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TIGHTNESS OF SYNCHRONOUS PROCESSES

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Tightness of Synchronous Processes

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Abstract

Let $X = \{X(t) : t \geq 0\}$ be a positive recurrent synchronous process (PRS), that is, a process for which there exists an increasing sequence of random times $\tau = \{\tau(k)\}$ such that for each k the distribution of $\theta_{\tau(k)} \circ X = \{X(t + \tau(k)) : t \geq 0\}$ is the same and the cycle lengths $T_n = \tau(n+1) - \tau(n)$ have finite first moment. Whereas the ergodic properties of such processes are well known in the literature, the same is not so for the distributional properties of either the marginals $X(t)$ or more generally the shifted processes $\theta_s X = \{X(s+t) : t \geq 0\}$ in function space. In the present paper we show that these distributions are in fact tight. In contrast to classical regenerative processes ~~we also show~~ that the standard types of regularity assumptions (non-lattice cycle length distribution, mixing) do not ensure weak convergence to steady-state for a PRS. Applications are given in the context of one-dependent regenerative (od-R) processes. These arise in the queueing models that motivated this paper.

Keywords: synchronous process, tightness, stationary process

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1. Preliminaries

Throughout this paper, $X = \{X(t) : t \geq 0\}$ will denote a stochastic process with a complete separable metric state space $\mathcal{S}$ and having paths in the space $\mathcal{D} = \mathcal{D}_{\mathcal{S}}[0, \infty)$ of functions $f : R_+ \to \mathcal{S}$ that are right continuous and have left hand limits. $\mathcal{D}$ is endowed with the Skorohod topology and is a complete separable metric space. $(\Omega, \mathcal{F}, P)$ will denote the underlying probability space and we view X as a random element of $\mathcal{D}$. Let Δ denote an arbitrary fixed element not in the set $\mathcal{S}$. We then endow $S \stackrel{\text{def}}{=} \mathcal{S} \cup \{\Delta\}$ with the one-point compactification topology.

Definition 1.1. *X is said to be a* **synchronous** *process with respect to the random times $0 = \tau(-1) \leq \tau(0) < \tau(1) < \cdots$ (with $\lim_{n\to\infty} \tau(n) = \infty$ a.s.) if $\{X_n : n \geq 2\}$ forms a stationary sequence in the space $\mathcal{D}_S^{\infty}$, where*

$$X_n(t) = \begin{cases} X(\tau(n-1)+t), & \text{if } 0 \leq t < T_n; \\ \Delta, & \text{if } t \geq T_n. \end{cases}$$

$T_n \stackrel{\text{def}}{=} \tau(n) - \tau(n-1)$ is called the n^{th} cycle length, X_n is called the n^{th} cycle and we refer to $(\tau(n))$ as the synch-times for X with counting process $N(t) = \max\{n > 0 : \tau(n) \leq t\}$.

The important point here is that at the random times $\tau(k)$, $X(t)$ and its future probabilistically start over. However, in contrast to classical regenerative processes, the future is not necessarily independent of any of the past $\{\tau(1), \ldots, \tau(k); X(s) : 0 \leq s \leq \tau(k)\}$. In particular τ does not (in general) form a renewal process and hence the renewal equation does not apply to synchronous processes.

Definition 1.2. *A synchronous process X is called* **non-delayed** *if $\tau(0) = 0$ a.s.;* **delayed** *otherwise. It is called* **positive recurrent** *if $E(T_1) < \infty$;* **null recurrent** *otherwise. $\lambda \stackrel{\text{def}}{=} \frac{1}{E(T_1)}$ is called the* **rate** *of the synch times.*

From now on, PRS will be used to abbreviate *positive recurrent synchronous process.*

Other names have been given to a synchronous process; for example Serfozo [1972] refers to them as semi-stationary processes. In Rolski[1981] they arise as *Palm* versions of stationary processes (associated with point processes). Closely related to this is the general theory of stationary marked point processes. In any case, the ergodic properties of synchronous processes are well known in the literature. We state several such results the proofs of which can be found in, for example Baccelli and Bremaud [1987], Daly and Vere-Jones[1988], Franken et al [1981], Glynn and Sigman [1989], Rolski[1981] and Serfozo[1972].

Let $\theta_t : \mathcal{D}_S \to \mathcal{D}_S$ denote the *shift operator* $(\theta_t x)(s) = x(t+s)$.

Theorem 1.1. *Suppose X is a PRS and $f : \mathcal{D}_S \to \Re$ is measurable. Let $J_n = J_n(f) \stackrel{\text{def}}{=} \int_{\tau(n-1)}^{\tau(n)} f(\theta_t \circ X)dt$. If $J_0(|f|) < \infty$ a.s. and if either $f \geq 0$ a.s. or $E\{J_1(|f|)\} < \infty$ then*

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t f(\theta_s \circ X)ds = \frac{E\{J_1|\mathcal{I}\}}{E\{T_1|\mathcal{I}\}} \quad \text{a.s.} \tag{1.1}$$

where $\mathcal{I}$ denotes the invariant σ-field associated with $\{(X_n, T_n)\}$.

Let P^0 denote the probability measure under which X is non-delayed, that is, $P^0(X \in A) = P(\theta_{\tau(1)} \circ X \in A)$.

Corollary 1.1. *Under the conditions of Theorem 1.1, if in addition $\mathcal{I}$ is trivial (every set has probability 0 or 1) then $\{J_n, T_n : n \geq 1\}$ is ergodic and hence a.s.*

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t f(\theta_s \circ X)ds = \frac{E\{J_1\}}{E\{T_1\}} = \lambda \int_0^\infty P^0(\theta_s \circ X \in A; \tau(1) > s)ds. \tag{1.4}$$

Under these circumstances, X is called ergodic.

The following Corollary follows from (1.1) by an elementary application of Fubini's Theorem and the Bounded Convergence Theorem.

Corollary 1.2. *Under the hypothesis of Theorem 1.1, if in addition f is bounded then*

$$\overline{\mu}_t(f) \stackrel{\text{def}}{=} \frac{1}{t}\int_0^t Ef(\theta_s \circ X)ds \longrightarrow \pi(f) \stackrel{\text{def}}{=} E\left\{\frac{E\{J_1|\mathcal{I}\}}{E\{T_1|\mathcal{I}\}}\right\}. \tag{1.5}$$

π above defines a measure on $\mathcal{D}$ and (for reasons given below in Proposition 1.1) is called the stationary probability measure for X. In particular, by choosing $f = 1_A$ (an indicator function), we have $\overline{\mu}_t(A) \to \pi(A)$ for each Borel set A of $\mathcal{D}$; thus the Cesaro averaged distributions converge weakly.

Proposition 1.1. *Let π be the stationary measure of a PRS X. Then under π, $\theta = (\theta_s)$ is measure preserving on $\mathcal{D}$, that is, for each Borel set A, $\pi(A) = \pi(\theta_{-s}A)$ for all $s \geq 0$. In particular, if X has distribution π, then X is time stationary, that is, $\theta_t X$ has the same distribution for each $t \geq 0$.*

Let P^* denote the probability measure under which X has distribution π, that is, $P^*(X \in A) = \pi(A)$. From (1.4) we obtain for an *ergodic* synchronous process that

$$P^*(X \in A) = \lambda \int_0^\infty P^0(\theta_s \circ X \in A; \tau(1) > s)ds. \tag{1.6}$$

If X is positive recurrent but not ergodic then the RHS of (1.6) still defines a measure on $\mathcal{D}$ (but not necessarily the same as the π from (1.5)). In fact, more can be said:

Proposition 1.2 *For a PRS the RHS of (1.6) defines a measure on $\mathcal{D}$ (in general, not the same as π) under which $\theta = (\theta_s)$ is measure preserving.*

Proof: Clearly the RHS of (1.6) defines a probability measure on $\mathcal{D}$. Call this measure ψ. Then

$$\begin{aligned}
\psi(\theta_{-t}A) &= \lambda \int_0^\infty P^0(\theta_{t+s} \circ X \in A; \tau(1) > s)ds \\
&= \lambda E\{\int_0^{\tau(1)} 1_A(\theta_{t+s}X)ds\} \\
&= \lambda E\{\int_t^{\tau(1)+t} 1_A(\theta_s X)ds\} \\
&= \psi(A) - \lambda E\{\int_0^t 1_A(\theta_s X)ds - \int_{\tau(1)}^{\tau(1)+t} 1_A(\theta_s X)ds\}.
\end{aligned}$$

The result follows since (by the definition of synchronous) the last two integrals above have the same distribution. ■

Let X be PRS with steady-state distribution π. One might expect to obtain weak convergence, as $t \longrightarrow \infty$, of the measures $\nu_t \stackrel{\text{def}}{=} P(X(t) \in \cdot)$ or $\mu_t = P(\theta_t X \in \cdot)$ by placing some further regularity assumptions on X such as a non-lattice cycle length distribution and/or mixing cycles. Unfortunately, as the following example shows, one must be very careful in asserting stronger modes of convergence for a PRS than the *Cesaro* type obtained from Theorem 1.1 or Corollary 1.2.

Example (1) *A PRS having both a spread-out cycle length distribution and mixing cycles that does not converge weakly.* Let $B(t)$ denote the time until the next integer point strictly after time t. This is actually the forward recurrence time for a renewal process $\{t_n\}$ with $t_n = n$, $n \geq 0$. The steady-state marginal distribution of B is $Unif(0,1)$. Also, since B is regenerative (it regenerates at times t_n), $B(t)$ converges in the Cesaro sense to $Unif(0,1)$ regardless of initial conditions.

Let (U_n) be i.i.d. $\sim Unif(.5, 1)$ and define $\tau(n) = n + U_n$. In particular $n + .5 < \tau(n) < n + 1$, $n \geq 0$, and the cycle lengths $T_n \stackrel{\text{def}}{=} \tau(n) - \tau(n-1) = U_n - U_{n-1} + 1$ are *one-dependent*, that is, T_n depends upon T_{n+1} but is independent of T_{n+k}, $k \geq 2$. Moreover the cycle length distribution has an absolutely continuous component and hence is non-lattice and spread-out. Clearly B is PRS wrt τ and the cycles of B are also one-dependent. In particular they are mixing, that is, $P(X_n \in A, X_{k+n} \in B) - P(X_n \in A)P(X_{k+n} \in B)$ tends to 0 as k tends to ∞ for all sets A, B. Now consider the delayed version (with respect to τ); $B(0) = 1$. In this case $B(n) = 1$, $n \geq 1$. Weak convergence is therefore impossible since if $B(t)$ converges weakly then its weak limit must be $Unif(0, 1)$ (the same as its Cesaro limit) and all convergent subsequences $B(s_k)$ must converge to $Unif(0, 1)$ also. In fact we now will show that the non-delayed version (with respect to τ) $X(t) = B_{\tau_1 + t}$ also does not converge weakly. To this end simply observe that at times $s_n = (2n+1)/2$ we have $.5 < X(s_n) < 1$ and hence no mass occurs on $(0, .5)$. But, as before, $X(t)$ converges to $Unif(0, 1)$ in the Cesaro sense, since X is actually the same as B with the random initial condition $B(0) = \tau(1)$.

Given a synchronous process X define a new process $\tilde{X}$ by $\tilde{X}(t) = \theta_t \circ X$.

Proposition 1.3 *If X is synchronous then $\tilde{X}$ is synchronous with the same synch times as X. The paths of $\tilde{X}$ are continuous; in particular they lie in $\mathcal{D}_{\mathcal{D}_S}$.*

Proof: The state space of $\tilde{X}$ is the complete separable metric space $\mathcal{D}_S$. Moreover, the sample paths of $\tilde{X}$ are actually continuous, that is, if $x \in \mathcal{D}$ and $s \longrightarrow t$ then $\theta_s x \longrightarrow \theta_t x$ in the Skorohod topology of $\mathcal{D}$ (see Lemma1.1 of Rolski[1981])). It is immediate from the definition of synchronous process for X that the distribution of $\tilde{X}$ (and its future) starts over at the synch times of X. ■

Remark (1.1): In the case of a discrete time process $\{X(k) : k \geq 0\}$, one can convert to continuous time by defining $X(t) = X([t])$.

2. Tightness

Although weak convergence of a PRS can not be obtained in general (and placing conditions on the cycles does not appear to help), we do have

Theorem 2.1. *A PRS is tight, that is, for each $\epsilon > 0$ there exists a compact set $K(\epsilon) \subset \mathcal{S}$ such that*

$P(X(t) \in K(\epsilon)) > 1-\epsilon$ for all $t \geq 0$. In fact $\{\theta_t \circ X\}$ is tight, that is, for each $\epsilon > 0$ there exists a compact set $C(\epsilon) \subset \mathcal{D}$ such that $P((\tilde{X})(t) \in C(\epsilon)) > 1-\epsilon$ for all $t \geq 0$.

Proof: Let ψ denote the measure from Proposition 1.2. Let $F(x)$ denote the cdf for $\tau(1)$. Fix $\epsilon > 0$ and then choose an $a = a(\epsilon) > 0$ such that $F(a) \leq \epsilon\lambda/2$ (Recall that we are assuming that the cycle lengths are strictly positive). From (1.6) it follows that

$$\psi(A) \geq \lambda \int_0^a P^0(\theta_s \circ X \in A; \tau(1) > s)ds \geq \lambda \int_0^a P^0(\theta_s \circ X \in A; \tau(1) > a)ds. \tag{2.7}$$

Observe that

$$\begin{aligned} P^0(\theta_s \circ X \in A) &= P^0(\theta_s \circ X \in A; \tau(1) > a) + P^0(\theta_s \circ X \in A; \tau(1) \leq a) \\ &\leq P^0(\theta_s \circ X \in A; \tau(1) > a) + F(a). \end{aligned}$$

Substituting the above into (2.7) we obtain

$$\psi(A) \geq \lambda \int_0^a P^0(\theta_s \circ X \in A)ds - aF(a),$$

and hence

$$\int_0^a P^0(\theta_s \circ X \in A)ds \leq \lambda^{-1}\psi(A) + a\epsilon/2. \tag{2.8}$$

For each $u \geq 0$ and each compact set B of $\mathcal{S}$ define $A(B,u) = \{x \in \mathcal{D} : x(t) \in B; t \in [u, u+a]\}$. By the compact containment condition (see Ethier and Kurtz[1986], remark 7.3, page 129) there exists a compact set $K_1 = K_1(\epsilon, a)$ in $\mathcal{S}$ such that

$$\psi(A(K_1, u)) > 1 - \lambda a\epsilon/2. \tag{2.9}$$

Moreover, by stationarity of X under ψ, K_1 doesn't depend upon u. For any set A let $\overline{A}$ denote the complement of the set. From (2.8) and (2.9) we obtain

$$\begin{aligned} \int_0^a P^0(\theta_s \circ X \in \overline{A}(K_1,u))ds &\leq \lambda^{-1}\psi(\overline{A}(K_1,u)) + a\epsilon/2 \\ &\leq a\epsilon. \end{aligned} \tag{2.10}$$

But $u + a \in s + [u, u+a]$ for each $s \in [0,a]$ and hence for each $s \in [0,a]$ $P^0(X(u+a) \in \overline{K_1}) \leq P^0(\theta_s \circ X \in \overline{A}(K_1,u))$. Substituting into (2.10) yields

$$P^0(X(u+a) \in \overline{K_1}) \leq \epsilon, \quad u \geq 0,$$

or equivalently

$$P^0(X(t) \in K_1) > 1 - \epsilon, \quad t \geq a. \tag{2.11}$$

Using the compact containment condition again there exists a compact set $K_2 = K_2(\epsilon, a)$ such that $P^0(X \in A(K_2, 0)) > 1 - \epsilon$; in particular $P^0(X(t) \in K_2) > 1 - \epsilon$ for all $t \in [0, a]$. Thus for $K_\epsilon = K_1 \cup K_2$ we obtain $P^0(X(t) \in K_\epsilon) > 1 - \epsilon$ for all $t \geq 0$. Thus we have shown that the marginal distributions of a non-delayed PRS are in fact tight. Moreover, by Proposition 1.3 we also obtain tightness of the non-delayed $\tilde{X}$. To handle the delayed case, suppose X is delayed, fix $\epsilon > 0$ and choose an M large enough so that $P(\tau(0) > M) \leq \epsilon$. Then for any compact set K if $t > M$ then

$$\begin{aligned}
P(X(t) \in \overline{K}) &\leq P(X(t) \in \overline{K}; \tau(0) \leq M) + \epsilon \\
&\leq P(X(\tau(0) + t - s) \in \overline{K}, \quad \text{some} \quad s \in [0, M]) + \epsilon \\
&= P^0(X(t - M + s) \in \overline{K}, \quad \text{some} \quad s \in [0, M]) + \epsilon \\
&= P^0(X(u + s) \in \overline{K}, \quad \text{some} \quad s \in [0, M]) + \epsilon.
\end{aligned} \tag{2.12}$$

where $u = t - M$. But now we are dealing with the non-delayed version of $\tilde{X}$ which we just showed was tight; thus for any $\delta > 0$ we can choose a compact set of paths $C(\epsilon) \subset \mathcal{D}$ such that for all $t \geq 0$, $P^0(\theta_t \circ X \in C(\epsilon)) > 1 - \delta$. Using this fact together with the compact containment condition, it follows the last probability in (2.12) can be made arbitrarily small (uniformly over $u \geq 0$) for appropriate compact sets $K \subset \mathcal{S}$. This is because the complement of the event

$$\{X(u + s) \in \overline{K}, \quad \text{some} \quad s \in [0, M]\}$$

is the event

$$\{X(u + s) \in K, \quad \text{for all} \quad s \in [0, M]\}.$$

For $t \leq M$ we can use the compact containment condition on X over the time interval $[0, M]$ to obtain a compact set K such that $P(X(t) \in K) > 1 - \epsilon$ for all $t \in [0, M]$. The proof is now complete. The last assertion of our theorem follows by applying our result to synchronous process $\tilde{X}$ (of Proposition 1.3). ■

Corollary 2.1 *Suppose X is a PRS and let $Y(t) = t_{N(t)+1} - t$ denote the time until the next synch time after time t. Then Y is tight.*

Proof: Y is easily seen to be a PRS with the same synch times as X. ■

Corollary 2.2 *If X is a PRS and f is continuous mapping from $\mathcal{S}$ into a complete separable metric space $\mathcal{S}_1$ then $\{f(X(t))\}$ is tight.*

Proof: $f(X(t))$ has paths in $\mathcal{D}_{\mathcal{S}_1}$ and is PRS wrt the same synch times as X. ■

Corollary 2.2 may fail if the path regularity of $\mathcal{D}$ is not enforced. The importance of this is that measurable functions of a PRS X need not be tight since $f(X(t))$ may no longer have paths in $\mathcal{D}$.

Example (2) *A functional of a PRS that is not tight.* Let $B(t)$ be the forward recurrence time for the deterministic renewal process $\{t_n\}$; $t_n = n$ $(n \geq 0)$. Define $f(x) = 1/x$, $x \in (0,1]$. Then $f(B(t))$ does not have a limit from the left at any integer point; in fact for each n, $f(B(t)) \longrightarrow \infty$ as $t \longrightarrow n$. In particular, $f(B(t))$ is not tight.

3. Applications to queueing models

In Sigman [1989], a variety of queueing models were shown to have representations in continuous time as a one-dependent regenerative process (od-R). An od-R process X is a synchronous process for which the cycles are one dependent, that is X_n is dependent upon X_{n+1} but is independent of $\{X_{n-k}; k \geq 2\}$. In particular, a positive recurrent od-R process is an ergodic PRS.

Together with Theorem 2.1 we now can obtain a variety of new tightness results for such things as queue length. We present two such results as an illustration.

One large class of models are those that can be represented at exogenous arrival epochs as a Harris recurrent Markov chain $C = \{C_n\}$. As shown in Sigman [1989] these models inherit a one-dependent regenerative structure (od-R) from C when represented as a process Z in continuous time. Our first example is the simplest non-trivial example of this type; the classic FIFO $GI/GI/c$ queue (see for example, page 493 of Wolff[1989]). The interarrival time and service time distributions are only assumed to have finite first moment (no non-lattice or spread-out assumptions!). λ and μ denote the arrival and service rate respectively. $Q(t)$ denotes the number of customers waiting in the queue (not in service) at time t. $Y(t)$ is the c-tuple of residual service times. $B(t)$ denotes the forward recurrence time of the exogenous renewal process of arrivals. $\vec{K}(t)$ denotes a list of the service times of all customers waiting in the queue at time t. $V(t)$ denotes the *total work in system process*, that is, the sum of all remaining service times of all customers in the system at time t.

Proposition 3.1. *For a FIFO $GI/GI/c$ queue, if $0 < \lambda < c\mu$ then regardless of initial conditions the queue length process $Q(t)$ and the total work in system process $V(t)$ are tight.*

Proof: From Proposition 8.1 of Sigman [1989] it follows that the process $Z(t) = (Q(t), Y(t), B(t), \hat{K}(t))$ is a positive Harris Recurrent Markov process and hence is positive recurrent od-R (Theorem 2 of Sigman [1989]). Both $V(t)$ and $Q(t)$ are continuous functionals of Z and hence for any fixed inital state each forms an (ergodic) PRS (with the same synch times as Z) and hence is tight. ■

We mention that the same kind of result above can be derived for open queueing networks having i.i.d. exogenous interarrival times (general distribution, finite fiorst moment), i.i.d. service times (general distributions, finite first moment) and Markovian routing. These are sometimes called *Open Jackson Networks with general i.i.d. input*.

Our second example is a single server queue with input (the marked point process of arrival and service times) *governed* by a Harris recurrent Markov process (HRMP) (see section 7 of Sigman [1989]). The idea here is that the queue inherits the od-R structure of its input. λ denotes the long run arrival rate and μ the long run service rate: $\rho \stackrel{\text{def}}{=} \lambda/\mu$. We do not assume the FIFO discipline; any work conserving discipline is allowed.

Proposition 3.2. *For a single server queue with input governed by a positive HRMP, if $0 < \rho < 1$ then regardless of initial conditions the queue length process $Q(t)$ and the total work in system process $V(t)$ are tight.*

Proof: Analogous to the proof of Proposition 3.1. from Proposition 7.1 of Sigman [1989], both $Q(t)$ and $V(t)$ are continuous functionals of a positive HRMP and hence PRS. ■

> *Remark(3.1)*: It is known that in general, $Q(t)$ and $V(t)$ for the above models do not converge weakly to their steady-state distributions. For the FIFO $GI/GI/c$, if the interarrival time distribution is spread-out then weak convergence is obtained (in fact in total variation).

References

[1] Baccelli, F. and Bremaud, P. (1987). *Palm Probabilities and Stationary Queues*. Lecture Notes in Statistics No. 41. S ..inger Verlag, Berlin.

[2] Daly, D. and Vere-Jones, D. (1988). *An Introduction to the Theory of Point Processes*. Springer Verlag, New York.

[3] Ethier, S. and Kurtz, T. (1986). *Markov Processes, Characterization and Convergence*. Wiley series in Probability and Statistics. John Wiley and Sons.

[4] Franken, P., Koenig, D., Arndt, U., Schmidt, V.(1981). *Queues and Point Processes*. Akademie Verlag, Berlin.

[5] Glynn, P. and Sigman, K. (1989). *Regenerative Processes*. In preparation.

[6] Rolski, T. (1981) *Stationary Random Processes Associated With Point Processes*. Springer Verlag, New York.

[7] Serfozo, R. (1972). Semi-Stationary Processes. *Z. Wahrscheinlichkeitstheorie verw. Geb.* **23**, 125-132.

[8] Sigman, K. (1989). One-Dependent Regenerative Processes And Queues In Continuous Time. *Math of OR* (to appear Fall 89).

[9] Wolff, R. W. (1989). *Stochastic Modeling And The Theory Of Queues*. Prentice Hall, Englewood Cliffs, New Jersey.

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Abstract

Let $X = \{X(t) : t \geq 0\}$ be a *positive recurrent synchronous process (PRS)*, that is, a process for which there exists an increasing sequence of random times $\tau = \{\tau(k)\}$ such that for each k the distribution of $\theta_{\tau(k)} \circ X = \{X(t + \tau(k)) : t \geq 0\}$ is the same and the cycle lengths $T_n = \tau(n+1) - \tau(n)$ have finite first moment. Whereas the ergodic properties of such processes are well known in the literature, the same is not so for the distributional properties of either the marginals $X(t)$ or more generally the shifted processes $\theta_s X = \{X(s+t) : t \geq 0\}$ in function space. In the present paper we show that these distributions are in fact tight. In contrast to classical regenerative processes we also show that the standard types of regularity assumptions (non-lattice cycle length distribution, mixing) do not ensure weak convergence to steady-state for a PRS. Applications are given in the context of one-dependent regenerative (od-R) processes. These arise in the queueing models that motivated this paper.

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