

AD-A211 646

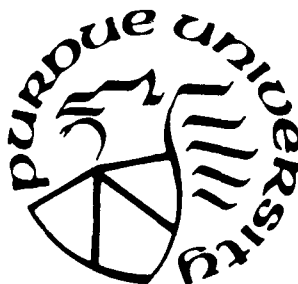
Empirical Bayes Estimation of Binomial Parameter
with Symmetric Priors*

by

TaChen Liang
Department of Mathematics
Wayne State University
Detroit, MI 48202

Technical Report # 89-13

PURDUE UNIVERSITY



DEPARTMENT OF STATISTICS



This document has been approved
for public release and sale; the
distribution is unlimited.

89 8 21 141

①

Empirical Bayes Estimation of Binomial Parameter
with Symmetric Priors*

by

TaChen Liang
Department of Mathematics
Wayne State University
Detroit, MI 48202

Technical Report # 89-13

Department of Statistics
Purdue University

June 1989

∞

* This research was supported in part by the Office of Naval Research Contract N00014-88-K-0170 and NSF Grants DMS-8606964, DMS-8702620 at Purdue University.

△

This document has been approved
for public release and sale; the
distribution is unlimited.

EMPIRICAL BAYES ESTIMATION OF BINOMIAL PARAMETER WITH SYMMETRIC PRIORS

TaChen Liang
Department of Mathematics
Wayne State University
Detroit, MI 48202



Accession For	
NTIS GRA&I	☒
NTIS	☐
Unannounced	☐
Distribution	
For	
Distribution	
Availability Codes	
Avail and/or	
Dist	Special
A-1	

ABSTRACT

This paper deals with the problem of estimating the binomial parameter via the nonparametric empirical Bayes approach. This estimation problem has some surprising phenomenon that estimators which are asymptotically optimal in the usual empirical Bayes sense do not exist (Robbins (1956, 1964)). However, as pointed out by Liang (1984) and Gupta and Liang (1986), it is possible to construct asymptotically optimal empirical Bayes estimators if the unknown prior is symmetric about the point $1/2$. In this paper, assuming symmetric priors a monotone empirical Bayes estimator is constructed by using the isotonic regression method. This estimator is asymptotically optimal in the usual empirical Bayes sense. The corresponding rate of convergence is investigated and shown to be at least of order $n^{-1/2}$, where n is the number of past observations at hand.

Key Words and Phrases: Bayes estimator, empirical Bayes, asymptotically optimal, rate of convergence, isotonic regression, symmetric prior.

1. INTRODUCTION

Consider a sequence of Bernoulli process consisting of N trials. Let p denote the probability of success for each trial and Y stand for the number of successes among the total N trials. Then Y follows a binomial distribution with probability function $f(y|p) = \binom{N}{y} p^y (1-p)^{N-y}$, $y = 0, 1, \dots, N$. Suppose that the parameter p is a realization of a random variable P having a prior distribution G . Thus, under the squared error loss, given $Y = y$, the Bayes estimator of p is the posterior mean of P given by

$$\varphi_G(y) = \frac{\int_0^1 p f(y|p) dG(p)}{\int_0^1 f(y|p) dG(p)} = \frac{w(y)}{h(y)} \quad (1.1)$$

where $h(y) = \int_0^1 p^y (1-p)^{N-y} dG(p)$ and $w(y) = \int_0^1 p^{y+1} (1-p)^{N-y} dG(p)$. Also, $f_G(y) = \binom{N}{y} h(y)$ is the marginal probability function of Y . The minimum Bayes risk is $r(G) \equiv r(G, \varphi_G) = E[(\varphi_G(Y) - P)^2]$.

When the prior distribution G is unknown, many authors, based on the past observations, treated this estimation problem via the empirical Bayes approach of Robbins (1956, 1964). For details, the reader is referred to Liang and Huang (1988), Vardeman (1978) and the related references. However, as pointed out by Robbins (1956, 1964), this estimation problem has some surprising phenomenon that estimators which are asymptotically optimal in the usual empirical Bayes sense do not exist. This is due to the fact that the function $w(y)$ cannot be consistently estimated when the prior distribution G is completely unknown. To remedy this deficiency, Robbins (1956) suggested taking one more observations at each stage and proposed an estimator which is asymptotically optimal in a modified sense. Gupta and Liang (1989) treated this estimation problem through the parametric empirical Bayes approach assuming the prior to be a member of beta distribution family with unknown hyperparameters and then using the past observations to estimate

the unknown hyperparameters. Liang (1984) and Gupta and Liang (1986) have pointed out that if the unknown prior is symmetric about the point $\frac{1}{2}$, it is possible to construct asymptotically optimal empirical Bayes estimators for the binomial parameter p . However, no estimators were proposed.

In this paper, we deal with this estimation problem through the nonparametric empirical Bayes approach assuming symmetric priors. A monotone empirical Bayes estimator is constructed by using the isotonic regression method. This estimator is asymptotically optimal in the usual empirical Bayes sense. The corresponding rate of convergence is investigated and shown to be at least of order n^{-1} where n is the number of past observations at hand.

2. CONSTRUCTION OF EMPIRICAL BAYES ESTIMATORS

For each $j = 1, 2, \dots$, let (Y_j, P_j) be a pair of random variables where Y_j is observable but P_j is not. Conditional on $P_j = p_j$, Y_j has a binomial probability function $f(y|p_j) = \binom{N}{y} p_j^y (1 - p_j)^{N-y}$, $y = 0, 1, \dots, N$. It is assumed that $P_j, j = 1, 2, \dots$, are independently distributed with common unknown prior distribution G . Therefore, $Y_j, j = 1, 2, \dots$, are iid with marginal probability function $f_G(y)$. Let $\underline{Y}_n = (Y_1, \dots, Y_n)$ denote the n past observations and $Y_{n+1} = Y$ the current random observation. In the empirical Bayes estimation case, an estimation φ_n for the present problem is a function based on a sequence of past observations $\underline{Y}_n$ and the present observation $Y = y$. We investigate this estimation problem under the following assumption.

Assumption A: The prior distribution G is symmetric about the point $\frac{1}{2}$, and N is an even number.

Under Assumption A, we have the following lemma which describes the relationship

between $w(y)$ and $h(y)$.

Lemma 2.1. Under Assumption A, we have

- a) $w\left(\frac{N}{2}\right) = \frac{1}{2}h\left(\frac{N}{2}\right)$.
- b) $w(x) = w(N - x - 1)$ for $x = 0, 1, \dots, N - 1$.
- c) $w(x) + w(N - x) = h(x) = h(N - x)$, $x = 0, 1, \dots, N$.
- d) $w(x) + w(x + 1) = h(x + 1)$, $x = 0, 1, \dots, N - 1$.
- e) $\varphi_G(x) = 1 - \varphi_G(N - x)$, $x = 0, 1, \dots, N$, and
- f) $w(x) = \sum_{i=0}^{x-\frac{N}{2}} h(x-i)(-1)^i - (-1)^{x-\frac{N}{2}} h\left(\frac{N}{2}\right)/2$, $x = \frac{N}{2}, \dots, N$.

Proof: Straight computation.

For each $y = 0, 1, \dots, N$, define

$$f_n(y) = f_n(N - y) = \begin{cases} \frac{1}{2n} \sum_{j=1}^n I_{\{y, N-y\}}(Y_j) & \text{if } y \neq \frac{N}{2}, \\ \frac{1}{n} \sum_{j=1}^n I_{\{y\}}(Y_j) & \text{if } y = \frac{N}{2}. \end{cases}$$

$$h_n(y) = f_n(y) / \binom{N}{y}, \text{ and}$$

$$\begin{cases} w_n(y) = \sum_{i=0}^{y-\frac{N}{2}} h_n(y-i)(-1)^i - (-1)^{y-\frac{N}{2}} h_n\left(\frac{N}{2}\right)/2 & \text{if } N \geq y \geq \frac{N}{2}, \\ w_n(y) = h_n(y) - w_n(N - y) & \text{if } 0 \leq y \leq \frac{N}{2} - 1. \end{cases}$$

Both $h_n(y)$ and $w_n(y)$ are unbiased estimators of $h(y)$ and $w(y)$, respectively, $y = 0, 1, \dots, N$. Thus, it is intuitive to use $\frac{w_n(y)}{h_n(y)}$ as an estimator for $\varphi_G(y) \equiv \frac{w(y)}{h(y)}$. However, this naive estimator may have serious deficiencies. First, $h_n(y)$ may be equal to zero and thus, the function $\frac{w_n(y)}{h_n(y)}$ is not well defined. Second, it is possible that the value of $\frac{w_n(y)}{h_n(y)}$ may be greater than 1 or less than 0, while $0 \leq \varphi_G(y) \leq 1$ for all $y = 0, 1, \dots, N$. Hence, in the following, we seek a better estimator. The following lemma states the monotone

properties of the functions $\varphi_G(y)$, $h(y)$ and $w(y)$. These properties may suggest a way how to construct reasonable empirical Bayes estimators.

Lemma 2.2. a) For any prior distribution G , $\varphi_G(y)$ is an increasing function of y , $y = 0, 1, \dots, N$.

b) Under Assumption A, both $h(y)$ and $w(y)$ are increasing in y for $y = \frac{N}{2}, \dots, N$.

Based on the monotone properties described in Lemma 2.2, we let $\{\tilde{h}_n(x)\}_{x=\frac{N}{2}}^N$ be the isotonic regression of $\{h_n(x)\}_{x=\frac{N}{2}}^N$ with equal weights and from Lemma 2.1, define $\tilde{w}_n(x) = \sum_{i=0}^{x-\frac{N}{2}} \tilde{h}_n(x-i)(-1)^i - (-1)^{x-\frac{N}{2}} \tilde{h}_n(\frac{N}{2})/2$, for $\frac{N}{2} \leq x \leq N$. Thus, $\tilde{h}_n(x)$ is nondecreasing in x for $\frac{N}{2} \leq x \leq N$, and by this nondecreasing property, $\tilde{w}_n(x) \geq 0$ for $\frac{N}{2} \leq x \leq N$. However, $\tilde{w}_n(x)$ may still not possess the nondecreasing property. Thus, we let $\{w_n^*(x)\}_{x=\frac{N}{2}}^N$ be the isotonic regression of $\{\tilde{w}_n(x)\}_{x=\frac{N}{2}}^N$ with equal weights and from Lemma 2.1, define $h_n^*(x) = w_n^*(x-1) + w_n^*(x)$ for $\frac{N}{2} + 1 \leq x \leq N$ and $h_n^*(\frac{N}{2}) = 2w_n^*(\frac{N}{2})$. By the nondecreasing property of $w_n^*(x)$, $h_n^*(x)$ is nondecreasing in x for $\frac{N}{2} \leq x \leq N$. Now, for $\frac{N}{2} \leq x \leq N$, define

$$\varphi_n(x) = \begin{cases} \frac{w_n^*(x)}{h_n^*(x)} & \text{if } h_n^*(x) \neq 0, \\ \frac{1}{2} & \text{if } h_n^*(x) = 0. \end{cases}$$

Since $\varphi_n(x)$ may be not a nondecreasing function of x for $\frac{N}{2} \leq x \leq N$, we consider the isotonic regression $\{\varphi_n^*(x)\}_{x=\frac{N}{2}}^N$ of $\{\varphi_n(x)\}_{x=\frac{N}{2}}^N$ with equal weights. Also, for $0 \leq x \leq \frac{N}{2} - 1$, define $\varphi_n^*(x) = 1 - \varphi_n^*(N - x)$. Now one can see that $\varphi_n^*(x)$ is nondecreasing in x for $x = 0, 1, \dots, N$. We propose using $\varphi_n^*(x)$ as an estimator of $\varphi_G(x)$, $x = 0, 1, \dots, N$.

Remark 2.1. By the nondecreasing property of $w_n^*(x)$ on x , $x = \frac{N}{2}, \dots, N$, $\varphi_n(x) \geq \frac{1}{2}$ for all $x \geq \frac{N}{2}$ and hence, $\varphi_n^*(x) \geq \frac{1}{2}$ for $x \geq \frac{N}{2}$. Also, $h_n^*(x) = 0$ iff $w_n^*(x) = 0$ iff $\tilde{w}_n(y) = 0$ for all $\frac{N}{2} \leq y \leq x$ iff $\tilde{h}_n(y) = 0$ for all $\frac{N}{2} \leq y \leq x$ iff $h_n(y) = 0$ for all $y = N - x, \dots, x$, where $x \geq \frac{N}{2}$.

3. ASYMPTOTIC OPTIMALITY

Let $\psi_n(y)$ denote an empirical Bayes estimator based on the current observation y and the past data $Y_n = (Y_1, \dots, Y_n)$. Let $r(G, \psi_n)$ denote the conditional Bayes risk (conditional on Y_n) of the estimator ψ_n and $Er(G, \psi_n)$ the associated overall Bayes risk where the expectation E is taken with respect to Y_n . Since $r(G)$ is the minimum Bayes risk, $r(G, \psi_n) - r(G) \geq 0$ and therefore $Er(G, \psi_n) - r(G) \geq 0$. The nonnegative difference $Er(G, \psi_n) - r(G)$ is often used as a measure of the optimality of the empirical Bayes estimator ψ_n .

Definition 3.1. A sequence of empirical Bayes estimators $\{\psi_n\}_{n=1}^{\infty}$ is said to be asymptotically optimal in E at least of order β_n relative to the prior distribution G if $Er(G, \psi_n) - r(G) \leq O(\beta_n)$ where $\{\beta_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that $\lim_{n \rightarrow \infty} \beta_n = 0$.

The usefulness of empirical Bayes estimators in practical applications clearly depend on the convergence rates for which the risks of the successive estimators approach the minimum Bayes risk. In the following, the performance of the proposed empirical Bayes estimators $\{\varphi_n^*\}$ is evaluated on basis of the rates of convergence of the nonnegative difference $Er(G, \varphi_n^*) - r(G)$. Without loss of generality, we assume that $G(0) < \frac{1}{2}$ to exclude the extreme case. In the following, all the computations are made under Assumption A.

Lemma 3.1. For each $y = \frac{N}{2} + 1, \dots, N$, suppose that $h_n^*(y) > 0$. Then for $0 < t < \min(1 - \varphi_G(y), \varphi_G(y) - \frac{1}{2})$,

$$|\varphi_n^*(y) - \varphi_G(y)| > t \Rightarrow (N+2) \sum_{x=\frac{N}{2}}^N |h_n(x) - h(x)|^2 > [h(N/2)t]^2.$$

Proof: $|\varphi_n^*(y) - \varphi_G(y)| > t \Rightarrow \varphi_n^*(y) - \varphi_G(y) > t$ or $\varphi_n^*(y) - \varphi_G(y) < -t$. By the definition of $\varphi_n^*(y)$, $w_n^*(x)$, $\tilde{w}_n(x)$, $\tilde{h}_n(x)$, Lemma 2.1 and Theorem 2.1 of Barlow, et. al. (1972), we

have:

$$\varphi_n^*(y) - \varphi_G(y) > t$$

$$\Rightarrow \varphi_n(x) - \varphi_G(y) > t \quad \text{for some } N/2 + 1 \leq x \leq y.$$

$$\Rightarrow w_n^*(x)[1 - \varphi_G(y) - t] - w_n^*(x-1)[\varphi_G(y) + t] > 0 \quad \text{for some } N/2 + 1 \leq x \leq y$$

$$\Rightarrow [w_n^*(x) - w(x)][1 - \varphi_G(y) - t] - [w_n^*(x-1) - w(x-1)][\varphi_G(y) + t] > h(N/2)t$$

$$\text{for some } N/2 + 1 \leq x \leq y$$

$$\Rightarrow [w_n^*(x) - w(x)] > h(N/2)t \text{ or } [w_n^*(x-1) - w(x-1)] < -h(N/2)t$$

$$\text{for some } N/2 + 1 \leq x \leq y$$

$$\Rightarrow \sup_{\frac{N}{2} \leq x \leq N} |w_n^*(x) - w(x)| > h(N/2)t$$

$$\Rightarrow \sum_{x=\frac{N}{2}}^N |w_n^*(x) - w(x)|^2 > [h(N/2)t]^2$$

$$\Rightarrow \sum_{x=\frac{N}{2}}^N |\tilde{w}_n(x) - w(x)|^2 > [h(N/2)t]^2,$$

$$\text{since } \sum_{x=\frac{N}{2}}^N [w_n^*(x) - w(x)]^2 \leq \sum_{x=\frac{N}{2}}^N (\tilde{w}_n(x) - w(x))^2, \text{ see Theorem 2.1 of Barlow, et. al. (1972).}$$

Now, by the definition of $\tilde{w}_n(x)$, we have, for each $x = \frac{N}{2}, \dots, N$,

$$\begin{aligned} & [\tilde{w}_n(x) - w(x)]^2 \\ &= \left[\sum_{i=0}^{x-\frac{N}{2}} [\tilde{h}_n(x-i) - h(x-i)](-1)^i - (-1)^{x-\frac{N}{2}} [\tilde{h}_n(N/2) - h(N/2)]/2 \right]^2 \\ &\leq 2 \sum_{x=\frac{N}{2}}^N [\tilde{h}_n(x) - h(x)]^2 \\ &\leq 2 \sum_{x=\frac{N}{2}}^N [h_n(x) - h(x)]^2 \end{aligned}$$

where the last inequality is again from Theorem 2.1 of Barlow, et. al. (1972).

Based on the above discussions, we conclude that

$$\varphi_n^*(y) - \varphi_G(y) > t \Rightarrow (N+2) \sum_{x=\frac{N}{2}}^N [h_n(x) - h(x)]^2 > [h(N/2)t]^2. \quad (3.1)$$

Analogous to the preceding discussion, under the assumption that $h_n^*(y) > 0$, we can obtain:

$$\varphi_n^*(y) - \varphi_G(y) < -t \text{ and } h_n^*(y) > 0 \Rightarrow (N+2) \sum_{z=\frac{N}{2}}^N [h_n(z) - h(z)]^2 > [h(N/2)t]^2 \quad (3.2)$$

Therefore, (3.1) and (3.2) together lead to the result of the lemma.

Remark 3.1. Note that for each $y = \frac{N}{2} + 1, \dots, N$, as $t > 1 - \varphi_G(y)$, $\{\varphi_n^*(y) - \varphi_G(y) > t\} = \phi$; also, as $t > \varphi_G(y) - \frac{1}{2}$, $\{\varphi_n^*(y) - \varphi_G(y) < -t\} = \phi$.

Lemma 3.2. For each $y = \frac{N}{2} + 1, \dots, N$ and $t > 0$,

$$P\{|\varphi_n^*(y) - \varphi_G(y)| > t \text{ and } h_n^*(y) > 0\} \leq \sum_{z=\frac{N}{2}}^N 2e^{-\frac{4nh^2(\frac{N}{2})(\frac{N}{z})^2 t^2}{(N+2)^2}}.$$

Proof: By Remark 3.1, $P\{|\varphi_n^*(y) - \varphi_G(y)| > t, h_n^*(y) > 0\} = 0$ if $t \geq \max(1 - \varphi_G(y), \varphi_G(y) - \frac{1}{2})$. Thus, as $0 < t < \max(1 - \varphi_G(y), \varphi_G(y) - \frac{1}{2})$, from Lemma 3.1,

$$\begin{aligned} & P\{|\varphi_n^*(y) - \varphi_G(y)| > t, h_n^*(y) > 0\} \\ & \leq P\left\{\sum_{z=\frac{N}{2}}^N [h_n(z) - h(z)]^2 > \frac{[th(\frac{N}{2})]^2}{N+2}\right\} \\ & \leq \sum_{z=\frac{N}{2}}^N P\left\{|h_n(z) - h(z)| > \frac{\sqrt{2}th(\frac{N}{2})}{N+2}\right\} \\ & = \sum_{z=\frac{N}{2}}^N P\left\{|f_n(z) - f_G(z)| > \frac{\sqrt{2}th(\frac{N}{2})(\frac{N}{z})}{N+2}\right\} \\ & \leq \sum_{z=\frac{N}{2}}^N 2e^{-\frac{4nh^2(\frac{N}{2})(\frac{N}{z})^2 t^2}{(N+2)^2}} \end{aligned}$$

where the last inequality is obtained from Theorem 1 of Hoeffding (1963).

The following theorem is our main result.

Theorem 3.1. Let $\{\varphi_n^*\}_{n=1}^\infty$ be the sequence of empirical Bayes estimators constructed in Section 2. Then, under Assumption A,

$$Er(G, \varphi_n^*) - r(G) \leq O(n^{-1}).$$

Proof: Straightforward computation leads to the following.

$$\begin{aligned}
0 &\leq Er(G, \varphi_n^*) - r(G) \\
&= \sum_{y=0}^N E[(\varphi_n^*(y) - \varphi_G(y))^2] f_G(y) \\
&= 2 \sum_{y=\frac{N}{2}+1}^N E[(\varphi_n^*(y) - \varphi_G(y))^2] f_G(y).
\end{aligned} \tag{3.3}$$

For each $y = \frac{N}{2} + 1, \dots, N$,

$$\begin{aligned}
&E[(\varphi_n^*(y) - \varphi_G(y))^2] \\
&= \int_0^{\max(1-\varphi_G(y), \varphi_G(y)-\frac{1}{2})} 2tP\{|\varphi_n^*(y) - \varphi_G(y)| > t, h_n^*(y) > 0\} dt \\
&\quad + (\varphi_G(y) - 1/2)^2 P\{h_n^*(y) = 0\}.
\end{aligned}$$

Now, from Remark 2.1,

$$\begin{aligned}
P\{h_n^*(y) = 0\} &= P\{f_n(x) = 0 \text{ for all } x = N - y, \dots, y\} \\
&= [1 - F_G(y) + F_G(N - y - 1)]^n \\
&= \exp(-n \ln(1 - F_G(y) + F_G(N - y - 1)))^{-1} \\
&\leq O(n^{-1}).
\end{aligned} \tag{3.4}$$

where $F_G(\cdot)$ is the marginal distribution function of Y . Also, from Lemma 3.2, and the fact that $\max(1 - \varphi_G(y), \varphi_G(y) - \frac{1}{2}) < \frac{1}{2}$ for $y \geq \frac{N}{2} + 1$, we have

$$\begin{aligned}
&\int_0^{\max(1-\varphi_G(y), \varphi_G(y)-\frac{1}{2})} 2tP\{|\varphi_n^*(y) - \varphi_G(y)| > t, h_n^*(y) > 0\} dt \\
&\leq \int_0^{\frac{1}{2}} 4t \sum_{x=\frac{N}{2}}^N e^{-\frac{4nh^2(\frac{N}{2})(\frac{N}{x})^2 t^2}{(N+2)^2}} dt \\
&\leq \frac{1}{n} \sum_{x=\frac{N}{2}}^N \frac{(N+2)^2}{2h^2 (\frac{N}{2}) (\frac{N}{x})^2} \\
&= O(n^{-1}).
\end{aligned} \tag{3.5}$$

From (3.4) and (3.5), we conclude that for each $y = \frac{N}{2} + 1, \dots, N$,

$$E[(\varphi_n^*(y) - \varphi_G(y))^2] \leq O(n^{-1}). \tag{3.6}$$

Since N is finite and fixed, (3.6) and (3.3) together complete the proof of the theorem.

ACKNOWLEDGEMENT

This research was supported in part by the Office of Naval Research Contract N00014-88-K-0170 at Purdue University.

BIBLIOGRAPHY

- Barlow, R.E., Bartholomew, D.J., Bremner, J.M. and Brunk, H.D. (1972). *Statistical Inference Under Order Restrictions*. Wiley, New York.
- Gupta, S.S. and Liang, T. (1986). Empirical Bayes rules for selecting good binomial populations. *Adaptive Statistical Procedures and Related Topics* (Ed. J. Van Ryzin), IMS Lecture Notes-Monograph Series, Vol. 8, 110-128.
- Gupta, S.S. and Liang, T. (1989). Selecting the best binomial population: parametric empirical Bayes approach. To appear in *J. Statist. Plann. Inference*.
- Hoeffding, W. (1963). Probability inequalities for sums of bounded random variables. *J. Amer. Statist. Assoc.*, **58**, 13-30.
- Liang, T. (1984). Some contributions to empirical Bayes, sequential and locally optimal subset selection rules. Ph.D. Dissertation, Department of Statistics, Purdue University, West Lafayette, Indiana.
- Liang, T. and Huang, W.-T. (1988). On a monotone empirical Bayes estimator of binomial parameter. Submitted for publication.
- Robbins, H. (1956). An empirical Bayes approach to statistics. *Proc. Third Berkeley Symp. Math. Statist. Probab.*, **1**, University of California Press, 157-163.
- Robbins, H. (1964). The empirical Bayes approach to statistical decision problems. *Ann. Math. Statist.*, **35**, 1-20.
- Vardeman, S.B. (1978). Bounds on the empirical Bayes and compound risks of truncated versions of Robbins's estimator of a binomial parameter. *J. Statist. Plann. Inference*, **2**, 245-252.

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release, distribution unlimited.		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Technical Report #89-13			7a. NAME OF MONITORING ORGANIZATION		
6a. NAME OF PERFORMING ORGANIZATION Purdue University		6b. OFFICE SYMBOL (If applicable)	7b. ADDRESS (City, State, and ZIP Code)		
6c. ADDRESS (City, State, and ZIP Code) Department of Statistics West Lafayette, IN 47907		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-88-K-0170, NSF Grant DMS-8606964 DMS-8702620			
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Office of Naval Research		8b. OFFICE SYMBOL (If applicable)	10. SOURCE OF FUNDING NUMBERS		
8c. ADDRESS (City, State, and ZIP Code) Arlington, VA 22217-5000		PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) EMPIRICAL BAYES ESTIMATION OF BINOMIAL PARAMETER WITH SYMMETRIC PRIORS (Unclassified)					
12. PERSONAL AUTHOR(S) TaChen Liang					
13a. TYPE OF REPORT Technical		13b. TIME COVERED FROM TO		14. DATE OF REPORT (Year, Month, Day) June 1989	
15. PAGE COUNT 11					
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Bayes estimator, empirical Bayes, asymptotically optimal, rate of convergence, isotonic regression, symmetric prior.		
19. ABSTRACT (Continue on reverse if necessary and identify by block number) This paper deals with the problem of estimating the binomial parameter via the nonparametric empirical Bayes approach. This estimation problem has some surprising phenomenon that estimators which are asymptotically optimal in the usual empirical Bayes sense do not exist (Robbins (1956, 1964)). However, as pointed out by Liang (1984) and Gupta and Liang (1986), it is possible to construct asymptotically optimal empirical Bayes estimators if the unknown prior is symmetric about the point 1/2. In this paper, assuming symmetric priors a monotone empirical Bayes estimator is constructed by using the isotonic regression method. This estimator is asymptotically optimal in the usual empirical Bayes sense. The corresponding rate of convergence is investigated and shown to be at least of order n^{-1} , where n is the number of past observations at hand.					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION Unclassified		
22a. NAME OF RESPONSIBLE INDIVIDUAL TaChen Liang, author Shanti S. Gupta, Principal Investigator			22b. TELEPHONE (Include Area Code) 313 577-2493; 317 494-6031		22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted.
All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED