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Exponential Modeling Using Combined Forward and Backward Prediction

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Chapter 1

Introduction

A sequence of complex data can be modeled using a sum of exponentials model. One approach to this problem is to assume this model for the data sequence, use forward or backward linear prediction along with total least squares to estimate the poles of the model, and then use a total least squares method to determine the amplitude associated with each pole. This approach usually involves overestimating the order of the model and then ignoring the extraneous poles which result. In order to better estimate the true poles, Singular Value Decomposition (SVD) can be performed on the linear prediction matrix, the number of singular values needed for the model can be kept, the rest of the singular values being set to zero, and then a new linear prediction matrix is formed.

If forward prediction is used then the extraneous poles which are estimated will appear inside the unit circle [1]. If the data sequence is a stable one then the true poles also appear inside the unit circle and thus the two can be confused. To avoid this problem backward prediction can be used. The extraneous poles will appear outside the unit circle and thus the

true poles can be identified as the ones appearing inside. But if the data is a sequence created from poles both inside and outside the unit circle, then neither forward nor backward prediction will allow for such an easy determination of extraneous poles.

One method for separating true poles from extraneous poles is to use both forward and backward prediction. The true poles will appear in both estimates, but the extraneous poles will lie inside the unit circle for one method and outside for the other. This report considers the use of such a method, and compares it with the standard forward and backward methods.

Chapter 2

Forward Prediction

Assume that N data points are generated from M complex exponentials with white Gaussian noise added.

$$y(n) = \sum_{k=1}^M a_k p_k^n + w(n) \quad n = 1, \dots, N. \quad (2.1)$$

Here, p_k is the k th pole and a_k is the amplitude associated with that pole. The forward linear prediction equations can then be set up as follows [1,2,3].

$$\begin{bmatrix} y(N-1) & \dots & y(N-L) \\ \vdots & & \vdots \\ y(L) & \dots & y(1) \end{bmatrix} \begin{bmatrix} \hat{b}_{f1} \\ \vdots \\ \hat{b}_{fL} \end{bmatrix} = - \begin{bmatrix} y(N) \\ \vdots \\ y(L+1) \end{bmatrix} \quad (2.2)$$

or

$$Y_f \hat{b}_f = -c_f \quad (2.3)$$

where L is the order of prediction, and $\hat{b}_f$ is the coefficient vector of the polynomial $\hat{B}_f(z)$ given by

$$\hat{B}_f(z) = 1 + \hat{b}_{f1}z^{-1} + \dots + \hat{b}_{fL}z^{-L} \quad (2.4)$$

The solution first involves forming $[c_f : Y_f]$, performing a Singular Value Decomposition (SVD) on it, truncating all but the first M singular values,

and then reforming to get a better estimate $[\hat{c}_f : \hat{Y}_f]$, since only M modes are kept. Now $\hat{b}_f$ can be solved for using the pseudoinverse of $\hat{Y}_f$,

$$\hat{b}_f = -\hat{Y}_f^+ \hat{c}_f. \quad (2.5)$$

Now the estimated poles can be determined from the zeros of $\hat{B}_f(z)$,

$$\hat{p}_{fi} = \text{root}_i(\hat{B}_f(z)) \quad i = 1, 2, \dots, L. \quad (2.6)$$

Once the poles have been determined, the amplitude equations can be formed.

$$\begin{bmatrix} \hat{p}_{f1}^1 & \dots & \hat{p}_{fL}^1 \\ \vdots & & \vdots \\ \hat{p}_{f1}^N & \dots & \hat{p}_{fL}^N \end{bmatrix} \begin{bmatrix} \hat{a}_{f1} \\ \vdots \\ \hat{a}_{fL} \end{bmatrix} = \begin{bmatrix} y(1) \\ \vdots \\ y(N) \end{bmatrix} \quad (2.7)$$

or

$$\hat{P}_f \hat{a}_f = d_f \quad (2.8)$$

Least squares can now be used to estimate the amplitudes.

$$\hat{a}_f = (\hat{P}_f^H \hat{P}_f)^{-1} \hat{P}_f^H d_f \quad (2.9)$$

A remaining issue is how the true poles are separated from the $M - L$ extraneous poles. It is known [1] that the extraneous poles will lie inside the unit circle. If the true poles lie outside the unit circle, then separation is easy. However, if the true poles lie inside the unit circle, it is sometimes difficult to separate them from extraneous poles. One method involves choosing a radius threshold, r_t , and accepting any pole whose radius exceeds r_t as a true pole. Another method involves estimating the amplitude of all L poles, and choosing the modes whose amplitudes or energies exceed some amplitude threshold. A method related to the latter one is to keep all L

poles, and see from the spectral plot how the true and extraneous modes are estimated: if extraneous modes have low amplitude, this is evident from the spectral plot. For the purpose of comparison, this method is most useful, as its performance is not affected by the choice of a threshold radius or amplitude; we will use this method in the examples presented.

Chapter 3

Backward Prediction

The backward linear prediction equations can be set up as follows.

$$\begin{bmatrix} y(2) & \dots & y(L+1) \\ \vdots & & \vdots \\ y(N-L+1) & \dots & y(N) \end{bmatrix} \begin{bmatrix} \hat{b}_{b1} \\ \vdots \\ \hat{b}_{bL} \end{bmatrix} = - \begin{bmatrix} y(1) \\ \vdots \\ y(N-L) \end{bmatrix} \quad (3.1)$$

or

$$Y_b \hat{b}_b = -c_b \quad (3.2)$$

where again L is the order of prediction, and $\hat{b}_b$ is the coefficients to be estimated of the polynomial for the inverses of the poles.

$$\hat{B}_b(z) = 1 + \hat{b}_{b1}z^{-1} + \dots + \hat{b}_{bL}z^{-L} \quad (3.3)$$

The solution first involves forming $[c_b : Y_b]$, performing an SVD on it, truncating all but the first M singular values, and then reforming to get a better estimate $[\hat{c}_b : \hat{Y}_b]$. Now $\hat{b}_b$ can be solved for using the pseudoinverse of $\hat{Y}_b$,

$$\hat{b}_b = -\hat{Y}_b^+ \hat{c}_b \quad (3.4)$$

Now the estimated poles can be determined as:

$$\hat{p}_{bi} = \frac{1}{\text{root}_i(\hat{B}_b(z))} \quad i = 1, 2, \dots, L \quad (3.5)$$

Once the poles have been determined, the amplitudes can be estimated by solving Equation 2.7.

In the backward method, true poles can be separated from extraneous poles in the same manner as for the forward method. In this case, extraneous poles appear outside the unit circle. If the data contains only modes with poles inside the unit circle, the radius threshold (with $r_t = 1$), where poles inside of r_t are kept rather than discarded, is an effective method of pole separation. However, if the true poles lie outside the unit circle, pole separation can be difficult.

Chapter 4

Using Both Results

For some data, such as radar target data, true poles can appear both inside and outside the unit circle. In this case, neither the forward method nor the backward method has a clear advantage from the pole selection point of view. However, both methods can be used together as a means of separating the poles. Such a method entails obtaining the forward and backward pole estimates separately, and comparing them. True poles will be estimated at (nearly) the same location for both methods, while extraneous poles will be different (they will be inside the unit circle for the forward method and outside the unit circle for the backward method).

For noisy data, the forward and backward estimates of a given true pole will in general be different. They are considered to be a true pole estimate if they are within a distance d of each other. The distance d should be chosen large enough to account for variances in the pole estimate, but small enough to avoid accepting extraneous poles. Thus, d should depend on the signal to noise ratio. When the distance criterion is met, the pole estimate is chosen to be the average of the forward and backward estimate. The corresponding

modal amplitudes are then computed as in Equation 2.7.

Chapter 5

Simulation Results

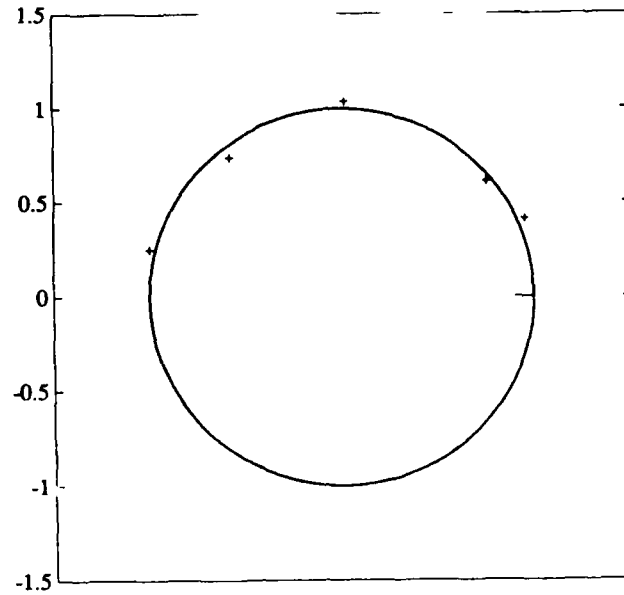
Twenty data points were created from five poles near the unit circle with various amplitudes. Their locations, amplitudes, and impulse response appear in Figures 5.1 and 5.2. The data was then corrupted twenty different times by zero mean complex Gaussian noise with a variance chosen such that the SNR was 4.9 dB. The poles and respective amplitudes were then estimated using forward prediction with $L = 10$ and $M = 5$. The results appear in Figures 5.3 and 5.4. Backward prediction was then used and the results appear in Figures 5.5 and 5.6. The combined method's results using a distance of acceptance of $d = .1$ appear in Figures 5.7 and 5.8.

Figures 5.9-5.14 are results for a SNR of 9.9 dB with everything else as before, and Figures 5.15-5.20 are for a SNR of 14.9 dB.

np= -999 dbsm
snr= 1003.89 db

POLES

3



np= -999 dbsm
snr= 1003.89 db

SPIKE PLOT

3

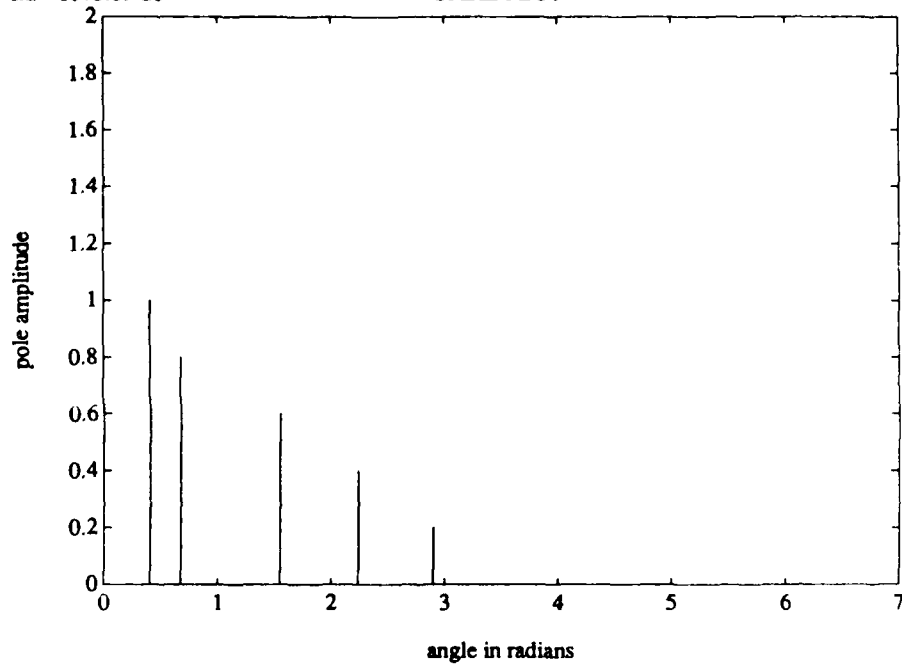


Figure 5.1: Actual Poles and Amplitudes

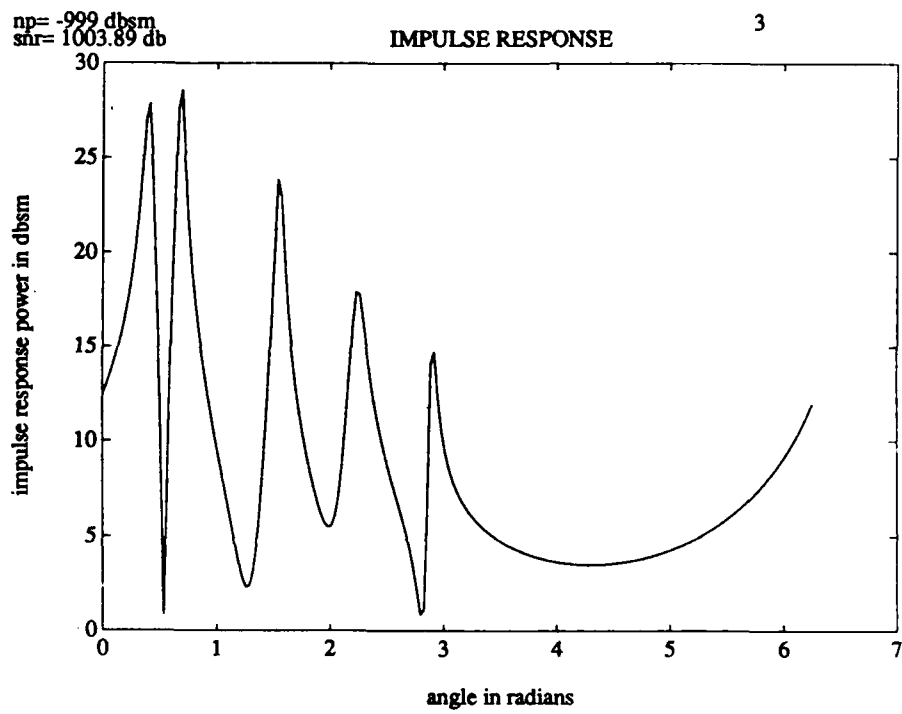


Figure 5.2: Actual Impulse Response

The results show that this method improves the discernment of the poles in both the z -plane plot and in the location of the peaks in the impulse responses and amplitude plots for high noise situations. The extraneous poles were eliminated very nicely. The cost, however, is the elimination of several true poles. This indicates that on a given trial all the true poles are not estimated. There is a need for several trials in order to make this procedure effective. There is also the question of what radius of acceptance to use; a larger one will reject fewer extraneous poles and accept more true ones, and vice versa. When the SNR was improved to 9.9 dB, the improvement in discernment of poles was smaller, and for 14.9 dB the backward prediction method was just as good as the combined method. The forward prediction was in all cases inferior. Thus for higher SNR's backward prediction is recommended, and for lower SNR's, when several trials are available, the use of the combined method is better.

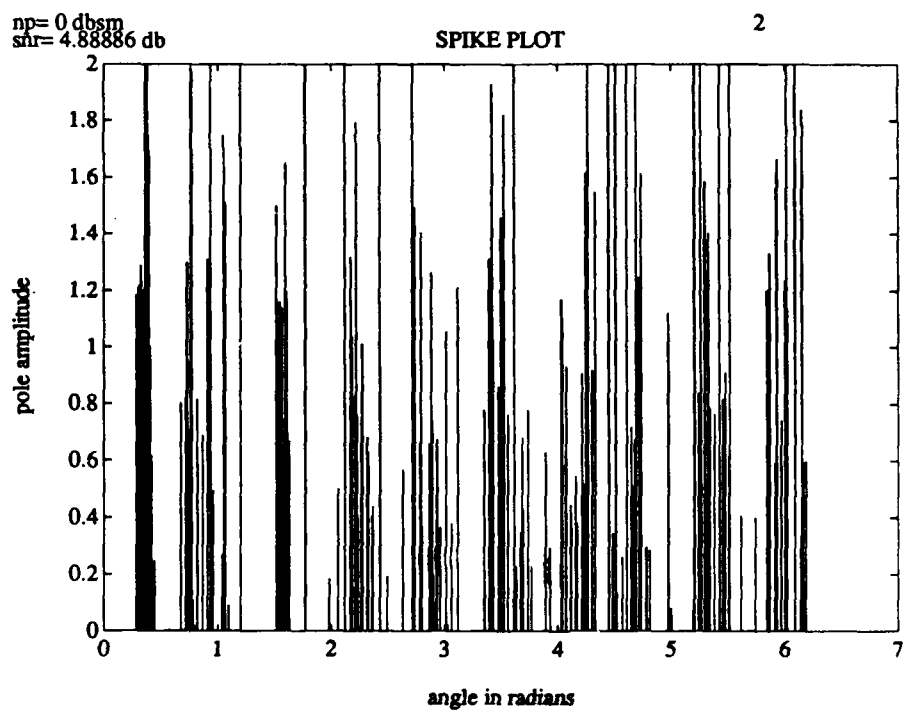
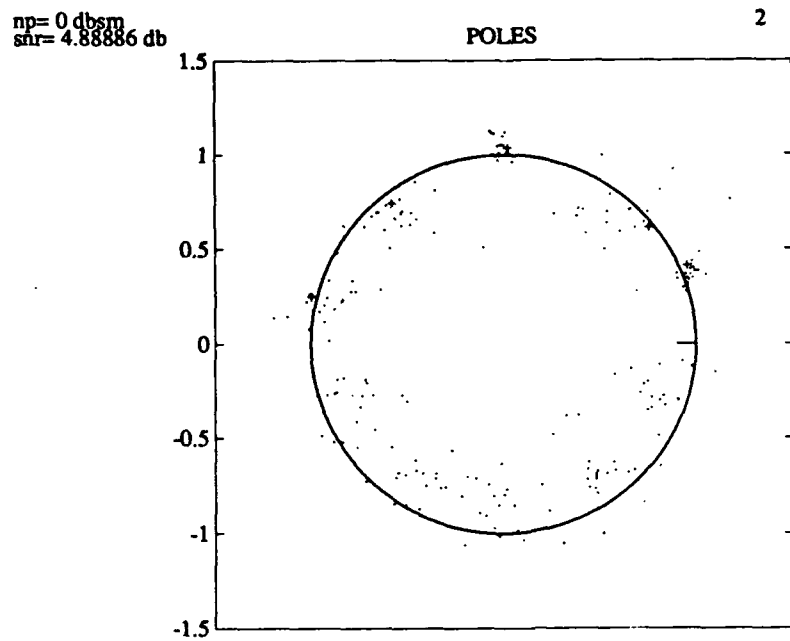


Figure 5.3: Estimated Poles and amplitudes for Forward Prediction, 4.9dB SNR

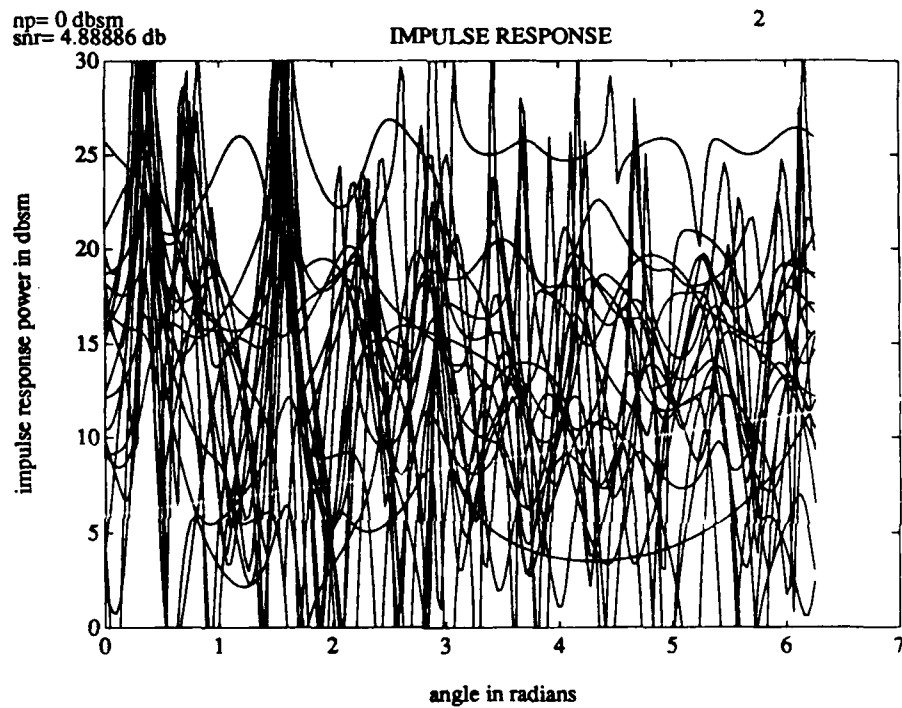


Figure 5.4: Estimated Impulse Response for Forward Prediction, 4.9dB SNR

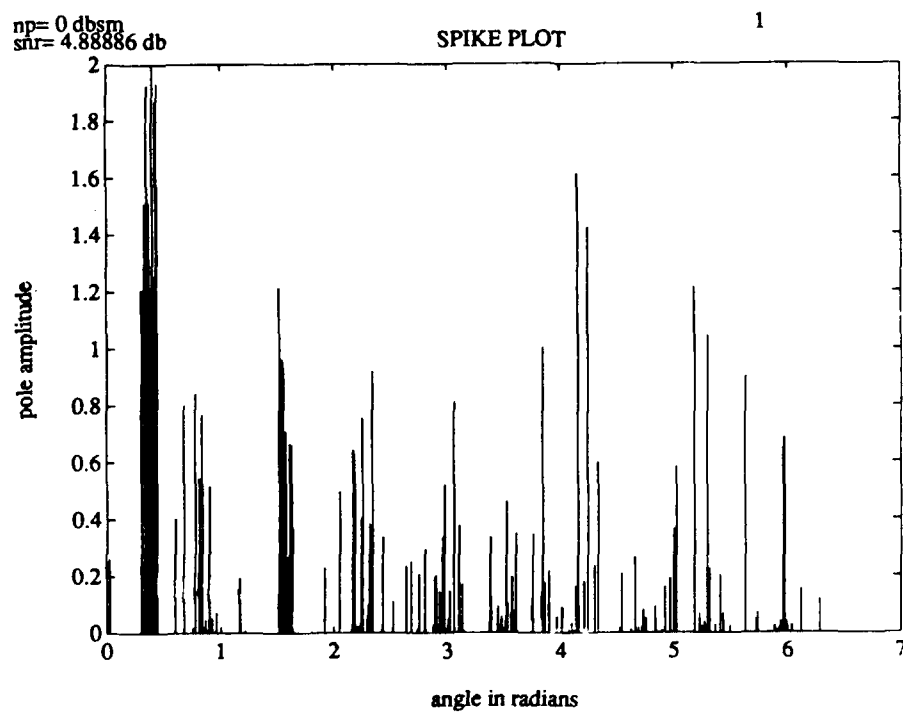
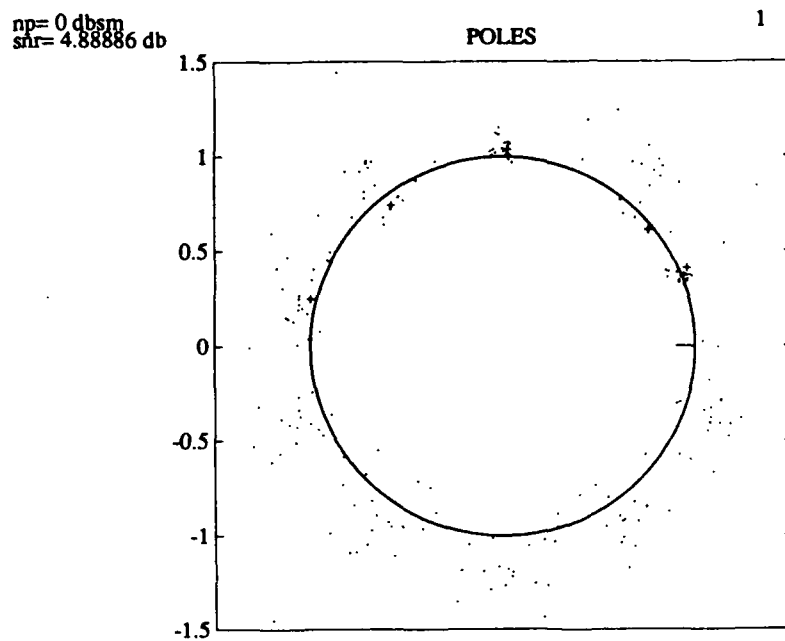


Figure 5.5: Estimated Poles and amplitudes for Backward Prediction, 4.9dB SNR

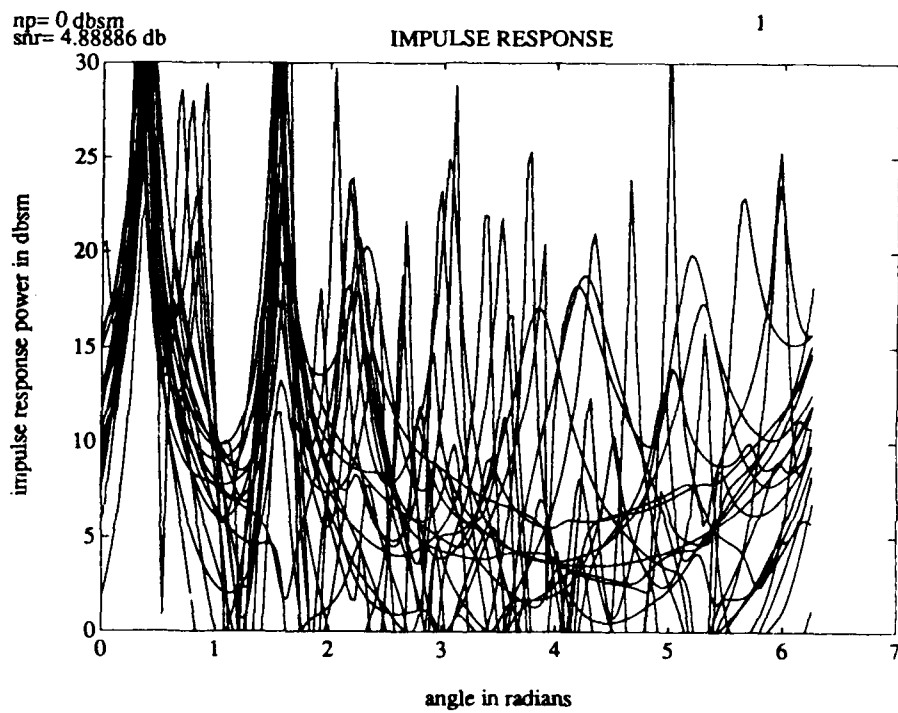


Figure 5.6: Estimated Impulse Response for Backward Prediction, 4.9dB SNR

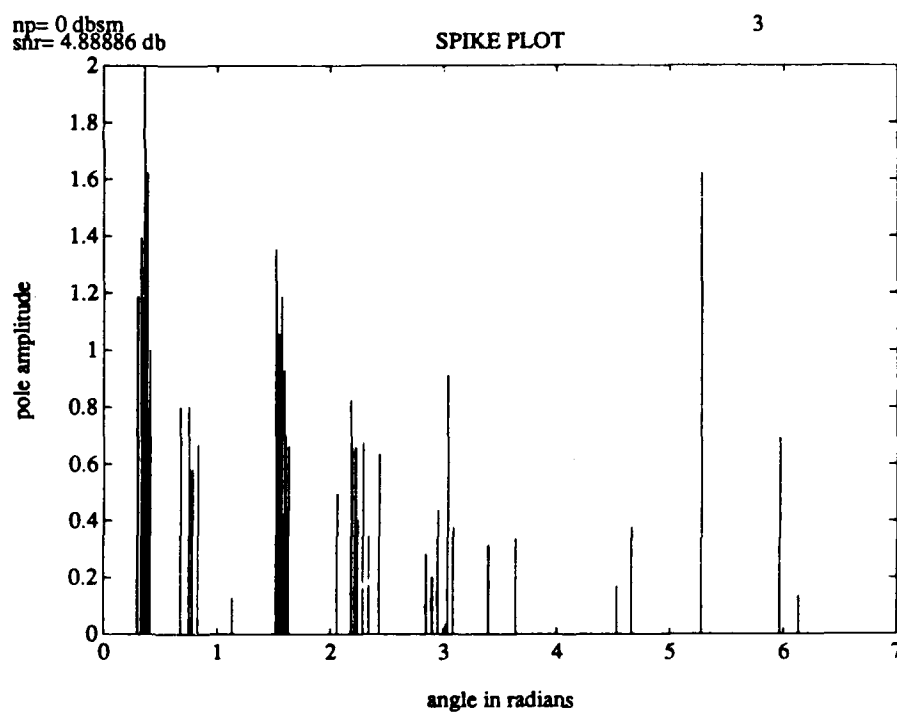
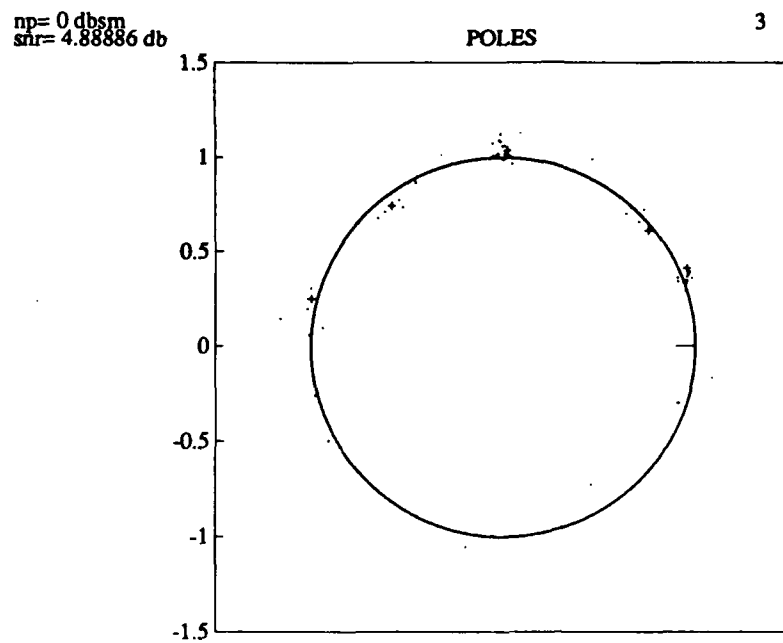


Figure 5.7: Estimated Poles and amplitudes for Dual Prediction, 4.9dB SNR

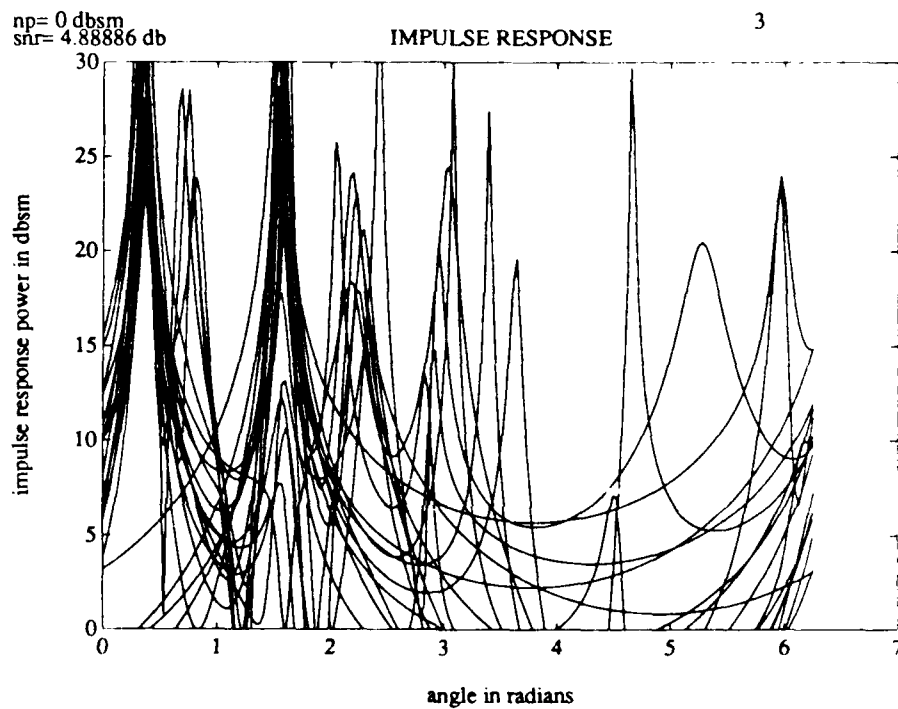


Figure 5.8: Estimated Impulse Response for Dual Prediction, 4.9dB SNR

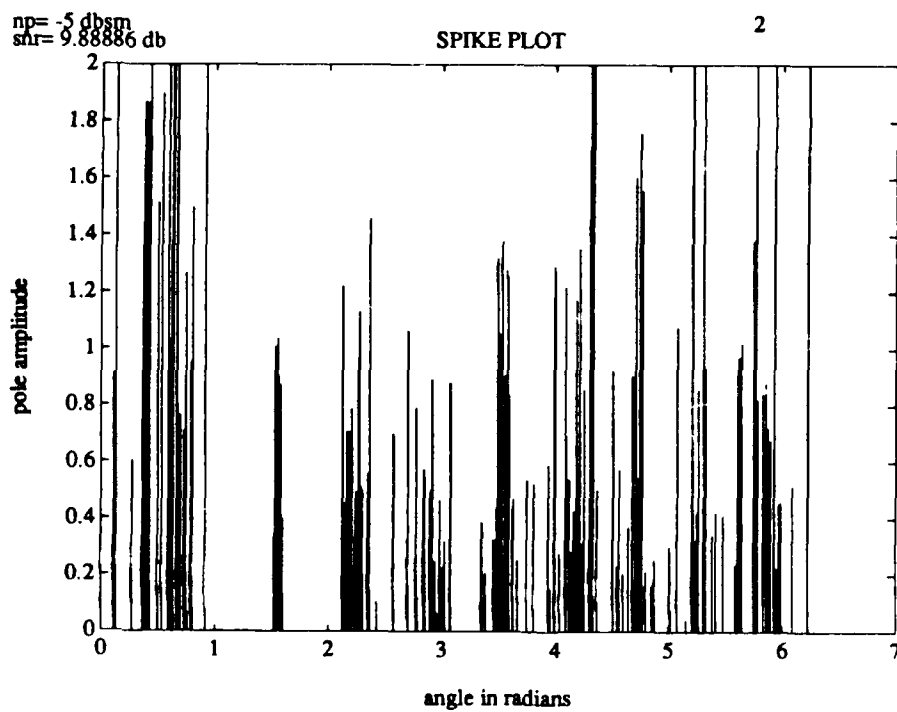
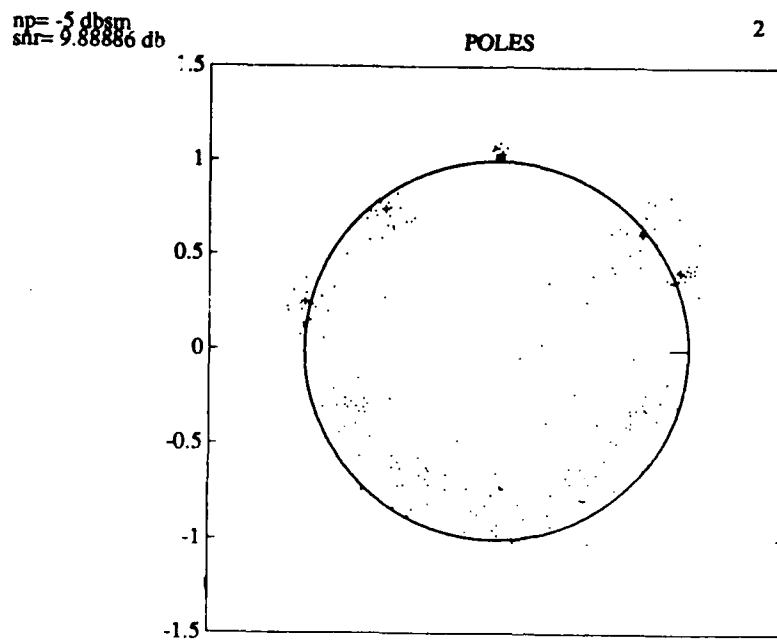


Figure 5.9: Estimated Poles and amplitudes for Forward Prediction, 9.9dB SNR

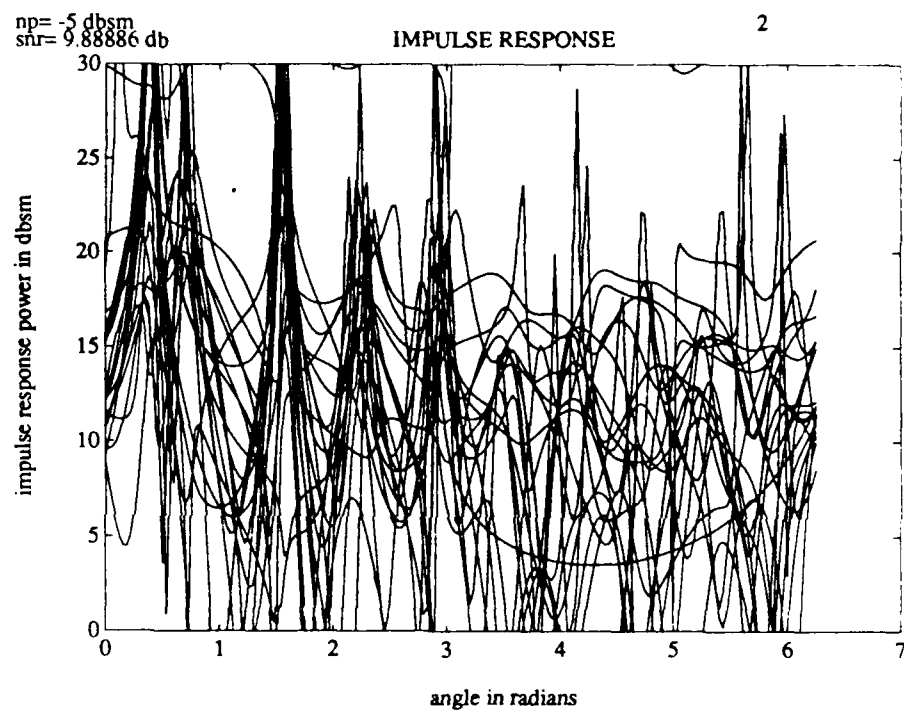


Figure 5.10: Estimated Impulse Response for Forward Prediction, 9.9dB SNR

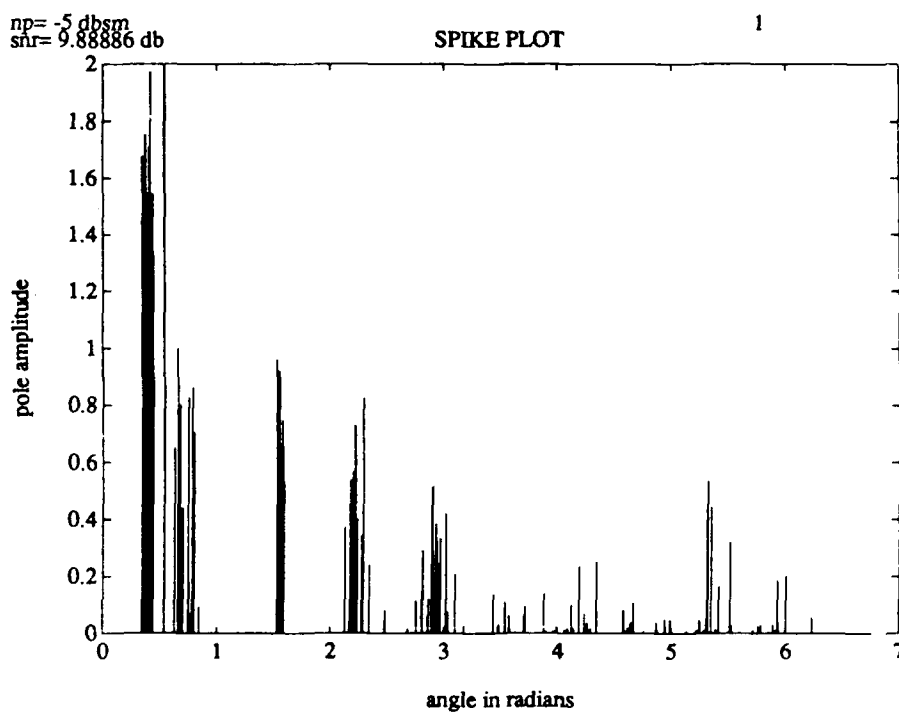
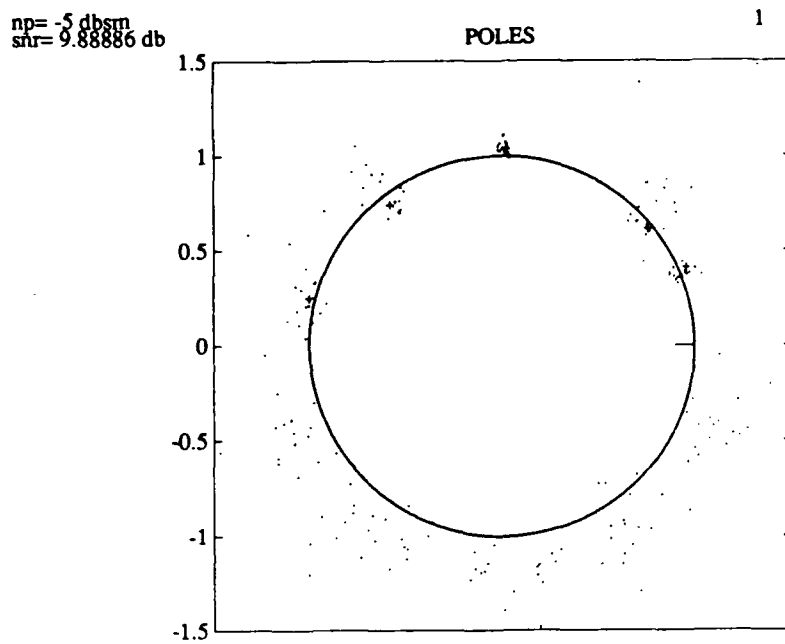


Figure 5.11: Estimated Poles and amplitudes for Backward Prediction, 9.9dB SNR

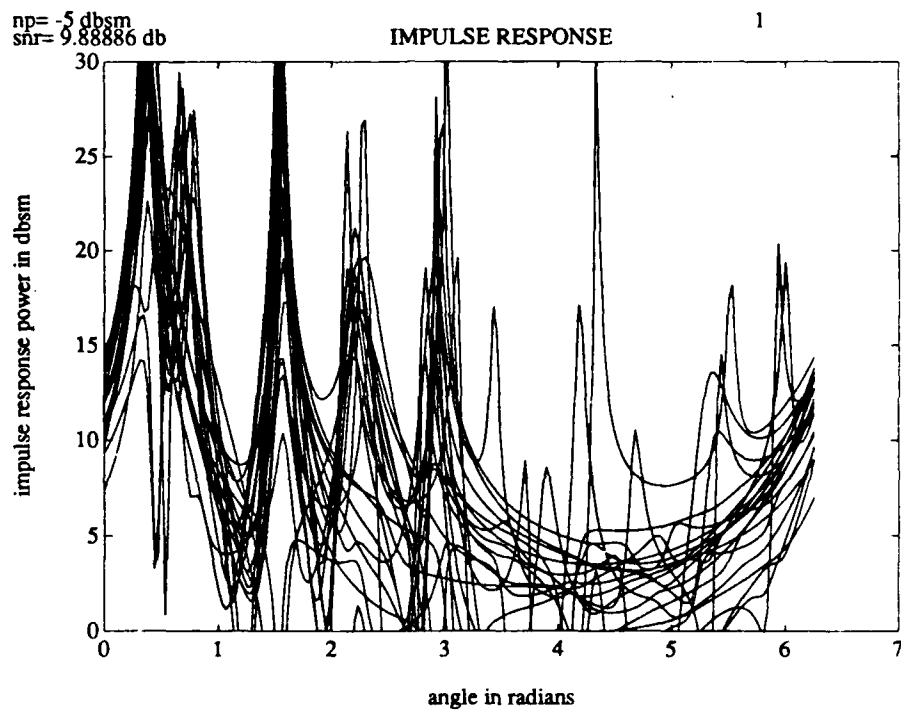


Figure 5.12: Estimated Impulse Response for Backward Prediction, 9.9dB SNR

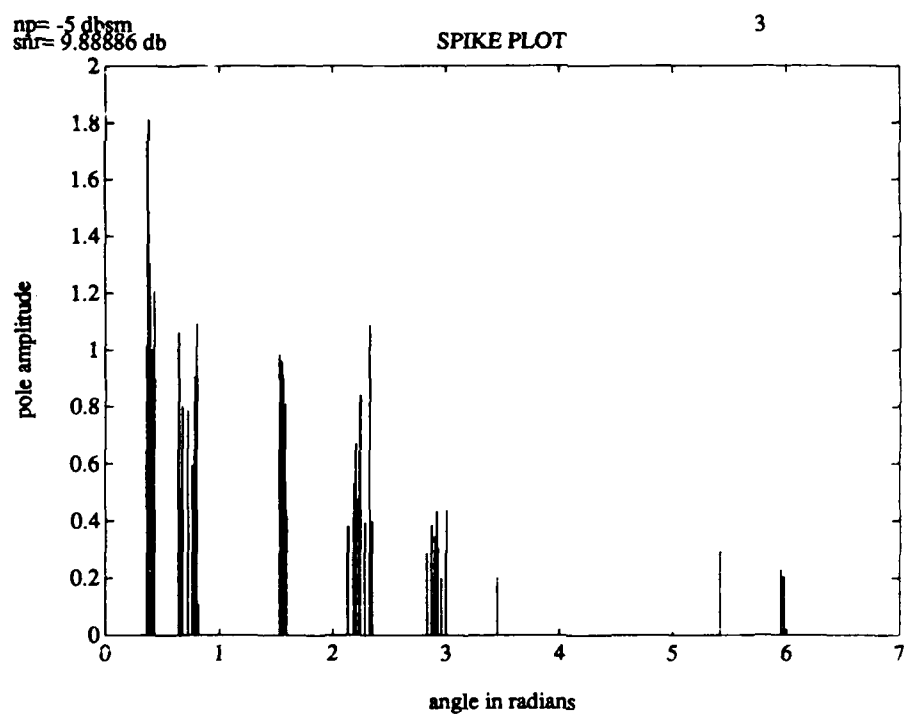
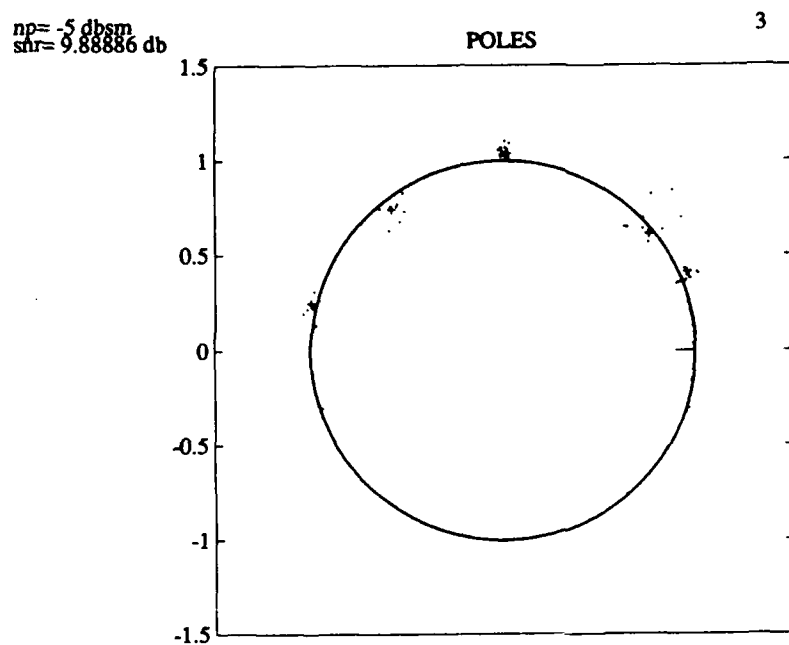


Figure 5.13: Estimated Poles and amplitudes for Dual Prediction, 9.9dB SNR

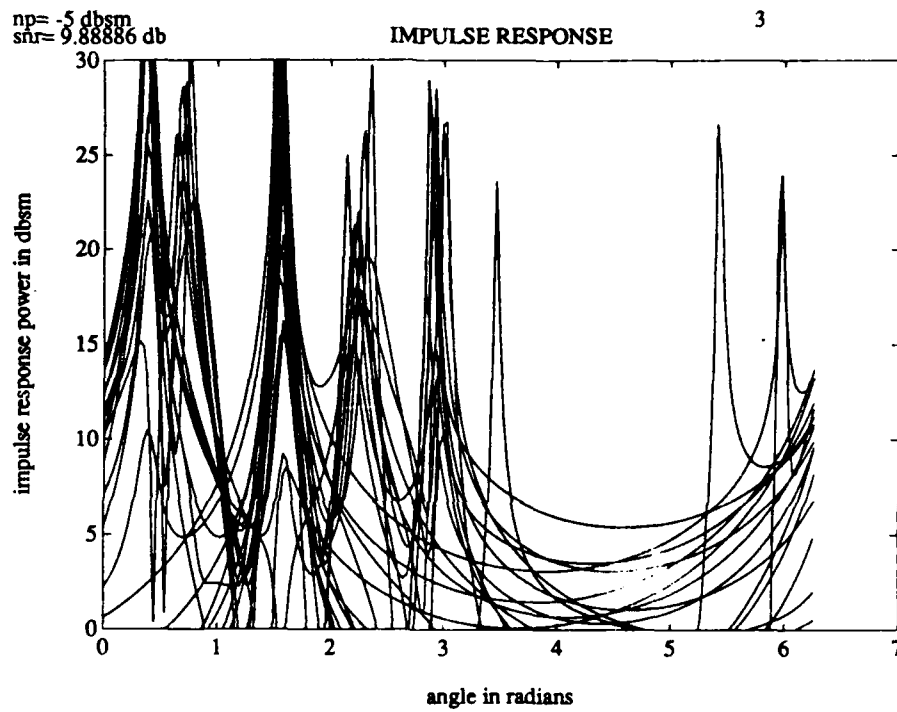
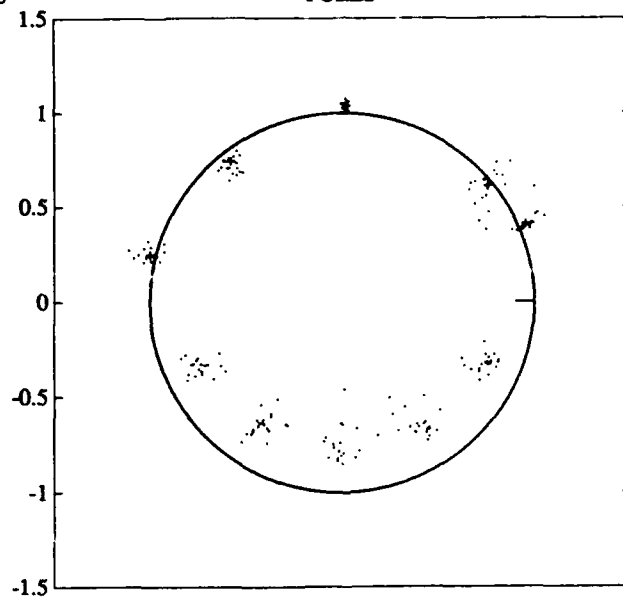


Figure 5.14: Estimated Impulse Response for Dual Prediction, 9.9dB SNR

np= -10 dbsm
snr= 14.8889 db

POLES

2



np= -10 dbsm
snr= 14.8889 db

SPIKE PLOT

2

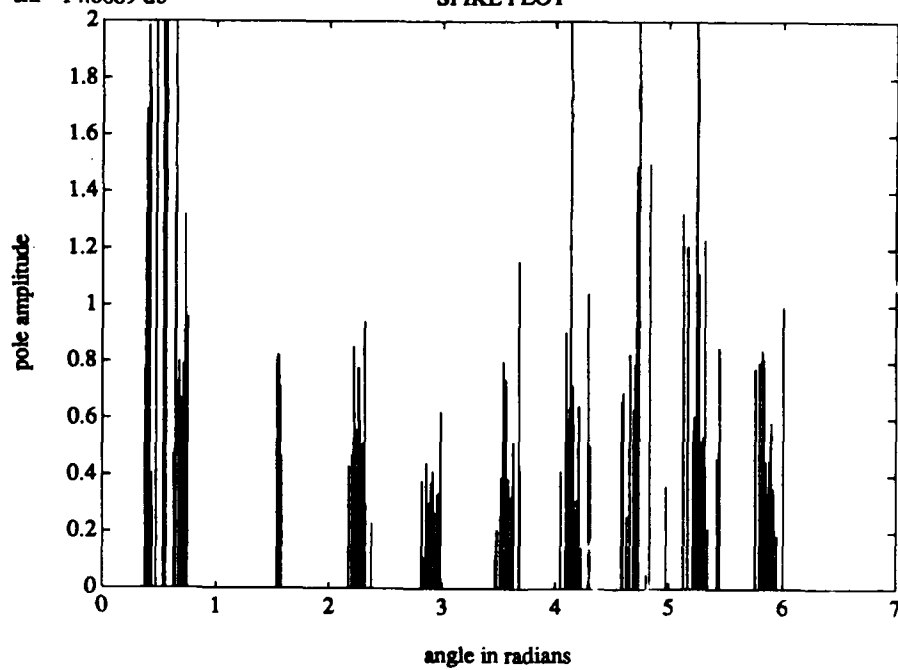


Figure 5.15: Estimated Poles and amplitudes for Forward Prediction, 14.9dB SNR

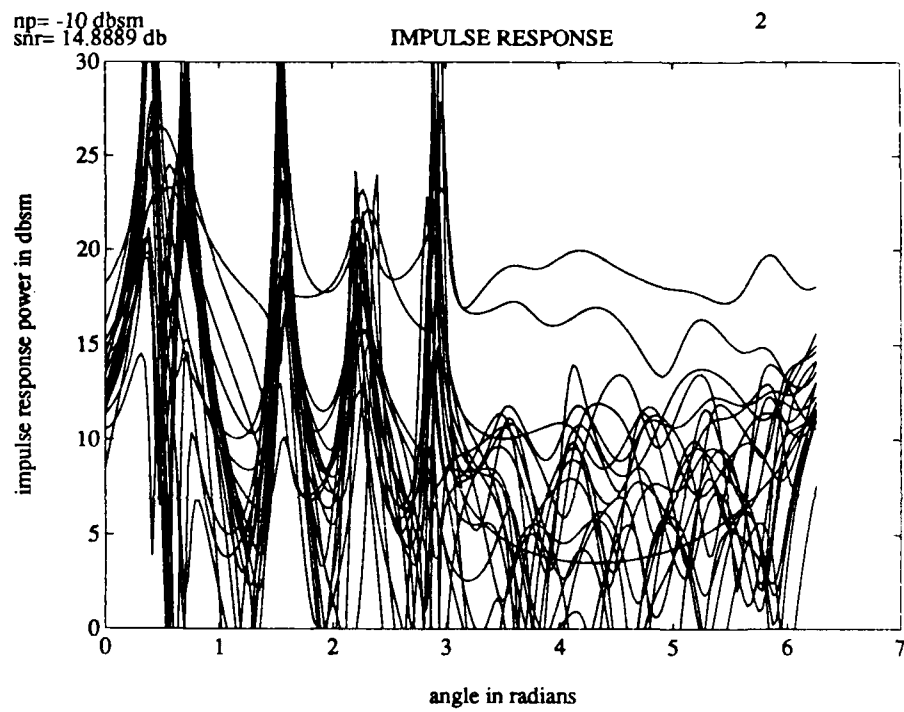
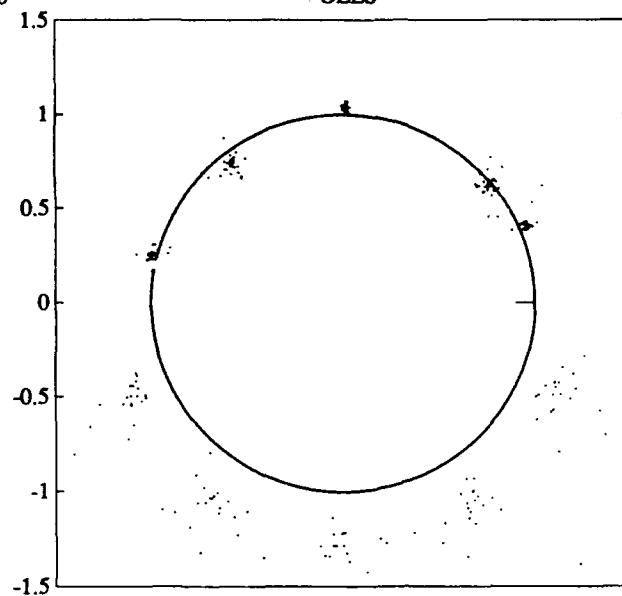


Figure 5.16: Estimated Impulse Response for Forward Prediction, 14.9dB SNR

np= -10 dbsm
snr= 14.8889 db

POLES

1



np= -10 dbsm
snr= 14.8889 db

SPIKE PLOT

1

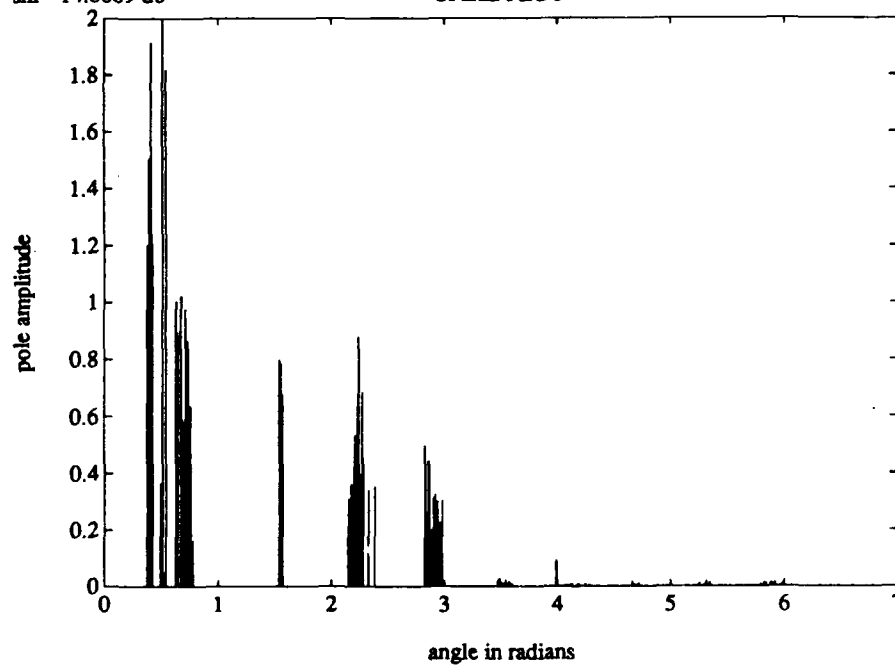


Figure 5.17: Estimated Poles and amplitudes for Backward Prediction, 14.9dB SNR

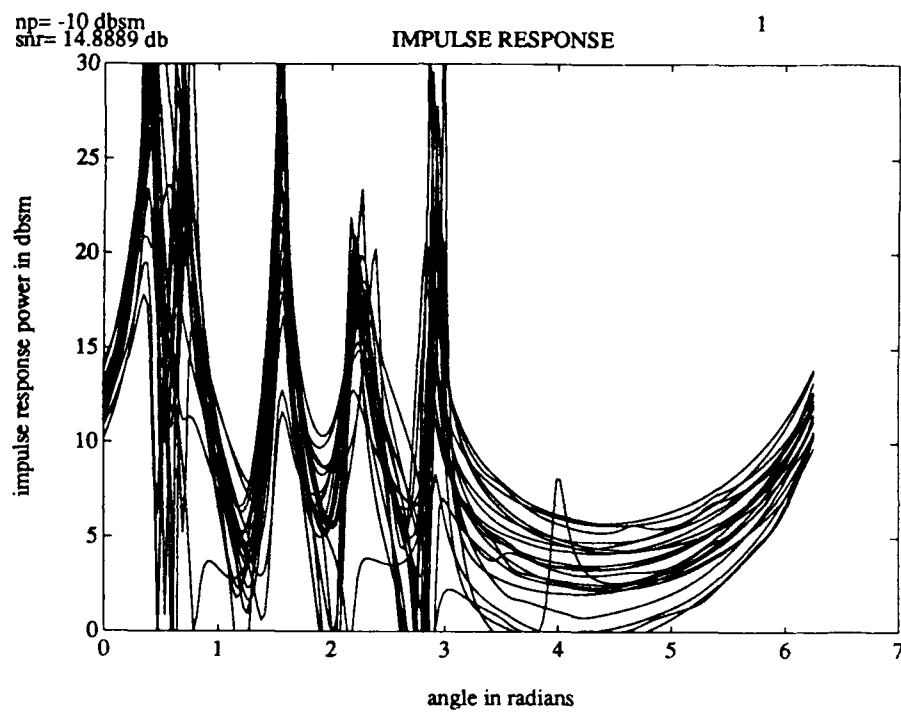


Figure 5.18: Estimated Impulse Response for Backward Prediction, 14.9dB SNR

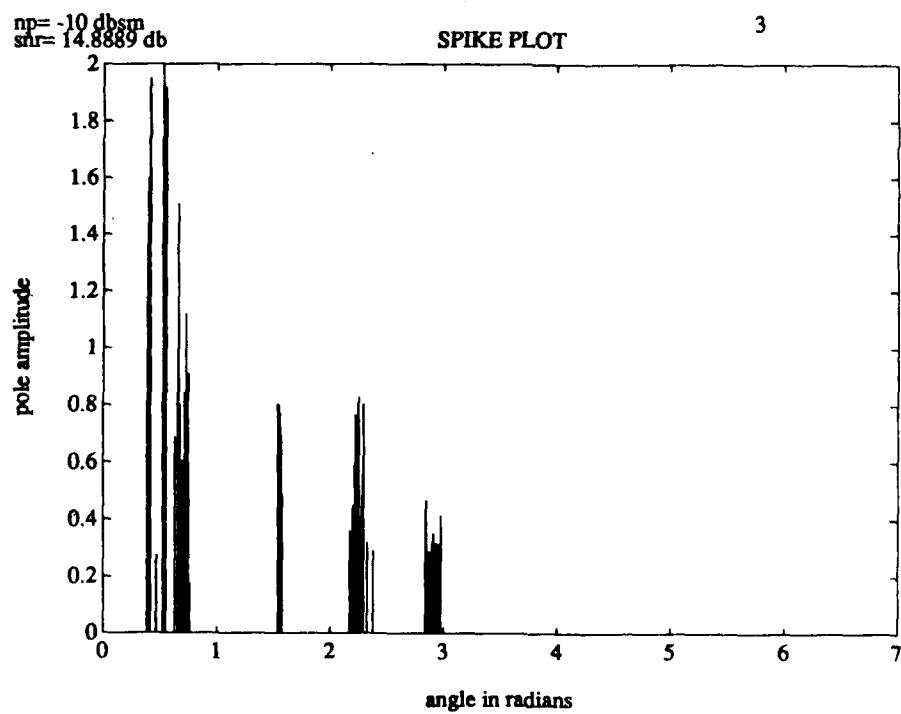
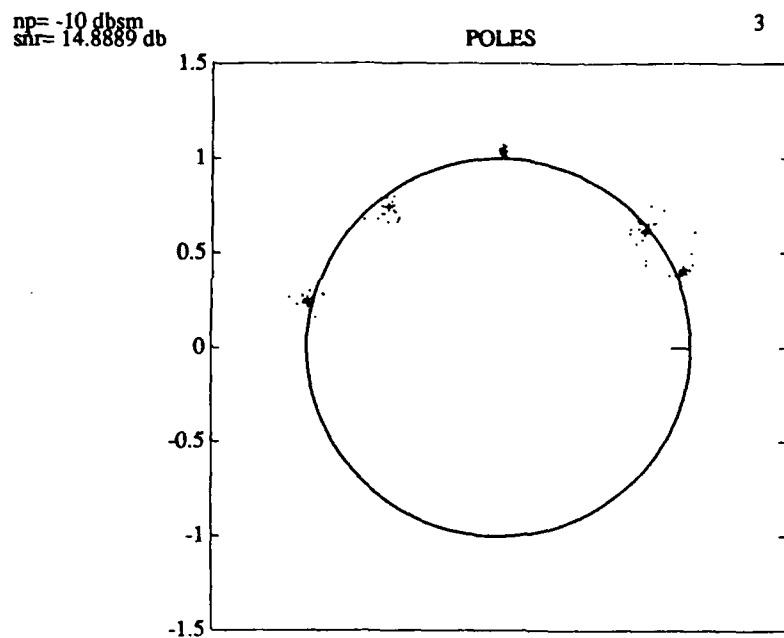


Figure 5.19: Estimated Poles and amplitudes for Dual Prediction, 14.9dB SNR

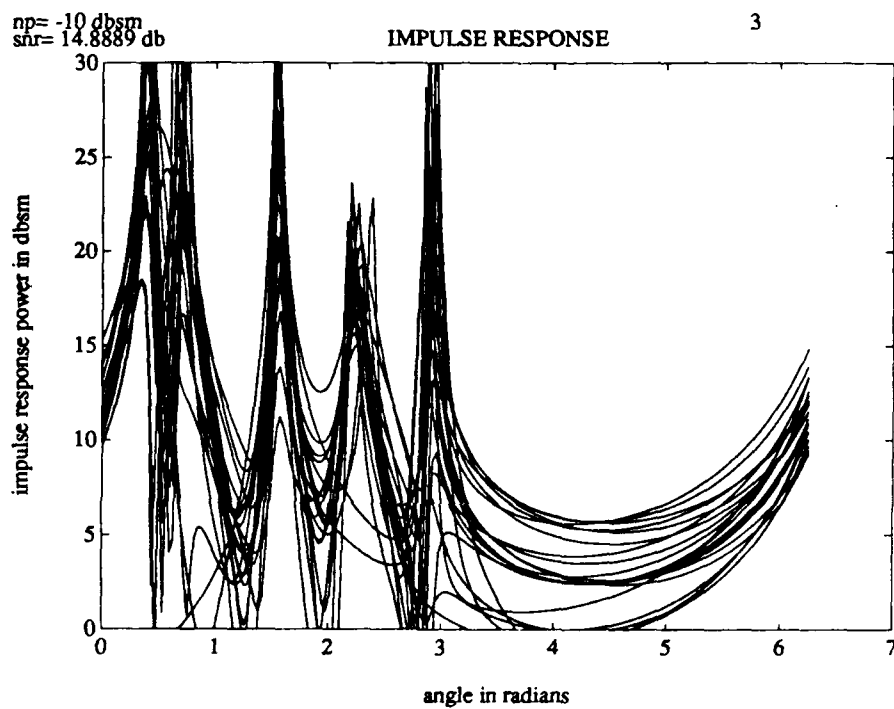


Figure 5.20: Estimated Impulse Response for Dual Prediction, 14.9dB SNR

Chapter 6

Summary

We considered a combined forward-backward method of estimating exponential signals in noise when the exponential modes may be both greater than one and less than one in magnitude (i.e., when the true poles lie both inside and outside the unit circle). We compared this method with the standard forward prediction and backward prediction methods. We found that when the signal to noise ratio was small, and when a number of independent measurements of a data set are available, the combined method works better than the standard methods. In other cases, the combined method gave about the same results as the backward prediction method.

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