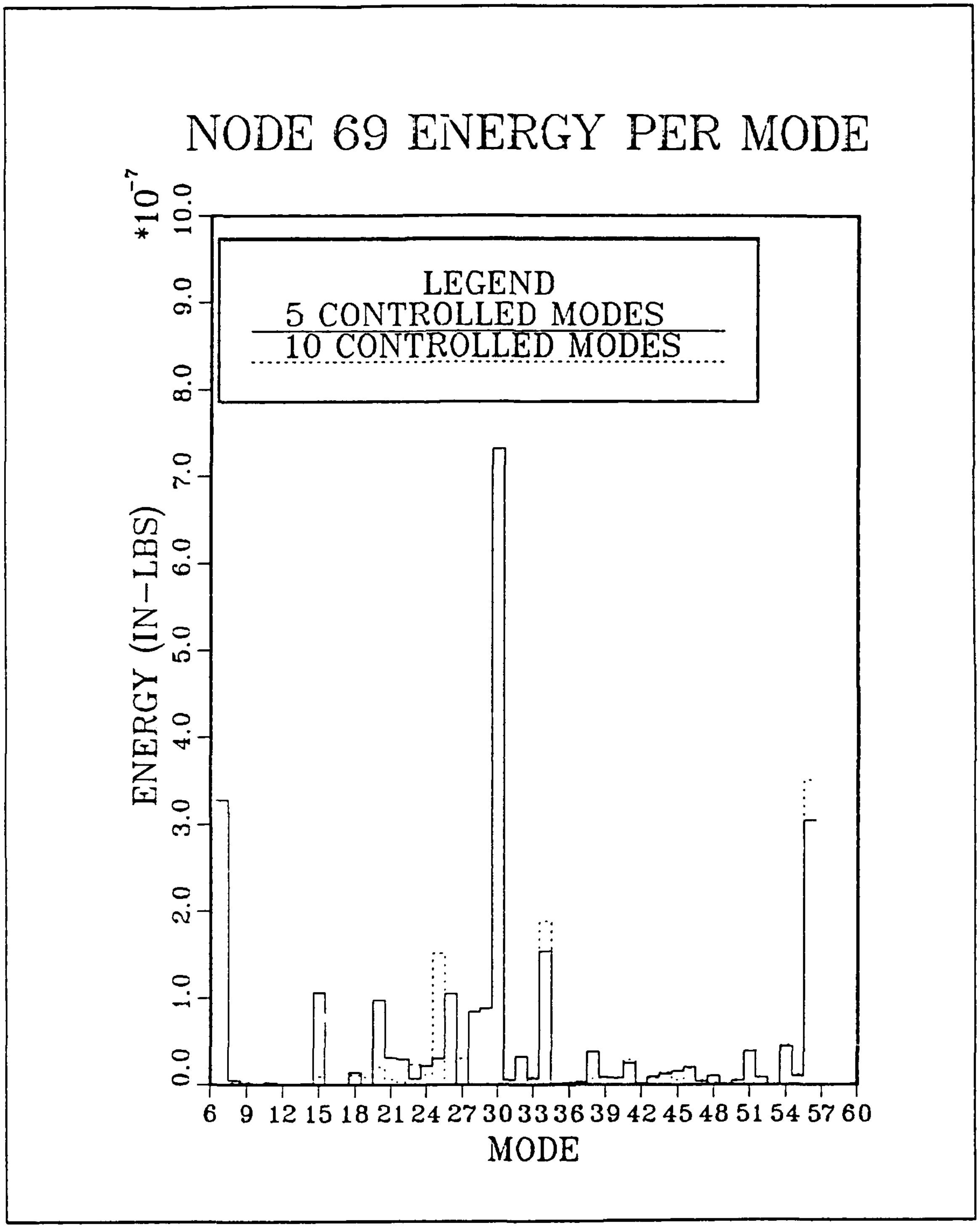

Figure 18. Five and ten controlled modes selected by modal energy, excitation at node 69.

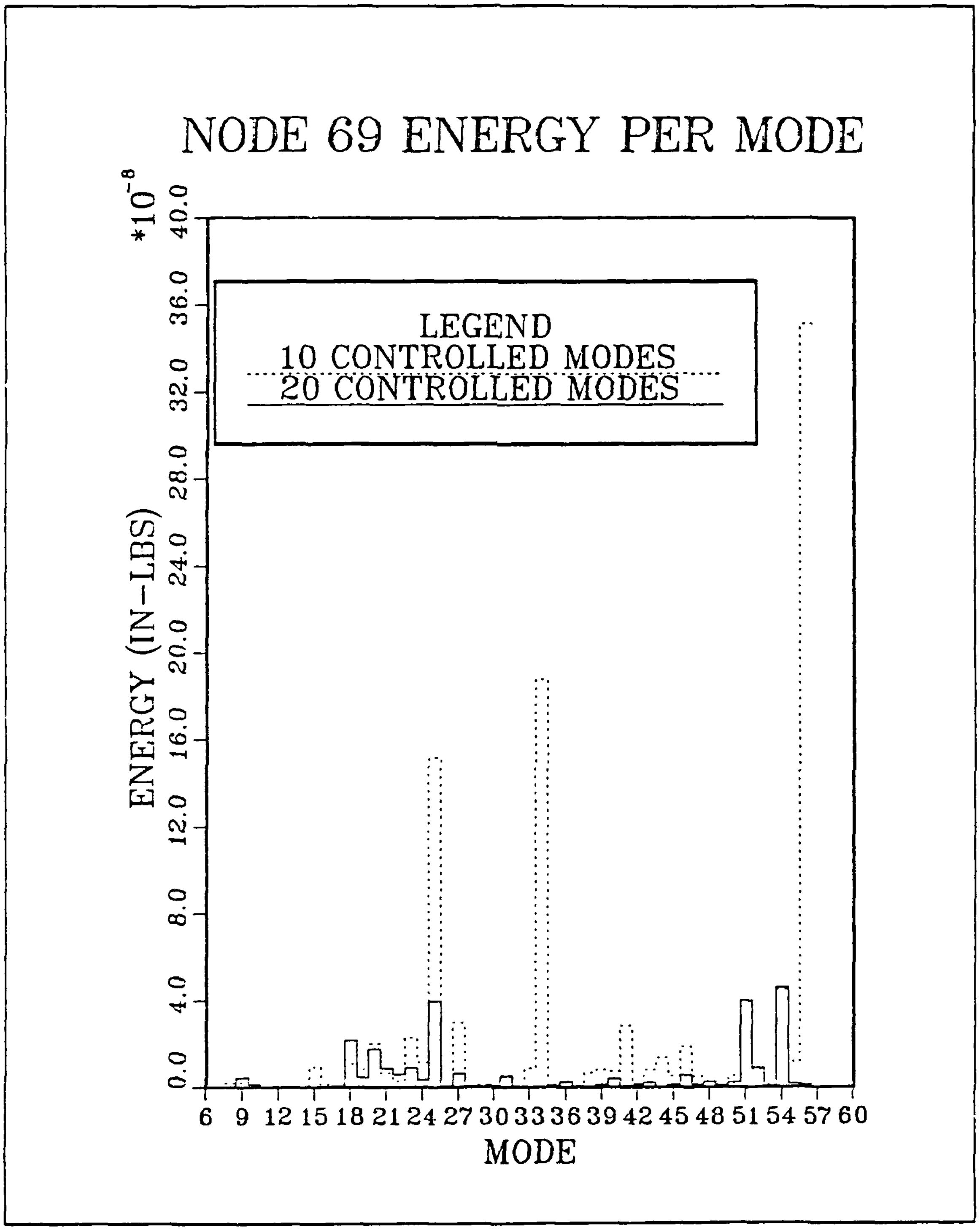

Figure 19. Ten and twenty controlled modes selected by modal energy, excitation at node 69.

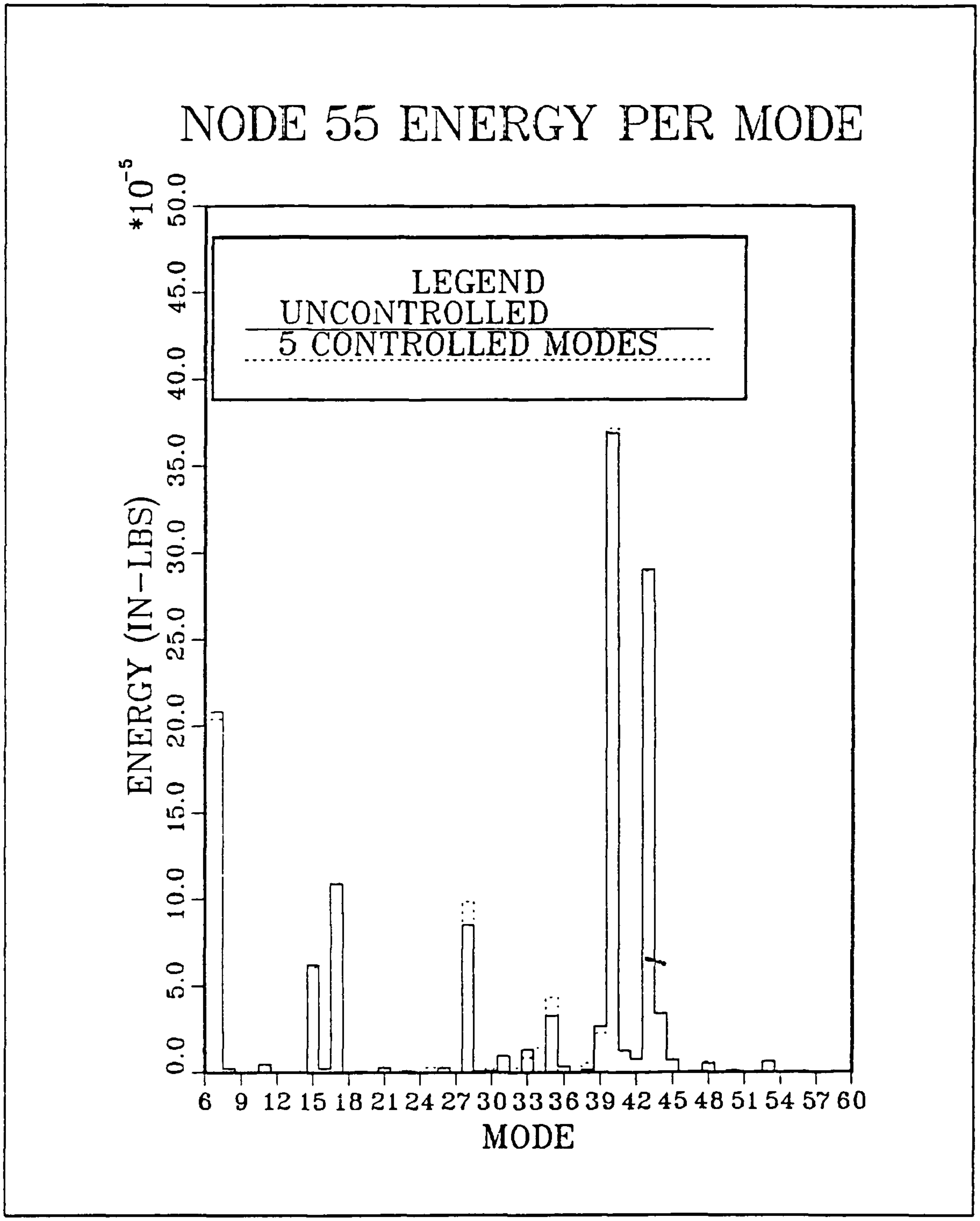

Figure 20. Five controlled modes selected by modal energy due to excitation at node 69, actual excitation at node 55.

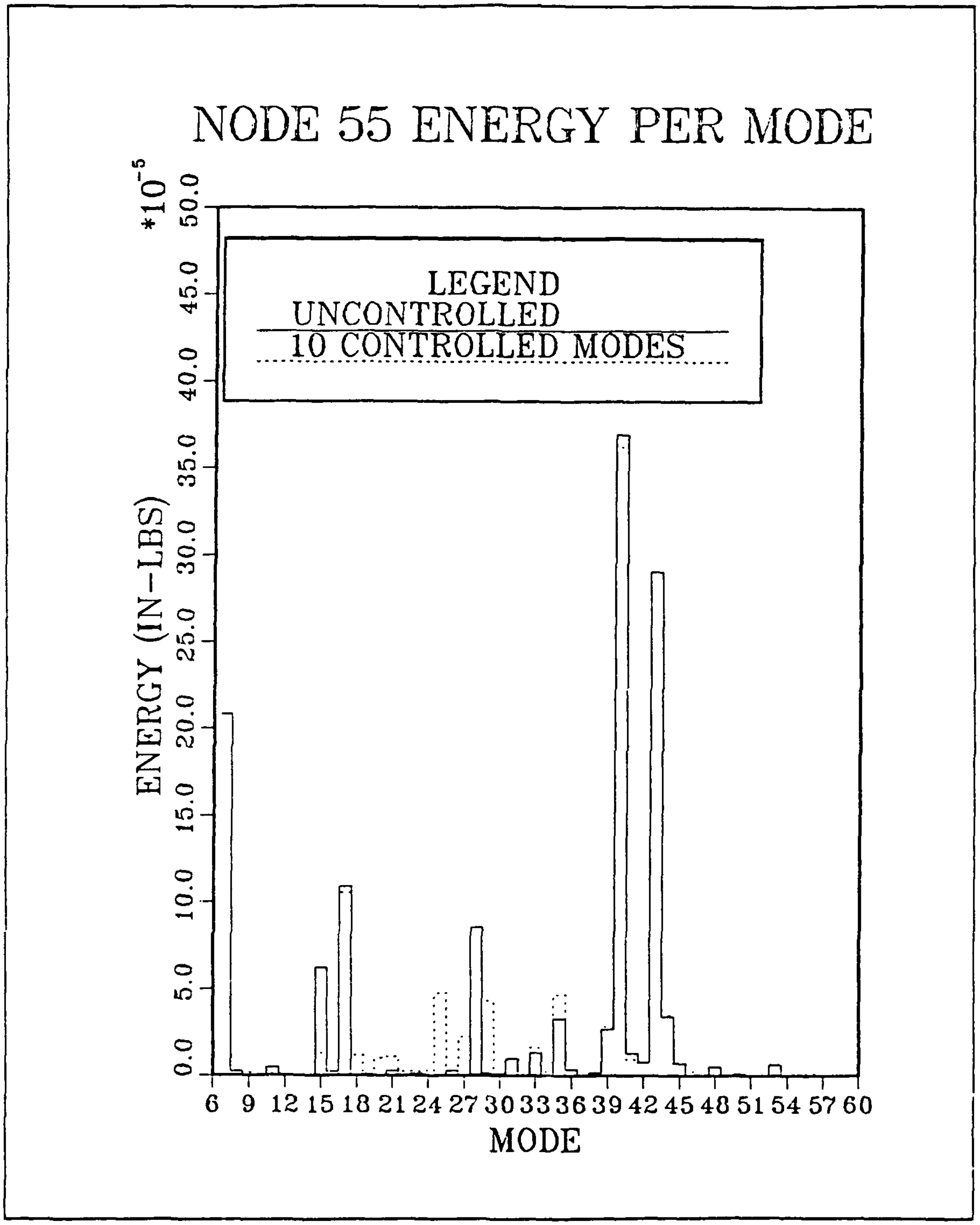

Figure 21. Ten controlled modes selected by modal energy due to excitation at node 69, actual excitation at node 55.

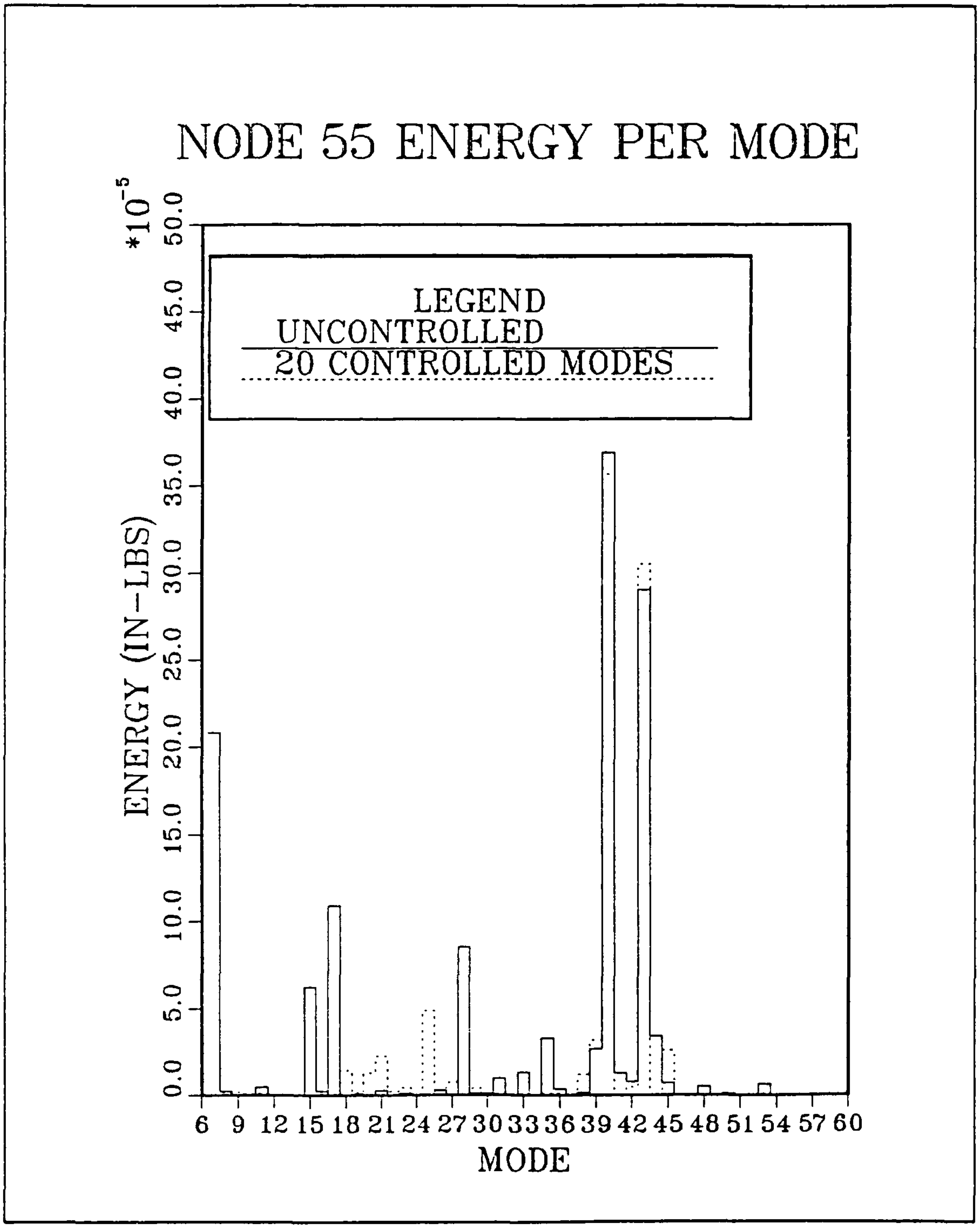

Figure 22. Twenty controlled modes selected by modal energy due to excitation at node 69, actual excitation at node 55.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

Results supported theoretical claim that the KL mode shapes converge to the natural mode shapes. For the space station example, a damping factor of 0.001 was required to make the approximation based on convergence good.

Preston discovered, see [Ref. 1: p. 46], that truncating the modes based on natural frequency yielded poor results. Modes with large coupling to the control system are not modeled when natural frequency is the method of truncation. Instead modes with very little coupling to the control system are modeled and in attempting to control these modes very large control torques are generated causing large excitations in the strongly coupled but unmodeled modes as seen in Figures 9 through 14.

The excitation of each mode depends on two factors: the natural frequency which determines the damping and the amplitude of the mode shape which determines how much excitation is received by each mode. The amplitude of the mode shape is the dominate factor which is evident from the open loop responses see Figures 7 and 8.

Truncating based on the open loop response, i.e., the Karhunen-Loeve model also yields poor results except when it is developed from the open loop response for the case of the disturbance torques being due to actuator noise, see Figure 8 on page 25. When truncating modes based on the open loop response, the coupling from the noise input is the dominate factor in selecting the modes to include in the model. In this case modes with large open loop excitation are controlled, but the control coupling still dominates yielding large excitation in the unmodeled modes (see Figures 15 through 17). When the disturbance torques are due to actuator noise, the modes with the largest open loop excitation are also the modes with the largest control coupling and the control system works well (see Figures 18 and 19).

The KL model based on the open loop response for disturbance torques due to actuator noise works well when applied to the space station with a disturbance torque applied at another location. As seen in Figures 20 through 22, the control system drives the modeled modes close to zero without exciting other modes to a significant degree. The problem with this configuration is that modes with large open loop excitation are not necessarily modeled and may be left excited by the control system.

B. DESIGN PROCEDURE

This design procedure is based on the above conclusions and is recommended for controlling vibration in LSS. First, the reduced order control model should be based on the KL mode shapes determined from the open loop response for disturbance torques due to actuator noise. Next, check the open loop response for disturbance torques at locations on the structure where noise inputs are most likely, e.g., the alpha joint. Modes strongly coupled to the disturbance input must be included in the KL model. This will determine the size of the KL (reduced order) model. If, after all this is done, the results are still unacceptable, then adding additional control elements should be considered. The placement of the additional control element(s) is determined by the node(s) with the largest modal amplitude(s) for the modes that remain excited by the initial control system. The process is repeated until an acceptable control is achieved.

APPENDIX A. SPACE STRUCTURE SIMULATION PROGRAM

This program simulates the dual keel space station described in Chapter I by implementing the model described in Chapter IV. The control described in Chapter IV is also simulated. For a detailed explanation of this program and its development see [Ref. 1].

```
C
C     ****************************************************************
C     *****          Space Structure Simulation Program          *****
C     *****                                                      *****
C     *****                  By William J. Preston               *****
C     ****************************************************************
C
C
C     ****************************************************************
C     *****                VARIABLE DECLARATIONS                 *****
C     ****************************************************************
C
      EXTERNAL EXCMS,RICDSD
      CHARACTER*6 NAM
      CHARACTER*1 AGAIN,CORECT,RAGAIN
      INTEGER NODE,MODE,KQ,EMODE,SMODE
      INTEGER CT,CF,KA,LOOP,PRNT,MODAL,V,COUNT,PRNTG
      INTEGER NF,NG,NH,NZ,I,K,M,CTADJ
      INTEGER IPVS(100),ITYPE(200)
C     ****************************************************
C     REAL TOTCST,RMODEN(7:100)
C     REAL*8 COSW1T,SINW1T,COST,CNTCST,ENERGY,RM
C     ****************************************************
      REAL RTOTAL,RMODEN(7:100)
      REAL*8 PHII(2,2,100),GAMMA(2,100),EGT,GMA,WN,W1,X1T,X2T
      REAL*8 PHI(188,188),B(188,3),BN(188,3),R(3,3),RR(3,3)
      REAL*8 RINV(3,3),RRINV(3,3),X1(7:100),X2(7:100),MODEN(7:100)
      REAL*8 COSW1T,SINW1T,COST,CNTCST,ENERGY,TOTCST,RM
      REAL*8 TCX,TCY,TCZ,DAMP,SAMPT,PI,SUM1,SUM2,SUM3,SUMC
      REAL*8 TNX,TNY,TNZ,IMPX,IMPY,IMPZ,IMPLSX,IMPLSY,IMPLSZ
      REAL LAMA(100),UGVEX(100,3),RNODE,RMODE,MIN,TIME,SAMPTM
      REAL UG69(100,3),UG23(100,3),UG55(100,3)
      REAL*8 H(100,100),G(100,100),L(3,100),BT(3,100)
      REAL*8 Z(200,200),W(200,200),ER(200),F(100,100),EI(200)
      REAL*8 SCALE(200),TEMP(100,3),TEMP1(3,100),WORK(100)
C     ****************************************************************
C
C     ****************************************************************
C     *****                 VARIABLE DEFINITIONS                 *****
C     ****************************************************************
C
```

```
C     LAMA = VECTOR OF THE SQUARE OF THE NATURAL FREQUENCIES
C     UGVEX = MODE POSITONS AND SLOPES OF THE NODAL POINTS
C     PHI = STATE TRANSITION MATRICIES FOR EACH MODE
C     GAMMA = INPUT TRANSITION MATRIX
C     A = DIAGONAL MATRIX CONSISTING OF PHI
C     B = INPUT MATRIX OF GAMMA AND CONTROL NODE SLOPES
C     BN = NOISE INPUT MATRIX OF GAMMA AND NOISE NODE SLOPES
C     DAMP = DAMPING FACTOR
C     SAMPT = SAMPLING TIME
C     IMPLSE = IMPULSE INPUT FUNCTION
C     TCX, TCY, TCZ = CONTROL TORQUE VALUES
C     IMPX, IMPY, IMPZ = AXIS IMPULSE NOISE VALUES
C     ENERGY = SYSTEM ENERGY COST VALUE FOR A GIVEN POINT IN TIME
C     CNTCST = SYSTEM CONTROL COST VALUE FOR A GIVEN POINT IN TIME
C     COST = TOTAL SYSTEM COST VALUE FOR A GIVEN POINT IN TIME
C     TOTCST = SYSTEM COST SUMMED OVER ALL TIME
C     MIN = NUMBER OF MINUTES SYSTEM WILL BE OBSERVED
C
C     **********      SAMPLE OF SPACEN EXEC FILE      **************
C
C     THIS FILE MUST BEGIN IN COLUMN 1 AND RUN WITH THE FOLLOWING
C     SEQUENCE FOR THE INITIAL RUN OF THE PROGRAM:
C
C             FORTVS2 SPACEN            (COMPILES PROGRAM)
C             SPACEN                   (LOADS AND RUNS PROGRAM)
C
C     SUBSEQUENT PROGRAM RUNS CAN ELIMINATE "FORTVS2 SPACEN" IF NO
C     CHANGES HAVE BEEN MADE TO THE PROGRAM.
C
C     CP DEF STOR 2M
C     FI 4 DISK KLAMA OUTPUT B (PERM
C     FI 30 DISK X1 OUTPUT A (RECFM F BLOCK 80 PERM
C     FI 31 DISK MODENG SPACEN A (RECFM F BLOCK 80 PERM
C     FI 32 DISK TORQUE OUTPUT A (RECFM F BLOCK 80 PERM
C     FI 33 DISK ENERGY OUTPUT A (RECFM F BLOCK 80 PERM
C     FI 34 DISK MDECST OUTPUT A (RECFM F BLOCK 80 PERM
C     FI 35 DISK COUNT INPUT A (RECFM F BLOCK 80 PERM
C     FI 40 DISK UTILITY DATA A (RECFM F BLOCK 80 PER
C     FI 41 DISK RUN DATA A (RECFM F BLOCK 80 PERM
C     FI 42 DISK KUG69 OUTPUT A (RECFM F BLOCK 80 PERM
C     FI 43 DISK KUG23 OUTPUT A (RECFM F BLOCK 80 PERM
C     FI 44 DISK KUG55 OUTPUT A (RECFM F BLOCK 80 PERM
C     LOAD SPACEN
C     START * NOXUFLOW
C
C
C     ***********************************************************
C
      PI = 4.0D0 * ATAN(1.0D0)
      SAMPT = 0.0
      DAMP = 0.0
      MODAL = 0
      IMPX = 0.0D0
      IMPY = 0.0D0
      IMPZ = 0.0D0
      NF = 100
```

```
      NG = 100
      NH = 100
      NZ = 200
C     *************
C
C     *****  NUMBER OF MINUTES THE SYSTEM WILL BE OBSERVED  *****
C
      MIN = 120.0
C
C     ***************************************************************
C     *****              SET LENGTH OF MODAL MODEL              *****
C     ***************************************************************
C
      MODAL = 56
C
C     ***************************************************************
C     *****                  READ LAMA MATRIX                   *****
C     ***************************************************************
C
      READ(4,1001) NAM
      READ(4,1002)(LAMA(I),I=1,100)
C
C     ***************************************************************
C     *****               SCREEN INTERACTION                    *****
C     ***************************************************************
C
C
C     *************          STARTING MODE NUMBER        **************
C
      SMODE = 7
C
C     ***************       NUMBER OF MODES TO SCAN      *************
C
      MODE =5
      EMODE = SMODE + MODE - 1
C
C     *************      NOISE INPUT POSITION       *****************
C
      NODE = 55
      AXIS = 1
C
C     **************           R MATRIX VALUE       ****************
C
      RM = 1E-12
C
C     *********°.01 FOR FULL SAMPLING TIME °.05 FOR REDUCED ******
C
      SAMPT = 0.01
C
C     ***************          DAMPING FACTOR         **************
C
      DAMP = 0.001D00
C
      DO 75 I = 1,100
         READ(42,1040) (UG69(I,K),K=1,3)
         READ(43,1040) (UG23(I,K),K=1,3)
```

```
         READ(44,1040) (UG55(I,K),K=1,3)
 75     CONTINUE
 C      *****************          BEGIN RUN            ***************
 
 *      DO 505 M = 1,3
 C
        WRITE (41,700) SMODE
        WRITE (41,701) MODE
        WRITE (41,706) EMODE
        WRITE (41,702) NODE
        WRITE (41,703) RM
        WRITE (41,704) SAMPT
        WRITE (41,705) DAMP
        WRITE (41,707) MIN
        WRITE (41,708) MODAL
 C
 C      *********     NOISE AXIS INPUT AND LOCATION      **********
 C
        IF(AXIS.EQ.1)THEN
           IMPX = 1.0D0/SAMPT
        ELSEIF(AXIS.EQ.2)THEN
           IMPY = 1.0D0/SAMPT
        ELSEIF(AXIS.EQ.3)THEN
           IMPZ = 1.0D0/SAMPT
        ELSEIF(AXIS.EQ.4)THEN
           IMPX = 1.0D0/SAMPT
           IMPY = 1.0D0/SAMPT
           IMPZ = 1.0D0/SAMPT
        ENDIF
 C
        COUNT = 0
 C
 C      *************      INITIALIZE MATRICIES       *****************
 C
        DO 40 I = 1,188
           DO 45 J = 1,188
              PHI(I,J) = 0.0
 45        CONTINUE
 40     CONTINUE
 C
        DO 60 I = 1,188
           DO 65 J = 1,3
              B(I,J) = 0.0
              BN(I,J) = 0.0
 65        CONTINUE
 60     CONTINUE
 C
        DO 70 K = 7,100
           X1(K) = 0.0
           X2(K) = 0.0
           MODEN(K) = 0.0
           RMODEN(K) = 0.0
 70     CONTINUE
 C
 C      *************************************************************
 C      *****            BEGIN MAIN PROGRAM                     *****
```

```
C     *****      ESTABLISH A, B AND B"NOISE" MATRICIES      *****
C     *************************************************************
C
      DO 600 I = SMODE,MODAL
         WN = DBLE(SQRT(LAMA(I)))
         GMA = DAMP*WN/2.0
         EGT = DEXP(-GMA*SAMPT)
         W1 = DSQRT((WN**2)-(GMA**2))
         COSW1T = DCOS(W1*SAMPT)
         SINW1T = DSIN(W1*SAMPT)
C
         IF(WN.EQ.0)THEN
              PHII(1,1,I) = EGT*COSW1T
              PHII(1,2,I) = SAMPT
              PHII(2,1,I) = 0
              PHII(2,2,I) = EGT*COSW1T
C
              GAMMA(1,I) = 0
              GAMMA(2,I) = 0
         ELSE
C
         PHII(1,1,I) = EGT*(COSW1T + (GMA*(W1**(-1)))*SINW1T)
         PHII(1,2,I) = (W1**(-1))*EGT*SINW1T
         PHII(2,1,I) = -(WN**2)*(W1**(-1))*EGT*SINW1T
         PHII(2,2,I) = EGT*(COSW1T - (GMA*(W1**(-1)))*SINW1T)
C
         GAMMA(1,I) = (WN**(-2))*(1.0D0-EGT*(COSW1T+(GMA/W1)*SINW1T))
         GAMMA(2,I) = (W1**(-1))*EGT*SINW1T
C
         ENDIF
C
600   CONTINUE
C
      V = 1
C
      DO 610 K = SMODE,MODAL
C
         PHI(V,V) = PHII(1,1,K)
         PHI(V,V+1) = PHII(1,2,K)
         PHI(V+1,V) = PHII(2,1,K)
         PHI(V+1,V+1) = PHII(2,2,K)
C
         B(V,1) = GAMMA(1,K)*DBLE(UG69(K,1))
         B(V,2) = GAMMA(1,K)*DBLE(UG69(K,2))
         B(V,3) = GAMMA(1,K)*DBLE(UG69(K,3))
         B(V+1,1) = GAMMA(2,K)*DBLE(UG69(K,1))
         B(V+1,2) = GAMMA(2,K)*DBLE(UG69(K,2))
         B(V+1,3) = GAMMA(2,K)*DBLE(UG69(K,3))
C
         V = V+2
C
610   CONTINUE
C
         DO 605 I = 1,100
            UGVEX(I,1) = 0.0
            UGVEX(I,2) = 0.0
```

```
              UGVEX(I,3) = 0.0
                IF(NODE.EQ.23) THEN
                   UGVEX(I,1) = UG23(I,1)
                   UGVEX(I,2) = UG23(I,2)
                   UGVEX(I,3) = UG23(I,3)
                ELSEIF(NODE.EQ.55) THEN
                   UGVEX(I,1) = UG55(I,1)
                   UGVEX(I,2) = UG55(I,2)
                   UGVEX(I,3) = UG55(I,3)
                ELSEIF(NODE.EQ.69) THEN
                   UGVEX(I,1) = UG69(I,1)
                   UGVEX(I,2) = UG69(I,2)
                   UGVEX(I,3) = UG69(I,3)
                ENDIF
605         CONTINUE
C
        V = 1
C
        DO 620 K = SMODE,MODAL
C
           BN(V,1) = GAMMA(1,K)*DBLE(UGVEX(K,1))
           BN(V,2) = GAMMA(1,K)*DBLE(UGVEX(K,2))
           BN(V,3) = GAMMA(1,K)*DBLE(UGVEX(K,3))
           BN(V+1,1) = GAMMA(2,K)*DBLE(UGVEX(K,1))
           BN(V+1,2) = GAMMA(2,K)*DBLE(UGVEX(K,2))
           BN(V+1,3) = GAMMA(2,K)*DBLE(UGVEX(K,3))
C
           V = V+2
C
620     CONTINUE
C
550     CONTINUE
C
C       ***********   ESTABLISH H, F AND R MATRICIES    **********
C
        DO 50 I = 1,NH
           DO 55 J = 1,NH
               H(I,J) = 0.0
               F(I,J) = 0.0
               G(I,J) = 0.0
               IF(I.LE.3)THEN
                 L(I,J) = 0.0
               ENDIF
55         CONTINUE
50      CONTINUE
C
        DO 61 I = 1,3
           DO 66 J = 1,3
                 R(I,J) = 0.0
                 RINV(I,J) = 0.0
66         CONTINUE
61      CONTINUE
        KQ = 1
        DO 80 K = SMODE,EMODE
           H(KQ,KQ) = DBLE(LAMA(K))
           H(KQ+1,KQ+1) = 1.0D0
```

```
         KQ = KQ+2
 80   CONTINUE
C
      K = 0
      DO 85 K = 1,3
         R(K,K) = RM
         RINV(K,K) = 1.0D0/RM
 85   CONTINUE
C
      DO 88 I = 1,2*MODE
         DO 89 J = 1,2*MODE
            F(I,J) = PHI(I,J)
 89      CONTINUE
 88   CONTINUE
C
C     *****   COMPUTE G MATRIX AS NEEDED BY RICDSD SUBR     *****
C
         CALL MATRAN(B,188,2*MODE,3,BT,3)
C
         CALL MATMUL(B,188,2*MODE,3,RINV,3,3,TEMP,NH)
C
         CALL MATMUL(TEMP,NH,2*MODE,3,BT,3,2*MODE,G,NG)
C
C     *************************************************************
C                  BEGIN RICCATI GAIN CALCULATIONS
C     *************************************************************
C
      CALL RICDSD(NF,NG,NH,NZ,2*MODE,4*MODE,F,G,H,Z,W,ER,EI,WORK,
     +            SCALE,ITYPE,IPVS)
C
      WRITE(41,*) ' '
      WRITE (41,130)
130   FORMAT (/' THE CLOSED LOOP EIGENVALUES ARE:'/)
      DO 140 I = 1,2*MODE
         WRITE (41,*) ER(I),EI(I)
140   CONTINUE
      WRITE (41,150) WORK(1)
150   FORMAT (/' CONDITION ESTIMATE IS:',D26.18)
C
C     ************      COMPUTE GAIN MATRIX  L      ************
C
         CALL MATMUL(BT,3,3,2*MODE,H,NH,2*MODE,TEMP1,3)
C
         CALL MATMUL(TEMP1,3,3,2*MODE,B,188,3,RR,3)
C
         DO 103 I = 1,3
              DO 104 J=1,3
                   RR(I,J)=R(I,J)+RR(I,J)
104           CONTINUE
103      CONTINUE
C
      CALL DLINDS(3,RR,3,RRINV,3)
C
      CALL MATMUL(RRINV,3,3,3,TEMP1,3,2*MODE,BT,3)
C
```

```
      CALL MATMUL(BT,3,3,2*MODE,PHI,188,2*MODE,L,3)
C
      WRITE(41,*) ' '
      WRITE(41,*) '  GAIN MATRIX   L '
      WRITE(41,*) ' '
      WRITE(41,*) '      ROW 1                    ROW 2                   ROW 3'
      DO 9155 I=1,2*MODE
              WRITE(41,1040) (L(J,I),J=1,3)
9155  CONTINUE
      WRITE(41,*) ' '
C
C     **********************************************************
C     *****         COMPUTATION OF TORQUES AND COSTS        *****
C     **********************************************************
C
9000  COUNT = 0
      TOTCST = 0.0D0
      TIME = 0.0
C
C     ****************************************************************
C     *****  SETS LOOP FOR THE NUMBER OF ITERATIONS NECESSARY    *****
C     *****  TO OBSERVE THE SYSTEM FOR DESIRED LENGTH OF TIME    *****
C     ****************************************************************
C
      LOOP = INT((MIN*60.0)/SAMPT)
      PRNT = INT(((MIN*60.0)/SAMPT)/300.0)
      PRNTG = INT(((MIN*60.0)/SAMPT)/1000.0)
C
      DO 200 N  = 0, LOOP
         TIME = DBLE(N)*SAMPT
C
         IF(N.EQ.0)THEN
            IMPLSX = IMPX
            IMPLSY = IMPY
            IMPLSZ = IMPZ
         ELSE
            IMPLSX = 0.0D0
            IMPLSY = 0.0D0
            IMPLSZ = 0.0D0
         ENDIF
C
C     ***********************************************************
C     ***********        CONTROL TORQUE EQUATIONS    ***********
C     ***********************************************************
C
      SUM1 = 0.0D0
      SUM2 = 0.0D0
      SUM3 = 0.0D0
C
      DO 210 CT = 1, MODE
         CTADJ = CT + (SMODE - 1)
         SUM1 = SUM1 + L(1,2*CT-1)*X1(CTADJ) + L(1,2*CT)*X2(CTADJ)
         SUM2 = SUM2 + L(2,2*CT-1)*X1(CTADJ) + L(2,2*CT)*X2(CTADJ)
         SUM3 = SUM3 + L(3,2*CT-1)*X1(CTADJ) + L(3,2*CT)*X2(CTADJ)
210   CONTINUE
C
```

```
C     *************    EVALUATION AT ZERO CONTROL     *************
C
C     IF(M.EQ.1)THEN
C        TCX = 0.0D0
C        TCY = 0.0D0
C        TCZ = 0.0D0
C     ELSE
         TCX = SUM1*(-1.0D0)
         TCY = SUM2*(-1.0D0)
         TCZ = SUM3*(-1.0D0)
C     ENDIF
C
C     IF(N.EQ.0)THEN
C        WRITE (32,*) 'IMPULSE X AXIS, IMPULSE Y AXIS, IMPULSE Z AXIS'
C        WRITE (32,*) ' '
C        WRITE (32,1040) IMPLSX, IMPLSY, IMPLSZ
C        WRITE (32,*) ' '
C        WRITE (32,*) 'CONTROL TORQUES TCX, TCY, TCZ'
C        WRITE (32,*) ' '
C     ENDIF
C
C        IF (N.LE.50) THEN
C            WRITE (32,2000) TIME, TCX, TCY, TCZ
C        ENDIF
C
C        IF (MOD(N,PRNTG).EQ.0) THEN
C            WRITE (32,2000) TIME, TCX, TCY, TCZ
C        ENDIF
C
C        IF (N.LE.20) THEN
C             WRITE(30,1035) TIME,X1(1),X1(5),X1(10),X1(15),X1(20)
C             WRITE(31,1035) TIME,X2(1),X2(5),X2(10),X2(15),X2(20)
C        ENDIF
C        IF (M.EQ.1) THEN
C         IF (MOD(N,PRNTG).EQ.0) THEN
C          WRITE(30,1036) TIME,X1(9),X1(10),X1(11),X1(12),X1(13),X1(14)
C           WRITE(30,1036) TIME,X1(1),X1(4),X1(14),X1(24),X1(34),X1(44)
C           WRITE(31,1035) TIME,X2(1),X2(5),X2(10),X2(15),X2(20)
C         ENDIF
C        ELSE
C         IF (MOD(N,PRNT).EQ.0) THEN
C           WRITE(30,1036) TIME,X1(1),X1(4),X1(24),X1(44),X1(74),X1(94)
C           WRITE(30,1036) TIME,X1(1),X1(4),X1(14),X1(24),X1(34),X1(44)
C           WRITE(31,1035) TIME,X2(1),X2(5),X2(10),X2(15),X2(20)
C         ENDIF
C        ENDIF
C
C
         IF (MOD(N,PRNTG).EQ.0) THEN
            COUNT = COUNT+1
         ENDIF
C
C     ***************************************************************
C     ********    SYSTEM COST FUNCTION CALCULATION        ********
C     ***************************************************************
C
```

```
      SUMC = 0.0D0
      ENERGY = 0.0D0
      CNTCST = 0.0D0
      COST = 0.0D0
C
      DO 230 CF = 7,MODAL
         MODEN(CF) = MODEN(CF)+(X1(CF)**2)*LAMA(CF)+X2(CF)**2
         SUMC = SUMC+(X1(CF)**2)*LAMA(CF)+X2(CF)**2
230   CONTINUE
C
C     ENERGY = SUMC
C     CNTCST = (TCX**2)*RM+(TCY**2)*RM+(TCZ**2)*RM
C     COST = ENERGY + CNTCST
C     TOTCST = TOTCST + COST
C
C        IF (MOD(N,PRNTG).EQ.0) THEN
C           WRITE(33,2000) TIME,ENERGY,CNTCST,COST
C        ENDIF
         IF(N.GE.(LOOP-50))THEN
            WRITE (34,3002) MODE,TOTCST
         ENDIF
C
C     ******************************************************************
C     *********            STATE UPDATE EQUATIONS           ***********
C     ******************************************************************
C
         DO 220 KA = 7,MODAL
         K = KA-6
C
         X1T=PHII(1,1,KA)*X1(KA)+PHII(1,2,KA)*X2(KA)+B((2*K-1),1)*TCX+
     +       B((2*K-1),2)*TCY+B((2*K-1),3)*TCZ+BN((2*K-1),1)*IMPLSX+
     +       BN((2*K-1),2)*IMPLSY+BN((2*K-1),3)*IMPLSZ
C
         X2T=PHII(2,1,KA)*X1(KA)+PHII(2,2,KA)*X2(KA)+B(2*K,1)*TCX+
     +       B(2*K,2)*TCY+B(2*K,3)*TCZ+BN(2*K,1)*IMPLSX+BN(2*K,2)*
     +       IMPLSY+BN(2*K,3)*IMPLSZ
C
         X1(KA) = X1T
         X2(KA) = X2T
C
220      CONTINUE
C
200   CONTINUE
C
      WRITE (35,3000) COUNT
      RTOTAL = TOTCST
C     WRITE (34,3002) MODE,RTOTAL
      WRITE (34,3002) MODE,TOTCST
C
      DO 235 K = 7,MODAL
         RMODEN(K) = MODEN(K)
         WRITE (31,3002) K, RMODEN(K)
235   CONTINUE
C
666   CONTINUE
C
```

```
*       IF(M.EQ.1)THEN
*          MODE=10
*          EMODE=16
*       ELSEIF(M.EQ.2)THEN
*          MODE=15
*          EMODE=21
*        ELSEIF(M.EQ.2)THEN
*           MODE=20
*           EMODE=26
*       ELSEIF(M.EQ.2)THEN
*          MODE=25
*          EMODE=31
*       ELSEIF(M.EQ.3)THEN
*          MODE=30
*          EMODE=36
*       ELSEIF(M.EQ.4)THEN
*          MODE=35
*          EMODE=41
*       IF(M.EQ.1)THEN
*          MODE=40
*          EMODE=46
*       ELSEIF(M.EQ.2)THEN
*          MODE=45
*          EMODE=51
*       ELSEIF(M.EQ.3)THEN
*          MODE=50
*          EMODE=56
*         ENDIF
*505    CONTINUE
C
C       ******************************************************************
C       *****                  FORMAT STATEMENTS                    *****
C       ******************************************************************
C
700     FORMAT (' ','STARTING MODE NUMBER:   ',I2)
701     FORMAT (' ','NUMBER OF MODES SCANNED:   ',I2)
702     FORMAT (' ','NOISE INPUT NODE:   ',I3)
703     FORMAT (' ','INITIAL R VALUE:   ',E12.4)
704     FORMAT (' ','SAMPLING TIME:   ',E12.4)
705     FORMAT (' ','DAMPING FACTOR:   ',E12.4)
706     FORMAT (' ','LAST CONTROLLED MODE:   ',I2)
707     FORMAT (' ','OBSERVATION TIME:   ',F5.1,' MINUTES')
708     FORMAT (' ','SIZE OF MODAL MODEL:   ',I3,' MODES')
1001    FORMAT(1X,A6)
1002    FORMAT(1X,8E15.8)
1004    FORMAT(1X,//)
1005    FORMAT(1X,60X,E11.5)
1008    FORMAT(1X,////)
1010    FORMAT(A1)
1035    FORMAT(' ',F7.2,2X,5(E12.6,2X))
1036    FORMAT(' ',F7.2,1X,6(E11.5,1X))
1040    FORMAT(' ',3(E15.8,5X))
1050    FORMAT(' ',4(E12.6,2X))
2000    FORMAT(1X,F7.2,3X,3(E15.8,3X))
2001    FORMAT(' ',T5,E15.8)
3000    FORMAT(I4)
```

```
3001  FORMAT(F7.2,2X,E12.5)
3002  FORMAT(I3,2X,E12.5)
C
599   STOP
      END
C
C     ************************************************************
C                  SUBROUTINE TO MATRIX MULTIPLY
C     ************************************************************
C
      SUBROUTINE MATMUL(M1,LD1,R1,C1,M2,LD2,C2,MP,LD3)
         INTEGER R1,C1,C2,LD1,LD2,LD3
         REAL*8 M1(LD1,1),M2(LD2,1),MP(LD3,1),SUM
C
         DO 650 I = 1,R1
              DO 660 J = 1,C2
                   SUM = 0.0D0
                 DO 670 K = 1,C1
                   SUM = SUM+M1(I,K)*M2(K,J)
670              CONTINUE
               MP(I,J) = SUM
660           CONTINUE
650      CONTINUE
      RETURN
      END
C
C     ************************************************************
C                 SUBROUTINE TO TRANSPOSE A MATRIX
C     ************************************************************
C
      SUBROUTINE MATRAN(MX,LDX,R1,C1,MT,LDT)
         INTEGER R1,C1,I,J,LDX,LDT
         REAL*8 MX(LDX,1),MT(LDT,1)
C
         DO 680 I = 1,R1
              DO 690 J = 1,C1
                   MT(J,I) = MX(I,J)
690           CONTINUE
680      CONTINUE
      RETURN
      END
```

APPENDIX B. KARHUNEN-LOEVE MODE PROGRAM

This program computes the Karhunen-Loeve mode shapes as discussed in Chapter IV. Subroutine GENRAA is called to generate the covariance matrix of the modal coordinates, $RAA = E[\alpha(t)\,\alpha^T(t)]$. One subroutine from the IMSL math library (DMRRRR) is used for matrix multiplication and two EISPACK subroutines are used to compute the eigenvalues and eigenvectors of the covariance matrix.

The subroutine GENRAA as noted above generates the covariance matrix, RAA, of the modal coordinates of the space station discussed in Chapter I. The algorithm:

- Computes the natural frequency, damping coefficient, γ, and damped frequency of each mode
- Computes the variances and covariances based on these parameters as given by equations 19, 20, 21

The KL mode shapes are found by solving the eigenvalue problem, equation 12.

$$RAA^{*}Q\,\phi = \lambda\,\phi \tag{35}$$

where the matrix Q is:

$$Q = \mathrm{diag}\left[\begin{bmatrix}\omega_7^2 & 0\\ 0 & 1\end{bmatrix},\begin{bmatrix}\omega_8^2 & 0\\ 0 & 1\end{bmatrix},\ldots,\begin{bmatrix}\omega_{56}^2 & 0\\ 0 & 1\end{bmatrix}\right] \tag{36}$$

and

- λ is the eigenvalue of the product, RAA * Q
- ϕ represents the orthonormal eigenvectors

RAA * Q is not a symmetric matrix so the eigenvalue problem is changed to:

$$[Q^{1/2}\,RAA\,Q^{1/2}]\;[Q^{1/2}\,\phi] = \lambda\,[Q^{1/2}\,\phi] \tag{37}$$

Let

$$\eta = Q^{1/2}\phi \text{ and } \phi = Q^{-1/2}\,\eta \tag{38}$$

and finally:

$$[Q^{1/2}\,RAA\,Q^{1/2}]\,\eta = \lambda\,\eta \tag{39}$$

The EISPACK routines for symmetric matrices are used to solve the eigenvalue problem, equation 39, and equation 38 is used to compute the KL mode shapes.

This program is run by executing the following EXEC file:

- FORTVS2 KLMODES2
- FI 4 DISK THESIS INPUT B (PERM
- FI 6 DISK RAA OUTPUT T (PERM
- FI 8 DISK EIGVEC OUTPUT T (PERM
- FI 10 DISK EIGVAL OUTPUT T (PERM
- FI 09 DISK EIGEN ERROR B (PERM
- FI 12 DISK NATMODES D001 B (PERM
- FI 13 DISK KLAMAQ OUTPUT B (PERM
- FI 14 DISK RAA OUTPUT (PERM
- FI 17 DISK NERR001 DATA A (PERM
- P DEF STOR 2M
- EXEC MATHPACK EISPACK
- EXEC TDISK 5 DIS
- LOAD KLMODES2 (START

```
      PROGRAM KLMODE
******************* Variable definitions***********************************
*     lama = vector of natural frequencies                                 *
*     ugvex= matrix of the modal amplitudes and modal slopes               *
*     natmod=vector containing the natural mode shapes that correspond    *
*            to the KL mode shapes                                         *
*     klama=vector of natural frequencies ordered according to th KL      *
*           mode shapes                                                    *
*     klmod=vector containing the largest eigenvectors in column of the   *
*           eigenvector matrix eigenvec                                     *
****************************************************************************
      CHARACTER*6 NAM
      DOUBLE PRECISION LAMA(100,1),UGVEX(684,100),D(100),E(100)
      DOUBLE PRECISION KLMOD(100),Y,N1,Q(100,100)
      DOUBLE PRECISION GAMA,DAMP,W(50),L(50),M(50),B2(1,50),Q2(100,100)
      DOUBLE PRECISION K(1,50),A,B,C,COEF,RAA(100,100),APB,APC,TEMP1
      DOUBLE PRECISION TEMP2,TEMP3,TEMP4,TEMP5,TEMP6,Q2RAA(100,100)
      DOUBLE PRECISION EIGVEC(100,100),Z(100,100),RAAQ(100,100)
      DOUBLE PRECISION Q2RQ2(100,100),Q2INV(100,100),KLAMA(50)
      INTEGER KK,LL,ZZ,MODR(100),MODC(100),MM,NATMOD(100),NUM
      DO 160 I=1,100
      DO 170 J=1,100
      Q2(I,J)=0.0D00
      Q2INV(I,J)=0.0D00
      RAA(I,J)=0.0D00
```

```
      Q(I,J)=0.0D00
170   CONTINUE
160   CONTINUE
      READ(4,1001) NAM
      READ(4,1002) (LAMA(I,1),I=1,100)
      READ(4,1001) NAM
      READ(4,1002) ((UGVEX(I,J),I=1,684),J=1,100)
C NODE IS THE DISTURBANCE INPUT LOCATION
C AND DILOC IS THE COMPUTED LOCATION OF THE MODAL SLOPE IN UGVEX
      NODE=69
      DISLOC = 4+6*(NODE-1)
C NUMODE IS THE NUMBER OF THE LAST MODE CONSIDERED
      NUMODE = 56
      CALL GENRAA(NUMODE,DISLOC,LAMA,UGVEX,W,L,M,B2,K,RAA)
      WRITE(16,1005)W(1),L(1),M(1),B2(1,1),K(1,1)
      LL=2
      KK=1
      DO 150 J=7,NUMODE
      Q2(KK,KK)=DSQRT(LAMA(J,1))
      Q2(LL,LL)=1.0D00
      Q2INV(KK,KK)=1/Q2(KK,KK)
      Q2INV(LL,LL)=1.0D00
      KK=KK+2
      LL=LL+2
150   CONTINUE
      CALL DMRRRR(100,100,Q2,100,100,100,Q2,100,100,100,Q,100)
      CALL DMRRRR(100,100,RAA,100,100,100,Q,100,100,100,RAAQ,100)
      DO 15 I=1,100
      WRITE(14,1007) RAA(I,I)
      WRITE(15,1007) Q2(I,I)
15    CONTINUE
      CALL DMRRRR(100,100,Q2,100,100,100,RAA,100,100,100,Q2RAA,100)
      CALL DMRRRR(100,100,Q2RAA,100,100,100,Q2,100,100,100,Q2RQ2,100)
      CALL TRED2(100,100,Q2RQ2,D,E,Z)
      CALL TQL2(100,100,D,E,Z,IERR)
      IF(IERR.NE.0)THEN
      WRITE(9,*)'ERROR=',I6
C     PAUSE
      ENDIF
      CALL DMRRRR(100,100,Q2INV,100,100,100,Z,100,100,100,EIGVEC,100)
      WRITE(6,1005)((RAA(I,J), J=1,100),I=1,100)
      WRITE(8,1006)((EIGVEC(I,J),J=1,100),I=1,100)
      WRITE(10,1007)(D(I),I=1,100)
      MM=1
      DO 10 I=100,1,-1
      KLMOD(MM)=0.0D00
      DO 20 J=1,100
      IF(DABS(KLMOD(MM)).LT.DABS(EIGVEC(J,I)))THEN
      KLMOD(MM)=EIGVEC(J,I)
      MODR(MM)=I
      MODC(MM)=J
      Y=(DBLE(MODC(MM)))/2.0D00
      I1=INT(Y)
      N1=Y-(DBLE(I1))
      IF(DABS(N1).NE.0.0D00)THEN
      NATMOD(MM)=(MODC(MM)+1)/2 + 6
```

```
      ELSE
      NATMOD(MM)=MODC(MM)/2 + 6
      ENDIF
      ENDIF
20    CONTINUE
      MM=MM+1
10    CONTINUE
      DO 30 I=1,100
      WRITE(12,100) MODR(I),MODC(I),NATMOD(I),KLMOD(I)
30    CONTINUE
      ZZ=50
      DO 180 I=1,100,2
      KLAMA(ZZ)=LAMA(NATMOD(I),1)
      ZZ=ZZ-1
180   CONTINUE
      WRITE(13,1005)(KLAMA(I),I=1,50)
      NUM=7
      DO 11 I=1,99,2
      IF(DABS(KLMOD(I)).GT.DABS(KLMOD(I+1)))THEN
      WRITE(17,200) NUM,DSQRT(2*(1-DABS(KLMOD(I))))
      ELSE
      WRITE(17,200) NUM,DSQRT(2*(1-DABS(KLMOD(I+1))))
      ENDIF
      NUM=NUM+1
11    CONTINUE
100   FORMAT(1X,I3,2X,I3,2X,I3,2X,E15.8)
200   FORMAT(1X,I2,2X,E15.8)
1001  FORMAT(1X,A6)
1002  FORMAT(1X,8E15.8)
1005  FORMAT(1X,5E15.8//)
1006  FORMAT(1X,5E15.8//)
1007  FORMAT(1X,5E15.8//)
      STOP
      END
      SUBROUTINE GENRAA (NUMODE,DISLOC,LAMA,UGVEX,W,L,M,B2,K,RAA)
      DOUBLE PRECISION LAMA(100,1),UGVEX(684,100)
      DOUBLE PRECISION GAMA,DAMP,W(50),L(50),M(50),B2(1,50)
      DOUBLE PRECISION K(1,50),A,B,C,COEF,RAA(100,100),APB,APC,TEMP1
      DOUBLE PRECISION TEMP2,TEMP3,TEMP4,TEMP5,TEMP6
      DAMP=0.001D00
      GAMA=DAMP/2.0D00
      DO 100 MODE=7,NUMODE
      W(MODE-6)=DSQRT(LAMA (MODE,1))
      L(MODE-6)=W(MODE-6)*DSQRT(1.0D00-GAMA*GAMA)
      M(MODE-6)=GAMA*W(MODE-6)
100   CONTINUE
           DO 110 J=7,NUMODE
           B2(1,J-6)=UGVEX(DISLOC+4*(J-1),J)
           K(1,J-6)=B2(1,J-6)/L(J-6)
C          K(1,J-6)=UGVEX(DISLOC,J)/L(J-6)
110        CONTINUE
      DO 130 I=1,50
      DO 140 J=1,50
      A=M(I)+M(J)
      B=L(I)-L(J)
      C=L(I)+L(J)
```

```
      COEF=K(1,I)*K(1,J)
      APB=2*(A*A+B*B)
      APC=2*(A*A+C*C)
      RAA(2*I-1,2*J-1)=COEF*(A/APB-A/APC)
      TEMP1=L(J)*B/APB+L(J)*C/APC
      TEMP2=M(J)*A/APB-A*M(J)/APC
      RAA(2*I-1,2*J)=COEF*(TEMP1-TEMP2)
      TEMP3=(L(I)*L(J)*A-L(I)*M(J)*B-L(I)*M(I)*B+M(I)*M(J)*A)/APB
      TEMP4=(L(I)*L(J)*A-L(I)*M(J)*C-L(I)*M(I)*C-M(I)*M(J)*A)/APC
      RAA(2*I,2*J)=COEF*(TEMP3+TEMP4)
      TEMP5=L(I)*C/APC-L(I)*B/APB
      TEMP6=M(I)*A/APB-M(I)*A/APC
      RAA(2*I,2*J-1)=COEF*(TEMP5-TEMP6)
140   CONTINUE
130   CONTINUE
      RETURN
      END
```

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