

2



RESEARCH AND DEVELOPMENT TECHNICAL REPORT
SLCET-TR-89-2

AD-A208 349

POLARIZATION MATRICES OF LITHIUM TANTALATE

ARTHUR BALLATO
ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY

APRIL 1989

DTIC
ELECTE
MAY 26 1989
S D

DISTRIBUTION STATEMENT

Approved for public release;
distribution is unlimited.

US ARMY
LABORATORY COMMAND
FORT MONMOUTH, NEW JERSEY 07703-5000

NOTICES

Disclaimers

The citation of trade names and names of manufacturers in this report is not to be construed as official Government indorsement or approval of commercial products or services referenced herein.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S) SLCET-TR-89-2			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
6a. NAME OF PERFORMING ORGANIZATION US Army Laboratory Command Electronics Tech & Devices Lab		6b. OFFICE SYMBOL (If applicable) SLCET-MA-A		7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State, and ZIP Code) Electronics Technology and Devices Laboratory ATTN: SLCET-MA-A Fort Monmouth, NJ 07703-5000				7b. ADDRESS (City, State, and ZIP Code)	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION		8b. OFFICE SYMBOL (If applicable)		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code)				10. SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO. 1L162705-		PROJECT NO. H94	TASK NO. K9
				WORK UNIT ACCESSION NO. DA303394	
11. TITLE (Include Security Classification) POLARIZATION MATRICES OF LITHIUM TANTALATE (U)					
12. PERSONAL AUTHOR(S) Arthur Ballato					
13a. TYPE OF REPORT Technical Report		13b. TIME COVERED FROM Jan 88 to Jan 89		14. DATE OF REPORT (Year, Month, Day) 1989 April	
15. PAGE COUNT 27					
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP			
09	01		Piezoelectric resonators; piezoelectric transducers;		
17	02		lithium tantalate; acousto-optics, tantalum compounds, lithium compounds, (lithium)		
19. ABSTRACT (Continue on reverse if necessary and identify by block number)					
In analytical treatments of piezoelectric/acoustic transducers, signal processors, and resonators, the electromechanical transduction mechanism is most often expressed in terms of the elements of the piezoelectric [e] or [d] matrices. Molecular interpretations of piezoelectricity, and especially electro-optical applications, usually involve polarization as the preferred variable, and consequently the alternative [a] and [b] matrices are of interest. The elements of these latter sets are calculated for lithium tantalate from measured elastopiezodielectric constants taken from the literature.					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION Unclassified		
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Arthur Ballato			22b. TELEPHONE (Include Area Code) (201) 544-2773		22c. OFFICE SYMBOL SLCET-MA A

DD Form 1473, JUN 86

Previous editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

CONTENTS

	Page
INTRODUCTION	1
CONSTITUTIVE EQUATION SETS	1
RELATIONS AMONG MATERIAL CONSTANTS	5
CALCULATION SEQUENCE	7
EXPLICIT FORMULAS FOR POINT GROUP 3m	10
INPUT VALUES FOR LI TA 03	17
OUTPUT VALUES FOR LI TA 03	17
CONCLUSIONS.	20
REFERENCES	20

TABLES

Table	Page
1. Symbols, Units, and Definitions	2
2. Relations among Material Constants	8
3. Further Relations among Material Constants	9
4. Elastopiezodielectric Matrices for Point Group 3m: The [e], [h], and [a] Sets	11
5. Elastopiezodielectric Matrices for Point Group 3m: The [d], [g], and [b] Sets	11
6. Isagric Elastic Stiffnesses	17
7. Piezoelectric Stress Constants	17
8. Dielectric Permittivities at Constant Strain	17
9. Elastic Stiffnesses	18
10. Elastic Compliances	18
11. Piezoelectric [e], [h], and [a] Values	18

Table	page
12. Piezoelectric [d], [g], and [b] Values	19
13. Dielectric (eps) Values	19
14. Dielectric (chi) Values	19
15. Dielectric (bet) Values	19
16. Dielectric (zet) Values	20

A-1

1	☒
2	☐
3	☐
4	
5	
6	
7	
8	
9	
10	



INTRODUCTION

Electromechanical transduction taking place via the piezoelectric effect is characterized phenomenologically by constitutive equations that relate the elastic and electric variables. These equations take a variety of forms, depending upon the choice of independent and dependent variables; the choice is normally dictated by the application. For example, piezoelectric resonators in the form of thickness mode plates are most easily treated using the isagrig elastic stiffnesses $[cE]$, the piezoelectric stress constants $[e]$, and the dielectric permittivities at constant strain $[(\epsilon)s]$.

Various measurement techniques yield values for the elements of a particular coefficient set more directly than those of another. The coefficients appearing in the different equation sets are, however, interrelated, so that once any one complete set is available, all the other sets of elements may be found. The most accurate and precise experimental results to date have been from plate resonator (resonance) and pulse-echo (transit-time) measurements. From the $[cE]$, $[e]$, and $[(\epsilon)s]$ matrices determined therefrom, those matrices representing material properties expressed in the other alternative forms may be calculated.

Electrooptical applications are becoming increasingly important. So also are treatments of piezoelectric and ferroelectric phenomena from the standpoint of molecular interactions. In both of these cases the constitutive equations using polarization as the independent electrical variable, rather than either electric intensity or displacement, assume greater importance than the sets traditionally used for transducer, signal processing, and resonator applications.

In this report we give the complete sets of linear constitutive equations relating elastic and electric fields. For each equation set the numerical values are computed for lithium tantalate, a highly piezoelectric ferroelectric with temperature-compensated orientations, from the measured $[cE]$, $[e]$, and $[(\epsilon)s]$ values of Smith and Welsh (Ref. 1). Coupling to the thermal field is neglected. Rationalized mks units are used throughout.

CONSTITUTIVE EQUATION SETS

Symbols and units for the quantities employed are given in Table 1. In terms of these, six constitutive equation sets are used. Of these, electric intensity, dielectric displacement, and polarization each appear in two sets as an independent variable. The sets are, in compressed matrix notation, as follows. A prime denotes transpose; $[I]$ is the unit matrix.

I. The Piezoelectric Stress Constant Set

$$\begin{aligned} [T] &= [cE] [S] - [e]' [E] & (1) \\ [D] &= [e] [S] + [(\epsilon)s] [E] & (2) \end{aligned}$$

TABLE 1. SYMBOLS, UNITS, AND DEFINITIONS.

QUANTITY	UNIT	SYMBOL/DEFINITION
Elastic stress	N/m ²	[T]
Elastic strain	-----	[S]
Electric intensity	V/m	[E]
Dielectric displacement	C/m ²	[D]
Dielectric polarization	C/m ²	[P]
Elastic compliance at constant [E], [D], [P]	m ² /N	[cE], [cD], [cP]
Elastic stiffness at constant [E], [D], [P]	N/m ²	[sE], [sD], [sP]
Dielectric permittivity at constant [T], [S]	F/m	[(eps)T], [(eps)S]
Dielectric constant, relative, at constant [T], [S]	-----	[(Kr)T], [(Kr)S] =[(eps)T]/(eps)o, [(eps)S]/(eps)o
Dielectric impermeability at constant [T], [S]	m/F	[(bet)T], [(bet)S] =[(eps)T] ⁽⁻¹⁾ , [(eps)S] ⁽⁻¹⁾
Dielectric impermeability, relative, at constant [T], [S]	-----	[(betr)T], [(betr)S] =[(bet)T]*(eps)o, [(bet)S]*(eps)o =[(Kr)T] ⁽⁻¹⁾ , [(Kr)S] ⁽⁻¹⁾
Dielectric susceptibility at constant [T], [S]	F/m	[(chi)T], [(chi)S] =[(Kr)T-I]*(eps)o, [(Kr)S-I]*(eps)o
Dielectric susceptibility, relative, at constant [T], [S]	-----	[(chir)T], [(chir)S] =[(chi)T]/(eps)o, [(chi)S]/(eps)o
Reciprocal dielectric susceptibility at constant [T], [S]	m/F	[(zet)T], [(zet)S] =[(chi)T] ⁽⁻¹⁾ , [(chi)S] ⁽⁻¹⁾
Reciprocal dielectric susceptibility, relative, at constant [T], [S]	-----	[(zetr)T], [(zetr)S] =[(zet)T]*(eps)o, [(zet)S]*(eps)o
Piezoelectric stress constant	C/m ²	[e]

TABLE 1. SYMBOLS, UNITS, AND DEFINITIONS. (continued)

QUANTITY	UNIT	SYMBOL/DEFINITION
Piezoelectric strain coefficient	$m/V = C/N$	[d]
Piezoelectric stress modulus	$N/C = V/m$	[h]
Piezoelectric strain constant	m^2/C	[g]
Piezoelectric polarization modulus	$V/m = N/C$	[a]
Piezoelectric polarization constant	m^2/C	[b]

Note: Square brackets, sic: [], denote matrices.

II. The Piezoelectric Strain Coefficient Set

$$\begin{aligned} [S] &= [sE] [T] + [d]' [E] & (3) \\ [D] &= [d] [T] + [(\epsilon)sT] [E] & (4) \end{aligned}$$

III. The Piezoelectric Stress Modulus Set

$$\begin{aligned} [T] &= [cD] [S] - [h]' [D] & (5) \\ [E] &= -[h] [S] + [(\beta)sS] [D] & (6) \end{aligned}$$

IV. The Piezoelectric Strain Constant Set

$$\begin{aligned} [S] &= [sD] [T] + [g]' [D] & (7) \\ [E] &= -[g] [T] + [(\beta)sT] [D] & (8) \end{aligned}$$

V. The Piezoelectric Polarization Modulus Set

$$\begin{aligned} [T] &= [cP] [S] - [a]' [P] & (9) \\ [E] &= -[a] [S] + [(\zeta)sS] [P] & (10) \end{aligned}$$

VI. The Piezoelectric Polarization Constant Set

$$\begin{aligned} [S] &= [sP] [T] + [b]' [P] & (11) \\ [E] &= -[b] [T] + [(\zeta)sT] [P] & (12) \end{aligned}$$

The electric variables are connected by the relation

$$[D] = (\epsilon)s_o * [E] + [P] \quad (13)$$

where $(\epsilon)s_o$ is the permittivity of free space, defined by

$$(\epsilon)s_o * (\mu)o * (c) * (c) = 1 ; \quad (14)$$

$(\mu)o$ is the permeability of free space, equal, by definition, to $4 \pi * 10^{(-7)}$, and (c) is the velocity of light in vacuo and, also by definition, is equal exactly to $2.99792458 * 10^8$ m/s.

From (13) the expressions for the remaining electric variables associated, respectively, with the six equation sets (1) to (12) may be found:

$$[P] = [e] [S] + [(\chi)s] [E] \quad (15)$$

$$[P] = [d] [T] + [(\chi)sT] [E] \quad (16)$$

$$[P] = (\epsilon)s_o * [h] [S] + [I - (\epsilon)s_o * (\beta)sS] [D] \quad (17)$$

$$[P] = (\epsilon)s_o * [g] [T] + [I - (\epsilon)s_o * (\beta)sT] [D] \quad (18)$$

$$[D] = -(\epsilon)_o * [a] [S] + [I + (\epsilon)_o * (\zeta)S] [P] \quad (19)$$

$$[D] = -(\epsilon)_o * [b] [T] + [I + (\epsilon)_o * (\zeta)T] [P] \quad (20)$$

RELATIONS AMONG MATERIAL CONSTANTS

The material constants are interrelated by the following formulas:

$$[cX] [sX] = [(\epsilon)Y] [(\beta)Y] = [I] \quad (21)$$

$$[(\chi)Y] [(\zeta)Y] = [(Kr)Y - (\chi)Y] = [I] \quad (22)$$

In (21) and (22), X = E, D, or P and Y = T or S.

$$\begin{aligned} [cD] - [cE] &= [h]' [e] = [e]' [h] \\ &= [h]' [(\epsilon)S] [h] = [e]' [(\beta)S] [e] \\ &= [a]' [e - h * (\epsilon)_o] = [e - h * (\epsilon)_o]' [a] \end{aligned} \quad (23)$$

$$\begin{aligned} [cP] - [cD] &= [h]' [a] * (\epsilon)_o = [a]' [h] * (\epsilon)_o \\ &= [h]' [(\epsilon)S] [(\zeta)S] [h] * (\epsilon)_o \\ &= [a]' [(\beta)S] [(\chi)S] [a] * (\epsilon)_o \\ &= [a - h]' [e] = [e]' [a - h] \end{aligned} \quad (24)$$

$$\begin{aligned} [cP] - [cE] &= [a]' [e] = [e]' [a] \\ &= [a]' [(\chi)S] [a] = [e]' [(\zeta)S] [e] \\ &= [h]' [e + a * (\epsilon)_o] = [e + a * (\epsilon)_o]' [h] \end{aligned} \quad (25)$$

$$\begin{aligned} [sE] - [sD] &= [d]' [g] = [g]' [d] \\ &= [d]' [(\beta)t] [d] = [g]' [(\epsilon)t] [g] \\ &= [b]' [d - g * (\epsilon)_o] = [d - g * (\epsilon)_o]' [b] \end{aligned} \quad (26)$$

$$\begin{aligned} [sD] - [sP] &= [b]' [g] * (\epsilon)_o = [g]' [b] * (\epsilon)_o \\ &= [g]' [(\epsilon)T] [(\zeta)T] [g] * (\epsilon)_o \\ &= [b]' [(\beta)T] [(\chi)T] [b] * (\epsilon)_o \\ &= [b - g]' [d] = [d]' [b - g] \end{aligned} \quad (27)$$

$$\begin{aligned} [sE] - [sP] &= [b]' [d] = [d]' [b] \\ &= [b]' [(\chi)T] [b] = [d]' [(\zeta)T] [d] \\ &= [g]' [d + b * (\epsilon)_o] = [d + b * (\epsilon)_o]' [g] \end{aligned} \quad (28)$$

$$\begin{aligned}
[(\text{zet})S] - [(\text{zet})T] &= [b] [a]' = [a] [b]' \\
&= [b] [cP] [b]' = [a] [sP] [a]' \quad (29)
\end{aligned}$$

$$\begin{aligned}
[(\text{chi})T] - [(\text{chi})S] &= [(\text{eps})T] - [(\text{eps})S] \\
&= [e] [d]' = [d] [e]' \\
&= [d] [cE] [d]' = [e] [sE] [e]' \quad (30)
\end{aligned}$$

$$\begin{aligned}
[(\text{bet})S] - [(\text{bet})T] &= [h] [g]' = [g] [h]' \\
&= [g] [cD] [g]' = [h] [sD] [h]' \quad (31)
\end{aligned}$$

$$[e] = [d] [cE] = [(\text{eps})S] [h] = [(\text{chi})S] [a] \quad (32)$$

$$[d] = [e] [sE] = [(\text{eps})T] [g] = [(\text{chi})T] [b] \quad (33)$$

$$\begin{aligned}
[h] = [g] [cD] &= [(\text{bet})S] [e] = [(\text{chi})S] [(\text{bet})S] [a] \\
&= [I - (\text{bet})S * (\text{eps})o] [a] \quad (34)
\end{aligned}$$

$$\begin{aligned}
[g] = [h] [sD] &= [(\text{bet})T] [d] = [(\text{chi})T] [(\text{bet})T] [b] \\
&= [I - (\text{bet})T * (\text{eps})o] [b] \quad (35)
\end{aligned}$$

$$\begin{aligned}
[a] = [b] [cP] &= [(\text{zet})S] [e] = [(\text{eps})S] [(\text{zet})S] [h] \\
&= [I + (\text{zet})S * (\text{eps})o] [h] \quad (36)
\end{aligned}$$

$$\begin{aligned}
[b] = [a] [sP] &= [(\text{zet})T] [d] = [(\text{eps})T] [(\text{zet})T] [g] \\
&= [I + (\text{zet})T * (\text{eps})o] [g] \quad (37)
\end{aligned}$$

Some alternative relations are the following:

$$\begin{aligned}
[a - h] &= [(\text{zet})S] [h] * (\text{eps})o \\
&= [(\text{bet})S] [a] * (\text{eps})o \quad (38)
\end{aligned}$$

$$\begin{aligned}
[b - g] &= [(\text{zet})T] [g] * (\text{eps})o \\
&= [(\text{bet})T] [b] * (\text{eps})o \quad (39)
\end{aligned}$$

$$[e + a * (\text{eps})o] = [(\text{eps})S] [a] \quad (40)$$

$$[d + b * (\text{eps})o] = [(\text{eps})T] [b] \quad (41)$$

$$[e - h * (\text{eps})o] = [(\text{chi})S] [h] \quad (42)$$

$$[d - g * (\text{eps})o] = [(\text{chi})T] [g] \quad (43)$$

Equations (21) to (43) result from equating like dependent variables in pairs selected from equations (1) to (12) and (15) to (20). Each

pair yields one equation in three variables, one mechanical and two electrical, or vice versa. Two other equations exist, again from (1) to (12) and (15) to (20), that contain the same three variables found in each paired equation. One of these auxiliary equations is used to eliminate one of the two variables of the same kind; the result is one equation in two variables, one electrical and one mechanical. These are now independent variables, so the coefficients must vanish; two relations between the material coefficients result. As an example, (3) and (7) both have [S] as dependent variable. Equating them produces one relation in [T], [E], and [D]; one of the electrical variables must be eliminated. This is done by using either (4) or (8); each contains the same three variables. If (8) is used to eliminate [E], one obtains $[sE - d'g - sD] [T] = [d'(\text{bet})T - g'] [D]$. Therefore, $[sE] - [sD] = [d]' [g]$ and $[g] = [(\text{bet})T] [d]$. Use of (4) instead of (8) leads to the equations $[sE] - [sD] = [g]' [d]$ and $[d] = [(\text{eps})T] [g]$. There are 36 pairs, six each equating [S] and [T], and eight each equating [E], [D], and [P]. The 72 relations contain many redundancies. Relations between the elastic, piezoelectric, and dielectric constants are shown schematically in Tables 2 and 3.

CALCULATION SEQUENCE

Using as input [cE], [e], and [(eps)S], one may compute the remaining quantities in a variety of ways. The following sequence is typical:

$$[sE] = [cE] \quad (-1) \quad (44)$$

$$[(\text{bet})S] = [(\text{eps})S] \quad (-1) \quad (45)$$

$$[d] = [e] [sE] \quad (46)$$

$$[h] = [(\text{bet})S] [e] \quad (47)$$

$$[(\text{eps})T] - [(\text{eps})S] = [e] [d]' \quad (48)$$

$$[(\text{eps})T] = [(\text{eps})S] + [e] [d]' \quad (49)$$

$$[(\text{bet})T] = [(\text{eps})T] \quad (-1) \quad (50)$$

$$[cD] - [cE] = [e]' [h] \quad (51)$$

$$[cD] = [cE] + [e]' [h] \quad (52)$$

$$[g] = [(\text{bet})T] [d] \quad (53)$$

$$[sE] - [sD] = [d]' [g] \quad (54)$$

$$[sD] = [sE] - [d]' [g] \quad (55)$$

$$[(\text{betr})S] = [(\text{bet})S] * (\text{eps})_0 \quad (56)$$

$$[(\text{zetr})S] = [(\text{betr})S] (I - (\text{betr})S) \quad (-1) \quad (57)$$

$$[(\text{zet})S] = [(\text{zetr})S] / (\text{eps})_0 \quad (58)$$

TABLE 2. RELATIONS AMONG MATERIAL CONSTANTS.

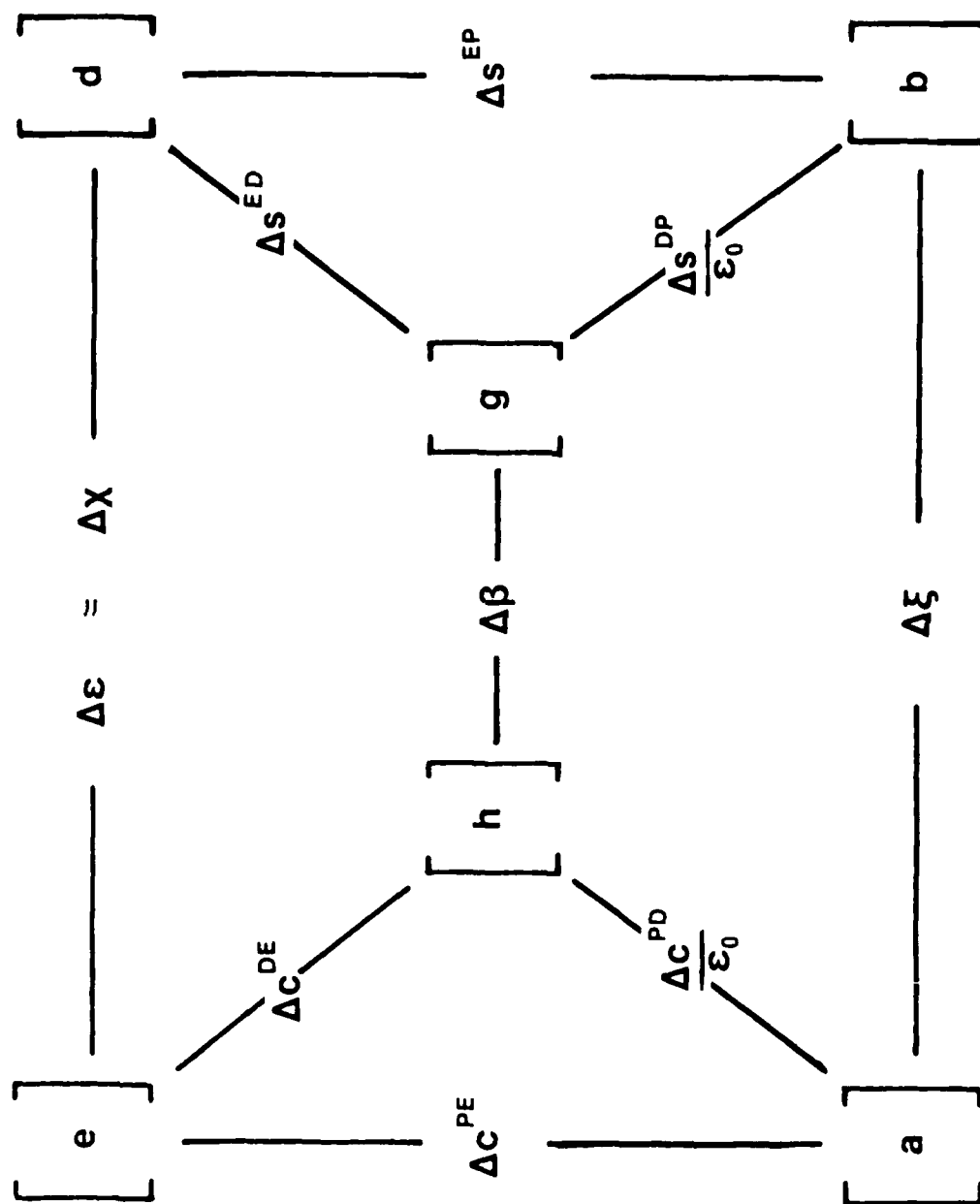
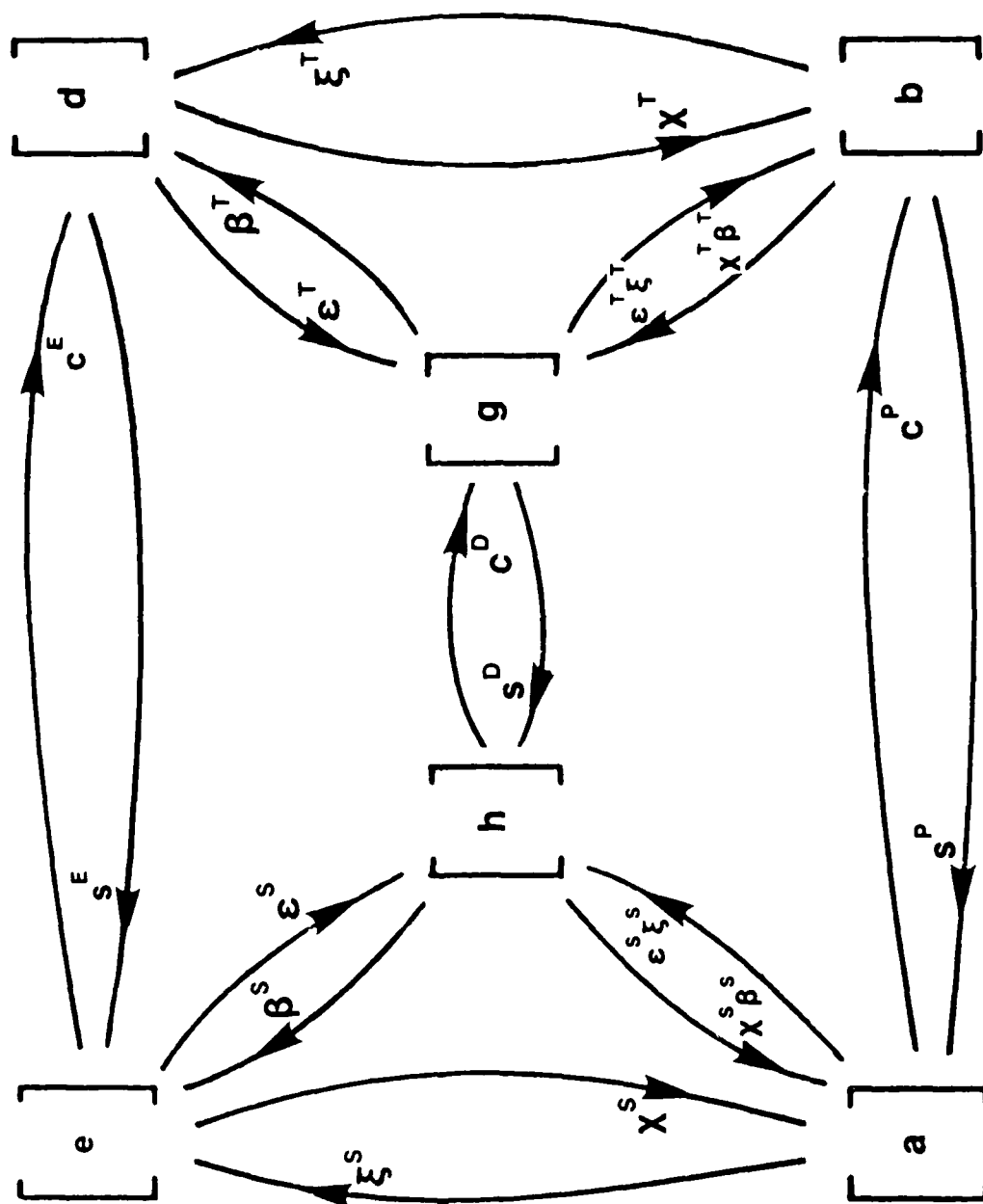


TABLE 3. FURTHER RELATIONS AMONG MATERIAL CONSTANTS.



$$[(\text{betr})T] = [(\text{bet})T] * (\text{eps})_o \quad (59)$$

$$[(\text{zetr})T] = [(\text{betr})T] [I - (\text{betr})T]^{(-1)} \quad (60)$$

$$[(\text{zet})T] = [(\text{zetr})T] / (\text{eps})_o \quad (61)$$

$$[(\text{chi})S] = [(\text{zet})S]^{(-1)} \quad (62)$$

$$[(\text{chi})T] = [(\text{zet})T]^{(-1)} \quad (63)$$

$$[a] = [(\text{zet})S] [e] \quad (64)$$

$$[b] = [(\text{zet})T] [d] \quad (65)$$

$$[cP] - [cE] = [e]' [a] \quad (66)$$

$$[cP] = [cE] + [e]' [a] \quad (67)$$

$$[cP] - [cD] = [a]' [h] * (\text{eps})_o \quad (68)$$

$$[sE] - [sP] = [d]' [b] \quad (69)$$

$$[sP] = [sE] - [d]' [b] \quad (70)$$

$$[sD] - [sP] = [g]' [b] * (\text{eps})_o \quad (71)$$

$$[(\text{bet})S] - [(\text{bet})T] = [h] [g]' \quad (72)$$

$$[(\text{chi})T] - [(\text{chi})S] = [(\text{eps})T] - [(\text{eps})S] \quad (73)$$

$$[(\text{zet})S] - [(\text{zet})T] = [a] [b]' \quad (74)$$

A number of these relations are used as checks. For example, $[(\text{bet})S]$ and $[(\text{bet})T]$ are known from (45) and (50), but the difference is recomputed in (72).

EXPLICIT FORMULAS FOR POINT GROUP 3m

Elastic:

The 6x6 elastic constant portions of Tables 4 and 5 partition into 4x4 and 2x2 submatrices. The 4x4 elastic stiffness and compliance submatrices are interrelated by formulas (75) to (93), taken from Cady (Ref. 2):

$$A = s_{33} * (s_{11} + s_{12}) - 2 * s_{13} * s_{13} \quad (75)$$

$$B = s_{44} * (s_{11} - s_{12}) - 2 * s_{14} * s_{14} \quad (76)$$

$$2 * c_{11} = s_{33} / A + s_{44} / B \quad (77)$$

$$2 * c_{12} = s_{33} / A - s_{44} / B \quad (78)$$

$$c_{13} = - s_{13} / A ; \quad c_{14} = - s_{14} / B \quad (79a), (79b) \quad (79)$$

TABLE 4. ELASTOPIEZODIELECTRIC MATRICES FOR POINT GROUP 3m:
THE [e], [h], AND [a] SETS.

11	12	13	14	00	00	]	00	-22	31	
						]				cE] e'
12	11	13	-14	00	00	]	00	22	31	-----]-----
						]				e] (eps)S
13	13	33	00	00	00	]	00	00	33	
						]				
14	-14	00	44	00	00	]	00	15	00	cD] h'
						]				-----]-----
00	00	00	00	44	14	]	15	00	00	h] (bet)S
						]				
00	00	00	00	14	66	]	-22	00	00	
						]				
00	00	00	00	15	-22	]	11	00	00	
						]				cP] a'
-22	22	00	15	00	00	]	00	11	00	-----]-----
						]				a] (zet)S
31	31	33	00	00	00	]	00	00	33	

$$66 = (11 - 12) / 2$$

Matrix entries show only subscripts.

TABLE 5. ELASTOPIEZODIELECTRIC MATRICES FOR POINT GROUP 3m:
THE [d], [g], AND [b] SETS.

11	12	13	14	00	00	]	00	-22	31	
						]				sE] d'
12	11	13	-14	00	00	]	00	22	31	-----]-----
						]				d] (eps)T
13	13	33	00	00	00	]	00	00	33	
						]				
14	-14	00	44	00	00	]	00	15	00	sD] g'
						]				-----]-----
00	00	00	00	44	14*2	]	15	00	00	g] (bet)T
						]				
00	00	00	00	14*2	66	]	-22*2	00	00	
						]				
00	00	00	00	15	-22*2	]	11	00	00	
						]				sP] b'
-22	22	00	15	00	00	]	00	11	00	-----]-----
						]				b] (zet)T
31	31	33	00	00	00	]	00	00	33	

$$66 = (11 - 12) * 2$$

Matrix elements show only subscripts.

$$c_{33} = (s_{11} + s_{12}) / A \quad (80)$$

$$c_{44} = (s_{11} - s_{12}) / B \quad (81)$$

$$c_{66} = (c_{11} - c_{12}) / 2 = s_{44} / (2 * B) \quad (82)$$

$$K = c_{33} * (c_{11} + c_{12}) - 2 * c_{13} * c_{13} \quad (83)$$

$$L = c_{44} * (c_{11} - c_{12}) - 2 * c_{14} * c_{14} \quad (84)$$

$$2 * s_{11} = c_{33} / K + c_{44} / L \quad (85)$$

$$2 * s_{12} = c_{33} / K - c_{44} / L \quad (86)$$

$$s_{13} = - c_{13} / K ; s_{14} = - c_{14} / L \quad (87a), (87b) \quad (87)$$

$$s_{33} = (c_{11} + c_{12}) / K \quad (88)$$

$$s_{44} = (c_{11} - c_{12}) / L \quad (89)$$

$$s_{66} = (s_{11} - s_{12}) = 2 * c_{44} / L \quad (90)$$

$$\det (4 \times 4) [s] = A * B \quad (91)$$

$$\det (4 \times 4) [c] = K * L \quad (92)$$

$$A * K = B * L = A * B * K * L = 1 \quad (93)$$

Formulas (75) to (93) hold for each set of constant electrical conditions: either E, D, or P constant.

$$[cD] - [cE] = [\text{del } cDE] = [e]' [h] = [h]' [e] \quad (23)$$

$$\text{del } cDE_{11} = + e_{22} h_{22} + e_{31} h_{31} \quad (94)$$

$$\text{del } cDE_{12} = - e_{22} h_{22} + e_{31} h_{31} \quad (95)$$

$$\text{del } cDE_{13} = + e_{31} h_{33} = + h_{31} e_{33} \quad (96)$$

$$\text{del } cDE_{14} = - e_{22} h_{15} = - h_{22} e_{15} \quad (97)$$

$$\text{del } cDE_{33} = + e_{33} h_{33} \quad (98)$$

$$\text{del } cDE_{44} = + e_{15} h_{15} \quad (99)$$

$$\text{del } cDE_{66} = + e_{22} h_{22} \quad (100)$$

$$\begin{aligned} [cP] - [cD] &= [\text{del } cPD] = [a]' [h] * (\text{eps})_0 \\ &= [h]' [a] * (\text{eps})_0 \end{aligned} \quad (24)$$

$$\text{del } cPD_{11} = (+ a_{22} h_{22} + a_{31} h_{31}) * (\text{eps})_0 \quad (101)$$

$$\text{del } cPD_{12} = (- a_{22} h_{22} + a_{31} h_{31}) * (\text{eps})_0 \quad (102)$$

$$\begin{aligned} \text{del cPD13} &= (+ a_{31} h_{33}) * (\text{eps})_o \\ &= (+ h_{31} a_{33}) * (\text{eps})_o \end{aligned} \quad (103)$$

$$\begin{aligned} \text{del cPD14} &= (- a_{22} h_{15}) * (\text{eps})_o \\ &= (- h_{22} a_{15}) * (\text{eps})_o \end{aligned} \quad (104)$$

$$\text{del cPD33} = (+ a_{33} h_{33}) * (\text{eps})_o \quad (105)$$

$$\text{del cPD44} = (+ a_{15} h_{15}) * (\text{eps})_o \quad (106)$$

$$\text{del cPD66} = (+ a_{22} h_{22}) * (\text{eps})_o \quad (107)$$

$$[cP] - [cE] = [\text{del cPE}] = [e]' [a] = [a]' [e] \quad (25)$$

$$\text{del cPE11} = + e_{22} a_{22} + e_{31} a_{31} \quad (108)$$

$$\text{del cPE12} = - e_{22} a_{22} + e_{31} a_{31} \quad (109)$$

$$\text{del cPE13} = + e_{31} a_{33} = + a_{31} e_{33} \quad (110)$$

$$\text{del cPE14} = - e_{22} a_{15} = - a_{22} e_{15} \quad (111)$$

$$\text{del cPE33} = + e_{33} a_{33} \quad (112)$$

$$\text{del cPE44} = + e_{15} a_{15} \quad (113)$$

$$\text{del cPE66} = + e_{22} a_{22} \quad (114)$$

From the del c13 entries we have the ratios

$$e_{31} / e_{33} = h_{31} / h_{33} = a_{31} / a_{33}. \quad (115)$$

From the del c14 entries we have the further ratios

$$e_{15} / e_{22} = h_{15} / h_{22} = a_{15} / a_{22}. \quad (116)$$

$$[sE] - [sD] = [\text{del sED}] = [d]' [g] = [g]' [d] \quad (26)$$

$$\text{del sED11} = + d_{22} g_{22} + d_{31} g_{31} \quad (117)$$

$$\text{del sED12} = - d_{22} g_{22} + d_{31} g_{31} \quad (118)$$

$$\text{del sED13} = + d_{31} g_{33} = + g_{31} d_{33} \quad (119)$$

$$\text{del sED14} = - d_{22} g_{15} = - g_{22} d_{15} \quad (120)$$

$$\text{del sED33} = + d_{33} g_{33} \quad (121)$$

$$\text{del sED44} = + d_{15} g_{15} \quad (122)$$

$$\text{del sED66} = + d_{22} g_{22} * 4 \quad (123)$$

$$[sD] - [sP] = [g]' [b] * (\epsilon s)o$$

$$= [b]' [g] * (\epsilon s)o \quad (27)$$

$$\text{del } sDP11 = (+ g22 b22 + g31 b31) * (\epsilon s)o \quad (124)$$

$$\text{del } sDP12 = (- g22 b22 + g31 b31) * (\epsilon s)o \quad (125)$$

$$\text{del } sDP13 = (+ g31 b33) * (\epsilon s)o$$

$$= (+ b31 g33) * (\epsilon s)o \quad (126)$$

$$\text{del } sDP14 = (- g22 b15) * (\epsilon s)o$$

$$= (- b22 g15) * (\epsilon s)o \quad (127)$$

$$\text{del } sDP33 = (+ g33 b33) * (\epsilon s)o \quad (128)$$

$$\text{del } sDP44 = (+ g15 b15) * (\epsilon s)o \quad (129)$$

$$\text{del } sDP66 = (+ g22 b22) * 4 * (\epsilon s)o \quad (130)$$

$$[sE] - [sP] = [\text{del } sEP] = [b]' [d] = [d]' [b] \quad (28)$$

$$\text{del } sEP11 = + d22 b22 + d31 b31 \quad (131)$$

$$\text{del } sEP12 = - d22 b22 + d31 b31 \quad (132)$$

$$\text{del } sEP13 = + d31 b33 = + b31 d33 \quad (133)$$

$$\text{del } sEP14 = - d22 b15 = - b22 d15 \quad (134)$$

$$\text{del } sEP33 = + d33 b33 \quad (135)$$

$$\text{del } sEP44 = + d15 b15 \quad (136)$$

$$\text{del } sEP66 = + d22 b22 * 4 \quad (137)$$

From the del s13 entries we have the ratios

$$d31 / d33 = g31 / g33 = b31 / b33. \quad (138)$$

From the del s14 entries we have the further ratios

$$d15 / d22 = g15 / g22 = b15 / b22. \quad (139)$$

Piezoelectric:

$$[d] = [e] [sE] \quad (33)$$

$$d15 = + e15 sE44 - e22 sE14 * 2 \quad (140)$$

$$d22 = + e22 (sE11 - sE12) - e15 sE14 \quad (141)$$

$$d_{31} = + e_{31} (s_{E11} + s_{E12}) + e_{33} s_{E13} \quad (142)$$

$$d_{33} = + e_{33} s_{E33} + e_{13} s_{E13} * 2 \quad (143)$$

$$[h] = [(bet)S] [e] \quad (34)$$

$$h_{15} = (bet)S_{11} e_{15} \quad (144)$$

$$h_{22} = (bet)S_{11} e_{22} \quad (145)$$

$$h_{31} = (bet)S_{33} e_{31} \quad (146)$$

$$h_{33} = (bet)S_{33} e_{33} \quad (147)$$

$$[g] = [(bet)T] [d] \quad (35)$$

$$g_{15} = (bet)T_{11} d_{15} \quad (148)$$

$$g_{22} = (bet)T_{11} d_{22} \quad (149)$$

$$g_{31} = (bet)T_{33} d_{31} \quad (150)$$

$$g_{33} = (bet)T_{33} d_{33} \quad (151)$$

$$[a] = [(zet)S] [e] \quad (36)$$

$$a_{15} = (zet)S_{11} e_{15} \quad (152)$$

$$a_{22} = (zet)S_{11} e_{22} \quad (153)$$

$$a_{31} = (zet)S_{33} e_{31} \quad (154)$$

$$a_{33} = (zet)S_{33} e_{31} \quad (155)$$

$$[b] = [(zet)T] [d] \quad (37)$$

$$b_{15} = (zet)T_{11} d_{15} \quad (156)$$

$$b_{22} = (zet)T_{11} d_{22} \quad (157)$$

$$b_{31} = (zet)T_{33} d_{31} \quad (158)$$

$$b_{33} = (zet)T_{33} d_{33} \quad (159)$$

Dielectric:

$$[(bet)Y] = [(eps)Y]^{(-1)} \quad (21)$$

$$(bet)Y_{11} = 1 / (eps)Y_{11} \quad (160)$$

$$(bet)Y_{33} = 1 / (eps)Y_{33} \quad (161)$$

$$[(\text{zetr})Y] = [(\text{betr})Y] [I - (\text{betr})Y]^{-1} \quad (162)$$

$$(\text{zet})Y_{11} = 1 / ((\text{eps})Y_{11} - (\text{eps})o) \quad (163)$$

$$(\text{zet})Y_{33} = 1 / ((\text{eps})Y_{33} - (\text{eps})o) \quad (164)$$

$$[(\text{eps})T - (\text{eps})S] = [\text{del } (\text{eps})] = [e] [d]' =$$

$$[(\text{chi})T - (\text{chi})S] = [\text{del } (\text{chi})] = [d] [e]' \quad (30)$$

$$\text{del } (\text{eps})_{11} = \text{del } (\text{chi})_{11} = + e_{15} d_{15} + e_{22} d_{22} * 2 \quad (165)$$

$$\text{del } (\text{eps})_{33} = \text{del } (\text{chi})_{33} = + e_{33} d_{33} + e_{31} d_{31} * 2 \quad (166)$$

$$[(\text{bet})S - (\text{bet})T] = [h] [g]' = [g] [h]' \quad (31)$$

$$\text{del } (\text{bet})_{11} = + h_{15} g_{15} + h_{22} g_{22} * 2 \quad (167)$$

$$\text{del } (\text{bet})_{33} = + h_{33} g_{33} + h_{31} g_{31} * 2 \quad (168)$$

$$[(\text{zet})S - (\text{zet})T] = [\text{del } (\text{zet})] = [a] [b]' = [b] [a]' \quad (169)$$

$$\text{del } (\text{zet})_{11} = + a_{15} b_{15} + a_{22} b_{22} * 2 \quad (170)$$

$$\text{del } (\text{zet})_{33} = + a_{33} b_{33} + a_{31} b_{31} * 2 \quad (171)$$

INPUT VALUES FOR LI TA 03

The values measured by Smith and Welsh (Ref.1) using the pulse-echo (transit-time) technique are as follows:

TABLE 6. ISAGRIC ELASTIC STIFFNESSES.

cE11	cE12	cE13	cE14	cE33	cE44	cE66
229.8	44.0	81.2	-10.4	279.8	96.8	92.9

Units: 10^9 N/m².

TABLE 7. PIEZOELECTRIC STRESS CONSTANTS.

e15	e22	e31	e33
2.72	1.67	-0.38	1.09

Units: C/m².

TABLE 8. DIELECTRIC PERMITTIVITIES AT CONSTANT STRAIN.

(eps)S11	(eps)S33
377.	379.

Units: 10^{-12} F/m.

OUTPUT VALUES FOR LI TA 03

The input values from Tables 6, 7, and 8 were used to compute the remaining elastic, piezoelectric, and dielectric quantities for lithium tantalate in the manner discussed in prior sections of this report. The results are given in Tables 9 to 16.

TABLE 9. ELASTIC STIFFNESSES.

	cE	cD	cP	del cDE	del cPE	del cPD
11	229.8	237.6	237.8	7.78	7.976	0.187
12	44.0	37.0	36.8	-7.02	-7.19	-0.169
13	81.2	80.1	80.1	-1.09	-1.12	-0.0261
14	-10.4	-22.4	-22.7	-12.0	-12.3	-0.291
33	279.8	282.9	283.0	3.13	3.21	0.0750
44	96.8	116.4	116.9	19.6	20.1	0.472
66	92.9	100.3	100.5	7.40	7.58	0.178

Units: 10^9 N/m².

TABLE 10. ELASTIC COMPLIANCES.

	sE	sD	sP	del sED	del sEP	del sDP
11	4.93	4.79	4.78	0.143	0.146	0.00283
12	-0.518	-0.424	-0.422	-0.0945	-0.0962	-0.00169
13	-1.28	-1.24	-1.23	0.0450	-0.0461	-0.00105
14	0.585	1.00	1.01	-0.419	-0.427	-0.00798
33	4.32	4.23	4.23	0.0832	0.0852	0.00195
44	10.46	8.98	8.95	1.48	1.51	0.0282
66	10.90	10.42	10.41	0.475	0.484	0.00905

Units: 10^{-12} m²/N.

TABLE 11. PIEZOELECTRIC [e], [h], AND [a] VALUES.

	e	h	a
15	2.72	7.21	7.39
22	1.67	4.43	4.54
31	-0.38	-1.00	-1.03
33	1.09	2.88	2.94

Units: e: C / m²; h and a: 10^9 V/m.

TABLE 12. PIEZOELECTRIC [d], [g], AND [b] VALUES.

	d	g	b
15	26.5	55.9	56.9
22	7.51	15.8	16.1
31	-3.07	-7.93	-8.11
33	5.68	14.7	15.0

Units: d: 10^{-12} m / V; g and b: 10^{-3} m²/C.

TABLE 13. DIELECTRIC (eps) VALUES.

	(eps)S	(eps)T	del (eps)TS
11	377.	474.	97.1
33	247.0	254.6	8.52

Units: 10^{-12} F/m.

del (eps)TS = del (chi)TS

TABLE 14. DIELECTRIC (chi) VALUES.

	(chi)S	(chi)T	del (chi)TS
11	368.	465.7	97.1
33	370.	479.	8.52

Units: 10^{-12} F/m.

del (chi)TS = del (eps)TS

TABLE 15. DIELECTRIC (bet) VALUES.

	(bet)S	(bet)T	del (bet)TS
11	2.65	2.11	-0.543
33	2.64	2.58	-0.0580

Units: 10^{-9} m/F.

TABLE 16. DIELECTRIC (zet) VALUES.

	(zet)S	(zet)T	del (zet)TS
11	2.72	2.15	-0.567
33	2.70	2.64	-0.0608

Units: $10^{(9)}$ m/F.

CONCLUSIONS

This report provides formulas interrelating the coefficients that appear in the several alternative sets of constitutive equations involving the elastic, piezoelectric, and dielectric properties of crystals. These are then specialized for crystals of class 3m; using measured values reported for lithium tantalate, numerical values of the elements of the polarization matrices are calculated.

REFERENCES

1. R. T. Smith and F. S. Welsh, "Temperature Dependence of the Elastic, Piezoelectric, and Dielectric Constants of Lithium Tantalate and Lithium Niobate," J. Appl. Phys., Vol. 42, No. 6, May 1971, pp. 2219-2230.
2. W. G. Cady, Piezoelectricity, McGraw-Hill, New York, 1946; Dover, New York, 1964.

ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY
MANDATORY DISTRIBUTION LIST
CONTRACT OR IN-HOUSE TECHNICAL REPORTS

15 Nov 88
Page 1 of 2

101 Defense Technical Information Center*
ATTN: DTIC-FDAC
Cameron Station (Bldg 5) (*Note: Two copies for DTIC will
Alexandria, VA 22304-6145 be sent from STINFO Office.)

483 Director
US Army Material Systems Analysis Actv
ATTN: DRXSY-MP
001 Aberdeen Proving Ground, MD 21005

563 Commander, AMC
ATTN: AMCODE-SC
5001 Eisenhower Ave.
001 Alexandria, VA 22333-0001

609 Commander, LABCOM
ATTN: AMSLC-CG, CD, CS (In turn)
2800 Powder Mill Road
001 Adelphi, Md 20783-1145

612 Commander, LABCOM
ATTN: AMSLC-CT
2800 Powder Mill Road
001 Adelphi, MD 20783-1145

680 Commander,
US Army Laboratory Command
Fort Monmouth, NJ 07703-5000
1 - SLCET-DD
2 - SLCET-DT (M. Howard)
1 - SLCET-DB
35 - Originating Office

681 Commander, CECOM
R&D Technical Library
Fort Monmouth, NJ 07703-5000
1 - ASQNC-ELC-I-T (Tech Library)
3 - ASQNC-ELC-I-T (STINFO)

705 Advisory Group on Electron Devices
201 Varick Street, 9th Floor
002 New York, NY 10014-4877

ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY
SUPPLEMENTAL CONTRACT DISTRIBUTION LIST
(ELECTIVE)

15 Nov 88
Page 2 of 2

205	Director Naval Research Laboratory ATTN: CODE 2627 001 Washington, DC 20375-5000	603	Cdr, Atmospheric Sciences Lab LABCOM ATTN: SLCAS-SY-S 001 White Sands Missile Range, NM 88002
221	Cdr, PM JTFUSION ATTN: JTF 1500 Planning Research Drive 001 McLean, VA 22102	607	Cdr, Harry Diamond Laboratories ATTN: SLCHD-CO, TD (In turn) 2800 Powder Mill Road 001 Adelphi, MD 20783-1145
301	Rome Air Development Center ATTN: Documents Library (TILD) 001 Griffiss AFB, NY 13441		
437	Deputy for Science & Technology Office, Asst Sec Army (R&D) 001 Washington, DC 20310		
438	HQDA (DAMA-ARZ-D/Dr. F.D. Verderame) 001 Washington, DC 20310		
520	Dir, Electronic Warfare/Reconnaissance Surveillance and Target Acquisition Ctr ATTN: AMSEL-EW-D 001 Fort Monmouth, NJ 07703-5000		
523	Dir, Reconnaissance Surveillance and Target Acquisition Systems Directorate ATTN: AMSEL-EW-DR 001 Fort Monmouth, NJ 07703-5000		
524	Cdr, Marine Corps Liaison Office ATTN: AMSEL-LN-MC 001 Fort Monmouth, NJ 07703-5000		
564	Dir, US Army Signals Warfare Ctr ATTN: AMSEL-SW-OS Vint Hill Farms Station 001 Warrenton, VA 22186-5100		
602	Dir, Night Vision & Electro-Optics Ctr CECOM ATTN: AMSEL-NV-D 001 Fort Belvoir, VA 22060-5677		