

ONE FILE COPY

1

AD-A210 274

ON LOWER CONFIDENCE BOUNDS FOR PCS
IN TRUNCATED LOCATION PARAMETER MODELS *

by

Shanti S. Gupta
Department of Statistics
Purdue University
West Lafayette, IN, USA

Lii-Yuh Leu
Graduate Institute of Statistics
National Central University
Chung-Li, Taiwan, R.O.C.

TaChen Liang
Department of Mathematics
Wayne State University
Detroit, MI 48202

Technical Report # 89-17C

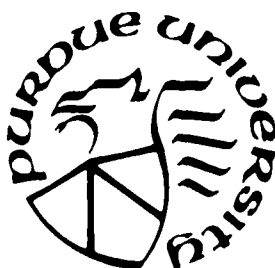
PURDUE UNIVERSITY

DTIC

ELECTE

JUL 14 1989

S D D



**CENTER FOR STATISTICAL
DECISION SCIENCES AND
DEPARTMENT OF STATISTICS**

DISTRIBUTION STATEMENT A

Approved for public release
Distribution Unlimited

89

V 18 OCT

1

ON LOWER CONFIDENCE BOUNDS FOR PCS
IN TRUNCATED LOCATION PARAMETER MODELS *

by

Shanti S. Gupta
Department of Statistics
Purdue University
West Lafayette, IN, USA

Lii-Yuh Leu
Graduate Institute of Statistics
National Central University
Chung-Li, Taiwan, R.O.C.

TaChen Liang
Department of Mathematics
Wayne State University
Detroit, MI 48202

Technical Report # 89-17C

DTIC
ELECTE
JUL 14 1989
S D D

COPY
INSPECTED
E

Department of Statistics
Purdue University

June, 1989

Accession For	
NTIS	CRA&I ☒
DTIC	TAB ☐
Unannounced ☐	
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	

* This research was supported in part by the Office of Naval Research Contract N00014-88-k-0170 and NSF Grants DMS-88/06964, DMS-8702620 at Purdue University. The research of Dr. Lii-Yuh Leu was also supported in part by the National Science Council, Republic of China.

△

DISTRIBUTION STATEMENT A
Approved for public release
Distribution Unlimited

ON LOWER CONFIDENCE BOUNDS FOR PCS
IN TRUNCATED LOCATION PARAMETER MODELS

Shanti S. Gupta
Department of Statistics
Purdue University
West Lafayette, IN 47907

Lii-Yuh Leu
Graduate Institute of Statistics
National Central University
Chung-Li, Taiwan, R.O.C.

TaChen Liang
Department of Mathematics
Wayne State University
Detroit, MI 48202

ABSTRACT

We are concerned with deriving lower confidence bounds for the probability of a correct selection in truncated location-parameter models. Two cases are considered according to whether the scale parameter is known or unknown. For each case, a lower confidence bound for the difference between the best and the second best is obtained. These lower confidence bounds are used to construct lower confidence bounds for the probability of a correct selection. The results are then applied to the problem of selecting the best exponential population having the largest truncated location-parameter. Useful tables are provided for implementing the proposed methods.

Key Words and Phrases: correct selection, probability of a correct selection, indifference zone, lower confidence bound, best population, truncated-location model, two-parameter exponential distribution.

1. Introduction

Let $X_{ij}, j = 1, \dots, n$, be a sample of size n from a population π_i , where $\pi_1, \dots, \pi_k$ are independently distributed with absolutely continuous cumulative distribution $G\left(\frac{x-\theta_i}{\beta}\right)$, $1 \leq i \leq k$, respectively, where $\beta > 0$, $-\infty < \theta_i < \infty, i = 1, \dots, k$ and $G(x) = 0$ if $x \leq 0$. Let $\underline{\theta} = (\theta_1, \dots, \theta_k)$ and let $\theta_{(1)} \leq \dots \leq \theta_{(k)}$ denote the ordered values of $\theta_1, \dots, \theta_k$. It is assumed that the exact pairing between the ordered and the unordered parameter is unknown. The population associated with $\theta_{(k)}$ is referred to as the best population. Assume that the experimenter is interested in the selection of the best population. For this purpose, let $X_i = \min(X_{i1}, \dots, X_{in})$. We denote the cumulative distribution and the density function of X_i by $F\left(\frac{x-\theta_i}{\beta}\right)$ and $f\left(\frac{x-\theta_i}{\beta}\right)$, respectively. In many situations, X_i can be a sufficient statistic for the parameter θ_i . The natural selection rule is to select the population yielding the largest X_i as the best population. Thus, a question which arises naturally is: Is the selected population actually the best? Or, more precisely, what kind of confidence statement can be made regarding this selection?

Let CS (correct selection) denote the event that the best population is selected. Thus, the probability of a correct selection (PCS) at $\underline{\theta}$ by applying the natural selection rule is:

$$\text{PCS}(\underline{\theta}) = \int_{x=0}^{\infty} \frac{k-1}{\pi} F\left(x + \frac{\theta_{(k)} - \theta_{(j)}}{\beta}\right) dF(x). \quad (1.1)$$

In general, to guarantee the probability of a correct selection, one needs to specify a positive number δ^* such that $\theta_{(k)} - \theta_{(k-1)} \geq \delta^*$, see Bechhofer (1954). Clearly, this indifference zone approach is formulated on the basis of designing an experiment. However, in a real situation, it may be hard to assign the value of δ^* such that $\theta_{(k)} - \theta_{(k-1)} \geq \delta^*$, since the parameter values $\theta_1, \dots, \theta_k$ are unknown. So that if the above assumption is not satisfied,

the probability of a correct selection cannot be guaranteed to be at least equal to the prespecified level. Parnes and Srinivasan (1986) have pointed out certain inconsistencies in the indifference zone formulation of certain selection problems. Also, see Fabian (1962) and Hsu (1981) for some possible ways to be out of this impasse.

Recently, retrospective analyses regarding the PCS have been studied by several authors. Olkin, Sobel and Tong (1976, 1982) have presented estimators of the PCS. Faltin and McCulloch (1983) have studied the small-sample properties of the Olkin-Sobel-Tong estimator of the PCS for the case when $k = 2$. Bofinger (1985) has discussed the nonexistence of consistent estimators of the PCS. Gutmann and Maymin (1987) have presented a procedure to test whether the selected population is the best. Anderson, Bishop and Dudewicz (1977) have given a lower confidence bound on the PCS in normal distribution models. Kim (1986) has presented a lower confidence bound on the PCS for the location-parameter model for the case where the underlying density function has the monotone likelihood ratio property and studied its application to normal model case. Gupta and Liang (1987) have derived a lower confidence bound for the PCS for the general location-parameter model and applied the result to normal populations.

In this paper, we are concerned with deriving lower confidence bounds for the probability of a correct selection for truncated location-parameter models. Two cases are considered according to whether the scale parameter β is known or unknown. For each of these, a lower confidence bound for the difference $(\theta_{(k)} - \theta_{(k-1)})/\beta$ is obtained and used to construct a lower confidence bound for the probability of a correct selection. The results are then applied to the problem of selecting the best two-parameter exponential

population having the largest truncated location parameter. Useful tables are provided for implementing the proposed procedures.

2. Lower Confidence Bound on PCS

In (1.1), replacing $\theta_{(k)} - \theta_{(i)}$ by $\theta_{(k)} - \theta_{(k-1)}$ for each $i = 1, \dots, k-2$, we have

$$\text{PCS}(\underline{\theta}) \geq \int_{x=0}^{\infty} \left[F \left(x + \frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \right) \right]^{k-1} dF(x). \quad (2.1)$$

Thus, if a lower confidence bound, say $\hat{\Delta}$, for $\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}$ can be obtained, then $\hat{P}_L = \int_{x=0}^{\infty} [F(x + \hat{\Delta})]^{k-1} dF(x)$ is a lower confidence bound for the $\text{PCS}(\underline{\theta})$. In the following, lower confidence bounds for the $\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}$ are derived for the two cases where the scale parameter β is either known or unknown.

Let $X_{[1]} \leq \dots \leq X_{[k]}$ denote the ordered statistics of $X_1, \dots, X_k$, and let $X_{(i)}$ denote the random observation associated with $\theta_{(i)}$, $i = 1, \dots, k$. For each fixed $\underline{\theta}$, let $\underline{\theta}^0 = (\theta_1^0, \dots, \theta_k^0)$ where $\theta_i^0 = -\infty$ for $i = 1, \dots, k-2$, and $\theta_i^0 = \theta_{(i)}$, for $i = k-1, k$. Then we obtain the following lemma which is analogous to Lemma 1 of Kim (1986).

Lemma 2.1. Let $f(y)$ be the density function of $F(y)$. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then, for each fixed $c > 0$,

a) $P_{\underline{\theta}}\{X_{[k]} - X_{[k-1]} > c\}$ is nonincreasing in $\theta_{(1)}$, and therefore,

b) $P_{\underline{\theta}}\{X_{[k]} - X_{[k-1]} > c\} \leq P_{\underline{\theta}^0}\{|X_{(k)} - X_{(k-1)}| > c\}$ for all $\underline{\theta}$.

Proof: Without loss of generality, we may assume that $\theta_1 \leq \dots \leq \theta_k$. Then, for each fixed

$c > 0$, straightforward computation leads to

$$\begin{aligned}
& P_{\theta} \{X_{[k]} - X_{[k-1]} > c\} \\
&= \sum_{i=1}^k P_{\theta} \{X_j \leq X_i - c \text{ for all } j \neq i\} \\
&= \int_{(\theta_k - \theta_1 + c)/\beta}^{\infty} \sum_{j=2}^k F\left(y + \frac{\theta_1 - \theta_j - c}{\beta}\right) dF(y) + \sum_{i=2}^{k-1} \int_{(\theta_k - \theta_i + c)/\beta}^{\infty} \sum_{\substack{j=1 \\ j \neq i}}^k F\left(y + \frac{\theta_i - \theta_j - c}{\beta}\right) dF(y) \\
&\quad + \int_{\max((\theta_{k-1} - \theta_k + c)/\beta, 0)}^{\infty} \sum_{j=1}^{k-1} F\left(y + \frac{\theta_k - \theta_j - c}{\beta}\right) dF(y).
\end{aligned}$$

Then,

$$\begin{aligned}
& \frac{\partial}{\partial \theta_1} P_{\theta} \{X_{[k]} - X_{[k-1]} > c\} \\
&= \sum_{j=2}^{k-1} \int_{(\theta_k - \theta_1 + c)/\beta}^{\infty} \beta^{-1} \left[\sum_{\substack{m=2 \\ m \neq j}}^k F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right) \right] \left[f\left(y + \frac{\theta_1 - \theta_j - c}{\beta}\right) f(y) - \right. \\
&\quad \left. f\left(y - \frac{c}{\beta}\right) f\left(y + \frac{\theta_1 - \theta_j}{\beta}\right) \right] dy \\
&\quad + \int_{(\theta_k - \theta_1 + c)/\beta}^{\infty} \beta^{-1} \left[\sum_{m=2}^{k-1} F\left(y + \frac{\theta_1 - \theta_m - c}{\beta}\right) \right] f\left(y + \frac{\theta_1 - \theta_k - c}{\beta}\right) f(y) dy \\
&\quad - \int_{\max((\theta_{k-1} - \theta_k + c)/\beta, 0)}^{\infty} \beta^{-1} \left[\sum_{m=2}^{k-1} F\left(y + \frac{\theta_k - \theta_m - c}{\beta}\right) \right] f\left(y + \frac{\theta_k - \theta_1 - c}{\beta}\right) f(y) dy \\
&\equiv I_1 + I_2 - I_3 \text{ (say).}
\end{aligned}$$

Note that under the assumption, $f(y + \frac{\theta_1 - \theta_j - c}{\beta}) f(y) \leq f(y - \frac{c}{\beta}) f(y + \frac{\theta_1 - \theta_j}{\beta})$ for all $y \geq \frac{\theta_k - \theta_1 + c}{\beta}$ since $y + \frac{\theta_1 - \theta_j - c}{\beta} \leq y - \frac{c}{\beta}$, $y + \frac{\theta_1 - \theta_j}{\beta} \leq y$. Thus, $I_1 \leq 0$. For the difference $I_2 - I_3$, we consider the following two cases.

Case 1. As $\theta_{k-1} - \theta_k + c > 0$, after changing variables,

$$\begin{aligned}
& I_2 - I_3 \\
&= \int_{(\theta_k - \theta_1 + c)/\beta}^{\infty} \beta^{-1} \left[\frac{k-1}{\pi} F \left(y + \frac{\theta_1 - \theta_m - c}{\beta} \right) \right] f \left(y + \frac{\theta_1 - \theta_k - c}{\beta} \right) f(y) dy \\
&\quad - \int_{(\theta_{k-1} - \theta_1 + c)/\beta}^{\infty} \beta^{-1} \left[\frac{k-1}{\pi} F \left(y + \frac{\theta_1 - \theta_m - c}{\beta} \right) \right] f \left(y - \frac{c}{\beta} \right) f \left(y + \frac{\theta_1 - \theta_k}{\beta} \right) dy \\
&= \int_{(\theta_k - \theta_1 + c)/\beta}^{\infty} \beta^{-1} \left[\frac{k-1}{\pi} F \left(y + \frac{\theta_1 - \theta_m - c}{\beta} \right) \right] \left[f \left(y + \frac{\theta_1 - \theta_k - c}{\beta} \right) f(y) - f \left(y - \frac{c}{\beta} \right) \right. \\
&\quad \left. f \left(y + \frac{\theta_1 - \theta_k}{\beta} \right) \right] dy \\
&\quad - \int_{(\theta_{k-1} - \theta_1 + c)/\beta}^{(\theta_k - \theta_1 + c)/\beta} \beta^{-1} \left[\frac{k-1}{\pi} F \left(y + \frac{\theta_1 - \theta_m - c}{\beta} \right) \right] f \left(y - \frac{c}{\beta} \right) f \left(y + \frac{\theta_1 - \theta_k}{\beta} \right) dy \\
&\leq 0
\end{aligned} \tag{2.2}$$

since at the right-hand side of (2.2), the first term is nonpositive, see the preceding arguments, and the second term is nonnegative.

Case 2. As $\theta_{k-1} - \theta_k + c \leq 0$, changing variables and following straight computation, we have

$$\begin{aligned}
& I_2 - I_3 \\
&= \int_{(\theta_k - \theta_1 + c)/\beta}^{\infty} \beta^{-1} \left[\frac{k-1}{\pi} F \left(y + \frac{\theta_1 - \theta_m - c}{\beta} \right) \right] f \left(y + \frac{\theta_1 - \theta_k + c}{\beta} \right) f(y) dy \\
&\quad - \int_{(\theta_k - \theta_1)/\beta}^{\infty} \beta^{-1} \left[\frac{k-1}{\pi} F \left(y + \frac{\theta_1 - \theta_m - c}{\beta} \right) \right] f \left(y - \frac{c}{\beta} \right) f \left(y + \frac{\theta_1 - \theta_k}{\beta} \right) dy \\
&= \int_{(\theta_k - \theta_1 + c)/\beta}^{\infty} \beta^{-1} \left[\frac{k-1}{\pi} F \left(y + \frac{\theta_1 - \theta_m - c}{\beta} \right) \right] \left[f \left(y + \frac{\theta_1 - \theta_k - c}{\beta} \right) f(y) - f \left(y - \frac{c}{\beta} \right) \right. \\
&\quad \left. f \left(y + \frac{\theta_1 - \theta_k}{\beta} \right) \right] dy \\
&\quad - \int_{(\theta_k - \theta_1)/\beta}^{(\theta_k - \theta_1 + c)/\beta} \beta^{-1} \left[\frac{k-1}{\pi} F \left(y + \frac{\theta_1 - \theta_m - c}{\beta} \right) \right] f \left(y - \frac{c}{\beta} \right) f \left(y + \frac{\theta_1 - \theta_k}{\beta} \right) dy \\
&\leq 0.
\end{aligned}$$

Based on the preceding discussion, it follows that $\frac{\partial}{\partial \theta_{(1)}} P_{\theta} \{X_{[k]} - X_{[k-1]} > c\} \leq 0$.

Therefore, $P_{\theta}\{X_{[k]} - X_{[k-1]} > c\}$ is nonincreasing in $\theta_{(1)}$. Part b) is a result of repeated application of part a). $\square$

2.1. The Scale Parameter β Known Case

Let $H(t)$ be the distribution function of $\frac{(X_1 - \theta_1) - (X_2 - \theta_2)}{\beta}$. Note that the distribution of $\frac{X_i - \theta_i}{\beta}$ is independent of θ_i and β . Thus the distribution $H(t)$ is independent of θ_1, θ_2 and β . For any real value t ,

$$H(t) = \begin{cases} \int_0^{\infty} F(y+t)dF(y) & \text{if } t \geq 0, \\ \int_{-t}^{\infty} F(y+t)dF(y) & \text{if } t < 0. \end{cases}$$

Also, $H(-t) = 1 - H(t)$ for all t . For each fixed $\alpha, 0 < \alpha < 1$, let $t_{\frac{\alpha}{2}}$ be the upper $\frac{\alpha}{2}$ -quantile of the distribution $H(t)$. By the symmetric property of $H(t)$, $t_{\frac{\alpha}{2}} > 0$. For this fixed α , define a nonnegative function $L_{\alpha}(t)$ on $[0, \infty)$ implicitly by

$$H(L_{\alpha}(t) - t) + H(-L_{\alpha}(t) - t) = \alpha \text{ for } t \geq t_{\frac{\alpha}{2}} \quad (2.3)$$

and $L_{\alpha}(t) = 0$ if $0 \leq t < t_{\frac{\alpha}{2}}$. One needs to prove that the function $L_{\alpha}(t)$ is well defined.

Lemma 2.2. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then, the function $L_{\alpha}(t)$ defined implicitly by (2.3) always exists.

Proof: Let $h(t)$ be the density function of $H(t)$. Then $h(t)$ is symmetric about the point 0. Under the assumption, one can see that $h(t)$ is unimodal and $h(t_1) > h(t_2)$ if $|t_1| < |t_2|$.

For each fixed $t \geq t_{\frac{\alpha}{2}}$, define the function $M(c)$ for $c \geq 0$ as follows: $M(c) = H(c - t) + H(-c - t)$. Then, $M'(c) = h(c - t) - h(-c - t) > 0$ for $c > 0$ since $|c - t| < c + t$. Thus, $M(c)$ is strictly increasing in c . Now, $M(0) = 2H(-t) \leq 2H(-t_{\frac{\alpha}{2}}) = \alpha$ since $t \geq t_{\frac{\alpha}{2}}$. Also, $\lim_{c \rightarrow \infty} M(c) = 1$. By the continuity and strictly increasing property of $H(t)$, there exists a

unique $c > 0$ such that $M(c) = H(c-t) + H(-c-t) = \alpha$. We then denote that c by $L_\alpha(t)$.

Thus, $L_\alpha(t)$ is well-defined. $\square$

Lemma 2.3. For given $0 < \alpha < 1$, the function $L_\alpha(t)$ is strictly increasing in t for $t \geq t_{\frac{\alpha}{2}}$.

Proof of the above lemma is straightforward.

Remark 2.1. a) For each fixed $\alpha, 0 < \alpha < 1$, by the definition of $L_\alpha(t)$, as $t \rightarrow \infty, L_\alpha(t) - t \rightarrow t_{1-\alpha}$ where $t_{1-\alpha}$ is the point such that $H(t_{1-\alpha}) = \alpha$. Since $t_{1-\alpha}$ is a fixed number, $L_\alpha(t) \rightarrow \infty$ as $t \rightarrow \infty$. Also, as $0 < \alpha \leq \frac{1}{2}, L_\alpha(t) - t < 0$ for all $t \geq t_{\frac{\alpha}{2}}$. This can be verified by noting that if $L_\alpha(t) \geq t$ for some $t \geq t_{\frac{\alpha}{2}}$, then, $\alpha = H(L_\alpha(t) - t) + H(-L_\alpha(t) - t) \geq H(0) + H(-L_\alpha(t) - t) > \frac{1}{2}$, which is a contradiction.

b) Since $L_\alpha(t)$ is strictly increasing in t for $t \geq t_{\frac{\alpha}{2}}$ and $L_\alpha(t) = 0$ for $0 \leq t < t_{\frac{\alpha}{2}}$, we may have a generalized inverse function of $L_\alpha(t)$ by letting $L_\alpha^{-1}(0) = t_{\frac{\alpha}{2}}$ and for $s > 0, L_\alpha^{-1}(s) = t$ if $L_\alpha(t) = s$. Note that $L_\alpha^{-1}(s)$ is strictly increasing in s .

In the following, we give a conservative $100(1 - \alpha)\%$ lower confidence bound for $\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta}$.

Theorem 2.1. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then, $P_{\underline{\theta}} \left\{ \frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} > L_\alpha \left(\frac{X_{[k]} - X_{[k-1]}}{\beta} \right) \right\} \geq 1 - \alpha$ for all $\underline{\theta}$.

Proof: By Lemma 2.1 and the definition of $L_\alpha(t)$, it follows that

$$\begin{aligned} & P_{\underline{\theta}} \left\{ \frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \leq L_\alpha \left(\frac{X_{[k]} - X_{[k-1]}}{\beta} \right) \right\} \\ &= P_{\underline{\theta}} \left\{ L_\alpha^{-1} \left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \right) \leq \frac{X_{[k]} - X_{[k-1]}}{\beta} \right\} \\ &\leq P_{\underline{\theta}_0} \left\{ L_\alpha^{-1} \left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \right) \leq \left| \frac{X_{(k)} - X_{(k-1)}}{\beta} \right| \right\} \\ &= H(L_\alpha(t_0) - t_0) + H(-L_\alpha(t_0) - t_0) \\ &= \alpha, \text{ by the definition of } L_\alpha(t), \end{aligned}$$

where $t_c = L_\alpha^{-1}(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta})$ and where θ^0 and $X_{(i)}$ are defined previously. Thus,

$$P_\theta \left\{ \frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} > L_\alpha \left(\frac{X_{[k]} - X_{[k-1]}}{\beta} \right) \right\} \geq 1 - \alpha \text{ for all } \theta. \quad \square$$

The following theorem is a direct consequence of Theorem 2.1.

Theorem 2.2. Let $\hat{P}_L = \int_0^\infty \left[F \left(y + L_\alpha \left(\frac{X_{[k]} - X_{[k-1]}}{\beta} \right) \right) \right]^{k-1} dF(y)$. Then, under the assumption that $\log f(y)$ is concave on $(0, \infty)$,

$$P_\theta \{ \text{PCS}(\theta) \geq \hat{P}_L \} \geq 1 - \alpha \text{ for all } \theta.$$

That is, $\hat{P}_L$ is an at least $100(1 - \alpha)\%$ lower confidence bound for the PCS (θ) .

2.2 The Scale Parameter β Unknown Case

When the scale parameter β is unknown, for each $i = 1, \dots, k$, let $T_i = T(X_{i1}, \dots, X_{in})$ be a nonnegative function of $X_{i1}, \dots, X_{in}$, which depends on $X_{i1}, \dots, X_{in}$ only through the difference $X_{ij} - X_i, j = 1, \dots, n$. That is, T is a location-invariant function. It is assumed that the function T is such that $T(cx_1, \dots, cx_n) = cT(x_1, \dots, x_n)$ for any positive value c . Also, let $S = S(T_1, \dots, T_k)$ be a nonnegative function of $T_1, \dots, T_k$ such that $S(ct_1, \dots, ct_k) = cS(t_1, \dots, t_k)$ for all $c > 0$. If for each $i = 1, \dots, k$, X_i is a complete sufficient statistic for the parameter θ_i , then T_i is independent of X_i since the distribution of T_i is independent of the parameter θ_i . Therefore, the distribution of S is independent of the parameters $\theta_1, \dots, \theta_k$, and S is independent of $(X_1, \dots, X_k)$. Also, by the preceding assumption, the distribution of $W = \frac{S}{\beta}$ is independent of the parameter β . Let $Q(w)$ denote the distribution of W .

For each fixed $0 < \alpha < 1$, let $t_{\frac{\alpha}{2}}^*$ be the point such that $\int_0^\infty H \left(-t_{\frac{\alpha}{2}}^* y \right) dQ(y) = \frac{\alpha}{2}$. Note that $t_{\frac{\alpha}{2}}^* > 0$. Define a nonnegative function $L_\alpha^*(t)$ on $[0, \infty)$ implicitly by

$$\int_0^\infty [H(L_\alpha^*(t) - ty) + H(-L_\alpha^*(t) - ty)] dQ(y) = \alpha \text{ for } t \geq t_{\frac{\alpha}{2}}^* \quad (2.4)$$

and $L_\alpha^*(t) = 0$ if $0 \leq t < t_{\frac{\alpha}{2}}^*$. Analogous to Lemmas 2.2 and 2.3, we have the following lemma.

Lemma 2.4. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then, $L_\alpha^*(t)$ always exists. Also, $L_\alpha^*(t)$ is strictly increasing in t for $t > t_{\frac{\alpha}{2}}^*$ and $L_\alpha^*(t_{\frac{\alpha}{2}}^*) = 0$.

Analogous to Remark 2.1.b, we let $L_\alpha^{*-1}(\cdot)$ be the generalized inverse function of $L_\alpha^*(\cdot)$. It should be noted that $L_\alpha^{*-1}(s)$ is strictly increasing in s for $s \geq 0$. Now, a conservative $100(1 - \alpha)\%$ lower confidence bound for $(\theta_{(k)} - \theta_{(k-1)})/\beta$ is given as follows.

Theorem 2.3. Assume that $\log f(y)$ is concave on $(0, \infty)$. Then,

$$P_{\underline{\theta}, \beta} \left\{ \frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} > L_\alpha^* \left(\frac{X_{[k]} - X_{[k-1]}}{S} \right) \right\} \geq 1 - \alpha \text{ for all } \underline{\theta} \text{ and } \beta.$$

Proof: By Lemma 2.1, it follows that

$$\begin{aligned} & P_{\underline{\theta}, \beta} \left\{ \frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \leq L_\alpha^* \left(\frac{X_{[k]} - X_{[k-1]}}{S} \right) \right\} \\ &= P_{\underline{\theta}, \beta} \left\{ L_\alpha^{*-1} \left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \right) \leq \frac{X_{[k]} - X_{[k-1]}}{S} \right\} \\ &= P_{\underline{\theta}, \beta} \left\{ L_\alpha^{*-1} \left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \right) \frac{S}{\beta} \leq \frac{X_{[k]} - X_{[k-1]}}{\beta} \right\} \\ &\leq P_{\underline{\theta}_0, \beta} \left\{ L_\alpha^{*-1} \left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \right) \frac{S}{\beta} \leq \left| \frac{X_{(k)} - X_{(k-1)}}{\beta} \right| \right\} \\ &= \int_0^\infty [H(L_\alpha^*(t_0) - t_0 y) + H(-L_\alpha^*(t_0) - t_0 y)] dQ(y) \\ &= \alpha \end{aligned}$$

where $t_0 = L_\alpha^{*-1} \left(\frac{\theta_{(k)} - \theta_{(k-1)}}{\beta} \right)$ and the last equality is obtained due to the definition of $L_\alpha^*(t)$. Thus, the proof of this theorem is complete. $\square$

Theorem 2.4. Let $\hat{P}_L^* = \int_0^\infty \left[F \left(y + L_\alpha^* \left(\frac{X_{[k]} - X_{[k-1]}}{S} \right) \right) \right]^{k-1} dF(y)$. Assume that $\log f(y)$ is concave in $(0, \infty)$. Then

$$P_{\underline{\theta}, \beta} \left\{ \text{PCS}(\underline{\theta}) \geq \hat{P}_L^* \right\} \geq 1 - \alpha \text{ for all } \underline{\theta} \text{ and } \beta.$$

3. Selecting the Best Exponential Population

Let $X_{ij}, j = 1, \dots, n$, be a sample of size n from a two-parameter exponential distribution with density function $g(x|\theta_i, \beta) = \beta^{-1} e^{-(x-\theta_i)/\beta} I_{(\theta_i, \infty)}(x), i = 1, \dots, k$, where the common scale parameter β may be either known or unknown. The best population is the one associated with the largest truncated location parameter $\theta_{(k)}$. For each $i = 1, \dots, k$, let $X_i = \min(X_{i1}, \dots, X_{in})$. Based on $X_1, \dots, X_k$, the natural selection rule selects the population yielding the largest sampled value $X_{[k]}$ as the best population. The corresponding PCS is:

$$\begin{aligned} \text{PCS}(\theta) &= \int_{y=0}^{\infty} \prod_{i=1}^{k-1} \left[1 - e^{-\left(y + \frac{n(\theta_{(k)} - \theta_{(i)})}{\beta}\right)} \right] e^{-y} dy \\ &\geq \int_{y=0}^{\infty} \left[1 - e^{-\left(y + \frac{n(\theta_{(k)} - \theta_{(k-1)})}{\beta}\right)} \right]^{k-1} e^{-y} dy. \end{aligned} \quad (3.1)$$

In order to find out a lower confidence bound for the PCS, we need to obtain a lower confidence bound for $\frac{n(\theta_{(k)} - \theta_{(k-1)})}{\beta}$. We consider two situations according to whether the common scale parameter β is known or unknown.

3.1 Lower Confidence Bound for PCS: β Known Case

Let $\mu_i = \frac{n\theta_i}{\beta}$ and $Y_i = \frac{nX_i}{\beta}$. Then $Y_i - \mu_i$ has an exponential distribution with density $f(y) = e^{-y} I_{(0, \infty)}(y)$. Let $H(t)$ be the distribution of $(Y_1 - \mu_1) - (Y_2 - \mu_2)$. Then,

$$H(t) = \begin{cases} 1 - \frac{1}{2}e^{-t} & \text{if } t \geq 0, \\ \frac{1}{2}e^t & \text{if } t < 0. \end{cases}$$

For fixed $\alpha \in (0, 1)$, let $t_{\frac{\alpha}{2}}$ denote the upper $\frac{\alpha}{2}$ -quantile of $H(t)$. Then, $t_{\frac{\alpha}{2}} = -\ln \alpha$. Define function $L_{\alpha}(t)$ on $(0, \infty)$ such that $H(L_{\alpha}(t) - t) + H(-L_{\alpha}(t) - t) = \alpha$ for $t \geq t_{\frac{\alpha}{2}}$, and $L_{\alpha}(t) = 0$ if $0 \leq t < t_{\frac{\alpha}{2}}$. Since higher confidence statement is always desirable, we

need to consider only $\alpha \in (0, \frac{1}{2})$. By Remark 2.1.b, $L_\alpha(t) - t < 0$ for all $t > 0$. Thus, straightforward computation gives that $L_\alpha(t) = \ell n [\alpha e^t + \sqrt{\alpha^2 e^{2t} - 1}]$. From Theorem 2.1, letting $\tilde{\Delta} = \frac{n(X_{[k]} - X_{[k-1]})}{\beta}$, we have

$$P_{\underline{\theta}} \left\{ \frac{n(\theta_{(k)} - \theta_{(k-1)})}{\beta} > L_\alpha(\tilde{\Delta}) \right\} \geq 1 - \alpha \text{ for all } \underline{\theta}.$$

Letting $\hat{P}_L = \int_{y=0}^{\infty} [1 - \exp(-y - L_\alpha(\tilde{\Delta}))]^{k-1} e^{-y} dy$, we then have:

$$P_{\underline{\theta}} \{ \text{PCS}(\underline{\theta}) \geq \hat{P}_L \} \geq 1 - \alpha \text{ for all } \underline{\theta}.$$

3.2 Lower Confidence Bound for PCS: β Unknown Case

When the common scale parameter β is unknown, let $S = \sum_{i=1}^k \sum_{j=1}^n \frac{(X_{ij} - X_i)}{k(n-1)}$. Then S is independent of $X_1, \dots, X_k$ and $\frac{k(n-1)S}{\beta}$ has a gamma distribution with shape parameter $m = k(n-1)$ and scale parameter 1. Let $Q_m(y)$ denote the distribution of $\frac{S}{\beta}$. For $0 < \alpha < 1$, let $t_{\frac{\alpha}{2}}^*$ be the point such that $\int_0^\infty H(-t_{\frac{\alpha}{2}}^* y) dQ_m(y) = \frac{\alpha}{2}$. Straightforward computation yields $t_{\frac{\alpha}{2}}^* = m \left(\alpha^{-\frac{1}{m}} - 1 \right)$. The function $L_\alpha^*(t)$ is then implicitly defined by

$$\int_0^\infty [H(L_\alpha^*(t) - yt) + H(-L_\alpha^*(t) - yt)] dQ_m(y) = \alpha \text{ for } t \geq t_{\frac{\alpha}{2}}^*$$

and $L_\alpha^*(t) = 0$ for $0 \leq t < t_{\frac{\alpha}{2}}^*$. Thus, for $t > t_{\frac{\alpha}{2}}^*$, $L_\alpha^*(t)$ is such that

$$\begin{aligned} & \int_0^{L_\alpha^*(t)/t} \left[1 - \frac{1}{2} e^{-L_\alpha^*(t) + yt} \right] dQ_m(y) \\ & + \int_{L_\alpha^*(t)/t}^\infty \frac{1}{2} e^{L_\alpha^*(t) - yt} dQ_m(y) + \int_0^\infty \frac{1}{2} e^{-L_\alpha^*(t) - yt} dQ_m(y) = \alpha. \end{aligned}$$

Also, from Theorem 2.3,

$$P_{\underline{\theta}, \beta} \left\{ \frac{n(\theta_{(k)} - \theta_{(k-1)})}{\beta} > L_\alpha^* \left(\frac{n(X_{[k]} - X_{[k-1]})}{S} \right) \right\} \geq 1 - \alpha \text{ for all } \underline{\theta} \text{ and } \beta.$$

Letting $\Delta^* = \frac{n(X_{[k]} - X_{[k-1]})}{S}$ and $\hat{P}_L^* = \int_0^\infty [1 - \exp(-y - L_\alpha^*(\Delta^*))]^{k-1} e^{-y} dy$, we then have

$$P_{\theta, \beta} \{ \text{PCS}(\theta) \geq \hat{P}_L^* \} \geq 1 - \alpha \text{ for all } \theta \text{ and } \beta.$$

The values of the function $L_\alpha^*(t)$ are given in Tables 1 and 2 for $\alpha = 0.05, 0.10$ and for selected values of m and $t \geq t_{\frac{\alpha}{2}}^*$. Note that when m is sufficiently large, $t_{\frac{\alpha}{2}}^* \approx -\ln \alpha$ and $L_\alpha^*(t) \approx \ln [\alpha e^t + \sqrt{\alpha^2 e^{2t} - 1}]$ for $t > t_{\frac{\alpha}{2}}^*$.

4. An Illustrative Example

We use the insulating fluid example (taken from Table 4.1, page 462 of Nelson (1982)) to illustrate the way to implement the proposed procedure. There are six groups of insulating fluid. The purpose is to identify which group of insulating fluid has the largest guaranteed life-time when subjected to high voltage stress. Ten items from each group are put in a life-test experiment which is subject to high voltage stress. It is assumed that the distribution of the life-time for each insulating fluid is a two-parameter exponential with common unknown scale parameter β . The times to breakdown in minutes is shown in the following.

Table 3. Times to Breakdown

Group	1	2	3	4	5	6
	1.89	1.30	1.99	1.17	8.11	2.12
	4.03	2.75	0.64	3.87	3.17	3.97
	1.54	0.00	2.15	2.80	5.55	1.56
	0.31	2.17	1.08	0.70	0.80	1.49
	0.66	0.66	2.57	3.82	0.20	8.71
	1.70	0.55	0.93	0.02	1.13	2.10
	2.17	0.18	4.75	0.50	6.63	7.21
	1.82	10.60	0.82	3.72	1.08	3.83
	9.99	1.63	2.06	0.06	2.44	1.34
	2.24	0.71	0.49	3.57	0.78	5.13
X_i	0.31	0.00	0.49	0.02	0.20	1.34

From Table 3, we obtain: $X_1 = 0.31, X_2 = 0.00, X_3 = 0.49, X_4 = 0.02, X_5 = 0.20$ and

$X_6 = 1.34$. According to the natural selection rule, we select Group 6 as the best group.

Then, a reasonable question is: what kind of confidence statement can be made regarding

the PCS? For this purpose, based on the above given data, we have $\frac{n(X_{[6]} - X_{[5]})}{s} = 3.8455$.

For different α values, the $100(1-\alpha)\%$ lower confidence bounds $\hat{P}_L^*$ of the PCS are computed

and given as follows:

α	0.05	0.10	0.15	0.20	0.25
$\hat{P}_L^*$	0.5356	0.7373	0.8166	0.8591	0.8856

Thus, we can claim, for example, that with at least 85 percent confidence, $PCS \geq \hat{P}_L^* =$

0.8166.

References

- Anderson, P.O., Bishop, T.A. and Dudewicz, E.J. (1977). Indifference zone ranking and selection: confidence intervals for true achieved $P(CD)$. *Commun. Statist.*, A(6), 1121-1132.
- Bechhofer, R.E. (1954). A single-sample multiple-decision procedure for ranking means of normal populations with known variances. *Ann. Math. Statist.*, 25, 16-39.
- Bofinger, E. (1985). On the non-existence of consistent estimators for $P(CS)$. *Amer. J. Math. Management Sci.*, 5, 63-76.
- Fabian, V. (1962). On multiple decision methods for ranking population means. *Ann. Math. Statist.*, 33, 248-254.
- Faltin, F. and McCulloch, C.E. (1983). On the small-sample properties of the Olkin-Sobel-Tong estimators of the probability of a correct selection. *J. Amer. Statist. Assoc.*, 78, 464-467.
- Gupta, S.S. and Liang, T. (1987). On a lower confidence bound for the probability of a correct selection: analytical and simulation studies. To appear in the Proceedings of the First International Conference in Statistical Computing.
- Gutmann, S. and Maymin, Z. (1987). Is the selected population the best? *Ann. Statist.*, 15, 456-461.
- Hsu, J.C. (1981). Simultaneous confidence intervals for all distances from the "best". *Ann. Statist.*, 9, 1026-1034.
- Kim, W.-C. (1986). A lower confidence bound on the probability of a correct selection. *J. Amer. Statist. Assoc.*, 81, 1012-1017.
- Nelson, W. (1982). *Applied Life Data Analysis*, Wiley, New York.
- Olkin, I., Sobel, M. and Tong, Y.L. (1976). Estimating the true probability of a correct selection for location and scale parameter families. Technical Report 110, Stanford University, Department of Statistics.
- Olkin, I., Sobel, M. and Tong, Y.L. (1982). Bounds for a k -fold integral for location and scale parameter models with application to statistical ranking and selection problem. *Statistical Decision Theory and Related Topics III*, Vol. 2 (Eds. S.S. Gupta and J.O. Berger), New York, Academic Press, pp. 193-212.
- Parnes, M. and Srinivasan, R. (1986). Some inconsistencies and errors in the indifference zone formulation of selection. *Sankhyā*, 48, 86-97.

Table 1. Values of $L_{\alpha}^*(t)$ for $\alpha = 0.05$

m	$t_{\frac{\alpha}{2}}^*(m)$	$t - t_{\frac{\alpha}{2}}^*(m)$															
		0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.550	0.600	0.650	0.700		
5	4.103	0.235	0.334	0.410	0.475	0.532	0.585	0.634	0.680	0.724	0.765	0.805	0.843	0.881	0.917		
6	3.885	0.247	0.351	0.432	0.500	0.561	0.617	0.669	0.718	0.764	0.808	0.851	0.892	0.932	0.970		
7	3.739	0.256	0.364	0.448	0.519	0.583	0.641	0.695	0.747	0.795	0.841	0.886	0.929	0.971	1.011		
8	3.634	0.263	0.374	0.461	0.534	0.600	0.660	0.716	0.769	0.819	0.867	0.914	0.958	1.002	1.044		
9	3.555	0.269	0.382	0.471	0.546	0.614	0.675	0.733	0.787	0.839	0.888	0.936	0.982	1.027	1.071		
10	3.493	0.274	0.389	0.479	0.556	0.625	0.688	0.747	0.802	0.855	0.906	0.954	1.002	1.048	1.092		
11	3.443	0.277	0.395	0.486	0.564	0.634	0.698	0.758	0.815	0.868	0.920	0.970	1.018	1.065	1.111		
12	3.403	0.281	0.399	0.492	0.571	0.642	0.707	0.768	0.825	0.880	0.932	0.983	1.032	1.080	1.127		
13	3.369	0.283	0.403	0.497	0.577	0.649	0.714	0.776	0.834	0.890	0.943	0.995	1.044	1.093	1.140		
14	3.340	0.286	0.407	0.501	0.582	0.654	0.721	0.783	0.842	0.898	0.952	1.004	1.055	1.104	1.152		
15	3.316	0.288	0.410	0.505	0.586	0.660	0.727	0.790	0.849	0.906	0.960	1.013	1.064	1.114	1.162		
16	3.295	0.290	0.412	0.508	0.590	0.664	0.732	0.795	0.855	0.913	0.968	1.021	1.072	1.123	1.172		
17	3.276	0.291	0.415	0.511	0.594	0.668	0.736	0.800	0.861	0.919	0.974	1.028	1.080	1.130	1.180		
18	3.259	0.293	0.417	0.514	0.597	0.672	0.740	0.805	0.866	0.924	0.980	1.034	1.086	1.137	1.187		
19	3.245	0.294	0.419	0.516	0.600	0.675	0.744	0.809	0.870	0.929	0.985	1.039	1.092	1.144	1.194		
20	3.232	0.295	0.420	0.518	0.602	0.678	0.747	0.812	0.874	0.933	0.990	1.044	1.098	1.149	1.200		
21	3.220	0.296	0.422	0.520	0.605	0.681	0.750	0.816	0.878	0.937	0.994	1.049	1.102	1.155	1.206		
22	3.209	0.297	0.423	0.522	0.607	0.683	0.753	0.819	0.881	0.941	0.998	1.053	1.107	1.159	1.211		
23	3.200	0.298	0.425	0.524	0.609	0.685	0.756	0.822	0.884	0.944	1.001	1.057	1.111	1.164	1.215		
24	3.191	0.299	0.426	0.525	0.611	0.687	0.758	0.824	0.887	0.947	1.005	1.061	1.115	1.168	1.220		
25	3.183	0.300	0.427	0.527	0.612	0.689	0.760	0.827	0.889	0.950	1.008	1.064	1.118	1.171	1.223		
30	3.150	0.303	0.432	0.532	0.619	0.697	0.769	0.836	0.900	0.961	1.020	1.077	1.132	1.187	1.240		
40	3.111	0.307	0.437	0.539	0.627	0.707	0.780	0.848	0.913	0.976	1.036	1.094	1.151	1.206	1.260		
50	3.087	0.309	0.441	0.544	0.633	0.713	0.786	0.856	0.921	0.985	1.045	1.104	1.162	1.218	1.273		
60	3.072	0.311	0.443	0.547	0.636	0.717	0.791	0.861	0.927	0.990	1.052	1.111	1.169	1.226	1.281		
120	3.033	0.315	0.449	0.554	0.645	0.727	0.802	0.873	0.941	1.006	1.068	1.129	1.188	1.246	1.303		
∞	2.996	0.319	0.455	0.562	0.654	0.737	0.814	0.886	0.955	1.021	1.085	1.147	1.208	1.267	1.325		

Table 1. Values of $L_{\alpha}^*(t)$ for $\alpha = 0.05$ (cont.)

m	$t_{\frac{\alpha}{2}}^*(m)$	$t - t_{\frac{\alpha}{2}}^*(m)$																
		0.750	0.800	0.850	0.900	0.950	1.000	1.050	1.100	1.150	1.200	1.250	1.300	1.350	1.400			
5	4.103	0.952	0.986	1.020	1.053	1.085	1.117	1.148	1.178	1.208	1.238	1.268	1.297	1.325	1.354			
6	3.885	1.008	1.045	1.081	1.116	1.150	1.184	1.218	1.251	1.283	1.316	1.347	1.379	1.410	1.440			
7	3.739	1.051	1.090	1.128	1.165	1.201	1.237	1.273	1.307	1.342	1.376	1.409	1.443	1.475	1.508			
8	3.634	1.085	1.126	1.165	1.204	1.242	1.279	1.316	1.353	1.389	1.424	1.459	1.494	1.529	1.563			
9	3.555	1.113	1.155	1.196	1.236	1.275	1.314	1.352	1.390	1.427	1.464	1.500	1.537	1.572	1.608			
10	3.493	1.136	1.179	1.221	1.262	1.303	1.343	1.382	1.421	1.459	1.497	1.535	1.572	1.609	1.646			
11	3.443	1.156	1.199	1.242	1.284	1.326	1.367	1.407	1.447	1.487	1.526	1.564	1.602	1.640	1.678			
12	3.403	1.172	1.217	1.261	1.304	1.346	1.388	1.429	1.470	1.510	1.550	1.589	1.628	1.667	1.706			
13	3.369	1.186	1.232	1.276	1.320	1.363	1.406	1.448	1.489	1.530	1.571	1.611	1.651	1.690	1.730			
14	3.340	1.199	1.245	1.290	1.335	1.378	1.422	1.464	1.506	1.548	1.589	1.630	1.671	1.711	1.751			
15	3.316	1.210	1.257	1.302	1.347	1.392	1.435	1.479	1.522	1.564	1.606	1.647	1.688	1.729	1.770			
16	3.295	1.220	1.267	1.313	1.359	1.404	1.448	1.492	1.535	1.578	1.620	1.662	1.704	1.745	1.786			
17	3.276	1.228	1.276	1.323	1.369	1.414	1.459	1.503	1.547	1.590	1.633	1.676	1.718	1.760	1.802			
18	3.259	1.236	1.284	1.331	1.378	1.424	1.469	1.514	1.558	1.602	1.645	1.688	1.731	1.773	1.815			
19	3.245	1.243	1.292	1.339	1.386	1.432	1.478	1.523	1.568	1.612	1.656	1.699	1.742	1.785	1.827			
20	3.232	1.250	1.298	1.346	1.394	1.440	1.486	1.532	1.577	1.621	1.665	1.709	1.753	1.796	1.839			
21	3.220	1.256	1.305	1.353	1.400	1.447	1.494	1.539	1.585	1.630	1.674	1.718	1.762	1.806	1.849			
22	3.209	1.261	1.310	1.359	1.407	1.454	1.500	1.547	1.592	1.637	1.682	1.727	1.771	1.815	1.858			
23	3.200	1.266	1.315	1.364	1.412	1.460	1.507	1.553	1.599	1.645	1.690	1.734	1.779	1.823	1.867			
24	3.191	1.270	1.320	1.369	1.418	1.465	1.513	1.559	1.605	1.651	1.697	1.742	1.786	1.831	1.875			
25	3.183	1.274	1.325	1.374	1.423	1.471	1.518	1.565	1.611	1.657	1.703	1.748	1.793	1.838	1.882			
30	3.150	1.291	1.343	1.393	1.442	1.491	1.540	1.588	1.635	1.682	1.729	1.775	1.821	1.867	1.912			
40	3.111	1.313	1.365	1.417	1.468	1.518	1.568	1.617	1.666	1.714	1.762	1.810	1.857	1.905	1.951			
50	3.087	1.327	1.380	1.432	1.484	1.535	1.585	1.635	1.685	1.734	1.783	1.831	1.880	1.928	1.976			
60	3.072	1.336	1.389	1.442	1.494	1.546	1.597	1.647	1.698	1.747	1.797	1.846	1.895	1.944	1.992			
120	3.033	1.358	1.413	1.468	1.521	1.574	1.627	1.679	1.730	1.782	1.833	1.883	1.934	1.984	2.034			
∞	2.996	1.382	1.438	1.494	1.549	1.603	1.657	1.711	1.764	1.817	1.870	1.922	1.974	2.026	2.078			

Table 1. Values of $L_{\alpha}^*(t)$ for $\alpha = 0.05$ (cont.)

m	$t_{\frac{1}{2}}^*(m)$	$t - t_{\frac{1}{2}}^*(m)$															
		1.450	1.500	1.550	1.600	1.650	1.700	1.750	1.800	1.850	1.900	1.950	2.000				
5	4.103	1.382	1.410	1.437	1.465	1.492	1.519	1.545	1.572	1.598	1.624	1.650	1.676				
6	3.885	1.471	1.501	1.531	1.560	1.590	1.619	1.648	1.677	1.705	1.734	1.762	1.790				
7	3.739	1.540	1.572	1.604	1.635	1.667	1.698	1.729	1.759	1.790	1.820	1.850	1.880				
8	3.634	1.596	1.630	1.663	1.696	1.729	1.762	1.794	1.826	1.858	1.890	1.922	1.953				
9	3.555	1.643	1.678	1.712	1.747	1.781	1.815	1.848	1.882	1.915	1.948	1.981	2.014				
10	3.493	1.682	1.718	1.754	1.789	1.824	1.859	1.894	1.929	1.963	1.998	2.032	2.066				
11	3.443	1.715	1.752	1.789	1.825	1.862	1.898	1.933	1.969	2.005	2.040	2.075	2.110				
12	3.403	1.744	1.782	1.819	1.857	1.894	1.931	1.967	2.004	2.040	2.076	2.113	2.148				
13	3.369	1.769	1.807	1.846	1.884	1.922	1.959	1.997	2.034	2.072	2.109	2.145	2.182				
14	3.340	1.791	1.830	1.869	1.908	1.946	1.985	2.023	2.061	2.099	2.137	2.174	2.212				
15	3.316	1.810	1.850	1.890	1.929	1.968	2.007	2.046	2.085	2.124	2.162	2.200	2.238				
16	3.295	1.827	1.868	1.908	1.948	1.988	2.028	2.067	2.106	2.145	2.184	2.223	2.262				
17	3.276	1.843	1.884	1.925	1.965	2.006	2.046	2.086	2.126	2.165	2.205	2.244	2.283				
18	3.259	1.857	1.898	1.940	1.981	2.022	2.062	2.103	2.143	2.183	2.223	2.263	2.302				
19	3.245	1.870	1.912	1.953	1.995	2.036	2.077	2.118	2.159	2.199	2.240	2.280	2.320				
20	3.232	1.881	1.923	1.966	2.007	2.049	2.091	2.132	2.173	2.214	2.255	2.296	2.336				
21	3.220	1.892	1.934	1.977	2.019	2.061	2.103	2.145	2.186	2.228	2.269	2.310	2.351				
22	3.209	1.901	1.945	1.987	2.030	2.072	2.115	2.157	2.198	2.240	2.282	2.323	2.364				
23	3.200	1.910	1.954	1.997	2.040	2.083	2.125	2.167	2.210	2.252	2.293	2.335	2.377				
24	3.191	1.919	1.962	2.006	2.049	2.092	2.135	2.177	2.220	2.262	2.304	2.346	2.388				
25	3.183	1.926	1.970	2.014	2.057	2.101	2.144	2.187	2.229	2.272	2.314	2.357	2.399				
30	3.150	1.958	2.003	2.048	2.092	2.137	2.181	2.225	2.269	2.313	2.356	2.400	2.443				
40	3.111	1.998	2.045	2.091	2.137	2.183	2.229	2.274	2.320	2.365	2.410	2.455	2.500				
50	3.087	2.023	2.071	2.118	2.165	2.212	2.258	2.305	2.351	2.398	2.444	2.490	2.536				
60	3.072	2.040	2.088	2.136	2.184	2.231	2.279	2.326	2.373	2.420	2.467	2.514	2.560				
120	3.033	2.084	2.133	2.183	2.232	2.281	2.330	2.379	2.428	2.477	2.526	2.574	2.623				
∞	2.996	2.129	2.180	2.232	2.283	2.334	2.385	2.436	2.486	2.537	2.588	2.638	2.689				

Table 2. Values of $L_{\alpha}^*(t)$ for $\alpha = 0.10$

m	$t_{\frac{\alpha}{2}}^*(m)$	$t - t_{\frac{\alpha}{2}}^*(m)$															
		0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.550	0.600	0.650	0.700		
5	2.924	0.252	0.358	0.440	0.510	0.572	0.629	0.682	0.732	0.780	0.825	0.869	0.911	0.952	0.992		
6	2.807	0.262	0.372	0.458	0.531	0.596	0.656	0.711	0.764	0.813	0.861	0.907	0.951	0.994	1.036		
7	2.726	0.270	0.383	0.471	0.547	0.614	0.676	0.733	0.787	0.839	0.888	0.936	0.982	1.027	1.070		
8	2.668	0.275	0.391	0.482	0.559	0.628	0.691	0.750	0.806	0.859	0.910	0.959	1.006	1.052	1.097		
9	2.624	0.280	0.398	0.490	0.568	0.639	0.703	0.764	0.821	0.875	0.927	0.977	1.026	1.073	1.119		
10	2.589	0.283	0.403	0.496	0.576	0.648	0.714	0.775	0.833	0.888	0.941	0.992	1.042	1.090	1.137		
11	2.561	0.286	0.407	0.502	0.583	0.655	0.722	0.784	0.843	0.899	0.953	1.005	1.055	1.104	1.152		
12	2.538	0.289	0.411	0.507	0.588	0.662	0.729	0.792	0.852	0.908	0.963	1.016	1.067	1.116	1.165		
13	2.519	0.291	0.414	0.511	0.593	0.667	0.735	0.799	0.859	0.916	0.972	1.025	1.076	1.127	1.176		
14	2.503	0.293	0.417	0.514	0.597	0.672	0.740	0.805	0.865	0.923	0.979	1.033	1.085	1.136	1.186		
15	2.489	0.295	0.419	0.517	0.601	0.676	0.745	0.810	0.871	0.929	0.986	1.040	1.093	1.144	1.194		
16	2.477	0.296	0.422	0.520	0.604	0.680	0.749	0.814	0.876	0.935	0.991	1.046	1.099	1.151	1.202		
17	2.466	0.297	0.423	0.522	0.607	0.683	0.753	0.818	0.880	0.940	0.997	1.052	1.105	1.157	1.208		
18	2.456	0.299	0.425	0.524	0.609	0.686	0.756	0.822	0.884	0.944	1.001	1.057	1.111	1.163	1.214		
19	2.448	0.300	0.427	0.526	0.611	0.688	0.759	0.825	0.888	0.948	1.005	1.061	1.115	1.168	1.220		
20	2.440	0.301	0.428	0.528	0.614	0.691	0.761	0.828	0.891	0.951	1.009	1.065	1.120	1.173	1.225		
21	2.434	0.301	0.429	0.529	0.615	0.693	0.764	0.831	0.894	0.954	1.013	1.069	1.124	1.177	1.229		
22	2.427	0.302	0.430	0.531	0.617	0.695	0.766	0.833	0.897	0.957	1.016	1.072	1.127	1.181	1.233		
23	2.422	0.303	0.431	0.532	0.619	0.696	0.768	0.835	0.899	0.960	1.019	1.075	1.131	1.184	1.237		
24	2.417	0.304	0.432	0.533	0.620	0.698	0.770	0.837	0.901	0.962	1.021	1.078	1.134	1.188	1.240		
25	2.412	0.304	0.433	0.534	0.621	0.699	0.772	0.839	0.903	0.965	1.024	1.081	1.136	1.191	1.244		
30	2.393	0.307	0.437	0.539	0.627	0.706	0.778	0.847	0.912	0.974	1.033	1.091	1.148	1.203	1.257		
40	2.370	0.310	0.441	0.544	0.633	0.713	0.787	0.856	0.922	0.985	1.046	1.105	1.162	1.218	1.273		
50	2.356	0.311	0.444	0.548	0.637	0.718	0.792	0.862	0.929	0.992	1.054	1.113	1.171	1.228	1.283		
60	2.347	0.313	0.446	0.550	0.640	0.721	0.796	0.866	0.933	0.997	1.059	1.119	1.177	1.234	1.290		
120	2.325	0.316	0.450	0.556	0.647	0.729	0.805	0.876	0.944	1.009	1.072	1.133	1.192	1.250	1.307		
∞	2.303	0.319	0.455	0.562	0.654	0.737	0.814	0.886	0.955	1.021	1.085	1.147	1.208	1.267	1.325		

Table 2. Values of $L_{\alpha}^*(t)$ for $\alpha = 0.10$ (cont.)

m	$t_{\frac{1}{2}}^*(m)$	$t - t_{\frac{1}{2}}^*(m)$															
		0.750	0.800	0.850	0.900	0.950	1.000	1.050	1.100	1.150	1.200	1.250	1.300	1.350	1.400		
5	2.924	1.030	1.068	1.105	1.142	1.178	1.213	1.248	1.282	1.316	1.349	1.382	1.415	1.447	1.479		
6	2.807	1.077	1.117	1.156	1.195	1.232	1.270	1.306	1.343	1.378	1.414	1.449	1.484	1.518	1.552		
7	2.726	1.113	1.154	1.195	1.235	1.275	1.313	1.352	1.390	1.427	1.464	1.501	1.537	1.573	1.608		
8	2.668	1.141	1.184	1.226	1.267	1.308	1.348	1.388	1.427	1.466	1.504	1.542	1.579	1.617	1.654		
9	2.624	1.164	1.208	1.251	1.294	1.335	1.377	1.417	1.458	1.497	1.537	1.576	1.614	1.653	1.691		
10	2.589	1.183	1.228	1.272	1.315	1.358	1.400	1.442	1.483	1.524	1.564	1.604	1.644	1.683	1.722		
11	2.561	1.199	1.244	1.289	1.334	1.377	1.420	1.463	1.505	1.546	1.588	1.628	1.669	1.709	1.749		
12	2.538	1.212	1.259	1.304	1.349	1.394	1.438	1.481	1.523	1.566	1.608	1.649	1.690	1.731	1.771		
13	2.519	1.224	1.271	1.317	1.363	1.408	1.452	1.496	1.540	1.582	1.625	1.667	1.709	1.750	1.791		
14	2.503	1.234	1.282	1.329	1.375	1.420	1.465	1.510	1.554	1.597	1.640	1.683	1.725	1.767	1.809		
15	2.489	1.243	1.291	1.339	1.385	1.431	1.477	1.522	1.566	1.610	1.654	1.697	1.740	1.782	1.824		
16	2.477	1.251	1.300	1.348	1.395	1.441	1.487	1.532	1.577	1.622	1.666	1.709	1.752	1.795	1.838		
17	2.466	1.258	1.307	1.355	1.403	1.450	1.496	1.542	1.587	1.632	1.676	1.720	1.764	1.807	1.851		
18	2.456	1.265	1.314	1.363	1.410	1.458	1.504	1.550	1.596	1.641	1.686	1.730	1.774	1.818	1.862		
19	2.448	1.270	1.320	1.369	1.417	1.465	1.512	1.558	1.604	1.650	1.695	1.739	1.784	1.828	1.872		
20	2.440	1.276	1.326	1.375	1.423	1.471	1.518	1.565	1.611	1.657	1.703	1.748	1.792	1.837	1.881		
21	2.434	1.280	1.331	1.380	1.429	1.477	1.524	1.571	1.618	1.664	1.710	1.755	1.800	1.845	1.890		
22	2.427	1.285	1.335	1.385	1.434	1.482	1.530	1.577	1.624	1.671	1.717	1.762	1.808	1.853	1.897		
23	2.422	1.289	1.339	1.389	1.439	1.487	1.535	1.583	1.630	1.676	1.723	1.769	1.814	1.859	1.904		
24	2.417	1.292	1.343	1.393	1.443	1.492	1.540	1.588	1.635	1.682	1.728	1.775	1.820	1.866	1.911		
25	2.412	1.296	1.347	1.397	1.447	1.496	1.544	1.592	1.640	1.687	1.734	1.780	1.826	1.872	1.917		
30	2.393	1.309	1.361	1.413	1.463	1.513	1.562	1.611	1.659	1.707	1.755	1.802	1.849	1.896	1.942		
40	2.370	1.327	1.380	1.432	1.484	1.535	1.585	1.635	1.685	1.734	1.782	1.831	1.879	1.927	1.974		
50	2.356	1.338	1.391	1.444	1.496	1.548	1.599	1.650	1.700	1.750	1.799	1.848	1.897	1.946	1.994		
60	2.347	1.345	1.399	1.452	1.505	1.557	1.609	1.660	1.710	1.761	1.811	1.860	1.910	1.959	2.008		
120	2.325	1.363	1.418	1.473	1.527	1.580	1.633	1.685	1.737	1.788	1.840	1.891	1.941	1.992	2.042		
∞	2.303	1.382	1.438	1.494	1.549	1.603	1.657	1.711	1.764	1.817	1.870	1.922	1.974	2.026	2.078		

Table 2. Values of $L_{\alpha}^*(t)$ for $\alpha = 0.10$ (cont.)

m	$t_{\frac{\alpha}{2}}^*(m)$	$t - t_{\frac{\alpha}{2}}^*(m)$															
		1.450	1.500	1.550	1.600	1.650	1.700	1.750	1.800	1.850	1.900	1.950	2.000				
5	2.924	1.511	1.543	1.574	1.605	1.636	1.666	1.697	1.727	1.757	1.787	1.817	1.846				
6	2.807	1.586	1.619	1.652	1.685	1.718	1.751	1.783	1.815	1.847	1.879	1.911	1.943				
7	2.726	1.644	1.679	1.714	1.748	1.783	1.817	1.851	1.884	1.918	1.952	1.985	2.018				
8	2.668	1.690	1.727	1.763	1.799	1.834	1.870	1.905	1.940	1.975	2.010	2.045	2.079				
9	2.624	1.729	1.766	1.803	1.840	1.877	1.914	1.950	1.986	2.022	2.058	2.094	2.130				
10	2.589	1.761	1.799	1.837	1.875	1.913	1.951	1.988	2.025	2.062	2.099	2.136	2.172				
11	2.561	1.788	1.827	1.866	1.905	1.944	1.982	2.020	2.058	2.096	2.134	2.171	2.209				
12	2.538	1.812	1.852	1.891	1.931	1.970	2.009	2.048	2.087	2.126	2.164	2.202	2.240				
13	2.519	1.832	1.873	1.913	1.953	1.993	2.033	2.073	2.112	2.151	2.190	2.229	2.268				
14	2.503	1.850	1.891	1.932	1.973	2.014	2.054	2.094	2.134	2.174	2.214	2.253	2.293				
15	2.489	1.866	1.908	1.949	1.991	2.032	2.073	2.113	2.154	2.194	2.234	2.274	2.314				
16	2.477	1.881	1.923	1.965	2.006	2.048	2.089	2.130	2.171	2.212	2.253	2.293	2.334				
17	2.466	1.893	1.936	1.978	2.021	2.062	2.104	2.146	2.187	2.229	2.270	2.311	2.351				
18	2.456	1.905	1.948	1.991	2.033	2.076	2.118	2.160	2.202	2.243	2.285	2.326	2.367				
19	2.448	1.915	1.959	2.002	2.045	2.088	2.130	2.172	2.215	2.257	2.299	2.340	2.382				
20	2.440	1.925	1.969	2.012	2.055	2.098	2.141	2.184	2.227	2.269	2.311	2.353	2.395				
21	2.434	1.934	1.978	2.022	2.065	2.108	2.152	2.195	2.237	2.280	2.323	2.365	2.407				
22	2.427	1.942	1.986	2.030	2.074	2.118	2.161	2.204	2.247	2.290	2.333	2.376	2.419				
23	2.422	1.949	1.994	2.038	2.082	2.126	2.170	2.213	2.257	2.300	2.343	2.386	2.429				
24	2.417	1.956	2.001	2.045	2.090	2.134	2.178	2.222	2.265	2.309	2.352	2.395	2.439				
25	2.412	1.962	2.007	2.052	2.097	2.141	2.185	2.229	2.273	2.317	2.361	2.404	2.448				
30	2.393	1.988	2.034	2.080	2.125	2.171	2.216	2.261	2.306	2.350	2.395	2.439	2.484				
40	2.370	2.022	2.069	2.116	2.163	2.209	2.256	2.302	2.348	2.394	2.440	2.486	2.532				
50	2.356	2.042	2.090	2.138	2.186	2.233	2.280	2.328	2.375	2.422	2.468	2.515	2.562				
60	2.347	2.056	2.105	2.153	2.201	2.249	2.297	2.345	2.393	2.440	2.487	2.535	2.582				
120	2.325	2.092	2.142	2.192	2.241	2.291	2.340	2.389	2.438	2.488	2.536	2.585	2.634				
∞	2.303	2.129	2.180	2.232	2.283	2.334	2.385	2.436	2.486	2.537	2.588	2.638	2.689				

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release, distribution unlimited.		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Technical Report #89-17			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
6a. NAME OF PERFORMING ORGANIZATION Purdue University		6b. OFFICE SYMBOL (if applicable)		7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State, and ZIP Code) Department of Statistics West Lafayette, IN 47907			7b. ADDRESS (City, State, and ZIP Code)		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Office of Naval Research		8b. OFFICE SYMBOL (if applicable)		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-88-k-0170, DMS-88-06964, DMS-8702620	
8c. ADDRESS (City, State, and ZIP Code) Arlington, VA 22217-5000			10. SOURCE OF FUNDING NUMBERS		
PROGRAM ELEMENT NO.		PROJECT NO.		TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) ON LOWER CONFIDENCE BOUNDS FOR PCS IN TRUNCATED LOCATION PARAMETER MODELS (Unclassified)					
12. PERSONAL AUTHOR(S) Shanti S. Gupta, Lii-Yuh Leu, and TaChen Liang					
13a. TYPE OF REPORT Technical		13b. TIME COVERED FROM TO		14. DATE OF REPORT (Year, Month, Day) June 1989	
15. PAGE COUNT 22					
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) correct selection, probability of a correct selection, indifference zone, lower confidence bound, best population, truncated-location model, two-parameter exponential distribution.		
FIELD	GROUP	SUB-GROUP			
19. ABSTRACT (Continue on reverse if necessary and identify by block number) We are concerned with deriving lower confidence bounds for the probability of a correct selection in truncated location-parameter models. Two cases are considered according to whether the scale parameter is known or unknown. For each case, a lower confidence bound for the difference between the best and the second best is obtained. These lower confidence bounds are used to construct lower confidence bounds for the probability of a correct selection. The results are then applied to the problem of selecting the best exponential population having the largest truncated location-parameter. Useful tables are provided for implementing the proposed methods.					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION Unclassified		
22a. NAME OF RESPONSIBLE INDIVIDUAL Shanti S. Gupta			22b. TELEPHONE (Include Area Code) (317) 494-6031		22c. OFFICE SYMBOL