

REPORT DOCUMENTATION PAGE

AD-A209 482

1b. RESTRICTIVE MARKINGS
None.

3. DISTRIBUTION AVAILABILITY OF REPORT

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

CWP-089

5. MONITORING ORGANIZATION REPORT NUMBER(S)

ARO 22971.3-65

6a. NAME OF PERFORMING ORGANIZATION
Center for Wave Phenomena
Colorado School of Mines6b. OFFICE SYMBOL
(If applicable)

7a. NAME OF MONITORING ORGANIZATION

U.S. Army Research Office

6c. ADDRESS (City, State, and ZIP Code)

Golden, CO 80401

7b. ADDRESS (City, State, and ZIP Code)

P.O. Box 12211
Research Triangle Park, NC 22709-22118a. NAME OF FUNDING / SPONSORING
ORGANIZATION
US Army Research Office8b. OFFICE SYMBOL
(If applicable)

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

DAAL03-86-K-0031

8c. ADDRESS (City, State, and ZIP Code)

P.O. Box 12211
Research Triangle Park, NC 27709

10. SOURCE OF FUNDING NUMBERS

PROGRAM
ELEMENT NO.PROJECT
NO.TASK
NO.WORK UNIT
ACCESSION NO.

11. TITLE (Include Security Classification)

Approximation Methods for Scattering From Rough Surfaces (unclassified)

12. PERSONAL AUTHOR(S)

Dr. John A. DeSanto

13a. TYPE OF REPORT
Final13b. TIME COVERED
FROM 2/1/86 TO 1/31/8914. DATE OF REPORT (Year, Month, Day)
March 30, 198915. PAGE COUNT
9

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD GROUP SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Scattering, rough surfaces, approximation methods.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

(See reverse side.)

SDTIC
ELECTE
JUN 21 1989
H

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT

☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION

Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL

Dr. John A. DeSanto

22b. TELEPHONE (Include Area Code)

(303)273-3036

22c. OFFICE SYMBOL

89 6 10 016

Block 19.

Abstract

Rigorous analytical methods in rough surface scattering lead to integral equations. In coordinate space these integral equations are written on the surface value of the field (Neumann, hard, TM boundary value problem), the surface value of the normal derivative of the field (Dirichlet, Soft, TE) or coupled equations on both (interface or penetrable surface or dielectric problem). An additional integration is necessary to find the scattered or transmitted field. In Fourier transform space the corresponding equations are on the scattering and/or transmission amplitudes with a direct interpretation as scattered or transmitted fields as a function of angle.

The integral equations express the full multiple scattering solution and this must be treated numerically. Truncation of the equations can lead to analytical results for single scattering (which is well known) and for double scattering. We describe here the approximations necessary to analytically evaluate to closed form the double scattering term. Specifically for plane wave incidence on a Gaussian distributed rough surface with a Gaussian correlation function we analytically evaluate the incoherent intensity for single scattering with correlated heights and slope, and for double scattering where the heights and slopes are uncorrelated.



Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By <i>per Form 50</i>	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
<i>A-1</i>	

CWP-089
March, 1989



**APPROXIMATION METHODS
FOR SCATTERING FROM
ROUGH SURFACES**

by

John A. DeSanto

**Center for Wave Phenomena
Department of Mathematics
Colorado School of Mines
Golden, Colorado 80401
Phone (303) 273-3557**

ABSTRACT

✓ Rigorous analytical methods in rough surface scattering lead to integral equations. In coordinate space these integral equations are written on the surface value of the field, (Neumann, hard, TM boundary value problem), the surface value of the normal derivative of the field (Dirichlet, Soft, TE), or coupled equations on both (interface or penetrable surface or dielectric problem). An additional integration is necessary to find the scattered or transmitted field. In Fourier transform space the corresponding equations are on the scattering and/or transmission amplitudes with a direct interpretation as scattered or transmitted fields as a function of angle.

The integral equations express the full multiple scattering solution and this must be treated numerically. Truncation of the equations can lead to analytical results for single scattering (which is well known) and for double scattering. We describe here the approximations necessary to analytically evaluate to closed form the double scattering term. Specifically for plane wave incidence on a Gaussian distributed rough surface with a Gaussian correlation function we analytically evaluate the incoherent intensity for single scattering with correlated heights and slopes, and for double scattering where the heights and slopes are uncorrelated.

1. INTRODUCTION

Several years ago we introduced k -space integral equations for scattering from a rough surface using Feynman-diagram and partial summation techniques [1-3]. Numerical solution of a truncated integral equation for the scattering amplitude of the coherent wave using Gaussian statistics on the surface illustrated the necessity of using multiple scattering techniques of this type to describe the scattering [4]. For the coherent field the results agreed well with experimental data into the regime of large roughness. A review of the specific work as well as a review of the field can be found in DeSanto and Brown [5].

One approximation used in this previous work was related to the specific way the transform equation arose from the coordinate-space equation. Recent work on backscatter enhancement [6,7] has prompted us to attempt to remove this approximation in order to have an exact version of the k -space equation, and to thereby enable us to study the single and double scattering approximations of the exact result. The latter define the boundary between possible analytical approximations and necessary numerical results in the sense that triple and higher order scattering must be done numerically. We briefly describe the successful result of this attempt for the Neumann or hard boundary value problem here.

2. PHASE MODULATION

For a plane wave incident at angle θ_i from the normal, the phase change (with no horizontal translation) upon reflection into angle θ_s from the rough surface $z = h(x_i)$ (with $x_i = (x, y)$) is given by

$$-h k \cos \theta_i - h k \cos \theta_s = (k_z^i - k_z) h, \quad (1)$$

where k is the wavenumber with incident z -component $k_z^i = -k \cos \theta_i$ and scattered z -component $k_z = k \cos \theta_s$. With an additional horizontal translation the full wave scattering function can be written as

$$\exp \left[-i(\underline{k} - \underline{k}^i) \cdot \underline{x}_h \right], \quad (2)$$

where $\underline{k}^i(\underline{k})$ is the incident (scattered) direction and $\underline{x}_h = (x, y, h)$ is a point on the surface. If we multiply this by the (non-unit) normal

$$n_m(x_i) = \delta_{m3} - \partial_{mi} h(x_i), \quad (3)$$

where ∂_{mi} is the transverse differential operator, and integrate over the transverse coordinates we get the function

$$A_m(\underline{k} - \underline{k}^i) = \int \int e^{-i[\underline{k} - \underline{k}^i] \cdot \underline{z}_h} n_m(\underline{x}_t) d\underline{x}_t, \quad (4)$$

which is termed the phase modulated amplitude spectrum. It describes all the scattering from initial state $\underline{k}^i$ into scattered state $\underline{k}$. It is also just the physical optics approximation for the scattering. It will occur again below in describing the multiple scattering in the integral equation and of course in the single and double scattering terms.

3. GREEN'S THEOREM AND THE SCATTERING AMPLITUDE

Using Green's theorem the field scattered from a rough surface, ψ^s , can be written at a point $\underline{x}$ as

$$\psi^s(\underline{x}) = \int \int \left\{ n'_m \partial'_m G(\underline{x}, \underline{x}'_n) \psi^s(\underline{x}'_h) - G(\underline{x}, \underline{x}'_h) N^s(\underline{x}'_h) \right\} d\underline{x}'_t \quad (5)$$

in terms of its value on the surface and the value of its normal derivative on the surface

$$N^s(\underline{x}_h) = n_m \partial_m \psi^s(\underline{x}_h), \quad (6)$$

where repeated subscripts are summed from one to three. Here G is the free space Green's function

$$G(\underline{x}, \underline{x}') = \frac{\exp \left[ik_0 |\underline{x} - \underline{x}'| \right]}{4\pi |\underline{x} - \underline{x}'|} \quad (7)$$

and n'_m is just $n_m(\underline{x}'_t)$. The asymptotic value of the Green's function ($R = |\underline{x}|$, $\underline{x}$ large)

$$G(\underline{x}, \underline{x}'_h) \sim \frac{e^{ik_0 R}}{4\pi R} e^{-i\underline{k} \cdot \underline{x}'_h}, \quad (8)$$

yields, using (5), the asymptotic value of the scattered field

$$\psi^s(\underline{x}) \sim \frac{e^{ik_0 R}}{4\pi R} F(\underline{k}), \quad (9)$$

where $F(\underline{k})$ is the scattering amplitude

$$F(\underline{k}) = - \int \int e^{-i \underline{k} \cdot \underline{x}_h} \left[N^s(\underline{x}_h) + i k_m n'_m \psi^s(\underline{x}_h) \right] d\underline{x}_i . \quad (10)$$

It appears as a quasi-Fourier transform over the surface field and normal derivative. The object is to find an integral equation on F and determine approximations from it. The actual scattering amplitude is found from (10) set on the energy shell, i.e. set $k_z = (k_0^2 - k_t^2)^{1/2}$.

4. NEUMANN PROBLEM

The Neumann (N) boundary value problem is given by the boundary condition

$$\begin{aligned} N^s(\underline{x}_h) &\doteq n_m \partial_m \psi^s(\underline{x}_h) \\ &= - n_m \partial_m \psi^i(\underline{x}_h) \\ &\doteq - N^i(\underline{x}_h) \end{aligned} \quad (11)$$

which states that the normal derivative of the total field ($\psi^i + \psi^s$) vanishes on the surface. Here ψ^i is the incident field. The result can be written from (10) as

$$F^N(\underline{k}) = F_1^N(\underline{k}) - i k_m \phi_m^s(\underline{k}) , \quad (12)$$

where

$$F_1^N(\underline{k}) = \int \int e^{-i \underline{k} \cdot \underline{x}_h} n'_m \partial'_m \psi^i(\underline{x}_h) d\underline{x}_i , \quad (13)$$

and

$$\phi_m^s(\underline{k}) = \int \int e^{-i \underline{k} \cdot \underline{x}_h} n'_m \psi^s(\underline{x}_h) d\underline{x}_i , \quad (14)$$

where F_1^N will be shown to describe single scattering (and is known since the incident field is known) and where the object will be to find an integral equation on $k_m \phi_m^s$, the remaining term in (12).

In order to do the latter it is first necessary to formulate an integral equation on ψ^s . The latter follows from the limit of (5) as $\underline{x} \rightarrow \underline{x}_h$, i.e. as $\underline{x}$ approaches a point on the surface, and the use of (11). The normal derivative of G in (5) has a discontinuity

on the surface, and the regularized integral equation can be written as

$$\psi^s(\underline{x}_h) = 2 \int \int G(\underline{x}_h, \underline{x}'_h) N^i(\underline{x}'_h) d\underline{x}'_i - 2 \int \int n'_m R_m(\underline{x}_h - \underline{x}'_h) \psi^s(\underline{x}'_h) d\underline{x}'_i, \quad (15)$$

which is a second kind equation with a regularized kernel R_m given by ($\tilde{G}$ is the Fourier transform of G)

$$R_m(\underline{x}) = [2\pi]^{-3} \int \int \int e^{i\underline{k} \cdot \underline{x}} \tilde{G}(\underline{k}) P_m(\underline{k}) d\underline{k}, \quad (16)$$

where

$$P_m(\underline{k}) = i \left[k_{mi} + \delta_{m3} K^2 P \left(\frac{1}{k_z} \right) \right], \quad (17)$$

with $K^2 = k_0^2 - k_i^2$. The symbol P represents the Cauchy principal value distribution.

We can then "Fourier" transform (15) using (14) to find an equation on ϕ_m^s and thus $k_m \phi_m^s$ to satisfy (12). The result can be written in the form

$$F_N(\underline{k}) = F_1^N(\underline{k}) + F_2^N(\underline{k}) + F_3^N(\underline{k}), \quad (18)$$

where F_1 is given by (13), and $F_2^N(\underline{k})$ is

$$F_2^N(\underline{k}) = -2ik_m \int \int e^{-i\underline{k} \cdot \underline{x}'_i} n'_m (S N^i)(\underline{x}'_h) d\underline{x}'_i, \quad (19)$$

where $S N^i$ is the single layer potential with density N^i given by

$$(S N^i)(\underline{x}) = \int \int G(\underline{x}, \underline{x}'_h) N^i(\underline{x}'_h) d\underline{x}'_i. \quad (20)$$

Given the incident field this function is also known. Its value on the surface is just the continuous limit of its value as the field point $\underline{x}$ approaches the surface ($\underline{x} \rightarrow \underline{x}_h$).

Finally $F_3^N(\underline{k})$ is

$$F_3^N(\underline{k}) = 2ik_m [2\pi]^{-3} \int \int \int A_m(\underline{k} - \underline{\beta}) \tilde{G}(\underline{\beta}) W(\underline{\beta}) d\underline{\beta}, \quad (21)$$

where W satisfies the integral equation

$$W(\underline{\beta}) = W^*(\underline{\beta}) - 2[2\pi]^{-3} P_m(\underline{\beta}) \int \int \int A_m(\underline{\beta} - \underline{\beta}') \tilde{G}(\underline{\beta}') W(\underline{\beta}') d\underline{\beta}' , \quad (22)$$

whose Born term is

$$W^*(\underline{\beta}) = 2 P_m(\underline{\beta}) \int \int e^{-i\underline{\beta} \cdot \underline{x}_i} n'_m(S N^i)(\underline{x}_i) d\underline{x}_i , \quad (23)$$

which equals $i F_2^N(\underline{\beta})$ on the energy shell $\left[\beta_z = (k_0^2 - \beta_i^2)^{1/2} \right]$.

5. PLANE WAVES

For plane wave incidence

$$\psi^i(\underline{x}) = \exp(i \underline{k}^i \cdot \underline{x}) , \quad (24)$$

we have, using (13), that

$$F_1^N(\underline{k}) = i k_m^i A_m(\underline{k} - \underline{k}^i) , \quad (25)$$

which describes single scattering (one A -function) from incident state $\underline{k}^i$ to scattered state $\underline{k}$. F_2^N can be written using (19) as

$$F_2^N(\underline{k}) = 2[2\pi]^{-3} k_m^i k_p^i \int \int \int A_m(\underline{k} - \underline{\beta}) \tilde{G}(\underline{\beta}) A_p(\underline{\beta} - \underline{k}^i) d\underline{\beta} , \quad (26)$$

which describes double scattering (two A -functions) from incident state $\underline{k}^i$ into all states $\underline{\beta}$, propagation in these states (with the propagator $\tilde{G}$), and rescattering into state $\underline{k}$. This is just the matrix element of the double scattering term between an incident plane wave state and scattered state $\underline{k}$. The function F_3^N in lowest order has three A -functions and describes triple and higher order scattering. The functions F_1^N and F_2^N appear to be the limiting case for analytical approximation methods. Triple and higher order scattering must be approached numerically. Note that in terms of the separation we described, *both single and double scattering terms are given explicitly in terms of the incident field*. This is also, of course, true for the more general case of non-plane wave incidence, although for the latter more integrals are involved.

6. KIRCHHOFF APPROXIMATION

It can easily be seen that the (single scattering) Kirchhoff approximation for plane wave incidence applied to (10) yields the result

$$F_{1K}^N(\underline{k}) = i(k_m^i - k_m) A_m(\underline{k} - \underline{k}^i) . \quad (27)$$

Comparison with the exact single scattering result (25) indicates that the Kirchhoff approximation gets some of the single scattering right, but then approximates the remaining true multiple scattering by a single scattering term involving the scattering state $\underline{k}$.

7. AVERAGES

There are two ways to determine average field or intensity quantities if the surface is random. One is to do a straightforward numerical solution of the deterministic equations for each member of the statistical ensemble of surfaces (Monte Carlo calculation), and then to form the averages of the desired quantities. For a one-dimensional surface this has been done in coordinate-space by Thorsos [8]. The method includes all orders of multiple scattering since it is the solution of an integral equation. The second way is to apply averaging operators to the equations to generate integral equations on the field average (coherent field or Dyson equation) or the average of the second (or higher order) moments of the field (Bethe-Salpeter, etc. equations). In k -space and using diagram methods we have previously described this procedure [1-3,5]. Here we adopt the latter philosophy but only applied to explicit representations of single and double scattering.

The averaging operator E is an integral operator whose dimensionality (and integration variables) is the number of random variables involved and whose kernel is the joint probability density function of all the random variables. We have expressed the scattering amplitudes in terms of A_m -functions, and each A_m -function contains three random variables, the height function h , and the two slope terms, h_x and h_y . Coherent averages are defined (here) as first taking the averages of the scattering amplitudes F_1^N or F_2^N and then forming the coherent intensity from the square of the magnitude of this result. For coherent single scattering we thus have three random variables, and for coherent double scattering, six. For Gaussian height distribution and Gaussian correlation function (or, in general, for any statistically homogeneous or spatially stationary process) it can be shown that the coherent averages for any order of multiple scattering yield intensities which are non-zero in only the specular direction. For incoherent scattering (here) we first form the magnitude squared of the scattering amplitudes and then apply the averaging operator. For incoherent single scattering we thus have six random variables (two A -functions) and for incoherent double scattering, twelve. The latter, of course, begins to get formidable.

In spite of the apparent multivariability, the restriction to Gaussian statistics results in only two-point moments (correlation function) on the surface heights and slopes. There is only one correlation function but because of the multivariability it has several combinations of coordinate variability. Many coordinate derivatives also occur (for height-slope correlations) and second derivatives (slope-slope correlations). It is tempting because of all this variability to carry out a partial integration of the A_m -functions. The result, neglecting the partially integrated term is

$$A_m(\underline{k}) \approx k_m(k_z)^{-1} A(\underline{k}) , \quad (28)$$

where

$$A(\underline{k}) = \int \int e^{-i\underline{k} \cdot \underline{x}_h} d\underline{x}_h , \quad (29)$$

and thus reducing the random variability of each A_m -function to a single random variable, the surface height. This essentially decorrelates heights and slopes. Attractive as this approximation is in terms of analysis, there is no understanding of its effect in terms of physical modelling, in particular for scattering processes involving large heights and slopes.

8. EXAMPLES

For a Gaussian process we list two examples of the incoherent intensity. One is for single scattering and one for double scattering. For single scattering the incoherent (I) intensity is

$$I_{II}(\underline{k}) = E |F_1(\underline{k})|^2 , \quad (30)$$

where for F_1 we give the result in such a way that it includes the exact single scattering result (25) or the Kirchhoff result (27). It is a single integral and includes all height slope correlations

$$I_{II}(\underline{k}) = 8\pi d^2 L^2 \int_0^\infty \rho \exp \left[- \left(k_0 c \sigma \right)^2 (1 - \mu(\rho)) \right] \sum_{j=1}^5 V_j(\theta, \rho) d\rho , \quad (31)$$

where d^2 is the ensonified surface area, L is the correlation distance, k_0 the wavenumber, σ the rms height, $c = \cos\theta + \cos\theta^i$, $\mu(\rho)$ the Gaussian correlation function (where ρ is dimensionless) and

$$V_1(\theta, \rho) = J_0(M\rho) K_3^2, \quad (32)$$

$$V_2(\theta, \rho) = 4\rho u(\rho) J_1(M\rho) c k_0 \sigma (\sigma/L) (K_1 \cos\theta + K_2 \sin\theta) K_3, \quad (33)$$

$$V_3(\theta, \rho) = 2\mu(\rho) J_0(M\rho) (\sigma/L)^2 (K_1^2 + K_2^2), \quad (34)$$

$$V_4(\theta, \rho) = -4\rho^2 \mu(\rho) \left[1 + (k_0 c \sigma)^2 \mu(\rho) \right] J_0(M\rho) (\sigma/L)^2 \left[K_1 \cos\theta + K_2 \sin\theta \right]^2 \quad (35)$$

and

$$V_5(\theta, \rho) = 4M^{-1} \rho \mu(\rho) \left[1 + (k_0 c \sigma)^2 \mu(\rho) \right] J_1(M\rho) \cdot (\sigma/L)^2 \left\{ (K_1 \cos\theta + K_2 \sin\theta)^2 - \frac{1}{2} (K_1^2 + K_2^2) \right\}, \quad (36)$$

where the J 's are Bessel functions and

$$M = |k_t - k_t^i| L. \quad (37)$$

Here if $K_j = k_j^i - k_j$ we have the Kirchhoff result from (27) and if $K_j = k_j^i$ the exact single scattering result from (25). With a Taylor expansion in the exponential the integrals can be evaluated using tables.

The second example is that of exact double scattering for a Gaussian process where in order to carry out the averaging it was necessary to carry out a partial integration of the A_m -functions using (28) (i.e. heights and slopes are not correlated). In addition the Taylor expansion mentioned to evaluate (31) is used. The result is

$$E I_2^N(\underline{k}) = 4\pi d^2 \left(\frac{L}{\sigma} \right)^2 \frac{(k_z^i - k_z)^2}{|\underline{k}^i - \underline{k}|^4} S S^i e^{-\Delta_j^2 L^2 / 4\sigma^2}, \quad (38)$$

where

$$\Delta_j = \left\{ (\underline{k}^i - \underline{k}) / (k_z^i - k_z) \right\}_j \quad (39)$$

$$\Delta_t = (k_t^i - k_t)/(k_z^i - k_z) , \quad (40)$$

(which is a transverse vector),

$$S^i = (k_m^i \Delta_m)^2 (\beta^{i2} - 4 \delta^i)^{-1/2} \quad (41)$$

$$\beta^i = 2(1 + \Delta_t^2)^{-1} (k_z^i \Delta_t^2 - k_t^i \cdot \Delta_t) \quad (42)$$

$$\delta^i = (k_z^i)^2 (1 + \Delta_t^2)^{-1} (\Delta_t^2 - 1 - 2 k_t^i \cdot \Delta_t / k_z^i) \quad (43)$$

and S is just S^i with k^i replaced by k . Although it has been possible to do the full evaluation it is under the restriction of uncorrelated heights and slopes. I regard this as a severe restriction and am presently attempting to overcome it.

REFERENCES

- [1] Zipfel, G.G. and J.A. DeSanto, "Scattering of a scalar wave from a random rough surface: a diagrammatic approach," J. Math. Phys. 13, 1903-1911 (1972).
- [2] DeSanto, J.A., "Scattering from a random rough surface: diagram methods for elastic media," J. Math. Phys. 14, 1566-1573 (1973).
- [3] DeSanto, J.A., "Green's function for electromagnetic scattering from a random rough surface," J. Math. Phys. 15, 283-288 (1974).
- [4] DeSanto, J.A. and O. Shisha, "Numerical solution of a singular integral equation in random rough surface scattering theory," J. Comp. Phys. 15, 286-292 (1974).
- [5] DeSanto, J.A. and G.S. Brown, "Analytical techniques for multiple scattering from rough surfaces," in Progress In Optics, E. Wolf ed., 23, 2-62 (1986).
- [6] Mendez, E.R. and K.F. O'Donnell, "Observation of depolarization and backscattering enhancement in light scattering from Gaussian random surfaces," Optics Comm. 61, 91-95 (1987).
- [7] O'Donnell, K.F. and E.R. Mendez, "Experimental study of scattering from characterized random surfaces," J. Opt. Soc. Am. A 4, 1194-1204 (1987).
- [8] Thorsos, E.I., "The validity of the Kirchhoff approximation for rough surface scattering using a Gaussian roughness spectrum," J. Acoust. Soc. Am. 83, 78-92 (1988).