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GPS Orbit Determination: Bootstrapping to Resolve Carrier Phase Ambiguity

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For Global Positioning System (GPS) satellite orbit determination, the most accurate observable available is carrier phase, differenced between observing stations and between satellites to cancel both transmitter and receiver related errors. For maximum accuracy, the integer cycle ambiguities of such observations must be resolved. To perform this ambiguity resolution, a bootstrapping strategy is effective. This strategy requires the tracking stations to have a wide ranging progression of spacings. Then, by conventional "integrated Doppler" processing of the observations from the most widely spaced stations, the orbits can be determined well enough to permit resolution of the ambiguities of the observations from the most closely spaced stations. The resolution of these ambiguities can reduce the uncertainty of the orbit determination enough to enable ambiguity resolution for more widely spaced stations, which will reduce the orbital uncertainty further, and enable ambiguity resolution for still more widely spaced stations, and so on. ~~We have tested~~ this strategy tested with two different tracking networks. In one ~~network~~, six stations had closest-pair spacings of 71, 245, 1500, ..., and 4000 km. Resolving ambiguities for the 71-km pair made it possible to do so for the 245-km pair. This limited ambiguity resolution reduced both the formal and the actual errors of GPS orbit determinations by a factor of two. In the second ~~network~~, twelve stations were arranged in a spiral with geometrically increasing spacings from 10 to 330 km. By bootstrapping, all ambiguities for baselines up to about 100 km long were resolved. The distance was limited by strong ionospheric variability. Still, orbit-determination uncertainty (3σ) was reduced to about 1:1,000,000. Improved handling of ionospheric effects in ambiguity resolution and the use of observations spanning more than one day should further reduce the uncertainty.

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INTRODUCTION

For the determination of Global Positioning System (GPS) satellite orbits, observations of pseudorange, Doppler frequency, and carrier phase are commonly used [see, *e.g., Beutler et al.*, 1986; *Lichten and Border*, 1987]. Of all available observables, carrier phase is potentially the most accurate, especially if observations are differenced between observing stations and between satellites to cancel transmitter- and receiver-related and other common-mode errors. Doubly differenced phase observations can be accurate at the millimeter level. However, the interpretation of phase observations in terms of range or position has an ambiguity, stemming from the periodic nature of the carrier wave.

Phase Ambiguity

The doubly differenced phase, $\Delta\Delta\phi_{kq}{}^{ij}$ for satellites i, j and stations k, q, is related to the doubly differenced range, $\Delta\Delta r_{kq}{}^{ij}$, by the equation

$$\Delta\Delta\phi_{kq}{}^{ij} = -(1/\lambda)\,\Delta\Delta r_{kq}{}^{ij} + N_{kq}{}^{ij}, \qquad (1)$$

in which λ is the carrier wavelength, generally known; and $N_{kq}{}^{ij}$ is an additive bias, initially unknown, but equal to an integer number of cycles. The bias $N_{kq}{}^{ij}$ is constant for an uninterrupted time-series of observations. This bias, which expresses the ambiguity of the phase observable, is known as the "ambiguity parameter" of the series.

Resolving the Ambiguity, or Fixing the Bias

Sometimes the integer value of the ambiguity parameter can be determined, so that it may be subtracted out, or otherwise accounted for. In either case it is said that the ambiguity of the series of observations is resolved, or that the bias is "fixed." Fixing the bias enhances the utility of the observations, as may be appreciated by considering that, in general, a series of observed values has some mean value, or average, plus a variation about the mean. Both the mean value and the variation about the mean contain potentially useful information about the orbits of the satellites and the positions of the tracking stations. Except for a factor of λ, the average doubly-differenced carrier phase equals the average doubly-differenced range. The variation of the phase equals the variation of this distance.

If the average of a series of phase observations includes an unknown bias, in other words if the bias has not been fixed, then the position- or range-related part of the average phase is unknown, and it is difficult to derive position information from it. However, when the bias is fixed, the position-related part of the average phase is known and can contribute to determining the satellite orbits and receiver positions.

In the usual method of bias-fixing, known as the "geometric" method [*Counselman et al.*, 1979; *Bossler et al.*, 1980; *Wübbena et al.*, 1986], ambiguity parameters and unknown position coordinates are estimated simultaneously by least-squares fitting to a collection of doubly-differenced phase observations. In effect, the variation of each series of observations about its mean serves to determine the position-related parameters. From these parameters, the satellite-to-station ranges are computed. The doubly differenced phase is computed from these ranges, and subtracted from the observed value. The average of the difference between the observed and the computed phase is an estimate of the bias. If this estimate is near an integer value and has sufficiently small uncertainty, then the correct integer value of the bias can be identified with confidence. The bias can be fixed.

Since 1982 we have been using an extension of this method in which every integer value in a finite interval surrounding the estimate of each ambiguity parameter is tested. A least-squares fit of all the non-ambiguity parameters to the observations is made for each set of ambiguity-parameter integers within these ranges. For each set, the sum of the squares of the post-fit differences between the observed and computed values of doubly differenced phase is computed. This sum, known as χ^2, which the fitting process minimizes, indicates the badness of the fit. The particular set of integer ambiguity-parameter values having the smallest χ^2 is identified. Confidence in the correctness of this identification is determined by the contrast between the associated value of χ^2, and the next-smallest value found.

For reliable bias-fixing, any error in the theoretically computed value of the phase observable must be small in comparison with one cycle of phase. This requirement can be met in practice if the distance between the receiving stations is small enough, and if the orbits of the satellites are known well enough.

Interstation Distance; Progressive Spacing

The sensitivity of the doubly differenced phase to satellite orbital position increases with increasing interstation distance, or "baseline length." Thus, orbital uncertainty interferes with bias-fixing, and fixing biases is more difficult, the longer the baseline. On the other hand, longer baselines may be better for orbit determination precisely because of their greater sensitivity. How can we increase the baseline length, and still be able to fix biases? One method, proposed by Counselman [1987], is to arrange the stations with a wide-ranging progression of spacings. If the ratio of the largest to the smallest spacing is sufficient, then the time-variation of the doubly differenced phases from the longest baselines is sufficient to determine the orbits accurately enough that the biases of the observations from the shortest baselines can be fixed. With these biases fixed, the uncertainty of the orbit determination is smaller, so that longer baseline biases may now be fixed. The process may be continued until no more biases can be fixed.

APPLICATIONS TO TWO LARGE-RATIO NETWORKS

This method of fixing biases in orbit-determining has been tested with dual-frequency carrier phase observations of GPS satellites from two tracking networks: one of transcontinental extent, the other about one-tenth as wide, each with a large ratio of longest to shortest baseline length.

Transcontinental Network Test

The transcontinental network comprised six tracking stations whose spacings ranged from 71 km to 4000 km. (See Fig. 1.) Observations of five satellites (NAVSTAR numbers 3, 4, 6, 8, and 9) spanning a total of six hours were available from these stations on each of three days (1985 April 1, 2, and 3). We treated each day separately in order to obtain three somewhat independent test results. On each day we determined the five satellites' orbits, first without fixing biases, then applying the baseline-by-baseline, stepwise procedure to fix the doubly differenced phase biases. A preliminary report of these continental-network test results was made by Abbot and Counselman [1987], and a detailed description is in press [*Counselman and Abbot,* 1989]. We summarize the test procedure and results here.

Test Procedure

We began by determining the satellite orbits by the usual method of least-squares fitting to the doubly differenced phases with all bias parameters free (unconstrained). Using these orbits, we fixed the biases for the shortest baseline, which was 71 km long. With these biases fixed, we re-determined the orbits and fixed the biases for the next-longer baseline (OV-Moj), which was 245 km long. With the biases of these two baselines fixed, we made a final determination of the orbits. The remaining baselines in our six-station network were all too long (over 1500 km) for their biases to be fixed.

Simultaneously with the orbital parameters and those biases which had not been fixed, we adjusted an atmospheric delay parameter for each station, the position coordinates of three stations (FtD, Moj, and ML), and clock epoch offsets for three stations. No external constraint was ever placed on any adjusted parameter.

We weighted all observations equally in these least-squares adjustments. Since some systematic errors (*e.g.*, those due to unmodeled atmospheric refraction) are known to increase with increasing baseline length, it could be argued that too little weight was given to the observations from the shortest baselines. However, the fact that the shorter baselines were not more heavily weighted does tend to strengthen our conclusion (see below), that satellite orbit determination can be enhanced substantially by including quite a short baseline in a tracking network of very long baselines.

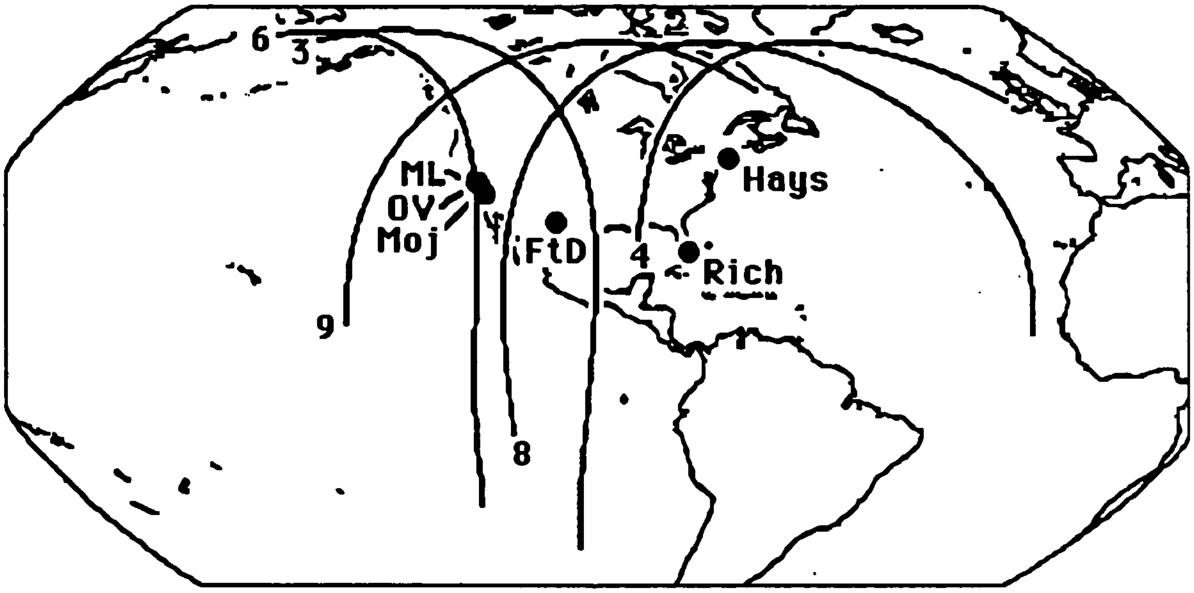


Fig. 1. The ground tracks of the five satellites and the six stations of the transcontinental tracking network are shown. The portion of each satellite's orbit which was observed is shown with the NAVSTAR number (3, 4, 6, 8, 9) marking the beginning of the track. ML = Mammoth Lakes, CA; OV = Owens Valley, CA; Moj = Mojave, CA; FtD = Ft. Davis, TX; Rich = Richmond (near Miami), FL; Hays = Haystack Observatory, near Westford, MA. The ML–OV distance was 71 km; OV–Moj, 245 km; OV-FtD, about 1500 km; OV–Hays, about 4000 km.

We compared the estimated uncertainties of the orbital position determinations with and without bias fixing. We also compared the actual errors, which we determined by reference to more accurate orbit-determinations based on additional observations, and an additional type of observation. These comparisons showed that the short-baseline bias-fixing had reduced both the estimated and the actual errors of the orbital position determinations, consistently, by about a factor of two.

To measure the effect of fixing biases for the 71-km baseline on the ability to fix biases for the 245-km baseline, we used χ^2 statistics as described above. We found a substantial improvement on each of the three days.

Test Results: 1. Uncertainties of Orbital Position Estimates

The estimated uncertainties of the orbital positions of all five satellites, determined with and without fixing the biases of the observations from the two shortest baselines, are shown in Figure 2. Our "estimated uncertainty" is three times the square root of the relevant diagonal element of the inverse of the coefficient matrix of the least-squares normal equations, multiplied by the square root of the sum of the postfit residuals, and divided by the number of degrees of freedom. In other words, our estimated uncertainty is three times the scaled formal standard error. We include the factor of three to account, empirically, for systematic error. The greatest of the uncertainties ($3\sigma_x$, $3\sigma_y$, $3\sigma_z$) of the three Cartesian coordinates of each satellite, at a time near the middle of the observing span on April 1, is shown. Very similar results were obtained for April 2 and 3.

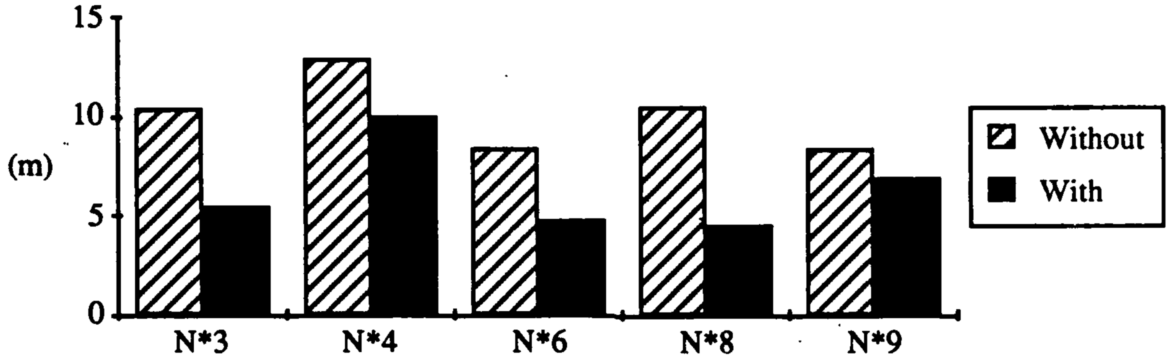


Fig. 2. Estimated uncertainty of orbital position (greatest of $3\sigma_x$, $3\sigma_y$, $3\sigma_z$) for each satellite (NAVSTAR 3, ..., 9), estimated **without**, and **with**, fixing the short-baseline (71- & 245-km) doubly-differenced phase biases at integer-cycle values. In either case, all longer-baseline biases were free, and all observations from all six stations were included with equal weights in the least-squares estimation.

Test Results: 2. Actual Errors of Orbital Position Estimates

To assess the actual — as opposed to the estimated — errors of our orbit determinations, we compared them with a determination by King *et al.* [1986]. We believe [*Abbot et al.*, 1986; *Bock et al.*, 1986] that the errors of the comparison orbits are smaller than 2 parts in 10^7 — *i.e.*, under 5 meters in orbital position.

The orbital position differences between the comparison determination and the two test determinations, one done without, and one done with fixing biases for the short baselines, are shown in Figure 3. For each satellite, the greatest magnitude of the vector position difference occurring at any time during the observations on April 2, is plotted. The differences are consistent with the estimated uncertainties which were plotted in Figure 2.

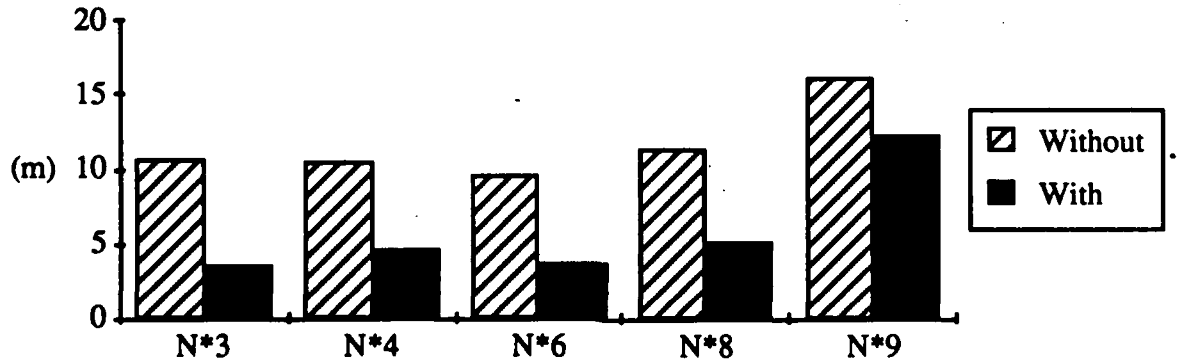


Fig. 3. Actual error of orbital position (peak magnitude of vector error) for each satellite (NAVSTAR 3, ..., 9) determined **without**, and **with**, fixing the short-baseline (71- & 245-km) doubly-differenced phase biases at integer-cycle values. In either case, all longer-baseline biases were free and all observations from all six stations were included with equal weights in the least-squares estimation.

Test Results: 3. Enhancement of Ability to Resolve Ambiguity

Fixing the biases of the observations from one baseline makes it easier to fix the biases of the observations from another baseline. This "bootstrap" principle is the key to the progressively-spaced-station method of orbit determination. To predict the ability to determine uniquely the integer values of bias parameters, and thus to fix them reliably, we use a normalized "χ^2 contrast" statistic given by

$$\text{Contrast} = [\,(\chi_1^2/\chi_0^2) - 1\,] \cdot [\,N_{d.o.f.}^{1/2}\,], \qquad (2)$$

where χ_0^2 is the smallest and χ_1^2 is the next-smallest value of χ^2 found in the integer bias search, and $N_{d.o.f.}$ is the number of degrees of freedom of χ^2. If one makes the usual ridiculous assumption of unbiased independent errors, one can show that this statistic has nearly a normal distribution. In actual practice, we have found that contrast less than 2 is definitely not sufficient for reliable bias-fixing; a value of 3 indicates that the biases are probably, but not certainly fixable; and a value greater than 4 implies a high level of confidence.

On each of the three days of our transcontinental-network test, all the biases (both L1 and L2) for the shortest baseline, the 71-km line between OVRO and Mammoth Lakes, could be fixed with high confidence even when no biases were fixed for any other baseline. The same could not be said for any longer baseline. However, if the biases for the 71-km line were fixed, then all the biases of the 245-km (OVRO-Mojave) baseline could be fixed. The χ^2 contrast statistics for the 245-km baseline, with and without bias-fixing on the 71-km line, are shown for each day in Figure 4.

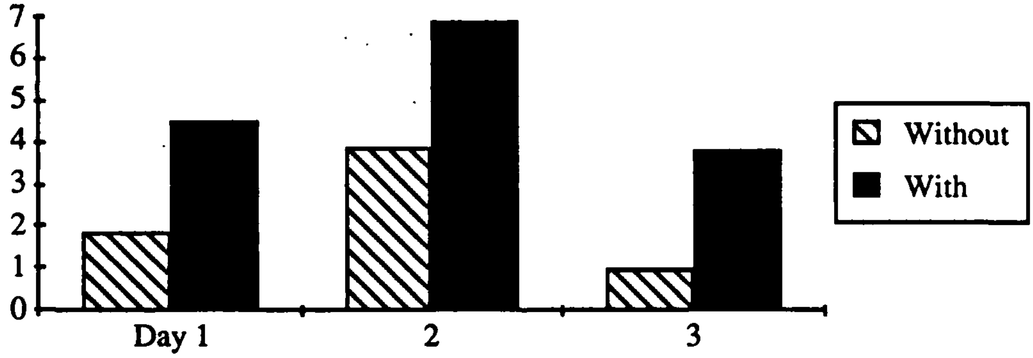


Fig. 4. For each day of the test, χ^2 contrast statistics measuring the ability to determine integer bias values for the 245-km baseline are shown, computed **without**, and **with**, fixed-integer biases for the 71-km baseline.

Summary of Transcontinental-Network Test Results

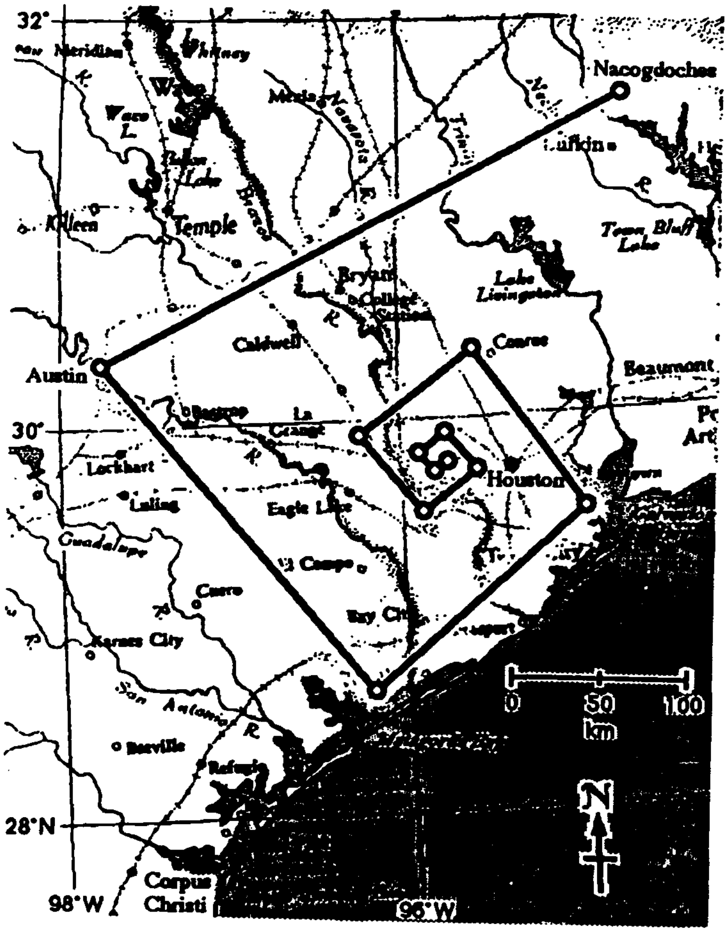
The test results show that orbit-determination accuracy can be enhanced by including quite short baselines in a tracking network of very long baselines. In our transcontinental network, the length of one baseline was less than 2 percent of the overall network size. We were able to resolve the ambiguities of the doubly differenced carrier phase observations from this baseline because it was so short. Resolving these ambiguities improved the satellite orbit determinations, and made possible the resolution of the ambiguities of a 3.5-times-longer baseline. As a result, orbit-determination accuracy was improved by a factor of two.

This result is remarkable, in view of how short our ambiguity-resolved baselines were. We believe that, given a denser and more regularly spaced progression of tracking-station spacings, we could have carried the ambiguity-resolution further, and would have improved the orbit-determination accuracy further.

The Nautilus Network

In order to be able to test this prediction, during the recent Global Orbit Tracking Experiment (GOTEX) [*International Association of Geodesy,* 1988] we observed the GPS satellites with a "Nautilus" network comprising twelve stations in a logarithmic spiral array. The interstation baseline lengths increased in geometric progression from 10 to 330 km, and each baseline was nearly orthogonal to its shorter and longer neighbors. (See Fig. 5.)

Fig. 5. The Nautilus network in southeastern Texas.



We intend to combine the tracking data from the Nautilus network with data from several other, more widely spaced, stations which participated in GOTEX (including the Haystack, Richmond, and Mojave stations shown in Fig. 1) in order to have a very wide range of interstation spacings for study purposes. Since the other stations' data has not yet been received, just a few results from a preliminary analysis of the twelve Nautilus stations' data are presented here.

Nautilus Data Analysis

These stations observed six GPS satellites (NAVSTAR numbers 3, 4, 6, 8, 9, and 10) for a total span of six hours each day from Nov. 1 to Nov. 14, 1988. As of this writing we have done a cursory analysis of all 14 days' data, and a more detailed analysis of the data from just one day, Nov. 8, which we believe is typical.

For this preliminary analysis, accurate position coordinates were available *a priori* for only one station, at Austin, TX. To define a coordinate-system origin, we fixed the position of this station. To define the orientation of our coordinate system in the absence of any other fiducial stations, we applied *a priori* knowledge of the satellite orbital elements. In our weighted-least-squares parameter estimation we included a pseudo-observation of the semi-major axis (a), the eccentricity (e), the inclination (i), and the longitude of the ascending node (Ω) of each orbit, with independent error standard deviations (expressed fractionally for a, absolutely for e, and in radians for i and Ω) of 1.8×10^{-6}. These standard deviations are equivalent to across-track position error standard deviations of about 45 m (3σ uncertainty = 133 m). For the argument of pericenter (ω) and the mean anomaly (M), we included pseudo-observations with independent error standard deviations of 3.5×10^{-6} radian, equivalent to an along-track position error standard deviation of about 125 m (3σ uncertainty = 375 m).

The analysis procedure was similar to that employed for the transcontinental network, except that all stations' observations were not uniformly weighted. Phase measurement error standard deviations were assumed to be proportional to baseline length, and the observation weights were proportional to the inverse squares of these standard deviations. The measurement error standard deviations were also scaled such that the postfit χ^2 per degree of freedom was unity. (This scaling ensured that the orbital-element pseudo-observation standard deviations would be preserved.) Three position coordinates for each station but Austin, an atmospheric zenith delay parameter for each station but the central one, and clock-synchronization parameters for three stations, were estimated simultaneously with six orbital elements for each satellite. Aside from the inclusion of orbital-element pseudo-observations, no constraint was applied to any parameter. The estimated uncertainties (3σ) of the resulting orbital position estimates, before and after fixing biases, are shown in Fig. 6.

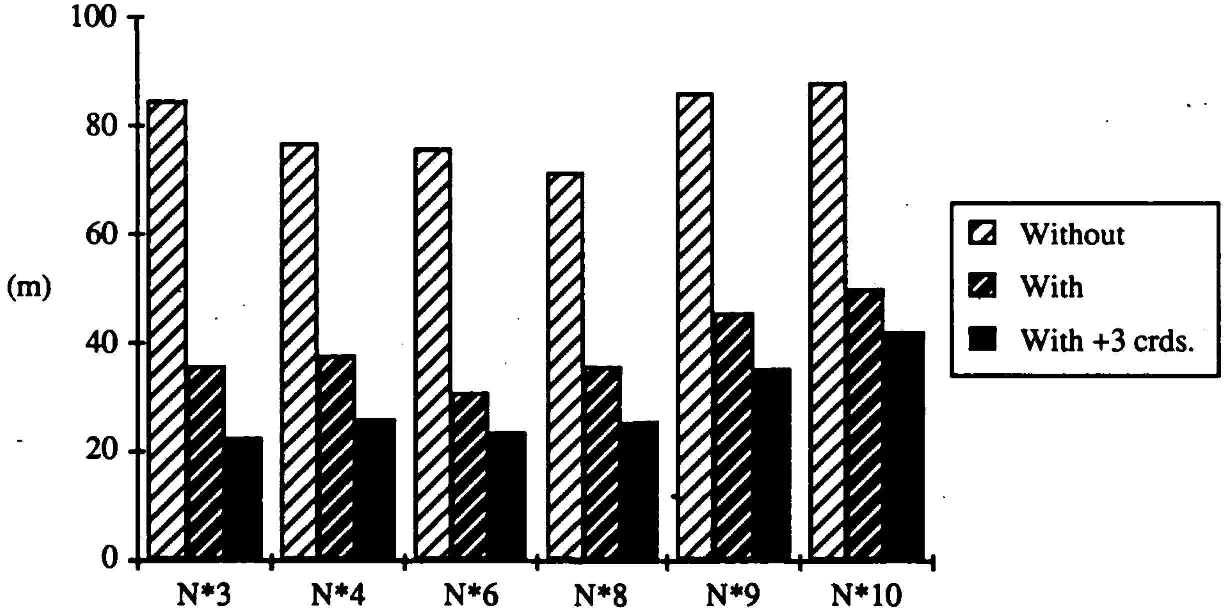


Fig. 6. Estimated uncertainty (greatest of $3\sigma_x$, $3\sigma_y$, $3\sigma_z$) of the orbital position of each satellite (NAVSTAR 3, ..., 10), determined from the Nautilus tracking data using prior knowledge of one station position and *a priori* knowledge of the orbits with 3-σ uncertainties of 133 m across-track and 375 m along-track: first **without** fixing any biases; next **with** biases fixed for the inner eight stations; finally with these biases fixed and three other station coordinates ("**+3 crds.**") fixed in order to reduce coordinate-frame orientation uncertainty (see text).

Results: Ability to Fix Biases

The L1 and L2 phase biases could not be fixed (reliably) for any baseline beyond those formed by the innermost eight stations, *i.e.*, for any baseline longer than about 100 km. It will be recalled that in our analysis of the 1985 transcontinental-network experiment we fixed biases for a much longer, 245-km, baseline. Several factors account for the reduced baseline-length limit in the 1988 experiment. Most important was very greatly increased ionospheric refraction, about 20 times greater for the 1988 November observations than for the 1985 observations. The increase is only partially explained by the different time of day (post- rather than pre-sunrise). Mainly, the increased level of solar activity in 1988-89 is responsible. The baseline-length limit for bias-fixing was also reduced because the 1988 observations were made with code-independent receivers, for which the ambiguity spacing is $\lambda/2$; whereas the 1985 observations were made with GPS P-code receivers, for which the ambiguity spacing is λ. We are aware that the bias-fixing algorithm currently used in our software does not handle ionospheric refraction properly, and believe it is capable of substantial improvement. We expect that software enhancements currently underway will enable us to fix biases for the entire Nautilus network.

Reduction of Orbital Uncertainty

Fig. 6 shows that fixing biases for just the inner eight stations of the Nautilus network reduces the orbital uncertainties by about a factor of two, from about 80 m to about 40 m. In either case, it should be noted that these uncertainties are substantially smaller than those of the pseudo-observations which were applied to constrain coordinate-system orientation. The biases-fixed uncertainties are actually so small, that the residual uncertainty in coordinate-system orientation is significant.

A simple numerical experiment was performed to show the effect of reducing the uncertainty in coordinate-system orientation. In addition to the three coordinates of the Austin station, which had been fixed to define a coordinate origin, three other coordinates were fixed to provide an orientation constraint: the heights of the Nacogdoches and Matagorda Bay stations, and the latitude of the Nacogdoches station. The resulting orbital uncertainties, shown in Fig. 6, were of the order of 20-40 m. Eventually we will have accurately determined the positions of the Nautilus stations and will be able to hold them fixed.

CONCLUSIONS

Both tests — the first with the transcontinental network, and the second with the much smaller Nautilus network — show that the accuracy of satellite orbit determination from doubly differenced phase observations is enhanced substantially if the integer-cycle ambiguities of the observations are resolved, that is, if the biases are fixed at the proper integer values. The tests also show that an effective method of fixing these biases is *bootstrapping*. In this method, conventional integrated-Doppler processing of the observations from the most widely spaced stations in the tracking network determines the orbits well enough that the ambiguities for closely spaced stations can be resolved. Resolving these ambiguities reduces the uncertainty of the orbit determination enough to enable ambiguity resolution for more widely spaced stations, which further reduces the orbital uncertainty.

With just a few hours' observations from the relatively small Nautilus network, which was just a few hundred kilometers across but was configured to facilitate bias-fixing, GPS satellite orbits were determined with estimated uncertainties of the order of 1 part in 10^6. With enhanced bias-fixing algorithms and the use of observations spanning more than one day, smaller uncertainties should be obtained.

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