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Estimation of time to maximum supercooling during dynamic frazil ice formation

Steven F. Daly and Kathleen D. Axelson

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Time to maximum supercooling is a parameter that can be easily measured during experiments on the dynamic, nonequilibrium stage of frazil ice formation. Mercier has determined an analytical expression for the time to maximum supercooling that depends only on the four basic system parameters: the rates of heat loss, seeding, turbulent dissipation and secondary nucleation. Mercier's analytical expression is applied to a number of experiments in which the time to maximum supercooling was measured. In each of the experiments, the heat loss rate and turbulent dissipation rate were reported or could be determined from the experiment description. The secondary nucleation was set at the value of 4×10^{10} nuclei/erg suggested by Mercier, and the seeding rate optimized to reproduce the experimental results. An inverse relationship was found between the coldroom temperature at which the experiment was conducted and the seeding rate. The optimized seeding rates varied from 2.7 to 7.5×10^{-5} crystals/cm^2 s. The implications for frazil ice formation in rivers and streams are discussed.

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PREFACE

This report was prepared by Steven F. Daly and Kathleen D. Axelson, Research Hydraulic Engineers, of the Ice Engineering Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory. Funding was provided by the Office of the Chief of Engineers, Directorate of Civil Works, under CWIS 32397, *Ice Control at Intakes*.

The report was technically reviewed by Dr. S. Colbeck and Dr. G. Ashton (both of CRREL). The authors acknowledge the work done by Dr. R.S. Mercier, which forms the basis of this report. In addition, they acknowledge the experimenters whose work is described in this report.

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Estimation of Time to Maximum Supercooling During Dynamic Frazil Ice Formation

STEVEN F. DALY AND KATHLEEN D. AXELSON

INTRODUCTION

Frazil ice is the major form of ice in northern rivers and streams. It has a significant negative impact on winter navigation, hydropower production and the use of water intakes, and contributes to ice jam flooding. Three rather general stages of frazil evolution can be identified. The first is the dynamic nonequilibrium stage, characterized by supercooled water, turbulent flow, rapid growth of disk-shaped crystals and the creation of new crystals by secondary nucleation. The second stage follows the first in time and is the evolution and transport stage. The third stage is characterized by stationary floating ice covers.

The first stage has been comprehensively described and reproduced in the laboratory, and can be quantitatively discussed through the use of a number continuity equation and a heat balance equation (Daly 1984). These equations are dimensionally incompatible and strongly nonlinear. Simplified forms of these equations are used with success to predict the steady-state performance of industrial crystallizers. However, the dynamic stage of frazil ice is inherently a nonsteady process and has resisted analysis.

Mercier (1984) developed a global model to describe reactive transport of dissolved and suspended solids in lakes, rivers and coastal areas. In an extension of the work of Daly (1984), Mercier applied his model to frazil ice. In addition, Mercier suggested certain simplifications that could be applied in limited situations. To explain the sensitivity of the global model to variations in the system parameters, for example, Mercier determined an analytical expression for the time required for a turbulent mass of water undergoing constant heat loss and a constant seeding rate to reach the point of maximum supercooling. The expression retains the basic physics and produces an analytical result amenable to simple numerical solution. In this paper we will describe the development of this analytical expression, report the numerical solution, review laboratory experiments in which the time to reach maximum supercooling was measured, explore the influence of the system parameters, apply the expression to the laboratory results, and discuss the implications for frazil growth in rivers and streams.

THEORY

The analytical expression proposed by Mercier assumes a homogeneous, isotropic water body. Initial frazil production is assumed to be caused by seed crystals, whose size is near the critical radius, introduced into the water at a constant seeding rate. The value of the critical radius can be calculated using thermodynamic principles (Forest and Sharma 1987) and is estimated to be about 0.4 μm for the range of supercooling found in rivers and streams (Mercier 1984). The expression assumes a completely mixed system, with uniform temperature and seed crystals quickly transported into suspension throughout the water column. Additional crystals are produced by secondary nucleation. It is assumed that secondary nucleation is dominated by new crystals created through collisions of large crystals by the action of turbulent shear. The size of the new crystals is assumed to be close to the critical radius.

The thermal energy balance is determined in the following manner. The water is supercooled by a constant heat loss rate per unit volume. Above the equilibrium temperature, any seed crystals introduced will melt. Below this temperature, seed crystals can grow and produce new crystals by secondary nucleation. It is assumed that as the ice crystals grow, the latent heat of fusion released is negligible before the point of maximum supercooling is reached. At the point of maximum supercooling, the rate of latent heat of fusion released equals the heat loss rate. After this point, the water temperature rises as the frazil ice growth continues. The point of the maximum increase in temperature corresponds to the maximum rate of frazil growth. Figure 1 illustrates this process.

A convenient parameter to characterize frazil ice growth is the time until maximum supercooling t_s. For particles less than 100 μm in size (which is a good assumption for frazil crystals before the maximum supercooling), frazil ice crystal growth prior to maximum supercooling may be described by the following equation

$$\frac{dr}{dt} = \frac{hT}{\rho_i L} \tag{1}$$

where r = crystal radius
t = time
h = heat transfer coefficient
T = supercooled water temperature
ρ_i = density of ice
L = latent heat of fusion.

A good approximation for the heat transfer coefficient (Daly 1984) is

$$h \approx \frac{k}{r} \tag{2}$$

where k is the thermal conductivity of water. Assuming that the latent heat released by the growing frazil is negligible up until the time of maximum supercooling, Mercier described

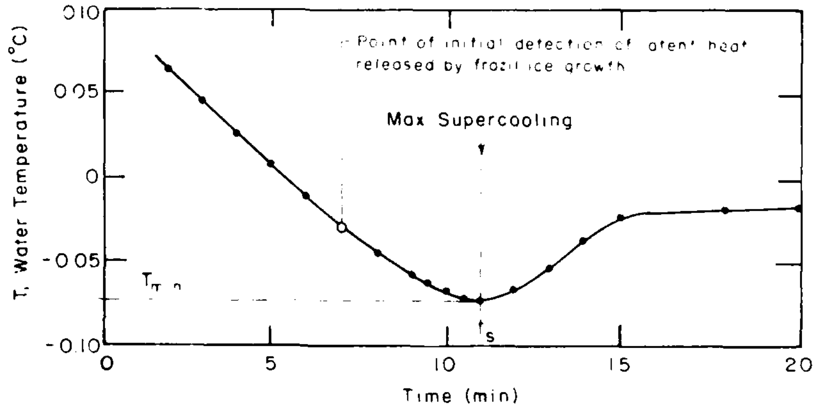


Figure 1. Typical curve of water temperature versus time during frazil ice growth.

the temperature of the supercooled water by the following equation

$$T = \frac{Q\,t}{\rho\, C_p} \tag{3}$$

where Q = heat loss rate per unit volume
C_p = heat capacity of the water
ρ = density of water.

Substituting eq 3 into eq 1, and assuming that the critical radius r_c is approximately 0, we find the crystal radius r at any time t using the equation

$$r(t) = \left(\frac{kQ}{\rho\rho_i\, LC_p}\right)^{1/2} t = \lambda t \tag{4}$$

where λ is the coefficient of crystal growth. The general equation for secondary nucleation is (Daly 1984)

$$I_T = Z E_T \tag{5}$$

where I_T = number of secondary nuclei produced per unit time per unit volume
Z = number of crystals produced per unit of collision energy
E_T = total rate of energy transfer by the various mechanisms of collision.

The mechanisms can include collisions caused by fluid shear, turbulent shear and buoyancy, and collisions with boundaries. For the present case, it is assumed that only collisions between two crystals are important, and eq 5 may be written in the form

$$I_T(t) = Z\int_{r_c}^{\lambda t}\int_{r_j}^{\lambda t} c(r_i, r_j)\,dr_i\,dr_j \tag{6}$$

where r_i and r_j are the radii of the two particles colliding and $c(r_i,r_j)$ is the collision function (Mercier 1984).

Taking secondary nucleation into account, Mercier gives a simplified particle number density distribution as follows

$$g(r,t) = \begin{cases} \left[I_T\left(t - \frac{r}{\lambda}\right) + I_v\right]/\lambda & r_c \le r \le \lambda t \\ 0 & r > \lambda t \end{cases} \tag{7}$$

where $g(r,t)$ is the crystal number density distribution and I_v is the crystal seeding rate per volume of water. To approximate the number of crystals produced per unit collision energy, it is assumed that the primary cause of collisions is turbulent shear, and that collisions of the largest particles (where $r \sim \lambda t$) will be the most important. At $r \sim \lambda t$, $g(r,t)$ will be approximately I_v/λ. Assuming $r_c \sim 0$, this approximation yields for secondary nucleation

$$g(r,t) = I_T(t) = 0.018 Z\rho_i \left(\frac{\varepsilon}{\nu}\right)^{3/2}\left(\frac{I_v}{\lambda}\right)^2 \left[1.67 - 0.45 \ln\lambda t\right](\lambda t)^{10} \tag{8}$$

where ε is the turbulent energy dissipation rate and ν is the kinematic viscosity of water (see Mercier [1984] for details). Noting that the mass of ice can be written

$$M_i(t) = \rho_i \int_0^{\lambda t} \frac{4}{3}\pi r^3 g(r,t)\,dr \tag{9}$$

and given the approximation of the number density distribution of eq 7, after substituting from eq 8, the mass of ice is

$$M_i(t) = \frac{\pi}{3}\rho_i \lambda^3 I_v t^4 + \left(\frac{4}{3}\pi\right)^2 (0.018)\, Z\rho_i^2 \left(\frac{\varepsilon}{\nu}\right)^{3/2} I_v^2 \lambda^{11} (4.3\times10^{-4})\, t^{14}. \tag{10}$$

At the time of maximum supercooling,

$$\frac{dM_i}{dt} = \frac{Q}{L}. \tag{11}$$

Then, by finding the derivative of eq 10 and solving for t_s, an expression for the time until maximum supercooling is

$$t_s^3(1+\zeta t_s^{10}) = \frac{0.24Q}{\rho_i L \lambda^3 I_v} \tag{12a}$$

where ζ is a coefficient defined as

$$\zeta = 4.5\times 10^{-4} \rho_i Z \left(\frac{\varepsilon}{\nu}\right)^{3/2} I_v \lambda^8 \tag{12b}$$

and λ is defined in eq 4. Equation 12 is an analytical expression for the time to maximum supercooling in terms of four system parameters: heat loss rate, crystal seeding rate per unit volume, turbulent energy dissipation rate and number of crystals produced per unit collision energy. Two of the parameters, heat loss rate and turbulent energy dissipation rate, can be measured and are controlled in every frazil ice experiment. The seeding rate is much more difficult to control and measure. The number of crystals produced per unit energy is a material property that has been estimated for other materials, but can only be estimated by numerical experiment for frazil ice.

The suitability of this analytical expression may be verified as the time to maximum supercooling has been reported for several different experiments. So, we solved eq 12 and evaluated the relative influence of the parameters. The results were compared to the experimental data as shown below.

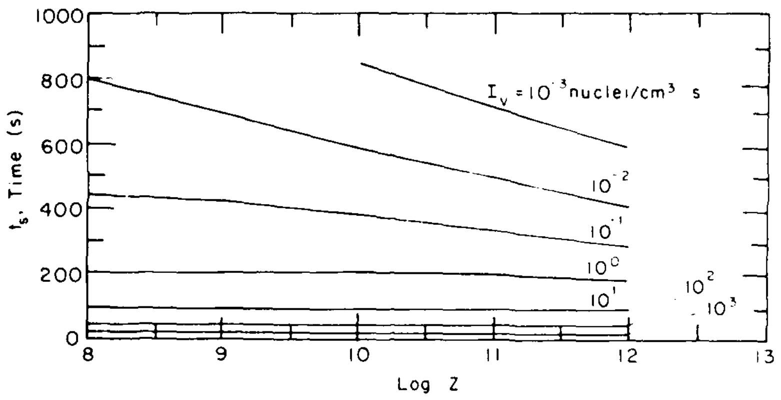
SENSITIVITY OF ANALYTICAL SOLUTION TO SYSTEM PARAMETERS

Using a computer program that we developed, we solved eq 12 for t_s. Heat loss rate, crystal seeding rate per unit volume, number of crystals produced per unit collision energy and turbulent energy dissipation rate were varied. Values of the other parameters used are shown in Table 1. To verify the accuracy of the program, the results were compared to those reported by Mercier. The comparison showed that the program performed satisfactorily.

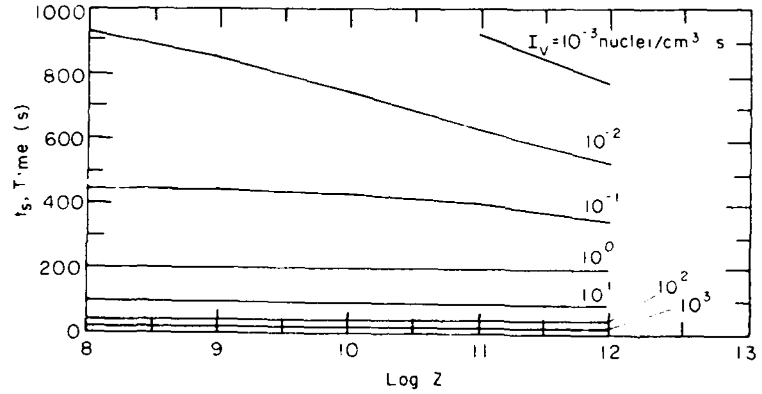
Table 1. Values of parameters used to determine sensitivity of the analytical solution to system parameters (after Batchelor 1967).

Parameter	*Symbol*	*Value*
Density of ice	ρ_i	0.92 g/cm^3
Density of water	ρ	1 g/cm^3
Latent heat of fusion	L	330 J/g
Thermal conductivity	k	5.6×10^{3} J/cm s °C
Specific heat capacity	C_p	4.217 J/g °C
Kinematic viscosity	ν	1.787×10^{-2} cm^2/s

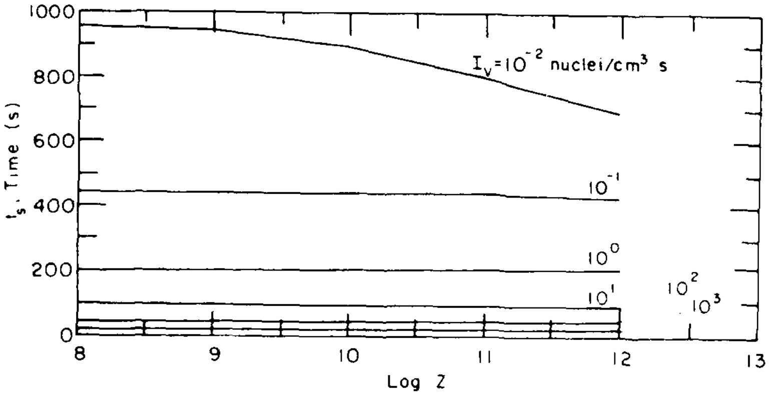
Following verification, we evaluated the relative influence of each parameter on the time to maximum supercooling by varying each over several orders of magnitude, but within a realistic range. The results are shown in Figure 2 for a constant heat loss rate, with seeding rate, turbulent energy dissipation rate and number of crys-



a. $\varepsilon = 100\ cm^2/s^3$.



b. $\varepsilon = 10\ cm^2/s^3$.



c. $\varepsilon = 1\ cm^2/s^3$.

Figure 2. Effects of varying system parameters on the time to maximum supercooling (t_s) *for a heat loss rate Q of* 10^{-4} *J/cm³ s and three turbulent energy dissipation rates* ε.

tals produced per unit collision energy varying as indicated. The time to maximum supercooling is dominated by the crystal seeding rate I_s and heat loss rate Q. The number of crystals produced per unit collision energy Z was less of a factor unless the crystal seeding rate was low.

The number of crystals produced per unit collision energy has a greater effect on time to maximum supercooling at higher turbulent energy dissipation rates (high ε). This is reasonable since more collisions would be expected at higher turbulence and hence secondary nucleation would play a greater role in frazil ice formation, particularly if the initial seeding rate is small.

EXPERIMENTAL DATA

Frazil ice experiments have been conducted under a variety of conditions, and time to maximum supercooling has been reported for several experiments. In each experiment, although the heat loss rate and turbulent energy dissipation are set by the experimenter, the values of these parameters have not often been reported. However, based on the information provided, it is possible to calculate these values as described below.

The heat loss rate may be determined from the rate of temperature decline of the turbulent water prior to the formation of frazil. To determine the turbulent energy dissipation rate, the Reynolds number of the flow in the experiment is computed from the average velocity and a characteristic length. For pipe flow, the diameter of the pipe is the characteristic length, and for open channel flow, the hydraulic radius is the characteristic length. The friction factor is determined from the Reynolds number of the flow using an estimated surface roughness (Chow 1959). For pipe flow, the turbulent energy dissipation rate may be found using

$$\varepsilon = \frac{f^{3/2}\bar{u}^3}{2d} \tag{13}$$

where f = friction factor
$\bar{u}$ = average velocity
d = pipe diameter.

For open channel flow, the turbulent energy dissipation rate is a function of the friction velocity u_*, which is the product of the friction factor and the mean velocity

$$u_* = \bar{u}\left(\frac{f}{8}\right)^{1/2} \tag{14}$$

The turbulent energy dissipation rate is then

$$\varepsilon = \frac{u_*^3}{KR_h}\left[\ln\left(\frac{u_* R_h}{\nu}\right) - 1\right] \tag{15}$$

where K is von Karman's constant, generally set equal to 0.4, and R_h is the hydraulic radius.

Table 2 contains a summary of data from the frazil ice experiments described below. Figure 3 depicts the range of values for heat loss rate and turbulent energy dissipation rate found from the experimental data.

Michel (1963) reportedly carried out over 80 frazil ice growth experiments in an outdoor recirculating flume constructed of a Plexiglas channel, 30.5 cm wide by 30.5 cm deep by 6.7 m in length, connected to a 15-cm-diameter Plexiglas pipe. A 20-cm-diameter variable speed pump with cast iron casing provided circulation. Many of the tests were done at

Table 2. Summary of experimental data.

Experimenter	*Description of experiment*	*Test no.*	t_s *(min)*	Q *(J/cm³ s)*	ε *(cm²/s³)*
Michel (1963)	Outdoor recirculating flume.	27	3.3	1.2×10^{-3}	33
Carstens (1966)	Recirculation flume in −10°C coldroom; propeller in tank in -10°C coldroom.	6A	4	1.19×10^{-3}	13
		5A	8	3.9×10^{-4}	—
Hanley and Michel (1977)	Cylindrical tank with paddles in coldroom. Varied coldroom temperature and velocity.	−2°C	38.1	0.9×10^{-4}	3
		−5°C	21.6	1.9×10^{-4}	3
		−10°C	14.7	3×10^{-4}	3
		−20°C	8.1	7.1×10^{-4}	3
Tsang and Hanley (1985)	Warm air jacket around tank, except test C, which was in recirculating flume. Varied salinity. Seeded with shavings of ice. Average coldroom temp. −1 °C.	A2 (48‰)	118.8	1.9×10^{-3}	2
		B1–3 (23‰)	43.6	2×10^{-3}	2
		B2–3 (11‰)	36.4	2×10^{-3}	2
		B3–4 (fresh)	18.1	1.9×10^{-3}	2
		C–4 (ocean)	123.9	1.1×10^{-3}	0.7
Mueller (1978)	Supercooled water first in agitation tank in warm room.	E08	—	—	1375
		E09	—	—	4667

night, and air temperature ranged from −32 to −6.7°C. Water was pumped into the flume and then circulated. Water temperature was measured every 15 seconds by a differential thermometer, reportedly accurate to ± 0.0025°C. Data were reported for only one test, in Michel's Figure 2. The water temperature decline rate was reported and we calculated the heat loss rate from this. Assuming that the experiment was dominated by the turbulence that occurred immediately downstream of the pump, we calculated the turbulent energy dissipation rate for this flow area. The reported average flow was 375 gal./min (0.025 m^3/s), yielding a velocity in the pipe of 73 cm/s, a Reynolds number of 8.3×10^4 and a friction factor of 0.023. The turbulent energy dissipation rate was then found using eq 13. Time to maximum supercooling was measured from the figure.

A recirculating oval flume, 20 cm wide by 30 cm deep by 600 cm long, located in a −10°C coldroom, was used by Carstens (1966) to study frazil ice. Water depth was 20 cm. The bottom and sides of the acrylic flume were insulated, and cooling of the water was aided by a fan blowing along the straight portion of the flume. Circulation in the flume was induced by a variable speed propeller. Water temperature was measured by a hand-held mercury thermometer marked to 0.01°C, immersed 5 to 10 cm. In a typical test, the water temperature decline rate was measured and the heat loss rate was then determined. For the experiments reported, the average flow velocity was 50 cm/s, yielding a Reynolds number of 1.8×10^4 and friction factor of 0.019. We calculated the turbulent energy dissipation rate using eq 15. Unfortunately, for several experiments shown in Carsten's Figure 5a, the turbulent energy dissipation rate could not be calculated because the velocity of the water was not reported. However, the time to maximum supercooling could be determined from this figure and is included for comparison.

Hanley and Michel (1977) conducted frazil ice experiments in a stainless steel tank, 120 cm in diameter and 76 cm deep, at air temperatures of −2, −5, −10 and −20°C. Water depth in the tank was not reported. Paddles attached to an axis and located 25 cm above the bottom of the tank induced rotational velocity in the tank. For these experiments, the coldroom temperature was set, and then the paddles started. Water temperature was measured by a thermistor placed just below the water surface. We calculated the heat loss

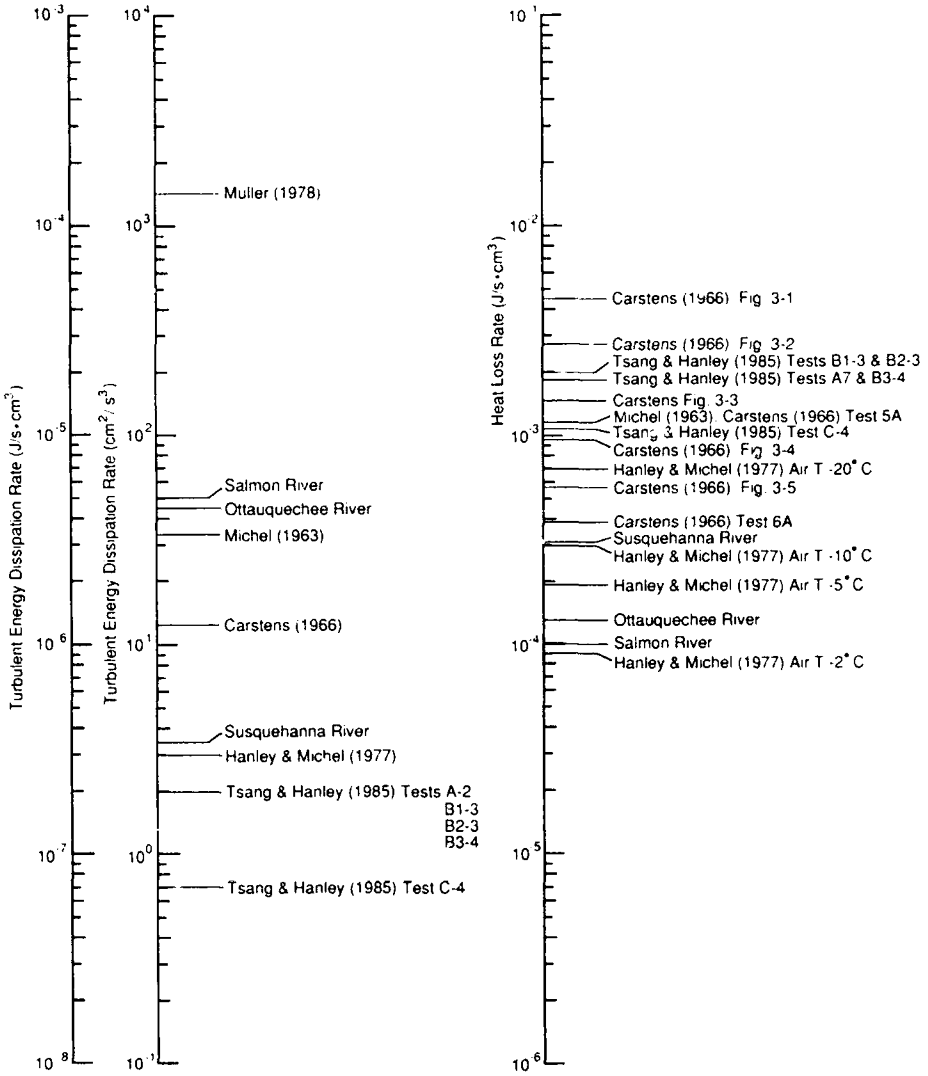


Figure 3. Estimated values of turbulent energy dissipation rate and heat loss rate for experimental data and three rivers.

rates from the reported water temperature decline rates. The results presented by Hanley and Michel were reportedly averaged over all velocities; the minimum velocity at which frazil growth was noted was 24 cm/s, and the maximum reported velocity was 73 cm/s. Assuming a water depth in the tank of 70 cm and an average velocity of 48.5 cm/s, we calculated the Reynolds number at 8.8×10^4 and estimated the friction factor to be 0.0145. Equations 13 or 15 yielded the same turbulent energy dissipation rate. Time to maximum supercooling was not reported. Total time of supercooling was reported, however, as was the time from the minimum temperature to the end of supercooling. We could estimate time to maximum supercooling by subtracting the latter from the former.

Frazil ice formation in fresh water, ocean water and artificial seawater of varying salinity was examined by Tsang and Hanley (1985). They placed a rectangular Plexiglas tank, 38 cm long by 25.5 cm wide by 15 cm deep, equipped with a stirrer at one end to provide turbulence, in a –15°C coldroom. A horizontal plate, 0.75 times the length of the tank, was placed at mid-depth to produce vertical recirculation currents, and a jacket of air, slightly above 0°C, was placed around the bottom and sides of the tank. Cooling was provided by a fan that pushed air across the surface of the water. Tsang and Hanley measured water temperature with a thermometer reportedly calibrated to 0.0001°C, with repeatability to 0.001°C. In these experiments, the water was first cooled to the previously selected temperature and then seeded either by a ball of ice or by scrapings from ice. We picked one sample from each of their five groups of experiments for analysis here. We calculated heat loss rates for each sample as we did for Hanley and Michel. All but the group C experiments took place in the tank described above, with a reported average velocity of 15 cm/s. Water depth was not reported and we assumed it to be 15 cm. For these experiments, we calculated the Reynolds number to be 3.9×10^3, with a resulting friction factor of 0.031. We estimated turbulent energy dissipation rates using eq 13. Group C (ocean water) tests took place in a recirculating flume, 15 cm wide by 13 cm deep, with a water depth of 11 cm, and average velocity of 15 cm/s. They reported the Reynolds number as 8.54×10^3. From this, we estimated the friction factor to be 0.025. The turbulent energy dissipation rate was estimated using eq 15. Time to maximum supercooling was reported.

Mueller (1978) studied the nucleation process of frazil ice using an agitating tank, 17.2 cm long by 12 cm wide by 20 cm deep, in a coldroom kept slightly above 0°C. The tank was surrounded by a jacket through which a coolant was circulated. Agitation was provided by a grid submerged in the tank. In these experiments, the water was supercooled to the desired degree, the coolant circulation was stopped, agitation was begun and the supercooled water seeded. Water depth ranged from 20 to 20.5 cm. He recorded temperature using linear type thermistors that were accurate to a reported ±0.02°C in air and ±0.002°C in water. Turbulent energy dissipation rates were reported. These experiments were not designed to allow the time to maximum supercooling to be determined as the water was not seeded until the maximum supercooling was achieved. We report the turbulent energy dissipation rates only to provide a comparison with the other experiments.

APPLICATION TO EXPERIMENTAL DATA

We applied the analytical expression to the experimental data cited above, assuming a constant number of crystals produced per unit energy of 4×10^{10} nuclei/erg as suggested by Mercier. Given the experimental values of heat loss rate and turbulent energy dissipation rate for each experiment (Table 2), we optimized the seeding rate to produce the reported times to maximum supercooling. The results are shown in Figure 4 and listed in Table 3. The optimized values of I_v, which range from 10^{-1} to 10^{-6} crystals/cm³ s, appear to be reasonable based on the experimental conditions.

A more consistent means of comparing the optimized seeding rates is to compare the results as a seeding rate per unit surface area, rather than a seeding rate per unit volume. The seeding rate per unit surface area can be simply calculated, given the design of the experimental apparatus. Michel's experiment had the highest optimized surface seeding rate (I_s) of 2.7 crystals/cm² s. Although the rate of seeding was not measured or considered, it is not difficult to imagine that conducting an experiment outdoors, as these were, would be a situation in which a large number of seed crystals could be present. The experiments of Carstens conducted in a large coldroom, had a comparably high value for the optimized surface seeding rate of 1 crystal/cm² s. The presence of Carstens' recirculating oval flume, driven by a propeller, in this coldroom could have contributed to the production of seed crystals. The optimized seeding rates for the experiments of Hanley and Michel are less than those of either Michel or Carstens. However, the results for these experiments show a consistent increase in seeding rate with a decrease in the coldroom air temperature. While this type of relationship has not been measured or reported elsewhere, such an inverse

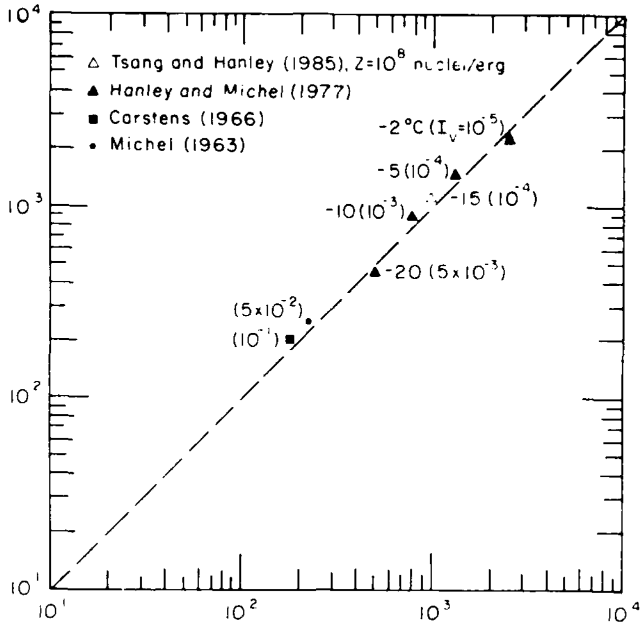


Figure 4. Calculated time versus reported time for experimental data using optimized seeding rate I_v.

Table 3. Comparison of experimental and computed results.

Source	Q ($J/cm^3\,s$)	ε (cm^2/s^3)	Experimental t_s (min)	Model t_s (min)	I_v $\left(\frac{crystals}{cm^3\,s}\right)$	I_v* $\left(\frac{crystals}{cm^2\,s}\right)$
Michel (1963)	1.2×10^{-3}	33	3.3	3.0	10^{-1}	2.7
Carstens (1966)	1.19×10^{-3}	13	4	3.7	5×10^{-2}	1.0
Hanley and Michel (1977)						
-2°C	9×10^{-4}	3	38.1	40.5	10^{-5}	7×10^{-4}
-5°C	1.9×10^{-4}	3	21.6	21.6	10^{-4}	7×10^{-3}
-10°C	3×10^{-4}	3	14.7	13.0	10^{-3}	7×10^{-2}
-20°C	7.1×10^{-4}	3	8.1	7.5	5×10^{-3}	3.5×10^{-1}
Tsang and Hanley (1985)						
48‰	1.9×10^{-3}	2	118		$< 10^{-6}$	
23‰	2×10^{-3}	2	43.6		$< 10^{-6}$	
11‰	2×10^{-3}	2	36.4		< 10-6	
fresh	1.9×10^{-3}	2	18.1	18.3	5×10^{-6}	7.5×10^{-5}
ocean	1.1×10^{-3}	0.7	124		$< 10^{-6}$	

* Michel, assumed depth—27 cm; Carstens, reported depth—20 cm; Hanley and Michel, assumed depth—70 cm; Tsang and Hanley assumed depth—15 cm.

relationship between seeding rate and room air temperature seems likely. Figure 5 is a plot of the coldroom air temperature and the calculated seeding rate for each experiment for which these values were reported or could be calculated. Air temperature for the experiment given in Michel was not reported; therefore, the average of reported air temperatures for all tests is plotted.

The optimized seeding rates for the experiments conducted by Tsang and Hanley are very low, less than 10^{-6} crystals/cm³ s. We do not know how much the design of their experiment contributed to these optimized seeding rates. The experiments were conducted in a box surrounded on all sides, except for the open top, by a warm air jacket. A horizontal

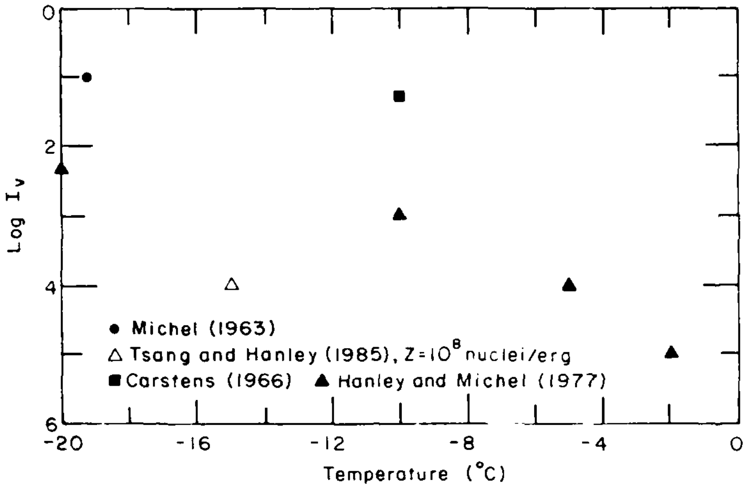


Figure 5. Plot of optimized seeding rate (I_v) versus experimental air temperatures.

baffle separated a top layer and a bottom layer. It may have been that the seed crystals were not present in their coldroom, or that the crystals were not fully mixed in their tank, or that the seed crystals melted near the walls of the tank.

Unfortunately, it is not really possible to know how realistic these optimized values are as there are no measured values reported either from the laboratory or the field. Osterkamp (1977) reports that hexagonal plate ice crystals ranging from 60 to 350 μm in size were observed on the air above a supercooled Alaskan stream. Their concentrations ranged from 6 to 6×10^4 crystals/m^3. It is possible to estimate a surface seeding rate from the concentration of crystals suspended in the air C_A as

$$I_s = C_A u_t \qquad 16)$$

where u_t is the terminal fall velocity of the ice crystals. Estimating a terminal fall velocity of the largest and smallest ice crystal as 17 cm/s and 5 cm/s, respectively (Pruppacher and Klett 1980), yields a seeding rate per unit surface area of 3×10^{-5} to 1.0 crystals/cm^2 s. This range encompasses the optimized surface seeding rates (Table 3) and provides a certain degree of support for them.

TIME TO MAXIMUM SUPERCOOLING IN NATURAL WATER BODIES

No field data on time to maximum supercooling exist for rivers. In fact, the temperatures of supercooled water are rarely reported. To investigate the implications of the analytical analysis for rivers and streams, we examined three frazil producing rivers: the Ottauquechee River in Vermont, the Susquahanna River in Pennsylvania, and a 160-km reach of the Salmon River near Salmon, Idaho. Using average values for channel slope, width, depth, discharge and Manning's n, we calculated the average velocity for each river assuming uniform flow. The heat loss rate was calculated assuming an average heat transfer rate of 20 W/m^2 °C for the Ottauquechee and Susquehanna Rivers and 10 W/m^2 °C for the Salmon River, as suggested by Zufelt.* The estimates of turbulent energy dissipation rate are

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* Personal communication with J. Zufelt, CRREL, 1988.

Table 4. Analytical solution applied to three frazil producing rivers, $Z = 4 \times 10^{10}$ nuclei/erg, I_v in crystals/cm³s.

			Time (min)			
River	$Q (J/cm^3 s)$	$\varepsilon (cm^2/s^3)$	$I_v = 0.001$	$I_v = 0.01$	$I_v = 0.1$	$I_v = 1$
Salmon	1×10^{-4}	50	13.8	9.6	6.3	3.4
Ottauquechee	3.28×10^{-4}	44	9.3	6.5	4.4	2.7
Susquehanna	1.31×10^{-4}	3.5	17.0	11.6	6.9	3.3

shown with the experimental data in Figure 3. Using the computer program, we calculated time to maximum supercooling. The results, shown in Table 4 for various seeding rates, indicate that time to maximum supercooling in rivers is quite short, on the order of minutes.

Considering the magnitude of the estimated times, field measurement of time to maximum supercooling will be quite difficult. For example, assuming a crystal seeding rate per unit volume of 10^{-1} crystals/cm³ s, the time to maximum supercooling for the Salmon River would be about 6 minutes. At an average winter velocity of about 1.73 m/s, supercooling would occur within a reach of 660 m. This length is about 0.4% of the 160-km reach of river in question. This may have implications for the placement of ice control structures that rely on adfreezing of frazil ice.

SUMMARY

Mercier (1984) presented a complex model of frazil ice growth. Our investigation examined a simplified analytical expression that described the time until maximum supercooling, a parameter often measured in laboratory experiments, in terms of four system parameters: heat loss rate, seeding rate, turbulent energy dissipation rate and the number of crystals produced per unit of collision energy. To explore the influence of the system parameters, a computer program to solve for time to maximum supercooling was developed. The program was verified using Mercier's data. Using a fixed value of the number of crystals produced per unit collision energy suggested by Mercier (4×10^{10} nuclei/erg), and the experimental values of heat loss rate and turbulent energy dissipation rate, we employed the program to optimize the seeding rate to produce the reported times to maximum supercooling. This process resulted in times close to those reported for reasonable values of seeding rate. An analysis of the data indicated that the seeding rate and heat loss rate have the most influence on time to maximum supercooling, while the number of crystals produced per unit energy and the turbulent energy dissipation rate have less influence.

We found an inverse relationship between the coldroom air temperature at which the experiment was conducted and the seeding rate. Using estimated values of heat loss rate and turbulent energy dissipation, we computed a range of values for time to maximum supercooling for the Salmon, Ottauqueechee and Susquehanna Rivers. The results indicate that time to maximum supercooling in rivers and streams may be very short, and that supercooled regions may be difficult to locate and measure.

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