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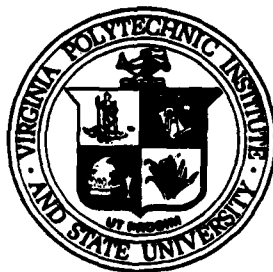
MODELING AND COMPUTATIONAL ALGORITHMS FOR
PARAMETER ESTIMATION AND OPTIMAL CONTROL OF
AEROELASTIC SYSTEMS AND LARGE FLEXIBLE
STRUCTURES

AFOSR GRANT AFOSR - 85 - 0287

for

30 September 1985 - 30 September 1988

INTERDISCIPLINARY CENTER FOR APPLIED MATHEMATICS



VIRGINIA POLYTECHNIC INSTITUTE

AND STATE UNIVERSITY

Blacksburg, Virginia 24061

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Interdisciplinary Center for Applied Mathematics
Department of Mathematics

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15 February 1989

FINAL TECHNICAL REPORT

on

MODELING AND COMPUTATIONAL ALGORITHMS FOR PARAMETER
ESTIMATION AND OPTIMAL CONTROL OF AEROELASTIC SYSTEMS AND
LARGE FLEXIBLE STRUCTURES

AFOSR GRANT AFOSR - 85 - 0287

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30 September 1985 - 30 September 1988

by

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I. PREFACE

This report contains a summary of the results partially supported under the Air Force Office of Scientific Research grant AFOSR - 85 - 0287 during the period September 30, 1985 to September 30, 1988. This project was concerned with the development of approximation schemes for control of distributed parameter systems. The principal investigators were Professors John A. Burns and Eugene M. Cliff.

During this three year period 26 research papers were completed. Also, 5 faculty (2 principal investigators - 3 post docs) and 8 graduates students were partially supported by the grant. Four of these students have received their Ph.D. degrees and the remaining students are expected to complete their work in the near future.

The basic goal of this project is the study of computational algorithms for control design of partial functional differential equations that model structural and fluid dynamic systems. We investigated several aspects of the development of computational algorithms for identification and control of distributed parameter systems. We also spent considerable effort on specific applications involving elastic, aeroelastic and viscoelastic systems. Progress was made on many of these problems. However, in this report we shall concentrate on the following major accomplishments. *(mgr)* ←

1. The development of "fast" Chandrasekhar type algorithms for optimal control problems and the application of this algorithm to flutter suppression of aeroelastic systems.
2. The development of well-posed state space models and computational schemes for an aeroelastic control system.
3. The development of a state space models and convergent approximation schemes for the control of a viscoelastic and thermoelastic vibrating systems.
4. Established the preservation of controllability under approximation for various finite element and finite difference approximations of parabolic systems. These studies also produced numerical measures of controllability that have direct implications on the conditioning of numerical control problems.
5. The development of computational procedures for time optimal and minimum effort maneuvering of flexible structures.

II. HIGHLIGHTS OF RESEARCH

In this section we discuss some of the work performed under the grant. Reprints of the papers will be forwarded to AFOSR as soon as they become available.

1. Chandrasekhar Algorithms.

We developed a fast algorithm for the general linear quadratic optimal control problem for distributed parameter systems governed by retarded functional differential equations. This algorithm combined a Chandrasekhar factorization technique with special "F-reduction" schemes to produce a fast computational scheme for computing feedback gains. Convergence of the algorithm was established and the algorithm was tested on a number of hereditary control problems (including an aeroelastic control problem). The results were extremely nice in that when the algorithm is compared to standard Riccati based methods, the factorization method always showed improved rates of convergence. **In many cases actual CPU time was reduced by an order of magnitude.** Moreover, in large problems (such as aeroelastic and viscoelastic control problems) the hybrid factorization-reduction scheme was the only algorithm that converged! These results are reported in [1] and [2].

2. State Space models for Aeroelastic Systems.

In order to develop computational methods for identification and control of aeroelastic systems, we devoted considerable effort to the analysis of well-posed models. For two dimensional, subsonic and incompressible flow the basic governing equation is a neutral functional differential equation with non-atomic difference operator. Until the present work, there did not exist any body of research capable of handling such equations. We established the well-posedness of this model and developed a more general theory for non-atomic equations. Although we constructed several convergent numerical schemes and most of these schemes are suitable for simulation and identification, they are often not very useful as a computational tool for control design. This remains an active area of research. The well-posedness results are presented in [3], [5] and [6] and the numerical scheme is given in [9]. More analysis of these types of models is needed before we can develop a real understanding of the problems associated with constructing practical computational algorithms that are suitable for control design.

3. Viscoelastic and Thermoelastic Vibrating Systems.

The development of practical control algorithms for large flexible space structures is highly dependent upon accurate internal damping models. In order to analyze various viscoelastic, thermoelastic and "fractional power" damping models, we developed state space models and computational algorithms that are based on the theory of Boltzmann materials. These models lead to partial functional differential equations and to the need for new computational algorithms.

We refined and combined several special algorithms in order to develop fast numerical algorithms for the linear quadratic optimal control problem for systems governed by partial-integro differential equations that model viscoelastic structures. These algorithms combine factorization techniques with special reduction schemes to produce a fast computational scheme for computing feedback gains. Convergence was established for several vibrating systems and these algorithms have been tested on a number of control problems. These results appear in papers [7], [8], [11], [16] and [23].

Among the most interesting aspects of this effort were the discoveries associated with the use of "standard" finite element/averaging numerical schemes for viscoelastic structures of Boltzmann type. This problem lead to two new discoveries that are somewhat surprising. First it appeared (see [7] and [8]) that a combined finite element/averaging scheme does not produce an unconditionally convergent scheme. If N denotes the number of "finite elements" in the spatial approximation and M denotes the number of approximations of the hereditary system, then it was shown in [8] that the finite element/averaging scheme is stable if

$$(S1) \quad M > NP$$

for some $p > 1$. This result implies that this scheme may be slow to converge and early numerical tests indicate that this is the case. Results by Ito and Fabiano at Brown show that condition (S1) is not necessary for stability. Moreover, they developed an improved version of the basic scheme that converges at extremely fast rates. Dr. Robert Miller investigated this question for more general viscoelastic structures. In particular, he has established a general framework for constructing state space models of viscoelastic and thermoelastic systems and applied the Fabiano-Ito scheme to these more general systems.

Boltzmann type models can be used to study a wide range of damping models in structures. This observation leads to a parameterization of damping models that is suitable for

parameter identification algorithms that would identify the "type" of damping in a given structure. H.T. Banks and R. Fabiano (past Ph. D. student supported under this grant) at Brown University have made use of this observation and produced many interesting new results. We feel that the early research supported by this grant will open several new approaches to damping modeling in structural control.

4. Preservation of System Properties Under Approximation.

We initiated a study of system theoretic properties preserved by finite dimensional approximations of distributed parameter systems. One aspect of this work is concerned with the problem of determining what types of approximation schemes lead to controllable finite dimensional models if it is given that the infinite dimensional system is exactly controllable (or approximately controllable, etc.)? More importantly, for computational algorithms the question of robustness of controllability (ie. how close is the approximate finite dimensional to an uncontrollable system?) can greatly affect any numerical algorithm used in control design. Dr. Gunther Peichl (a former Post Doctoral Fellow) and Dr. Burns have established several results for distributed parameter systems. Perhaps the most interesting aspect of this work concerns the great variance discovered in the robustness of various approximation schemes. For example, when applied to the heat equation the finite element scheme is two orders of magnitude more robust than finite difference schemes. This work appears in [15].

Our study of controllability measures for infinite-dimensional systems requires that one first understand these measures for finite dimensional systems. Finite-dimensional, linear [time-invariant] dynamical systems can be described by a system of ordinary differential equations of the form:

$$\dot{x}(t) = A x(t) + b u(t).$$

There is an associated geometrical interpretation involving a family of 'flows' on R^n . One geometrical question of interest is: given an initial point x_0 and a time T is there a control $u(\cdot): [0, T] \rightarrow R$ such that the solution $x(t; x_0, u)$ reaches the origin at time T [i.e. $x(T; x_0, u) = 0$]? If the system has this property for all x_0 in R^n , then we say the system is controllable. Note that since the system is linear and time-invariant the interval T is immaterial.

It is well-known that the system is controllable iff the rank of $[b, Ab, \dots, A^{n-1}b]$ is n . Indeed, the subspace

spanned by these vectors is exactly the set of initial data which can be controlled to the origin. One reason the idea of controllability is significant is that it is a necessary condition for solvability of important control problems. For example, the pole placement problem is that of finding a feedback matrix f so that the system with control given by $u = Kx$ has a specified set of [closed-loop] eigen-values. Such feedback might be used to speed up the rate at which solutions decay. Note that controllability is a yes or no issue; the system is either controllable or it isn't. In reality, of course, a system might be 'nearly uncontrollable'. The idea of a measure of controllability has been considered by many people.

The measure of controllability used in our studies is defined to be the distance between the given system and the nearest uncontrollable system. The distance is in terms of a matrix norm. This measure is useful because it indicates the numerical sensitivity of the pole-placement problem. That is if the measure is 'small' [a bad situation] then the gains needed to place the poles would be very sensitive to the data [i.e. the entries in A and b]. The problem of computing a value for the measure of controllability is a substantial one. For systems with certain symmetry properties [A is symmetric or skew-symmetric] Burns and Peichl ([13], [15] and [21]) have given simpler ways to compute this measure.

One widely used approach to control design for distributed-parameter systems is based on finite-dimensional approximations. The Trotter-Kato semigroup approximation theory provides the function-analytic setting that makes this idea work [provides some mathematical rigor]. It is natural then to ask how well various finite-dimensional approximations schemes do in preserving the measure of controllability. Burns and Peichl have done this for the heat equation [$y_t(t,x) = y_{xx}(t,x) + b(x) u(t)$]. Specifically, they compare standard finite-difference and finite-element approximations. For this example they were able to produce upper and lower bounds on the controllability measure. Numerical evidence suggests that these bounds are quite good.

4. Optimal Maneuvering of Flexible Structures.

Although the problem of stability enhancement for flexible structures is important, there is also a requirement to maneuver these structures in an efficient manner. The problem of time optimal slewing of certain flexible structures was considered in papers [19], [22] and [26]. In [19] and [22] the problem is treated with hard

bounds on the control torque. It was shown that for rest-to-rest maneuvers of certain models, the time-optimal torque history has an important symmetric property. This feature applies to the time-optimal problem and to certain other performance measures including some quadratic functionals. The results reported in [26] employ a soft bound on the control (integral norm squared), while [24] considers the related problem of minimum effort with time constraints. Finally, paper [20] reports on early numerical studies of time-optimal control for a parabolic system.

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