(2)

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

DIMENSIONAL ANALYSIS OF
THE QUANTIFIED JUDGEMENT MODEL

by

Jerome A. Clark

June, 1989

Thesis Advisor: Maurice D. Weir

Unclassified

Security Classification of this page

REPORT DOCUMENTATION PAGE

1a Report Security Classification Unclassified	1b Restrictive Markings
2a Security Classification Authority 2b Declassification/Downgrading Schedule	3 Distribution Availability of Report Approved for public release; distribution is unlimited.
4 Performing Organization Report Number(s)	5 Monitoring Organization Report Number(s)
6a Name of Performing Organization Naval Postgraduate School / 6b Office Symbol *(If Applicable)*	7a Name of Monitoring Organization Naval Postgraduate School
6c Address *(city, state, and ZIP code)* Monterey, CA 93943-5000	7b Address *(city, state, and ZIP code)* Monterey, CA 93943-5000
8a Name of Funding/Sponsoring Organization / 8b Office Symbol *(If Applicable)*	9 Procurement Instrument Identification Number
8c Address *(city, state, and ZIP code)*	10 Source of Funding Numbers Program Element Number / Project No / Task No / Work Unit Accession No

11 Title *(Include Security Classification)* Dimensional Analysis of the Quantified Judgement Model

12 Personal Author(s) Jerome A. Clark

13a Type of Report	13b Time Covered	14 Date of Report *(year, month,day)*	15 Page Count
Master's Thesis	From To	June 1989	84

16 Supplementary Notation The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 Cosati Codes		
Field	Group	Subgroup

18 Subject Terms *(continue on reverse if necessary and identify by block number)* Dimensional, Quantified Judgement Model, ~~QJM~~, ground combat, Dupuy, force strength,

19 Abstract *(continue on reverse if necessary and identify by block number*

Quantitative analysis, specifically in this thesis which studies the Quantified Judgment Model (QJM), has been used consistently as a method of analyzing ground combat. If the QJM model is to be used as a basis for making important ground combat decisions, then its internal mathematical consistency and military soundness must be firmly established. A universal requirement is that any model be both reasonable and valid, in which case the model itself must be able to withstand careful scrutiny. In the case of the QJM, a dimensional analysis to ensure mathematical consistency of the variables and submodels is one test of the reasonableness of the model. Dimensional analysis tests are applied in this thesis to examine the validity of the QJM. We also perform some analyses to determine how sensitive the outcomes predicted by the model are to the values of several of the coefficients appearing in its submodels. The final chapter presents our conclusions and recommendations for further investigation of the QJM.

20 Distribution/Availability of Abstract [X] unclassified/unlimited [ ] same as report [ ] DTIC users	21 Abstract Security Classification Unclassified
22a Name of Responsible Individual M.D. Weir	22b Telephone *(Include Area code)* (408) 646-2608 / 22c Office Symbol 53WC

DD FORM 1473, 84 MAR 83 APR edition may be used until exhausted security classification of this page

All other editions are obsolete Unclassified

Approved for public release; distribution is unlimited.

Dimensional Analysis of the Quantified Judgement Model

by

Jerome Adrian Clark
Captain, United States Marine Corps
B.S., United States Naval Academy, 1983

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
June 1989

Author: Jerome A. Clark
Jerome Adrian Clark

Approved by: Maurice D. Weir
Maurice D. Weir, Thesis Advisor

Bard K. Mansager
LTC B. Mansager USA, Second Reader

Peter Purdue
Peter Purdue,
Chairman, Department of Operations Research

K. T. Marshall
Kneale ~~T. Marshall,~~
Dean of Information and Policy ~~Sciences~~

ABSTRACT

Quantitative analysis, specifically in this thesis which studies the Quantified Judgment Model (QJM), has been used consistently as a method of analyzing ground combat. If the QJM model is to be used as a basis for making important ground combat decisions, then its internal mathematical consistency and military soundness must be firmly established. A universal requirement is that any model be both reasonable and valid, in which case the model itself must be able to withstand careful scrutiny. In the case of the QJM, a dimensional analysis to ensure mathematical consistency of the variables and submodels is one test of the reasonableness of the model. Dimensional analysis tests are applied in this thesis to examine the validity of the QJM. We also perform some analyses to determine how sensitive the outcomes predicted by the model are to the values of several of the coefficients appearing in its submodels. The final chapter presents our conclusions and recommendations for further investigation of the QJM.

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

A. DIMENSIONAL ANALYSIS OF MATHEMATICAL MODELS

In today's world of complex weapons and high technology, mathematical models provide especially powerful tools for military decision makers. They can be used both to predict a specific phenomenon and to explain what happens in a particular situation to give a better understanding of what specific outcomes will occur, or how likely they are to occur. A model may also aid the decision maker to define better which elements of a particular phenomenon are the most important and how they may affect the outcomes.

Because mathematical models are so significant to the military decision maker, it is important to ensure that any mathematical model that is used is reasonable and valid. A process that can sometimes be used to test for reasonableness is dimensional analysis. The many equations that may comprise a mathematical model may be mathematically inconsistent or incorporate improper or questionable relationships. Dimensional analysis can be a key process to identify such problems. One aspect of dimensional analysis is to evaluate each equation of a model to ensure that its dimensions are consistent.

Dimensional analysis is based on the assumption that all physical quantities have dimensions and that any functional relationship representing a variable must have a resulting dimension equal to that of the physical quantity symbolized by the variable. Before the dimensions of an equation are investigated , the variables that compose the equation must all be checked to ensure they are dimensionally sound. Some of the variables may represent a basic physical quantity with a known dimension (for instance, speed is equal to distance per unit time, yielding dimension LT^{-1}, where L represents length and T time.) . Other variables may be composed of more elaborate relationships requiring analysis. For example, if the variable y is a variable representing the expression F/μv where F represents force, μ is viscosity of a fluid, and v is the speed of an object in the fluid, then the dimension of y is given as in the following Figure 1. The notation [x] stands for the dimension of the variable x.[1]

$$[y] = \frac{[F]}{[\mu][v]} = \frac{\left(\frac{(\text{mass} \times \text{length})}{\text{time}^2}\right)}{\left(\frac{\text{mass}}{\text{length} \times \text{time}}\right)\left(\frac{\text{length}}{\text{time}}\right)}$$

$$= \text{length} ,$$

or given:

M = mass

L = length

T = time

$$[y] = \frac{[F]}{[\mu][v]} = \frac{M\,L\,T^{-2}}{(M\,L^{-1}\,T^{-1})(L\,T^{-1})}$$

$$= \frac{1}{L^{-1}} = L .$$

Figure 1. Dimensional Composition of a Variable

Two principles pertain to dimensions of variables representing physical quantities. The first principle asserts that whenever variables or expressions composed of variables representing physical quantities are set equal to one another, then the dimensions must be equal as well. In Figure 1, any variable set equal to the

[1]See Ref. 1: pp. 216-237, for a full discussion of dimensional analysis.

variable y, (which had a resulting dimension of length), must also have a dimension of length. The second principle asserts that when variables or products of variables are combined using addition, all terms in the sum must have the same dimension. Just as you cannot "add apples and oranges," components having different dimensions cannot be summed together. This principle is illustrated in Figure 2. An equation containing terms that are added together, but having different dimensions, is said to be <u>dimensionally incompatible</u> (hence incorrect).

Consider the equation:

$$F = mv + v^2$$

where:

$[m] = M$

$[v] = LT^{-1}$.

Therefore,

$[mv] = MLT^{-1}$, and

$[v^2] = (LT^{-1})(LT^{-1}) = L^2T^{-2}$.

However, $MLT^{-1} \neq L^2T^{-2}$, so the original equation is incorrect because mv and v^2 cannot be summed.

Figure 2. An Equation that is Dimensionally Incompatible

These two principles are applied in this thesis to investigate the reasonableness and mathematical consistency of the QJM.

B. THESIS GOAL AND OUTLINE

This thesis continues the work initiated in Captain Joseph Ciano's thesis, "The Quantified Judgment Model and Historic Ground Combat." In that thesis, Capt.

Ciano suggested several problems for further study on the Quantified Judgment Method of Analysis of Historical Combat Data (QJMA). A major component of the QJMA, called the Quantified Judgment Model (QJM), is a mathematical model used to assess historic ground battles. Capt. Ciano tested that model for real-world reasonableness by determining whether the major equations and submodels of the QJM were mathematically and militarily sound. Most of the primary equations in the QJM are composed of variables which themselves are equal to detailed secondary expressions. In order to test the major submodels of the QJM, Capt. Ciano assumed that the variables given by those secondary expressions were both dimensionally correct and sound. This thesis further evaluates the QJM by focusing on the validity of the particular Force Strength Equation, as well as other secondary equations within the model. The equations and various variables are examined to ensure that they are dimensionally consistent within the submodels and that they meet tests of reasonableness in relation to how they are combined and interpreted.

The thesis is organized into six chapters. Following an introduction in Chapter I, Chapter II describes briefly the QJM methodology for analyzing ground combat. It presents the major equations and submodels to give a clear understanding of how they are interpreted and interact. Chapter III introduces the Force Strength Equation and its various components. Force strength will be carefully defined and discussed. Each component of the Force Strength Equation is evaluated for dimensional consistency and reasonableness. Chapter IV then presents a sensitivity analysis on the Force Strength Equation based upon certain Operational Lethality Index variables (which will be defined.) Chapter V also evaluates the operational variables of the model. Finally, Chapter VI reviews the thesis results and suggests still further areas of study yet required on the QJM.

II. HISTORICAL GROUND COMBAT AND THE QUANTIFIED JUDGMENT MODEL

A. INTRODUCTION

This chapter summarizes the QJM and its equations. The focus is on how the QJM models historical ground combat.[2] The QJM was created by Col. T.N. Dupuy, U.S. Army Ret. and his colleagues at the Historical Evaluation and Research Organization. Dupuy developed the QJM to provide a basis for predicting battle outcomes. From Clausewitz's writings [Ref. 2], Dupuy extracted what he calls "Clausewitz's Law of Numbers". Dupuy quotes The Law of Numbers:

> If we... strip the engagement of all the variables arising from its purpose and circumstances, and disregard the fighting value of the troops involved (which is a given quantity), we are left with the bare concept of the engagement, a shapeless battle in which the only distinguishing factor is the number of troops on either side.
>
> These numbers, therefore, will determine victory. It is, of course, evident from the mass of abstractions I have made to reach this point that superiority of numbers in a given engagement is only one of the factors that determines victory. Superior numbers, far from contributing everything, or even a substantial part, to victory, may actually be contributing very little, depending on the circumstances.
>
> But superiority varies in degree. It can be two to one, or three or four to one, and so on, it can obviously reach the point where it is overwhelming.
>
> In this sense superiority of numbers admittedly is the most important factor in the outcome of an engagement so long as it is great enough to counterbalance all other contributing circumstances. It thus follows that as

---

[2]Since Captain Ciano does an excellent job of giving the reader a clear understanding of the QJM, this chapter follows the same format (without the analysis). The chapter is meant to review the QJM only, in order to facilitate better comprehension of the analysis performed in the following chapters.

many troops as possible should be brought into the engagement at the decisive point.

Whether these forces prove adequate or not, we will at least have done everything in our power. This is the first principle of strategy. In the general terms in which it in expressed here it would hold true for Greeks and Persians, for Englishmen and Mahrattas, for Frenchmen and Germans. [Ref.4: pp. 28-29]

From these excerpts, Dupuy formulated a mathematical equation which he felt represented the Law of Numbers (Figure 3). Performing analytical and mathematical refinement upon the equation to account for today's modern weapons technology, Dupuy formulated his QJM model.

$$\text{Outcome} = \frac{N_r \times V_r \times Q_r}{N_b \times V_b \times Q_b}$$

where:

N = numbers of troops

V = variable circumstances affecting a force in battle

Q = quality of force

r = red force subscript

b = blue force subscript

Figure 3. Clausewitz's Law of Numbers

B. MODEL DESCRIPTION

1. Combat Power Ratio

From the Law of Numbers equation, the combat power of each opposing force could be determined. However, to account for the great increase in weapons technology, Dupuy performed the following refinements to Clausewitz's equation:

1. Substituted Force Strength (S) for number of troops (N);
2. Elaborated and defined environmental and operational factors (OE) which represented the effect of the circumstances of combat on the force;
3. Substituted relative Combat Effectiveness Value (CEV) for troop quality.

The first two changes to the equation created the basic equation representing combat power of a force as shown in Figure 4.

Combat Power: $P = S \times OE \times Q$

where:

P = combat power of the force

S = force strength

OE = operational/environmental factors

Q = quality of troops

Figure 4. Combat Power Equation

Force Strength (S) is substituted for number of troop (N) to account for the many various weapons available to a force today. Because these weapons have so many different characteristics which influence their lethality, force strength is used to account for different lethalities among weapons and environmental and operational factors which influence each weapon's performance in battle. The second step of defining factors which represent the circumstances of combat on the force involves quantification of three major groups of variables: environmental, operational and behavioral (see Figure 5). Environmental variables are what Dupuy calls *factors that are caused by nature.* He defines operational variables as *factors which represent actions of combat forces* and are heavily influenced by commanders. Behavioral variables are those that represent *the nature of human participants in combat.*

ENVIRONMENTAL	OPERATIONAL	BEHAVIORAL
Terrain	Posture	Leadership
Weather	Mobility	Training
Season	Vulnerability	Experience
	Fatigue	Morale
	Surprise	Manpower Quality

Figure 5. Force Effects Variables

The combat power factors S, OE, and Q are derived from their own secondary equations which are introduced and explained later in this text. Figure 6 displays the approximate range of these factors.[3]

[3]These ranges were determined by Captain Ciano in [Ref. 3: pp. 16]. As he noted, the ranges are highly approximate and obtained from [Ref. 4: pp. 82,83] and [Ref. 5: pp. 228-230,234-239]. Theoretically, force strength (S) can range from zero to infinity. However, Figure 6 uses S values of 500 to 500,000.

FACTOR	LOWER RANGE	UPPER RANGE
S (force strength)	500	500,000
OE (operational/environmental)	0.25	4.0
Q (troop quality)	.25	5.0

Figure 6. Range of Combat Power Factors

In the next step, Dupuy compares the combat power of two opposing forces. The QJM calculates a *relative combat power* for two opposing forces by taking a ratio of the forces' combat powers. **Combat Power Ratio for the red force** is the ratio P(r)/P(b), representing the relative combat power of the red force, where P(r) and P(b) are the combat powers of the red and blue force. If the combat power ratio is greater than one, the red force would have a greater combat power than the blue force.

2. Actual Battle Results Ratio

Recall from the previous section that Dupuy performed three operations on Clausewitz's Law of Numbers:

1. Substituted Force Strength (S) for number of troops (N);
2. Elaborated and defined environmental and operational factors (OE) which represent the effect of the circumstances of combat on the force;
3. Substituted relative Combat Effectiveness Value (CEV) for troop quality (Q).

The third step is the process of determining how troop quality affects the outcome of battles. To do this, Dupuy compares the *theoretical* outcome of a battle with the actual results of that battle. As shown in Figure 7, the theoretical outcome is represented by a theoretical combat power ratio.

$$\textbf{Theoretical Combat Power Ratio} \equiv \frac{P'(b)}{P'(r)}$$

where:

$$P' = S \times OE$$

Figure 7. Theoretical Combat Power

This theoretical combat power ratio is the same as the combat power ratio presented in the previous section, with the exception that troop quality factors are omitted from the combat power of each force. The absence of troop quality factors in the theoretical combat power ratio is done on the basis that *the components of troop quality (leadership, morale, training, and luck/chance) are not directly measurable*. So, the theoretical combat power of a force is solely a measure of its strength in terms of weapons and operational/environmental factors. However, because these troop quality factors do affect battle outcomes, Dupuy compares the theoretical combat power ratio with actual battle results. The QJM determines actual battle results with the equation shown in Figure 8.

$$R = M + G + C$$

where:

R = battle results of the force

M = mission accomplishment

G = ability to gain or hold ground

C = effectiveness when casualties are incurred

Figure 8. Actual Battle Results Equation

The components of the actual battle results equation are defined by Dupuy as:

- Mission Accomplishment (M), an *expert* judgment of the extent to which a force accomplished its assigned or perceived mission.
- Spatial Effectiveness (G), a value representing the extent to which a force was able to gain or hold ground.
- Casualty Effectiveness (C), a value representing the efficiency of the force in terms of casualties, taking into consideration the strengths of the two sides, and the casualties incurred by both sides.

The mission accomplishment value is extracted from a table [Ref. 5: pp. 231] based upon the *expert* judgment given. The factors G and C are derived from their own secondary equations. The following values are the approximate ranges of the actual battle results factors.[4]

[4]The range of values for R, G and C are approximate and were provided to Capt. Ciano [Ref. 3] from T. N. Dupuy in a personal communication.

FACTOR	LOWER RANGE	UPPER RANGE
R (actual battle results)	-5.5	16.5
M mission accomplishment)	1	10
G (ability to gain or hold ground)	-3.0	3.0
C (casualty effectiveness)	-3.5	3.5

Figure 9. Range of Combat Power Factors

Once actual battle results are calculated for two forces, the QJM expresses the actual battle results of a battle as a ratio. This **Results Ratio** is defined to be R(r) / R(b) for the red force.

3. Combat Effectiveness Equation

In the previous section, we discussed how the theoretical combat power ratio and the actual battle results ratio were obtained from two opposing forces.

Theoretical Combat Power Ratio $\equiv \dfrac{P'(b)}{P'(r)}$, **where** $P' = S \times OE$

and

Actual Battle Results Ratio $\equiv \dfrac{R(b)}{R(r)}$

Figure 10. Theoretical Combat Power and Actual Battle Results Ratio for the Blue Force

From these two ratios, the final analytical stage of the model is performed. Recall that the last step of converting the Law of Numbers into the QJM was to *substitute* *relative Combat Effectiveness Value (CEV) for troop quality.* The model defines combat effectiveness (CEV) as:

$$\textbf{Combat Effectiveness (blue force):}\quad CEV(b) = \frac{\frac{R(b)}{R(r)}}{\frac{P'(b)}{P'(r)}} = \left[\frac{R(b)}{R(r)} \times \frac{P'(r)}{P'(b)}\right]$$

$$\textbf{Combat Effectiveness (red force):}\quad CEV(r) = \frac{\frac{R(r)}{R(b)}}{\frac{P'(r)}{P'(b)}} = \left[\frac{R(r)}{R(b)} \times \frac{P'(b)}{P'(r)}\right]$$

Figure 11. Combat Effectiveness Equation

The CEV is a comparison between the theoretical results (what should have happened, based on tangible factors) and the actual battle result (what did happen, due to the influence of tangible factors). The QJM's replacement of troop quality with CEV within the combat power ratio was found previously to be mathematically inconsistent by Capt. Ciano in his thesis. For consistency within the model, Capt. Ciano replaces the combat power equation with a *relative* combat power equation as given in Figure 12.[5]

---

[5]See Capt. Ciano's thesis [Ref. 3: pp. 33-40], to see how the relative combat power equation was derived. Dupuy uses combat power ratio [Ref. 5]; however, this original equation is inconsistent with the model.

$$\textbf{Relative Combat Power (blue force)}: \quad RCP(b) = \frac{S(b) \times OE(b) \times \sqrt{CEV(b)}}{S(r) \times OE(r)}$$

$$\textbf{Relative Combat Power (red force)}: \quad RCP(r) = \frac{S(r) \times OE(r) \times \sqrt{CEV(r)}}{S(b) \times OE(b)}$$

Figure 12. Relative Combat Power Equation

This chapter has demonstrated how Dupuy modeled the QJM based upon the Law of Numbers. The QJM consists of two submodels, the combat power ratio and the actual battle results ratio. For consistency, the combat power ratio is replaced by the relative combat power ratio. Using theoretical combat power (P') and actual battle results (R) values, the combat effectiveness (CEV) values for two opposing forces are obtained. These values are applied in the relative combat power ratio to complete the QJM model. The following chapter analyzes the force strength equation (S) and its variables for dimensional consistency and military reasonableness.

III. DECOMPOSITION OF FORCE STRENGTH VARIABLES

A. INTRODUCTION OF KEY FORCE STRENGTH RELATIONSHIPS

In the preceding chapter, the basic equations of the QJM were reviewed and discussed. Recall from that chapter, the combat power equation.

Combat Power: $P = S \times OE \times Q$

where:

P = combat power of the force

S = force strength

OE = operational/environmental factors

Q = quality of troops

Figure 13. Combat Power Equation

This chapter decomposes and analyzes force strength (S), a key factor within the combat power equation. Because force strength is a key variable within the QJM, it is very important that it be mathematically consistent and militarily sound. As shown in Figure 14, force strength is defined as the summation of the weapon system lethalities for eight major types of weapons systems, modified by environmental and operational factors.[6] The factors in the force strength equation are then discussed in turn.

[6] The equation given in Figure 14 is a modification of the equation given by Capt. Ciano [Ref. 3: pp. 45]. This new equation accurately accounts for the lethalities of different quantities of weapon types present within a force.

Force Strength (blue force): $S(b) = \sum_{i=1}^{8} Z_i(b) \times V_i(b)$

Force Strength (red force): $S(r) = \sum_{i=1}^{8} Z_i(r) \times V_i(r)$

where:

$$Z_i(b) = \sum_{j=1}^{m_i} n_j(b) \ \ OLI_j(b)$$

$$Z_i(r) = \sum_{j=1}^{m_i} n_j(r) \ \ OLI_j(r)$$

and,

S = force strength

Z = Operational Lethality Index of a weapon system category

V = weapon effects (terrain, weather, season, air superiority)

OLI = Operational Lethality of a specific type

n = the number of weapons of a specific type

m = the number of specific types of weapons within a category

i = the following eight QJM weapon system categories

1 = small arms

2 = machine guns

3 = heavy weapons

4 = anti-armor

5 = artillery

6 = air defense

7 = armor

8 = close air support

Figure 14. Force Strength Equation

B. OPERATIONAL LETHALITY INDEX (OLI)

Because of the vast array of weapons used by humankind in combat, there is a great range of lethality among the different types of weapons. In particular, the recent great leap in technology and science has created nuclear weapons with enormous destructive power. To account for the different degrees of lethality, Dupuy has assigned each weapon an Operational Lethality Index (OLI) value to represent its lethality. The factors discussed below are applied by Dupuy to account for the differing degrees of lethality among weapons. Since weapons are employed differently (based upon type), Dupuy bases all lethality measurements on a *standard theoretical, laboratory-like environment which could be common for all weapons;* thus, a controlled environment. He accomplishes this task by basing the lethality index of all weapons upon their performance against the same theoretical target. His theoretical target is a mass formation of men, each occupying 1 square meter each. The lethality index factors are divided into two groups: non-mobile weapons factors and mobile weapons factors.

1. Non-Mobile Weapons

The following OLI variables take into account those factors that influence the lethality of all non-mobile weapons. The first step in calculating the OLI value for a non-mobile weapon is to produce its Theoretical Lethality Index (TLI) value. This calculation is done by multiplying together the following factors:

- Rate of Fire (RF)
- Number of Potential Targets per Strike (PTS)
- Relative Incapacitating Effect (RIE)
- Effective Range (RN)
- Accuracy (A)

• Reliability (RL)

Then the TLIs are converted into OLIs by dividing by a Dispersion Factor. The remainder of this section analyzes each of these factors.

a. Rate of Fire (RF)

Rate of fire is the number of rounds per hour that a weapon can fire. Dupuy uses the unit of an hour because it provides enough time to permit consideration of sustained rates of fire. Dupuy assumes no weapon has a logistical problem during the one hour period. The following *rules of thumb* are given by Dupuy:

- For crew-served automatic weapons: RF = 4 × cyclic rate per minute
- For hand or shoulder automatic weapons: RF = 2 × cyclic rate per minute
- For aircraft-mounted automatic weapons: RF = 2 × cyclic rate per minute
- For must other weapons, Dupuy's graph [Fig 16] shows the normal rate of fire for most non-automatic weapons by caliber
- Mortar rate of fire is 1.2 × that value given in the graph for the mortar caliber.

Figure 15. Rules for Rate of Fire

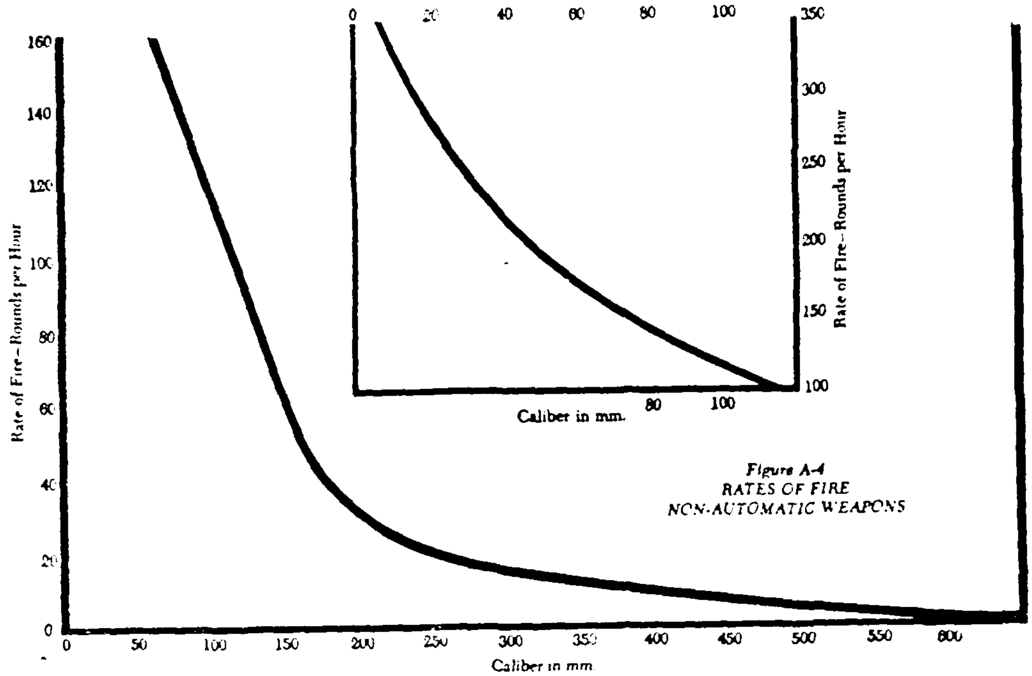

Figure 16. Dupuy's Rate of Fire based on Caliber

As defined by Dupuy [Ref. 5: pp. 20], RF has the dimension of rounds per hour. The first three rules of thumb describe RF in terms of cyclic rate. This gives RF the dimension of rounds per minute, conflicting with its original definition. To rectify this, the constants given in the equations for automatic weapons could be dimensional constants, with dimension of minutes per hour. We also concluded that these equations were only estimates at best. Defining RF in terms of cyclic rate will not produce accurate results for each of these automatic weapons. The true sustained rate per hour of weapons within each automatic weapons group will vary among each other. This variance will be due to different design characteristics (e.g., the width of a weapon's barrel affects how often the barrel must be changed which, in turn, affects the sustained rate of fire.). The graph of Dupuy's values for larger non-automatic weapons follows the general rule that *the larger the caliber of a weapon, the slower the rate of fire.* However, this rule is true only for the average rate of fire for all weapons at each particular caliber. For each caliber, different weapons will have a varied rate of fire based upon weapon type and weapon characteristics. Therefore, we conclude that the graph given by Dupuy is a rough estimate at best. It is better to use actual known sustained rates of fire if possible, with the graph being used as a backup means for determining a RF value. From this analysis, the following rules of thumb are proposed:

- For crew-served automatic weapons the sustained rate of fire **per hour** should be used. Otherwise, use the following estimate:
  RF = 4 (minute/hour) × cyclic rate per minute
- For hand or shoulder automatic weapons the sustained rate of fire **per hour** should be used. Otherwise, use the following estimate:
  RF = 2 (minute/hour) × cyclic rate per minute
- For aircraft-mounted automatic weapons the sustained rate of fire **per hour** should be used. Otherwise, use the following estimate:
  RF = 2 (minute/hour) × cyclic rate per minute
- For must other weapons, Dupuy's graph [Fig 16.] shows the normal rate of fire for most non-automatic weapons by caliber.
- Mortar rate of fire is 1.2 × that value given in the graph for the mortar caliber.

Figure 17. Revised Rules for Rate of Fire

b. Number of Potential Targets per Strike (PTS)

To account for the increased lethality of an area weapon, Dupuy gives all weapons throughout history a factor of lethality based upon their area of destruction (lethal area of burst) per round. PTS is the number of expected targets struck per round upon a formation of men, each of whom occupies one square meter. The PTS of weapons is based upon category.

- Individual weapons and light machine guns are usually limited to one target per strike.
- Machine guns with a caliber of 10-15 cms are assumed capable of hitting two targets per strike.
- Pre-high explosive artillery (i.e., 19th Century and earlier) are assigned 25 targets per round.
- High explosive weapons hit one target per square meter within the lethal area of burst. Dupuy determines PTS for these weapons from a graph [Ref. 5: pp. 193] based upon the caliber of the weapon.

A dimensional analysis of the PTS factor indicates that the dimensions for this factor are targets per round.

A dimensional analysis of the PTS factor gives

$$[\text{PTS}] = \frac{\text{targets}}{\text{meter}^2} \times \left[\begin{matrix}\text{lethal area} \\ \text{of burst}\end{matrix}\right], \quad \text{where} \quad \left[\begin{matrix}\text{lethal area} \\ \text{of burst}\end{matrix}\right] = \frac{\text{meter}^2}{\text{round}}$$

$$\text{which implies } [\text{PTS}] = \frac{\text{targets}}{\text{meter}^2} \times \frac{\text{meter}^2}{\text{round}} = \frac{\text{targets}}{\text{round}}$$

Figure 18. Dimensional Analysis of PTS Factor

This factor seems reasonable. The larger the lethal area of a weapon the larger the potential destruction it is capable of inflicting. However, we feel the PTS for high explosive weapons should be based upon the lethal area of the weapon's primary ammunition, not the caliber of the weapon.

c. Relative Incapacitating Effect (RIE)

RIE is the probability that a target struck by a particular weapon will be incapacitated. This variable is dimensionless and is multiplied by the other OLI factors. Any weapon more powerful than a light machine gun has a RIE factor of 1, leaving its OLI value unchanged. This factor just takes into account that smaller weapons are less likely to incapacitate a person than larger weapons, which is a reasonable assumption.

Example 1

WEAPON	RIE
Javelin	.4
Long Bow	.5
Springfield Rifle	.8
WWII Machine Gun	.8
M-60 Machine Gun	1.0
105 Howitzer	1.0

Discussion: Older, smaller weapons have a RIE factor less than one to indicate that these weapons are less than 100% probable of incapacitating a target they hit.

d. Effective Range (RN)

Range Factor: $RN = 1 \text{ (meter)} + \sqrt{.001 \times \text{Effective Range (meters)}}$

where:

$\text{Effective Range} = 90\% \text{ of maximum range}$

Figure 19. Range Factor Equation

This variable takes into account that the longer the range of a weapon, the more targets of opportunity it has. Dupuy further explains that:

> All enemies within the effective range of a weapon are forced to take some kind of passive or active countermeasures to protect themselves from the direct effect of the weapons employment within its effective range.

Dupuy gives an empirically derived formula for the range factor of weapons.[7] He established what he calls the *norm for range*.. This amounts to equating the length of a man's arm to one meter, called *normal range*. Based upon this, he expressed the range factors as defined in Figure 19. However, the dimensional analysis shown in Figure 20 illustrates that this equation is *mathematically inconsistent*.

[7]Dupuy also states that the effective range factor is represented by a second equation when computing this factor for tube artillery weapons. However, this second equation, $RN = .007 \times \sqrt{.01 \times \text{caliber}}$, is dimensionally different from the original equation. In addition, Dupuy stated that the equation for tube artillery weapons is particularly appropriate for AT and AH guns. This statement does not appear reasonable from a military standpoint. We noted that these weapons (direct fire) have a completely different flight profile than tube artillery (indirect fire). If the equation is appropriate for tube artillery, then it cannot be for AT and AA.

A dimensional analysis of the effective range factor gives the following:

$$[RN] = [1\ (\text{meter})] + \left[\sqrt{.001 \times \text{Effective Range}}\right],$$

where $[1(\text{meter})] = [\text{Effective Range}] = L\ (\text{length})$

This implies $[RN] = L + \sqrt{L} = L + L^{.5}$,

which is dimensionally incorrect.

Figure 20. Dimensional Analysis of Effective Range Factor

The range factor equation can be corrected mathematically by making the constant .001 a dimensional constant. Moreover, it may be surmised that Dupuy wanted to express the effective range factor as a value per each meter of range of a mobile machine. Therefore, we calculated the original expression over meters, causing it to be dimensionless. This new equation is shown in Figure 21.

Range Factor: $$RN = \frac{1\ \text{meter} + \sqrt{.001(\text{meters}) \times \text{Effective Range (meters)}}}{1\ \text{meter}}$$

where:

Effective Range = 90% of maximum range

Figure 21. Proposed Range Factor Equation

This proposed equation is mathematically consistent as shown in Figure 22 and appears to be reasonable. The greater the range of a weapon, the larger its range factor, thereby increasing the weapon's potential lethality.

A dimensional analysis of the proposed effective range factor gives:

$$[RN] = \left[\frac{1 \text{ meter} + \sqrt{.001(\text{meters}) \times \text{Effective Range (meters)}}}{1 \text{ meter}}\right]$$

where $[1(\text{meter})] = [\text{Effective Range}] = L$ (length)

$$\text{Thus, } [RN] = \frac{L + \sqrt{L \times L}}{L} = \frac{L + L}{L} = \frac{L}{L} = L^0$$

Figure 22. Dimensional Analysis of Effective Range Factor

The following example illustrates the proposed equation.

Example 2

WEAPON	EFFECTIVE RANGE (meters)
Weapon 1	5000
Weapon 2	9000

For Weapon 1,

$$\textbf{Range Factor:} \quad RN = \frac{1 \text{ meter} + \sqrt{.001(\text{meters}) \times \text{Effective Range (meters)}}}{1 \text{ meter}} = 3.236$$

Fcr Weapon 2,

Range Factor: $RN = \dfrac{1\text{ meter} + \sqrt{.001(\text{meters}) \times \text{Effective Range (meters)}}}{1\text{ meter}} = 4.0$

Discussion: The new equation gives dimensionless values for the range factor. Because weapon 1 has a smaller range than weapon 2, its effective range factor is smaller. Thus weapon 1 has a smaller potential lethality due to its smaller range.

e. Accuracy (A)

The accuracy factor is strictly a judgmental factor which is dimensionless. Weapons are given estimated probabilities that they will hit a target given the weapon is aimed correctly at the target. This factor assumes a perfect shot and excludes the inability of the user of the weapon to correctly aim the weapon. The following general rules apply to this factor:

- High muzzle velocity weapons have higher accuracy.
- Automatic weapons, mortars, rockets and free-flight missiles tend to be relatively inaccurate.
- Weapons having electronic guidance systems have higher accuracy than those that do not.

This factor appears plausible. The more accurate a weapon, the more deadly it is. However, Dupuy establishes that the values for this factor are obtained from official estimates of accuracy at mean battlefield ranges found in official manuals or other sources. If a source is unavailable, then the factor is estimated. This does not account for the different methods used by the sources to obtain an estimate. There is also no range ascribed to this factor, which is critical since the factor is multiplicative.

f. Reliability (RL)

The reliability factor is also a subjective and dimensionless factor. It accounts for the fact that mechanical weapons are not totally reliable. They have failures such as misfires, duds, and jamming. The more often a weapon fails to function properly because of mechanical failure, the less potential lethality it has. This factor is certainly reasonable. Suppose a force has 100 weapons of a certain type and 10 of them are apt to malfunction due to mechanical failure. Then the plausible lethality of the force should reflect only 90% lethality of 100 perfectly working weapons. Though this factor is logical, Dupuy does not clarify how values for reliability are obtained. He claims that the values are *obtained through reliability information from official or other sources*. Different sources have different standards upon which reliability is based. We feel that if the model is sensitive to the reliability factor, then this method of obtaining the value is too general.

g. Dimensional Analysis of the TLI Equation for Non-Mobile Weapons

In the previous sections, each variable comprising the TLI equation for Non-Mobile Weapons was examined for mathematical consistency and reasonableness. Figure 23 shows how the variables originally compose the TLI equation.

Theoretical Lethality Index:

TLI = RF × PTS × RIE × RN × A × RL

where:

RF = Rate of Fire

PTS = Number of Potential Targets per Strike

RIE = Relative Incapacitating Effect

RN = Effective Range

A = Accuracy

RL = Reliability

Figure 23. Lethality Index Equation for Non-Mobile Weapons

As defined by Dupuy [Ref. 4: pp. 84], theoretical lethality index is expressed in *casualties per hour*. Based upon this definition, when all the applicable variables are incorporated into the equation, the dimensions of TLI values should be the number of casualties per unit of time that a given weapon can inflict. As shown in Figure 24, a dimensional analysis of the OLI equations gives the desired dimensions.[8]

[8]The dimensions given for the effective range factor come from our proposed effective range factor.

A dimensional analysis of the TLI equation gives

$$[TLI] = [RF] \times [PTS] \times [RIE] \times [RN] \times [A] \times [RL] .$$

Substitution of $[RF] = \frac{\text{rounds}}{\text{hour}}$, $[PTS] = \frac{\text{targets}}{\text{round}}$, $[RIE] = \frac{\text{casualty}}{\text{target}}$,

and $[RN] = [A] = [RL] =$ dimensionless

implies that

$$[TLI] = \frac{\text{rounds}}{\text{hour}} \times \frac{\text{targets}}{\text{round}} \times \frac{\text{casualty}}{\text{target}} = \frac{\text{casualty}}{\text{hour}}$$

Figure 24. Dimensional Analysis of TLI Equation

The analysis in Figure 24 supports the modification of the proposed effective range equation we previously presented. The use of Dupuy's original equation would produce the same numerical result; however, the TLI dimensions would be incorrect.

h. Dispersion Factor (DI) and the OLI Equation

The dispersion factor is used to convert TLI values into OLI values. This factor considers the effect that increased lethality of weapons has upon the dispersion of a force. Dupuy claims that over the course of history, the continually increasing lethality of weapons has been accompanied by an increasing dispersion of forces in battle. The following values are assigned to this factor by Dupuy based on major historical time periods.

ERA	DI
Ancient Armies	1
Napoleonic Era	20
American Civil War	25
World War I	250
World War II	3,000
1970's	4,000
1980's	5,000

Figure 25. Dispersion Factors

Dupuy firmly establishes [Ref. 5] that throughout time, there has been an increasing dispersion of forces. However, though there has been a general trend of increasing dispersion, this factor cannot be based on time era alone. There are many factors (such as terrain, training, mobility, etc.) that influence the dispersion of a force. These factors are defined by Dupuy as circumstantial variables, which are applied elsewhere in the model. Many World War II battles fought on Pacific islands had forces that were less dispersed than forces that fought in World War I, due to circumstantial factors. However, based upon Dupuy's dispersion factor, any force in World War II would have twelve times the dispersion of a World War I force. Therefore, to ensure all factors that influence the dispersion of a force are accounted for, *the circumstantial variables as well as the dispersion factor should reflect their effects upon force dispersion.*

We found the dispersion factor inconsistent because its application to the TLI values creates the wrong dimensions for the OLI values. Recall that Dupuy defines weapon lethality as *the inherent capability of a given weapon to kill personnel, or to make material ineffective in a given amount of time.* Thus, the dimensions of the OLI values must be casualties over time. Division of TLI by a dispersion factor conflicts with this definition. Dupuy asserts that an OLI value is obtained from a TLI value by dividing the TLI by *the average number of square meters per man in contemporary deployments.* The following dimensional analysis shows that implementing the dispersion factor gives OLI the wrong dimensions.

A dimensional analysis of the OLI equation gives:

$$[\text{OLI}] = \left[\frac{\text{TLI}}{\text{DI}}\right],$$

where $[\text{TLI}] = \left[\frac{\text{casualty}}{\text{hour}}\right]$, and $[\text{DI}] = \left[\frac{\text{meter}^2}{\text{target}}\right]$.

which implies,

$$[\text{OLI}] = \frac{\frac{\text{casualty}}{\text{hour}}}{\frac{\text{meter}^2}{\text{target}}} = \frac{\text{casualty}}{\text{hour}} \times \frac{\text{target}}{\text{meter}^2} = \frac{\text{casualty-target}}{\text{hour-meter}^2}$$

Figure 26. Dimensional Analysis of Non-Mobile Weapons OLI Equation

Based on this analysis, the dispersion factor should be a dimensionless constant. Recall that Dupuy sets dispersion for PTS as one square meter occupancy per target. Therefore, the area dimension in the dispersion factor is already incorporated into the PTS factor. However, over time, the area associated with dispersion changes, so some dimensionless dispersion multiplier seems necessary to account for the time change. Nevertheless, a dispersion factor with respect to time alone *would be the same* for both the red and the blue forces. Therefore, in forming the relative combat power ratio, the dispersion factor would simply cancel from each term. For this reason, the dispersion factor can be dropped from the model. Therefore, in the next chapter, when a sensitivity analysis is performed on key factors, the dispersion factor is eliminated from the model. (Note: inclusion of the factor *does not alter* the final numerical result.)

2. Mobile Weapons Factors

The following factors take into account those variables which influence a mobile fighting machine. Because of their ability to maneuver in battle, these weapons have inherent characteristics which increase their lethality. The TLI of these machines is calculated by summing the TLI for each individual weapon of the machine and multiplying that number by both a mobility and a radius of action factor. Next a punishment factor is added to this number and the result is multiplied by a rapidity of fire effect factor, a fire control effect factor, and an ammunition supply effect, and (for aircraft) a ceiling effect. Dupuy applies these effects to all mobile machines except self-propelled artillery because *it is assumed that the enhancing effects are at least in part offset by increased vulnerability and decreased reliability due to mechanical considerations not affecting towed artillery.*

$$TLI = \left[\sum_{i=1}^{n} \left[TLI\,(i) \times BM \times RA \right] + PF \right] \times RFE \times FCE \times ASE \times CL$$

where:

TLI = Total Lethality Index

BM = Battlefield Mobility

RA = Radius of Action

PF = Punishment Factor

RFE = Rapidity of Fire Effect

FCE = Fire Control Effect

ASE = Ammunition Supply Effect

CL = Ceiling Effect

n = number of non-mobile weapons on mobile machine

i = non-mobile weapon on mobile machine

Figure 27. Lethality Index Equation for Mobile Weapons

The following text analyzes each component of the mobile weapons operational lethality index.

a. Battlefield Mobility (BM)

Battlefield mobility takes into account the effect that the speed of a machine has on its lethality (Figure 28).[9] The faster a machine can carry its weapons,

[9]Notational differences exist between the thesis and [Refs. 4,6]. Dupuy defines battlefield mobility factor as M, a symbol commonly used to represent mass in a dimensional analysis. This thesis uses the symbol BM. Notational changes are used to reflect better the meaning of the term to enhance the understanding by the reader.

the greater its potential lethality. The equation in Figure 28 is consistent with that observation: the faster the road speed of a machine, the greater is its battlefield mobility factor.

Battlefield Mobility Equation: $BM = 0.2 \times \sqrt{K}$

where

K = maximum road speed of a vehicle in kilometers per hour, or the maximum air speed of a aircraft in kilometers per hour, whichever is applicable.

Figure 28. Battlefield Mobility Factor

A dimensional analysis (Figure 29) of the equation gives the dimension Dupuy must assign to the factor.[10] It is possible that the dimension of this factor will turn out to be incompatible with the multiplication and addition of other OLI factors. If the dimensional analysis of the OLI equation reveals such a discrepancy, then the constant (.2) within the battlefield mobility equation may become a dimensional constant. Dupuy does not explain how this constant (0.2) was obtained for this equation. A sensitivity analysis will be performed in the next chapter to determine if the value of this constant is significant.

---

[10]Dupuy also states that the mobility factor is represented by a second equation when the range is expressed in miles per hour. However, that equation, $BM = \sqrt{.1} \times R(\text{mph})$, is not equivalent to the original equation. We will analyze the first equation because it is the first one presented by Dupuy [Ref. 5:pp. 22]. It is also consistent with other OLI factors which are expressed in terms of kph.

A dimensional analysis of the mobility factor gives:

$$[BM] = 0.2 \times [\sqrt{K}],$$

where $[K] = LT^{-1}$, L = distance, and T = time,

which implies,

$$[BM] = .2 \times \sqrt{\frac{L}{T}} = L^{.5}T^{-.5}.$$

Figure 29. Dimensional Analysis of Battlefield Mobility Factor

b. Radius of Action (RA)

Also referred to as *endurance* [Ref. 5: pp. 27], the radius of action factor assumes that the longer is the operational range of a machine, the greater will be its lethality. For a ground vehicle, this factor is how far the machine can travel with a full load of fuel over what Dupuy calls "average terrain." For an aircraft, the factor is the maximum distance the aircraft can travel to complete a round-trip mission. The mathematical equation representing this factor is given in Figure 30.

Radius of Action Equation: $RA = 0.08 \times \sqrt{R}$

where:

R = maximum range of a ground machine in kilometers, or maximum radius of an aircraft mission in kilometers, whichever applies.

Figure 30. Radius of Action Factor

The farther a machine can carry its weapons, the greater its potential lethality. The equation in Figure 30 is consistent with that relationship. The farther the range of a machine, the greater should be its radius of action factor. A dimensional analysis (Figure 31) of the equation gives the dimensions we believe Dupuy assigns to the factor.[11] As with the battlefield mobility factor, it is possible that the dimensions of this factor will be incompatible with the multiplication and addition of other OLI factors. If the dimensional analysis of the OLI equation reveals a discrepancy, then the constant (0.08) within the radius of action equation may become a dimensional constant. Dupuy does not explain how this constant (0.08) was derived. A sensitivity analysis will be performed in the following chapter.

A dimensional analysis of the mobility factor gives:

$$[RA] = 0.08 \times [\sqrt{R}],$$

where $[R] = L$,

which implies $[RA] = 0.08 \times \sqrt{L} = L^{.5}$

Figure 31. Dimensional Analysis of Radius of Action Factor

[11]Dupuy also states that the range factor is represented by a second equation when the range is expressed in miles per hour. However, this equation, $RA = \sqrt{.01} \times R(mi)$, is not algebraically equivalent to the original equation. We will use the first equation because it is the first one presented by Dupuy [Ref. 5: pp. 22]. It is also consistent with other OLI factors which are expressed in terms of kph.

c. Punishment Factor (PF)

The QJM considers that the less vulnerable a weapon is to an opposing force's lethality, the greater will be that weapon's lethality. The ability to absorb punishment as a result of armor is reflected in the punishment factor. Dupuy bases the ability to take punishment solely on a fighting machine's weight, reasoning that the heavier is the armor, the more damage it can sustain. The punishment factor is defined in Figure 32.[12]

Punishment Factor: $$PF = \frac{\text{weight (tons)}}{4} \times \sqrt{2 \times \text{weight (tons)}}$$

$$= \frac{\text{weight}^{1.5}}{2\sqrt{2}}$$

Figure 32. Punishment Factor Equation

The value obtained by the punishment factor is then added to the product of all other OLI values for the weapons the machine carries multiplied by the mobility factor and the radius of action.

[12] The equation given in Figure 31 conflicts with the equation originally given by Dupuy [Ref. 5: pp. 23] for this factor. Dupuy's equation has a multiplicative constant of 3,000 which he defines as a reflection of the effect of troop dispersion in relation to firepower. Troop dispersion is later accounted for in the dispersion factor (DI) and therefore should be extracted from the equation.

$$TLI' = \sum_{i=1}^{n}\left[TLI\,(i) \times BM \times RA\right] + PF$$

where:

TLI' = Lethality Index constructed thus far

OLI = Operational Lethality Index

BM = Battlefield Mobility Factor

RA = Radius of Action Factor

PF = Punishment Factor

n = number of non-mobile weapons on machine

i = non-mobile weapon

Figure 33. Punishment Factor Application to TLI

We find the equation in Figure 33 to be unreasonable. First, as weight increases for a weapon, the punishment factor increases at a growing rate because the second derivative of PF with respect to weight is positive. If it is logical that the ability to absorb punishment is based on weight, then it should be at a *decreasing* rate. Otherwise, it would seem reasonable that a force would sacrifice all other weapon's performance factors, such as speed and endurance, for more weight. Second, there is no upper bound on the punishment factor. Yet there has to be a point where increased armor would not increase lethality. Another problem we observe is that PF is a function *solely* of weight. But the design of a fighting machine will vary the percentage of weight that is actually applied to armor protection. Finally, it would seem that the PF factor should be multiplicative, not additive. We next discuss that the addition of this factor to TLI is mathematically incorrect from a dimensional point of view.

A dimensional analysis of the TLI equation based upon the addition of PF gives:

$$[TLI] = \left[\sum_{i=1}^{n} TLI'\ (i)\right] \times [BM] \times [RA] + [PF]$$

where $[TLI'] = \left[\sum_{i=1}^{n} TLI(i)\right] = \frac{\text{casualty}}{\text{hour}}$, $[BM] = \frac{\text{kilometers}^{.5}}{\text{hour}^{.5}}$,

and $[RA] = \text{kilometers}^{.5}$.

Also,

$$[PF] = [TLI'] = \frac{\text{casualty}}{\text{hour}}.$$

Figure 34. Dimensional Analysis of Original TLI Equation

The analysis in Figure 34 supports our claim that the addition of the punishment factor is inconsistent. The resulting dimensions of Dupuy's equation in Figure 32 for punishment factor is weight to the 3/2 power, but this is not equal to casualties per hour as required by Figure 34.. Nor, can we establish a reasonable alternative equation for the punishment factor to produce these latter dimensions. Moreover, the PF terms cannot be added to the product of TLI $\times$ BM $\times$ RA terms, which fail to have the dimension of casualties per hour. Therefore, we conclude that the addition of the punishment factor is not supported by a dimensional analysis, and the multiplication of this factor within the TLI equation should be examined. It appears reasonable that multiplication of this factor with the others is more logical. If two weapons have different OLI's, and if certain armor protection doubles the lethality of one weapon, it should double the lethality of the other weapon as well. An analysis of the OLI equation for mobile-weapons will be performed later in the text and may provide insight to a reasonable formulation and application of the punishment factor.

d. Armored Vehicle Factors

The following three factors are dimensionless and apply to armored vehicles only.

(1) Rapidity of Fire Effect (RFE)

Dupuy bases this factor on his theory that *the speed with which the principal weapon of an armored vehicle can be fired and reloaded has a significant bearing on its survival in combat against other armored vehicles or antitank weapons*. The values for this factor are derived by Dupuy [Ref. 5: pp. 24] based upon the principal weapon's sustained hourly rate of fire. Dupuy does not explain how this factor was derived. The RFE is a dimensionless factor and seems to duplicate the rate of fire (RA) factor already used to account for the increased lethality of a principal weapon having a faster rate of fire. (Recall that RA is also based on a weapon's sustained hourly rate of fire). Therefore, it is not clear why Dupuy includes this factor in the calculations of OLI values since its value could be incorporated into the RA factor.

(2) Fire Control Effect (FCE)

Dupuy defines this factor as "*a judgment factor representing practical fire control effectiveness, not necessarily sophisticated equipment*". (This factor is ill defined. We also do not understand what the phrase "not necessarily sophisticated equipment" is in reference to, or what that has to do with fire control effectiveness.) He further asserts *that the values for tanks operational in 1973 have been related to the assessed fire control effectiveness of the US M60A1 tank, to which a seemingly arbitrary value of .9 has been assigned.* The characteristics of fire control effectiveness, upon which the factor is based, are not given. A formula does not appear to apply, and there are no factor values given for tanks other than the US M60A1. Therefore, a relationship identifying the fire control effectiveness factor has not been clearly defined and established.

(3) Ammunition Supply Effect (ASE)

This factor accounts for the amount of ammunition an armored vehicle can carry. The values for this factor are obtained from a graph given by Dupuy, where the values are based upon the amount of ammunition the vehicle can carry divided by the sustained rate of fire in an hour for the principal weapon. This is a reasonable factor. The greater the amount of ammunition the machine can carry, the more rounds it can shoot, thereby increasing its lethality.

e. Ceiling Effect (CL)

A reasonable factor that accounts for the operational ceiling of aircraft. The model assigns the value 1 to this factor if the operational ceiling is 30,000 feet. For every 1000 foot drop in the ceiling from 30,000 feet, the factor is decreased by 0.02; for every 1000 foot increase, the factor is increased by 0.005. Though this factor is reasonable, we recommend a slight modification to improve the model. Since the optimal operational altitude of aircraft varies among type, a value of 1 should be assigned for each type of aircraft if its operational ceiling equals its optimum altitude. Variance of the operational ceiling from the aircraft's optimal altitude could be taken into account as stated before.

f. Dimensional Analysis of the TLI Equation for Mobile Weapons

In the previous sections, each variable comprising the TLI equation for mobile weapons was examined for mathematical consistency and reasonableness. As with non-mobile weapons, Dupuy applies the dispersion factor to convert the TLI values to OLI values. As with the non-mobile weapon's OLI equation, we maintain that the dispersion factor should be dimensionless. Recall how the variables originally compose the TLI equation (Figure 27).

As with non-mobile weapons, the theoretical lethality index of mobile weapons is *expressed in casualties per hour.* Based upon this definition, when all the applicable variables are incorporated into the equation, the dimensions of TLI values should once again be the number of casualties per unit of time a weapon can inflict. Because the factors RFE, FCE, ASE, and CL are dimensionless, a dimensional analysis of the original TLI equation is identical to the one performed previously in the section on the punishment factor (Figure 34.). In that analysis of the punishment factor we argued that the punishment factor should be multiplicative. The following analysis of the TLI equation attempts to derive the appropriate application and dimensions of the punishment factor. The next figure presents a dimensional analysis of the TLI equation when the punishment factor is applied through multiplication.

$$[TLI] = \left[\sum_{i=1}^{n} TLI\ (i) \right] \times [BM] \times [RA] \times [PF]$$

where $[TLI] = \left[\sum_{i=1}^{n} TLI\ (i) \right] = \dfrac{\text{casualty}}{\text{hour}}$, $[BM] = \dfrac{\text{kilometers}^{.5}}{\text{hour}^{.5}}$,

$[RA] = \text{kilometers}^{.5}$ and $[PF]$ is unknown.

The dimension equation demands that $[BM] \times [RA] \times [PF]$ be dimensionless.

Figure 35. Dimensional Analysis of the TLI Equation for Mobile Weapons

There are two possible ways for the combination, [BM] × [RA] × [PF], to be dimensionless. Either all three factors are dimensionless, or the factors BM and RA maintain their original dimensions, giving PF the following dimension:

$$[PF] = \frac{\text{hour}^{.5}}{\text{kilometers}}$$

In the latter case, the resulting dimension is inconsistent with the original definition of the punishment factor in Figure 31. Nor, can we identify a correct relationship that makes this dimension for PF seem reasonable. The other alternative, making all three factors dimensionless, is examined next.

In our previous analyses of the BM and RA equations, we suggested that the constants within those equations could possibly be required to be dimensional constants. Our analysis in Figure 36 shows the dimensions required of those constants in order for the factors BM and RA to be dimensionless. The requirement that the constants become dimensional suggests that some factors affecting the TLI equation have been omitted. These missing factors are "incorporated into" the constants 0.2 and 0.08 giving them the correct dimensions, indicated in Figure 36, in order that the BM and RA factors be dimensionless.

Battlefield Mobility Equation: $$BM = 0.2\left(\frac{\text{hour}^{.5}}{\text{kilometers}^{.5}}\right) \times \sqrt{K}$$

and

Radius of Action Equation: $$RA = 0.08\left(\text{kilometers}^{-.5}\right) \times \sqrt{R}$$

Figure 36. Proposed BM and RA Equations

Then if PF is also a dimensionless factor, the TLI equation would be mathematically sound. However, before the dimensions given in Figure 36 can be applied to the constants 0.2 and 0.08, further research would have to be performed to justify this application. We conclude that the punishment factor as originally given by Dupuy is inconsistent in the TLI equation.

Next, an examination of two Soviet weapons is performed to compare the results of the non-mobile weapon equation with that of the mobile weapon equation. An examination of OLI values given by Dupuy [Ref. 5: pp. 226-227] for Soviet weapons found the calculation of mobile OLI values inconsistent with the model. The following examples will examine the OLI calculation of a non-mobile weapon (Soviet D-30 Howitzer) and a mobile weapon (Soviet T-54 Tank), and then compare the two values for consistency. Though we have shown that the dispersion factor has no effect on the model, a dispersion factor of 4,000 will be used in the examples for consistency with Dupuy's calculations. The first example illustrates the OLI calculation for the non-mobile Soviet D-30 Howitzer.

Example 3

OLI CACULATION FOR THE D-30 SOVIET HOWITZER

Characteristics

Caliber (mm)............122

Range (km)............15,000

OLI Factors

RF........................120	RN........................4.87
PTS....................1975	A............................0.9
RIE..........................1	R............................0.9

Discussion: Based on Dupuy's Non-Mobile OLI equation, the OLI value for this weapon should be :

$$OLI = \frac{RF \times PTS \times RIE \times RN \times A \times RL}{DI}$$

$$= \frac{120 \times 1975 \times 1 \times 4.87 \times .9 \times .9}{4000} = 234$$

This value is equal to the value obtained by Dupuy.

The next example replicates the exact values given by Dupuy for the mobile Soviet T-54 Tank.

Example 4

DUPUY'S OLI CACULATION FOR THE SOVIET T-54 TANK

Characteristics

Speed (km/hr) 33

Range (km)...............400

Weight (tons).............36

OLI Factors

Composite OLI's.......343.2	RFE.......................0.92
BM.........................1.15	FCE.........................0.9
RA...........................1.6	ASE..........................68
PF...........................76	AM.........................1.05

Discussion: Based on Dupuy's OLI factors, the OLI equation gives the following results:

$$\text{OLI} = \left[\left(\text{Composite OLI's} \times \text{BM} \times \text{RA}\right) + \text{PF}\right] \times \text{RFE} \times \text{FCE} \times \text{ASE} \times \text{AM}$$

$$= \left[\left(343.2 \times 1.15 \times 1.6\right) + 76\right] \times .92 \times .9 \times 68 \times 1.05 = 405$$

Note that the dispersion factor is already included in the composite OLI's for non-mobile weapons: here in the value 343.2.

A comparison of the OLI values for the howitzer and the tank gives a *realistic assessment* of the two weapons. That is, the tank is about twice as lethal as the howitzer, which seems reasonable. Nevertheless, a discrepancy for mobile weapons still occurs in Dupuy's calculations of OLI values for WWI and WWII tanks [Ref.5:

pp. 26-27]. The constant to account for troop dispersion is 3,000 for both groups of tanks. However, the dispersion for the WWI era was given as 250 by Dupuy [Ref, 5: pp. 28], contradicting the value of 3000. Based on this finding, and the other problems we identified for the PF, the mobile weapons OLI equation is inconsistent and requires further development and refinement.

IV. SENSITIVITY ANALYSIS OF THE MODEL BASED ON KEY OLI FACTORS

The previous chapter introduces the force strength equation and its components. A critical element of force strength, namely, the OLI value, was analyzed. Each factor comprising the OLI value was evaluated for mathematical consistency and military reasonableness. It is not clear how Dupuy obtained several of the OLI factors. This chapter presents a sensitivity analysis of how changes in those factors influence the outcome of two theoretical opposing forces. Sensitivity is measured according to how changes in these factors affect the relative combat power equation of two opposing forces. To focus on changes caused by a particular OLI factor, all other factors are set equal to one. This procedure temporarily eliminates the effect of CE (that is, CEV) and operational/environment factors within the equation. Recall the equations:

$$\text{Relative Combat Power (blue force)}: \quad RCP(b) = \frac{S(b) \times OE(b) \times \sqrt{CEV(b)}}{S(r) \times OE(r)}$$

$$\text{Relative Combat Power (red force)}: \quad RCP(r) = \frac{S(r) \times OE(r) \times \sqrt{CEV(r)}}{S(b) \times OE(b)}$$

Figure 37. Relative Combat Power Equation

Setting OE = 1 and CE (=CEV) = 1 gives the following equations we use in our sensitivity analysis:

$$RCP(b) = \frac{S(b) \times 1 \times 1}{S(r) \times 1} = \frac{S(b)}{S(r)}$$

and

$$RCP(r) = \frac{S(r) \times 1 \times 1}{S(b) \times 1} = \frac{S(r)}{S(b)}.$$

This procedure reduces the RCP equation to a simple ratio of the two force strengths.

Due to the inconsistencies found in the mobile weapons OLI equation from the previous chapter, it was concluded that this equation did not pass the test for reasonableness. A sensitivity analysis of mobile OLI factors cannot be performed due to this failure. Therefore, this chapter will focus solely on non-mobile OLI factors. Nevertheless, it is recommended that once a reasonable equation for mobile weapon's OLI is established, a sensitivity analysis should be performed on both the battlefield mobility and radius of action factors.

A. NON-MOBILE OLI FACTORS

The non-mobile weapon's factors are analyzed here. A sensitivity analysis is performed on the effective range, accuracy, and reliability factors. The first step is to define the two forces. Dupuy has computed the value of these factors for Soviet weapons [Ref.5: pp. 226-227]. The two opposing forces have different compositions of weapons. The force compositions and their OLI values are given in Table 1. The OLI values for each weapon are computed from the OLI equation for non-mobile weapons (Figure 26) using the values provided by Dupuy for the non-mobile weapon factors. The OLI values of each weapon are then multiplied by the quantity of that weapon in each force. This produces the OLI total for that weapon within each force. Then the weapon totals for each type of weapon are summed together to produce the total OLI value of each force.

Weapon	OLI	Quantity		Total OLI	
		Red	Blue	Red	Blue
Rifle, AK47	1638	100	4000	163840	6552000
Machine Gun, RPK	3328	10	0	33280	0
Machine Gun, PKM	3887	0	100	0	388700
Howitzer, D30	934893	8	0	7479151	0
Force OLI Totals				7676271	6940700

TABLE 1. COMPOSITION OF OPPOSING FORCES FOR NON-MOBILE WEAPONS

Thus the values of the relative combat power equations are .904 and 1.11, respectively, for the blue and red force:

Relative Combat Power (blue force): $$RCP(b) = \frac{S(b)}{S(r)} = \frac{6940700}{7676271} = .904$$

Relative Combat Power (red force): $$RCP(r) = \frac{S(r)}{S(b)} = \frac{7676271}{6940700} = 1.11$$

Figure 38. Modified Relative Combat Power Calculations for Non-Mobile Weapons: CE = OE = 1.

1. Effective Range (RN)

The effective range is examined first to determine how sensitive the relative combat power equation is to this factor. Recall the effective range factor equation:

$$\textbf{Range Factor:} \quad RN = \frac{1\text{ meter} + \sqrt{.001(\text{meters}) \times \text{Effective Range (meters)}}}{1\text{ meter}}$$

where:

Effective Range = 90% of maximum range

Figure 39. Range Factor Equation

The first step in the sensitivity analysis is to examine the effect of small changes in the constants .001 and 1 on the outcome of the relative combat power equation. This is done by decreasing and increasing each constant separately by 25 percent of the constant's original value. Then, for each change, new force OLI totals are calculated by applying the changes to each weapon within the force. The results are then used to calculate the resulting relative combat power values to determine if any significant change occurs. Although slight changes occur in the intermediate calculations, the final results showed no significant effects and establish that the battle outcome is not sensitive to relatively small changes in the constants .001 and 1 appearing in RN.

Our next step is to analyze the original values themselves. Dupuy states that the constant 1 in the RN equation represents *a norm for ranges, the length of a man's arm, called Normal Range*. In a time when weapons have such tremendous range, we question the significance of a man's average arm length. Does the absence of this *norm for range*, or perhaps doubling it, have a significant effect on the battle outcome? The following tables are an analysis of these changes to the RN equation. The table shows the weapons within a force and their original OLI values. Then the original OLI values for each weapon of the force are listed. These values are followed by the new force OLI values produced by the following equations that are obtained when the *norm for range* is deleted and doubled:

Range Factor: $$RN = \frac{0 \text{ meter} + \sqrt{.001(\text{meters}) \times \text{Effective Range (meters)}}}{1 \text{ meter}}$$

Range Factor: $$RN = \frac{2 \text{ meter} + \sqrt{.001(\text{meters}) \times \text{Effective Range (meters)}}}{1 \text{ meter}}$$

The bottom line of each table gives the combined force OLI total, based on the original RN equation, the RN with norm for range absent, and the RN with norm for range doubled.

Weapon	OLI	Total OLI		
		Original Norm for Range	Zero Norm for Range	Norm for Range doubled
Rifle, AK47	1638	163840	98304	229376
Machine Gun, RPK	3328	33280	19968	46592
Howitzer, D30	934893	7479151	5947973	9019493
Red Force OLI Totals		7676271	6066245	9295461

TABLE 2. OLI FOR RED FORCE BASED ON CHANGES TO CONSTANT 1 WITHIN THE RN EQUATION

Weapon	OLI	Total OLI		
		Original Norm for Range	Zero Norm for Range	Norm for Range Doubled
Rifle, AK47	1638	6552000	3932160	9175040
Machine Gun, PKM	3887	388700	256061	522301
Blue Force OLI Totals		6940700	4188221	9697341

TABLE 3. OLI FOR BLUE FORCE BASED ON CHANGES TO CONSTANT 1 WITHIN THE RN EQUATION

Based on the total force OLI values obtained from the three different equations, the resulting relative combat powers are given in the next table.

Norm for Range	Relative Combat Power	
	Blue Force: RCP(b):	Red Force: RCP(r):
Zero	.690	1.44
Original Value	.904	1.11
Doubled	1.04	.959

TABLE 4. RESULTING RELATIVE COMBAT POWER DUE TO RN

As Table 4 shows, the relative combat power values are significantly affected by changes to the constant *norm for range*. Most notable is the change in the theoretical victor of the battle from the red force to the blue force when the *norm for range* is doubled. This battle analysis asserts that the value assigned to *norm for range* is critical to the predictions of the model. Therefore, *norm for range* should be established scientifically as an accurate and viable value for use within the model.

For the constant .001, Dupuy again fails to indicate how this factor is derived. To determine if the model is sensitive to this constant, we examine changes to it by one order of magnitude, i.e., 0.1 and 10. The same procedure is used to obtain values in the following tables, as with *norm for range*.

Weapon	OLI	Total OLI		
		.001	.0001	.01
Rifle, AK47	1638	163840	96622	376401
Machine Gun, RPK	3328	33280	19626	76456
Howitzer, D30	934893	7479151	3416674	20797759
Red Force OLI Totals		7676271	3532922	20797759

TABLE 5. OLI FOR RED FORCE BASED ON CHANGES TO CONSTANT .001 WITHIN THE RN EQUATION

Weapon	OLI	Total OLI		
		.001	.0001	.01
Rifle, AK47	1638	6552000	3864880	15056023
Machine Gun, PKM	3887	388700	214094	942857
Blue Force OLI Totals		6940700	4078974	15998880

TABLE 6. OLI FOR BLUE FORCE BASED ON CHANGES TO CONSTANT .001 WITHIN THE RN EQUATION

Based on the total force OLI values obtained from the three different equations, we obtain:

Constant Value	Relative Combat Power	
	Blue Force: RCP(b):	Red Force: RCP(r):
.0001	1.16	.866
.001	.904	1.11
.01	.769	1.30

TABLE 7. RESULTING RELATIVE COMBAT POWER BASED ON RN

As these values show, the relative combat power values are significantly affected by changes to this constant. Most notable is the change in the theoretical victor of the battle from the red force to the blue force when the constant is changed by a factor of 0.1. This battle analysis asserts that the value assigned to the constant is critical to the results of the model. Therefore, this constant should be established scientifically as an accurate and viable value for use within the model.

2. Accuracy (A)

The next factor to be examined for sensitivity is the accuracy factor. Recall that Dupuy defines this factor as *an estimated probability that a weapon will hit a target given that the weapon is aimed correctly at the target*. This factor excludes the inability of the user to aim the weapon correctly and assumes a perfect shot. Based on this definition, we feel it is quite possible that values assigned to this factor for a weapon could be incorrect. An estimate of 95% accuracy for a weapon could be off by 25% if the actual accuracy of the weapon is about 70%. The following analysis examines the affect to the relative combat power if a weapon's estimated accuracy is off by 25% of its actual accuracy. Utilizing the original battlefield situation, one by one, each weapon within the battle is given a new accuracy value, and a resulting relative combat power value is calculated (shown in Table 8). Then that weapon's accuracy value is returned to its original value before the next weapon is examined. For example, the original value for the accuracy factor of the AK-47 Rifle is 0.8. A 25% decrease in the value is 0.55, resulting in new red and blue OLI force totals of 7625071 and 4893200, respectively. Based on these new OLI force totals, resulting relative combat power equations are calculated (shown in Table 9). For the change to the AK47 Rifle accuracy value, the resulting values are .904 and 1.11 The following tables summarize the results for four different non-mobile weapons.

Weapon	Original Accuracy Value	New Accuracy Value	Red Force Total OLI	Blue Force Total OLI
Rifle, AK47	0.8	0.55	7625071	4893200
Machine Gun, RPK	0.8	0.55	7665871	6940700
Machine Gun, RPM	0.8	0.55	7676271	6819231
Howitzer, D30	0.9	0.65	5598729	6940700

TABLE 8. TOTAL FORCE OLI BASED ON CHANGES TO THE ACCURACY FACTOR

The resulting relative combat power values are:

Δ Change to:	Resulting Relative Combat Power	
	Blue Force: RCP(b)	Red Force: RCP(r)
No Weapons	.904	1.11
Rifle, AK47	.642	1.56
Machine Gun, RPK	.905	1.10
Machine Gun, RPM	.888	1.13
Howitzer, D30	1.24	.807

TABLE 9. RESULTING RELATIVE COMBAT POWER VALUES DUE TO ACCURACY FACTOR

As these values show, the relative combat power values can be significantly affected by changes to this variable. Changes in certain weapons (the howitzer in this example) are able to change the theoretical victor of the battle from the red force to the blue force when the accuracy variable is changed by 25%. Our analysis finds that the value assigned to this variable is critical to the outcome predicted by the model. Therefore, the value assigned to the accuracy factor should be established scientifically.

3. Reliability (RL)

The final non-mobile weapon OLI factor to be examined for sensitivity is the reliability factor. Recall that Dupuy defines this factor as *an estimated probability that a weapon will hit a target given that the weapon is aimed correctly at the target.* This factor excludes the inability of the user to aim the weapon correctly and assumes a perfect shot. Based on this definition, it is quite possible that values assigned to this factor for a weapon could be incorrect. An estimate of 95% reliability for a weapon could be off by 25% if the actual reliability of the weapon is 70%. As with accuracy, a sensitivity analysis is performed to examine the effect on the relative combat power if a weapon's estimated reliability is off by 25% of its actual accuracy. For the battle in our example, each weapon has a reliability value equal to that weapon's accuracy value. Therefore, the results of the reliability analysis are identical to the results of the accuracy analysis. That is, changes to this variable do significantly affect the relative combat power values. Changes in certain weapons (the howitzer in this example) are able to change the theoretical victor of the battle from the red force to the blue force when the reliability variable is changed by 25%. Our analysis reveals that the value assigned to this variable is critical to the outcome predicted by the model. Therefore, the value assigned to this variable should be established scientifically.

V. DECOMPOSITION OF OPERATIONAL VARIABLES

A. INTRODUCTION OF OPERATIONAL VARIABLES

In Chapter II, the basic equations of the QJM were introduced and discussed. Recall from that chapter the relative combat power equation (Figure 40).

Relative Combat Power (blue force) : $$RCP(b) = \frac{S(b) \times OE(b) \times \sqrt{CEV(b)}}{S(r) \times OE(r)}$$

Relative Combat Power (red force) : $$RCP(r) = \frac{S(r) \times OE(r) \times \sqrt{CEV(r)}}{S(b) \times OE(b)}$$

where:

S = force strength

OE = operational/environmental factors

CEV = combat effectiveness

Figure 40. Relative Combat Power Equation

This chapter analyzes the operational variables, which are key factors within the relative combat power equation. Dupuy defines operational variables as those factors which *represent actions of combat forces that influence the employment of the force and its weapons*. These factors are:

- Posture (U)
- Mobility (MO)
- Vulnerability (VU)
- Fatigue (B)
- Surprise (SU)

These variable are applied to the OE factor through multiplication:

$$OE = U \times MO \times VU \times B \times SU \times E\,, \quad \text{where } E = \text{environmental factors.}$$

A. POSTURE (U)

This factor accounts for the assumption that a defensive posture is stronger than an offensive posture. If a force (blue) is attacking another force (red) that is of equal combat power and in a fortified defensive position, then the red force has the advantage. Given all other factors being equal, the red force should be the victor. The following table gives the values of the posture factor based upon a force's stance.

POSTURE	
Attack	1.0
Defense (hasty)	1.3
Defense (prepared)	1.5
Defense (fortified)	1.6
Withdrawal	1.15
Delay	1.2

TABLE 10. POSTURE FACTORS

The posture factor is applied in the model as a multiplier of force strength. This factor appears to be reasonable. As the table shows, the more defensive the posture of a force, the larger is the factor, resulting in a greater force strength and combat power. However, though this factor is reasonable, Dupuy does not establish how its values are derived. Therefore, we will perform a sensitivity analysis on this factor to determine how sensitive the model results are to the posture values.

B. MOBILITY (MO)

Dupuy establishes the mobility factor to take into account the mobility of a force based upon three considerations:

1. The inherent characteristics of the force (i.e., how the force is composed.)

2. The degrading effects of environmental conditions (e.g., terrain and weather).

3. The relationship between the inherent mobility characteristics and environmental influences.

Dupuy expresses the inherent mobility of an attacking force with the equation in Figure 41. That is, M_a accounts for the first consideration.

$$M_a = \sqrt{\frac{\left[\dfrac{(N_a + 20J_a + W_{ia}) \times m_{ya}}{N_a}\right]}{\left[\dfrac{(N_d + 20J_d + W_{id}) \times m_{yd}}{N_d}\right]}}$$

where:

M = inherent force mobility

N = personnel strength

J = vehicular constant (∝ to non-fighting vehicles)

W_i = armored firepower

m_y = air superiority effect

d = defending force

a = attacking force

Figure 41. Inherent Force Mobility Equation

The inherent mobility of the defending force (M_d) is *always* set equal to one. We believe the inherent mobility equation to be unreasonable. Based on the equation in Figure 41, the inherent mobility of a force increases with the size of the force. We feel the opposite is true (at least beyond some lower limit). There is a real world tendency for larger forces to be less mobile than smaller forces (consider, for instance, moving all the equipment). Therefore, the equation should reflect a decrease in inherent mobility, vice increase, as the size of a force grows beyond some point. The equation also adds together three dimensionally different factors, N + 20J + W_i, in violation of one of our fundamental dimensional analysis principles.

The degrading effects of environmental conditions, terrain and weather, are reflected in the following tables.

TERRAIN FACTORS	WEATHER FACTORS
Rugged - Heavily Wooded.....0.4	Dry - Sunshine - Extreme Heat.0.9
Rugged - Mixed.................0.5	Dry - Sunshine - Temperate.....1.0
Rugged - Bare...................0.6	Dry - Sunshine - Extreme Cold.0.9
Rolling - Heavily Wooded.....0.6	Dry - Overcast - Extreme Heat..1.0
Rolling - Mixed.................0.8	Dry - Overcast - Temperate.....1.0
Rolling - Bare...................1.0	Dry - Overcast - Extreme Cold .0.9
Flat - Heavily Wooded.........0.7	Wet - Light - Extreme Heat.....0.9
Flat - Mixed......................0.9	Wet - Light - Temperate.........0.8
Flat - Bare, hard.................1.05	Wet - Light - Extreme Cold.....0.8
Flat Desert.......................0.95	Wet - Heavy - Extreme Heat....0.5
Rolling Dunes...................0.3	Wet - Heavy - Temperate........0.6
Swamp - jungle.................0.3	Wet - Heavy - Extreme Cold....0.5
Swamp - mixed or open........0.4	
Urban.............................0.7	

Figure 42. Terrain and Weather Mobility Factors

These factors do appear reasonable. As either terrain or weather conditions deteriorate, these multiplicative factors decrease in value.

The final step Dupuy performs to obtain total force mobility is to form a relationship between the inherent mobility of a force and the environmental influences, using the following equation. This equation is used to obtain the total mobility of the attacking force. The defending force mobility is always set equal to one.

$$MO_a = M_a - (1 - r_m \times h_m)(M_a - 1)$$

where:

MO = total force mobility

M = inherent force mobility

r_m = terrain effect

h_m = weather effect

Figure 43. Total Mobility Equation

For the equation in Figure 43 to be reasonable, it must reflect an increase in total force mobility when inherent mobility increases, and a decrease when environmental conditions deteriorate. To examine this criteria the equation in Figure 43 is rewritten.

$$\begin{aligned} MO_a &= M_a - (1 - r_m \times h_m)(M_a - 1) \\ &= M_a - \left[M_a - r_m h_m M_a - 1 + r_m h_m\right] \\ &= r_m h_m M_a + 1 - r_m h_m \\ &= r_m h_m (M_a - 1) + 1 \end{aligned}$$

Figure 44. Simplified Total Mobility Equation

As can be seen from Figure 44, the force mobility factor appears to be reasonable. An increase in inherent force mobility does cause an increase in total force mobility, and reduced environmental conditions do decrease total force mobility.

C. VULNERABILITY (VU)

The QJM's vulnerability factor reflects a force's vulnerability based upon its personnel strength, combat deployment exposure, relative firepower of the opposing force, air superiority, and increased exposure in amphibious and river crossing situations. The first step the QJM performs to determine vulnerability is to determine the inherent vulnerability of a force (Figure 45).

$$V = N \times c \times \left(\sqrt{\frac{S_e}{S_f}}\right) \times v_y \times v_r$$

where:

V = inherent friendly force vulnerability

N = friendly personnel strength

S = force strength

c = exposure effect = $\frac{u_v}{r_u}$

u_v = posture effect on vulnerability

r_u = terrain effect on vulnerability

v_y = air superiority effect on vulnerability

v_r = effect of amphibious and river crossings

f = friendly force

e = enemy force

Figure 45. Inherent Force Vulnerability

The inherent vulnerability is then related to force strength via the following equation:

$$VU_f = 1 - \left[\frac{V_f}{S_f}\right]$$

$$= 1 - \left[\frac{N \times c \times \left(\sqrt{\frac{S_e}{S_f}}\right) \times v_y \times v_r}{S_f}\right]$$

$$= 1 - \left[\frac{N \times c \times S_e^{.5} \times v_y \times v_r}{S_f^{1.5}}\right]$$

Figure 46. Force Vulnerability Equation

This variable appears reasonable with one exception. It should be noted that this variable is nonintuitive because a decrease in the vulnerability of a force (O_E) should reflect an increase in the vulnerability factor. This is because the factor is a multiplier. Thus, decreased vulnerability means increased combat power. This factor *does* reflect a decrease in combat power when enemy force strength increases, and an increase in vulnerability when the force strength of a force increases. The factor is also reasonable in that amphibious and river crossing situations, lack of air superiority, poor force posture, and bad terrain all increase the vulnerability of a force. However, as the personnel size of a force increases, the equation indicates a decrease in the vulnerability factor, resulting in a decrease in combat power. This disputes the age old adage of "strength in numbers."[13]

D. FATIGUE (B)

Fatigue is also called the *exhaustion factor* [Ref. 5: pp. 223]. It accounts for the effect fatigue has on a force's combat power. The idea is reasonable: a force that is fresh and well rested performs better than one that has been under continuous stress

[13]Note that the S factor only contains count of weapons in a force. It does not include the N term which counts personnel strength.

with little or no rest, resulting in a greater combat power. The value of the fatigue factor ranges from 0 to 1; the factor value being 1 for a fresh or fully rested unit. The value of the fatigue factor is decreased according to eight different rules established by Dupuy [Ref. 5: pp. 223]. At the conclusion of an engagement, a new exhaustion factor is calculated for each side based upon those rules. We conclude that the rules do account for circumstances that would affect fatigue of a force. However, the values assigned by rules are questionable. There are many circumstances and influences that would affect the fatigue of a force. We feel that constants cannot be assigned to these rules.

E. SURPRISE (SU)

The surprise factor is applied to indicate effects which occur when one force achieves tactical surprise over another. The effects of tactical surprise are applied to inherent force mobility and vulnerability by the following table.

	Surpriser's Inherent Mobility Characteristics (M_{sur})	Surpriser's Vulnerability (V_{sura})	Surprised's Vulnerability (V_{surd})
Complete Surprise	$\sqrt{5}$	.4	3
Substantial Surprise	$\sqrt{3}$	.6	2
Minor Surprise	$\sqrt{1.3}$	.9	1.2

Figure 47. Tactical Surprise Factors

Dupuy specifies that the factors in Figure 47 are applied to the inherent mobility and vulnerability factors once the level of surprise is defined. However, he does not specify how the surprise factors are applied to the equation. Therefore, it is recommended that clarification be given on these surprise factors so their effects and validity can be evaluated.

VI. CONCLUSIONS

Various mathematical models are used extensively by military leaders to analyze ground combat. The QJM is one such model which has received considerable recent attention from both the military and operations analysis community. This thesis continues the investigation initiated in Captain Joseph Ciano's thesis which examined the major equations and submodels of the QJM.

The present thesis further explored the QJM by focusing on the validity of the Force Strength Equation and other secondary equations within the model. A dimensional analysis was performed on the equation, its variables and submodels to check their dimensionally consistency and reasonableness. A sensitivity analysis was also performed on key variables to ascertain the importance and impact of assigned variable parameter values.

Examination of the Force Strength Equation and its variables (termed factors) determined that the equation does account for key factors contributing to force strength. However, dimensional analysis of the factors revealed that several of them are either mathematically inconsistent or militarily questionable. The correction of these deficiencies will certainly enhance the reasonableness of this model and could result in a more useful model for analyzing ground combat forces. However, it must be pointed out that the QJM model, as it presently stands, is seriously flawed. The discrepancies we found are summarized as follows:

A. DISCREPANCIES OF CERTAIN FORCE STRENGTH COMPONENTS

- **Rate of Fire (RF):** Rules of thumb given by Dupuy to determine RF were estimates. Such estimates are not necessary if actual sustained rates are known. Moreover, the original rules of thumb give RF the dimension of *rounds per* *minute*, conflicting with the original definition (see pp. 19). To correct these discrepancies, proposed rules of thumb were determined (Figure 17).

• **Potential Targets per Strike (PTS):** PTS for high explosive weapons should be based upon actual lethal area of the weapon, not the caliber of the weapon (see pp. 21).

• **Effective Range (RN):** The original equation attempts to add components having different dimensions, and does not express the factor as a value per each meter (see pp. 23). The following equation is suggested to replace it:

Range Factor: $$RN = \frac{1\text{ meter} + \sqrt{.001(\text{meters}) \times \text{Effective Range (meters)}}}{1\text{ meter}}$$

where:

Effective Range = 90% of maximum range

Figure 48. Proposed Range Factor Equation

• **Dispersion Factor (DI):** The dimensions of this factor were found to be inconsistent with the OLI equation. It was shown that this factor, if applied to the equation, should be dimensionless. However, because this factor is equivalent for *both* forces, it cancels from each term when applied through multiplication to the combat power ratio. Therefore, we suggest that the dispersion factor be dropped from the model (see pp. 29).

• **Punishment Factor (PF):** The original equation, which applies the punishment factor through addition, was found to be mathematically inconsistent. An alternative of applying this factor as a multiplier was examined for mathematical consistency. The examination revealed that this application could be achieved in one of two ways. First, the punishment factor could be given the following dimension:

$$[PF] = \frac{hour^{.5}}{kilometers}$$

However, the resulting dimension is inconsistent with the original punishment factor definition. Second, the factors BM, RA and PF could all be made dimensionless, yielding consistency (see pp. 36). But this approach requires justification also.

- **Battlefield Mobility (BM):** Based upon the discrepancies found with the punishment factor, the constant within the BM equation may be required to be a dimensional constant. However, this requirement indicates that some other factors affecting the TLI equation have been omitted (see pp. 32).

- **Radius of Action (RA):** Based upon the discrepancies found with the punishment factor, the constant 0.08 within the RA equation may be required to be a *dimensional constant*. However, this requirement would indicate that some factors affecting the TLI equation have been omitted (see pp. 34).

- **Ceiling Effect (CL):** Because different types of aircraft have different optimal operational altitudes, it is proposed that this factor be based upon variances from its optimal operational altitude, vice a set ceiling of 30,000 feet (see pp. 40).

- **Mobile Weapon OLI Equation:** Based on the inconsistency of the punishment factor as being additive, and other discrepancies found with Dupuy's OLI calculations for mobile weapons, we determined that this equation was mathematically inconsistent and requires reformulation (see pp. 40).

The next process performed on the model was a sensitivity analysis on key OLI factors. A sensitivity analysis was performed on these factors because Dupuy did not specify how these factors were derived. This analysis is intended to determine if the results of the model are sensitive to the values of these factors. If so, then accurate, scientific derivation of these factors is required. The results of this sensitivity analysis were as summarized below.

B. RESULTS OF THE SENSITIVITY ANALYSIS

• **Effective Range (RN):** The analysis established that the *norm for range* (namely 1), within the RN equation did not provide sensitivity for very small changes. The analysis, did reveal however, that the relative combat power values are significantly affected by changes to the constant of an order of magnitude. The same conclusion was drawn concerning the other constant, .001, within the equation. Therefore, the values for these constants should be firmly established scientifically (see pp. 50).

• **Accuracy (A):** The analysis examined how the relative combat power equation would be affected if an estimate of the accuracy factor assigned to a weapon was in error by 25%. The RCP equation changed significantly for changes to some, but not all, weapons. Nevertheless, that result did establish that the value assigned to this variable should be established scientifically (see pp. 54).

• **Reliability (RL):** The analysis was equivalent to the sensitivity analysis performed on the accuracy factor. Because the reliability value for each weapon had the same value as the accuracy value of that weapon, the results were identical to those obtained for the accuracy value. Thus, the values assigned to a weapon's reliability factor should be established through a careful scientific evaluation (see pp. 55).

• **Battlefield Mobility (BM):** A sensitivity analysis of this factor was not performed due to the inconsistencies found with the mobile weapon OLI equation. However, once that equation has been corrected, it is recommended that a sensitivity analysis of the constant (0.2) within the equation be performed.

• **Radius of Action (RA):** A sensitivity analysis of this factor was not performed due to the inconsistencies found with the mobile weapon OLI equation. However, once the equation has been corrected, it is recommended that a sensitivity analysis of the constant (0.08) within the equation be performed.

Our final analysis of the model was an examination of the QJM's operational variables. These are the variables that Dupuy defines as *representing the actions of combat forces that influence the employment of the force and its weapons*. Each of these variables are multipliers within the combat power equation.

C. FINDINGS OF OPERATIONAL VARIABLES

• **Posture (U):** This variable appears reasonable. Increased combat power is reflected for a better force posture. However, it is recommended that a sensitivity analysis be performed on this factor (see pp. 58).

• **Mobility (MO):** This variable was found to be inconsistent. The inherent mobility term within this variable reflects an increasing functional relationship between inherent mobility and force size: as force size increases, inherent mobility increases. We found this relationship unreasonable. The equation is also dimensionally inconsistent (because variables of different dimensions are added together, see pp. 58).

• **Vulnerability (VU):** This variable seems reasonable, with one exception. The equation reflects an increase in vulnerability as a force's personnel strength grows. This results in reduced combat power. We contend, however, that vulnerability *decreases* with increased personnel strength (see pp. 62).

• **Fatigue (B):** This factor appears reasonable. All other things being equal, a force with well rested troops or reinforcements has a higher combat power than a force with exhausted, shell-shocked troops (see pp. 64).

• **Surprise (SU):** The application of the effects of tactical surprise in the model are not well defined. Dupuy gives values for the effects of tactical surprise to inherent force mobility and vulnerability. However, it is never clearly defined just how these effects are applied within the model. Therefore, it is recommended that application of this factor within the model be clearly stated.

The analysis performed in this thesis better establishes exactly what military applications are practical for the QJM. Clearly the model is inappropriate to predict the outcomes of a battle. The model is too sensitive to values assigned to its constants to give realistic precise predictions. For this same reason, the model would be inappropriate for attempting to optimize the best combination of weapons for a battle situation. However, as a decision aid, once the model is corrected it can demonstrate certain trends in battles or successful units thereby enabling the identification of certain factors which may have a greater degree of influence on the battle field. This information could give military decision makers a greater understanding of some key factors to be carefully evaluated for prospects of greater performance and success on the battlefield.

The QJM provides a unique approach to analyzing ground combat. The model takes into account the many factors present on the battle field based on historical combat data. With reformulation of some of the submodels and continued refinement, the QJM should establish itself as an important, useful tool to the military community.

LIST OF REFERENCES

1. Giordano, F.R. and Weir, M.D., *A First Course in Mathematical Modeling*, Brooks/Cole Publishing Company, 1985.

2. Clausewitz, C., *On War*, Translated by Michael Howard and Peter Paret, Princeton University Press, 1976.

3. Ciano, J.F., *The Quantified Judgment Model and Historic Ground Combat*, Master's Thesis, Naval Postgraduate School, Monterey, California, September 1988.

4. Dupuy, T.N., *Understanding War, History and Theory of Combat*, Paragon House, 1987.

5. Dupuy, T.N., *Numbers, Predictions & War: The Use of History to Evaluate and Predict the Outcome of Armed Conflict*, Hero Books, 1985.

INITIAL DISTRIBUTION LIST

1.	Defense Technical Information Center Cameron Station Alexandria, VA 22304-6145	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, CA 93943-5002	2
3.	Dr. Samuel H. Parry, Code 55Py Department of Operations Research Naval Postgraduate School Monterey, CA 93943-5000	1
4.	Captain Wayne Hughes, USN, Ret., Code 55Hl Department of Operations Research Naval Postgraduate School Monterey, CA 93943-5000	1
5.	Dr. Maurice D. Weir, Code 55Wc Department of Applied Math Naval Postgraduate School Monterey, CA 93943-5000	1
6.	LTC Bard Mansager, USA, Code 55Ma Department of Operations Research Naval Postgraduate School Monterey, CA 93943-5000	1
7.	Commandant of the Marine Corps Code TE 06 Headquarters, U.S. Marine Corps Washington, D.C. 20360-0001	1
8.	CG MCDDC Attn: Dr. Branstein and History Branch Quantico, VA 22314-5001	2
9.	CG MCDDC Attn: Col. Breede and Intelligence Center Quantico, VA 22314-5001	2

10. Center for Strategic and International Studies 2
Attn: Dr. James Blackwell
1800 K Street, N.W.
Washington, D.C. 20006

11. Data Memory Systems, INC. 1
Attn: Colonel Trevor N. Dupuy, USA, Ret.
10392 Democracy Lane
Fairfax, VA 22030

12. The Military Conflict Institute 1
Attn: Donald S. Marshall
2842 Ashby Avenue
Berkeley, CA 94705-2341

13. Dr. Robert Helmbold 1
4505 Avamere Street
Bethesda, MD 20814

14. Deputy Undersecretary of the Army 1
for Operations Research
Room 2E261, Pentagon
Washington, D.C. 20310

15. Commander 1
U.S. Army TRADOC Analysis Command
Attn: ATRC-ZD (Mr. Bauman)
Fort Leavenworth, KS 66027-5200

16. Director 1
Attn: Mr. E.B. Vandiver III
U.S. Army Concepts Analysis Agency
Bethesda, MD 20814

17. Bell Hall Library 1
U.S. Army Combined Arms Center
Fort Leavenworth, KS 66027

18. Richard L. Humphrey 1
Naval Surface Warfare Center
10901 New Hampshire Avenue
Silver Spring, MD 20903-5000

19. Captain Jerome A. Clark, USMC 2
Mathematics Department
U. S. Naval Academy
Annapolis, MD 21402-5000