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NRL Memorandum Report 6522

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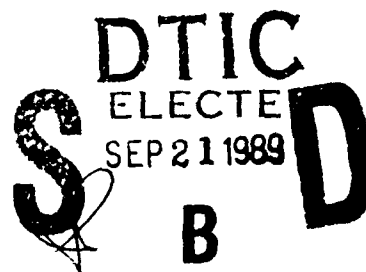
Cavity Eigenmodes for the NIST/NRL Free Electron Laser

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September 5, 1989



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89 9 20 132

REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b RESTRICTIVE MARKINGS		
2a SECURITY CLASSIFICATION AUTHORITY			3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.		
2b DECLASSIFICATION/DOWNGRADING SCHEDULE			5 MONITORING ORGANIZATION REPORT NUMBER(S)		
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Memorandum Report 6522			5 MONITORING ORGANIZATION REPORT NUMBER(S)		
6a NAME OF PERFORMING ORGANIZATION Naval Research Laboratory		6b OFFICE SYMBOL (If applicable) Code 4790	7a. NAME OF MONITORING ORGANIZATION		
6c. ADDRESS (City, State, and ZIP Code) Washington, DC 20375-5000			7b. ADDRESS (City, State, and ZIP Code)		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION ONR		8b. OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8c. ADDRESS (City, State, and ZIP Code) Arlington, VA 22217			10 SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO	PROJECT NO 9M0214	TASK NO WORK UNIT ACCESSION NO
11. TITLE (Include Security Classification) Cavity Eigenmodes for the NIST/NRL Free Electron Laser					
12. PERSONAL AUTHOR(S) Riyopoulos,* S. and Tang, C.M.					
13a. TYPE OF REPORT Interim		13b. TIME COVERED FROM _____ TO _____		14. DATE OF REPORT (Year, Month, Day) 1989 September 5	
15. PAGE COUNT 44					
16 SUPPLEMENTARY NOTATION *Science Applications Intl. Corp., McLean, VA 22102					
17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Free electron lasers		
			Resonator modes		
19 ABSTRACT (Continue on reverse if necessary and identify by block number)					
The cavity transfer matrix for the round trip of a light pulse in the optical resonator of the National Institute of Standards and Technology (NIST)/Naval Research Laboratory (NRL) Free Electron Laser (FEL) oscillator is derived. The cavity eigenmodes and the corresponding eigenvalues are obtained, using an expansion in Gaussian-Laguerre vacuum modes, by numerical diagonalization. The fractional power loss per eigenmode, caused by the finite sizes of the cavity mirrors and apertures are determined. It is found that the losses are very small over the entire wavelength regime of operation.					
20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS			21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a NAME OF RESPONSIBLE INDIVIDUAL C.M. Tang			22b TELEPHONE (Include Area Code) (202) 767-4148		22c OFFICE SYMBOL Code 4790

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CAVITY EIGENMODES FOR THE NIST/NRL FREE ELECTRON LASER

I. INTRODUCTION

The NIST(formerly NBS)/NRL FEL oscillator powered by a CW 185 MeV racetrack microtron is currently under construction¹⁻³. A simplified model of the resonator cavity, consisting of the wiggler vacuum chamber and the two mirrors, is shown schematically in Fig. 1. Since stimulated emission takes place predominantly along the electron beam path, the vector potential of the radiation A_R is expressed in terms of the free space eigenmodes $A_R(r) = A_{mp}(r) \exp(-i\omega t) e_{mp} + cc$ of the paraxial wave equation⁴, where e_{mp} is the polarization vector,

$$A_{mp}(r) = \frac{u_{mp}(r;W)}{\left(1 + \frac{z^2}{b^2}\right)^{1/2}} e^{ik\left(z + \frac{(x^2 + y^2)}{2R(z)}\right)} e^{i\zeta_{mp}(z)}, \quad (1)$$

$k = \omega/c$ is the wave number and ω is the frequency. In Eq. (1) the exponent $k[z + (x^2 + y^2)/2R(z)]$ contains the phase variation on the wavelength scale $\lambda = 2\pi/k$, with spherical wavefronts of curvature $1/R(z) = z/(z^2 + b^2)$. The slow phase variation is given by $\zeta_{mp}(z) = (2m + p + 1) \tan^{-1}(z/b)$. The spot size of the radiation envelope is $W(z) = w(1 + z^2/b^2)^{1/2}$, where the distance z is measured from the position of the waist $w = (2b/k)^{1/2}$. The amplitude squared of the mode drops by 1/2 over a distance equal to the Rayleigh length b (also known as confocal parameter). Most of the radiation is confined within a cone parametrized by the diffraction angle $\theta_d = W/z \approx (\lambda/b\pi)^{1/2}$. For given wavelength λ , any two of the four parameters R, W, b, z determine a mode uniquely.

The amplitude profile $u_{mp}(r;W)$ contains the transverse spatial variation, equivalent to a small $k_\perp$, perpendicular to the z -direction. In cylindrical coordinates (r, θ, z) , where $\tan\theta = x/y$, $r = (x^2 + y^2)^{1/2}$, $u_{mp}(r, \theta; W)$ takes the form

$$u_{mp}^{\pm}(r, \theta; W) = a_{mp} \begin{pmatrix} \cos p\theta \\ \sin p\theta \end{pmatrix} \xi^{p/2} L_m^p(\xi) e^{-\frac{\xi}{2}}, \quad \xi = \frac{2r^2}{W^2}, \quad (2)$$

where + (-) signifies cosine (sine) poloidal dependence, $L_m^p(\xi)$ are the associated Laguerre polynomials and the normalizing factor $a_{mp} = [1/(1+\delta_{p0})\pi W^2]^{1/2} [m!/(m+p)!]^{-1/2}$.

In the presence of conducting walls the vacuum expansion (1) still provides the best representation, because (a) the chamber inner radius is much larger than the radiation spot size and (b) the transit time through the cavity and the length of the light pulse itself are too short to allow multiple reflections on the walls and set up cavity eigenmodes. It can also be argued that the small fraction of the radiation reflected from the wall, acting as a perfect conductor for grazing incidence, is lost out of the resonator. The main effect of the chamber, therefore, comes from the two edge apertures, where the radiation spot size is maximum. The resonator is then modeled by a sequence of four optical elements, i.e., two apertures and two mirrors.

The radiation profile is altered after each encounter with an optical element. A pure incident mode $A_{mp}(r)$ will, in general, be partially transformed into different modes. This is caused by the finite size of the apertures, and, in addition, by spherical aberration and surface

imperfections in case of the mirrors. Consider the incoming radiation to a given optical element as consisting of various modes (m,p) of the same curvature $R_1(z)$. Both incident and reflected radiation are expanded into eigenmodes, respectively as follows,

$$A^i(r_i) = \sum_{m,p} c_{mp}^i A_{mp}(r_i) ,$$

$$A^o(r_o) = \sum_{n,q} c_{nq}^o A_{nq}(r_o) . \quad (3)$$

The relation among the incident and reflected expansion coefficients c_{mp}^i and c_{pq}^o is written as

$$c^o = R c^i , \quad (4a)$$

or

$$c_{nq}^o = \sum_{m,p} R_{nq}^{mp} c_{mp}^i , \quad (4b)$$

where R_{nq}^{mp} are the elements of the reflection matrix R .

The radiation profile at the end of the round trip inside the resonator will relate to the original profile through the resonator transfer matrix M ,

$$M = T_2 R_2 T_2 T_1 R_1 T_1 , \quad (5)$$

where T_i and R_i are the transmission matrices through the chamber apertures and the reflection matrices from the mirrors 1 and 2 respectively. The cavity eigenmodes C_j with eigenvalues v_j are given by

$$M C_j = v_j C_j . \quad (6)$$

Since M is generally nondiagonal, the eigenmodes are mixtures of vacuum modes (1).

The electron beam is an active medium that changes the radiation profile during amplification in each passage. If G is the amplification matrix, then a steady state exists finally if the matrix equation

$$G M C_s = g_s C_s \quad (7)$$

has solutions with $|g_s| = 1$. A steady state need not be an eigenmode of the empty resonator; this could happen only in the case of equal amplification g_a for each eigenmode, i.e., $G = g_a I$ where I is the identity matrix. In an FEL, a different gain is associated with each free space eigenmode. However, in cases when the off-diagonal elements of both G and M become vanishingly small, both the resonator modes and the final steady states approach the pure vacuum modes (1).

In this paper we first study the vacuum performance of the optical cavity. The detailed cavity mode structure in terms of vacuum modes and the associated eigenvalues are obtained by numerical diagonalization of the cavity matrix M . The fractional power loss η_j per cavity mode per trip is found from the magnitude of the eigenvalue

$$\eta_j = 1 - |v_j|^2. \quad (8)$$

The profiles for the cavity modes are also obtained utilizing the expansion coefficients of C_j into the vacuum modes. We also examine the eigenmode structure of the combined gain-transfer matrix $G M$ in cases of small gain.

II. TRANSFER MATRIX FOR A SINGLE OPTICAL ELEMENT

The reflection of Gaussian light beams from mirrors was studied in some detail in Ref. 5 for arbitrary angle of radiation incidence. In the limit of normal incidence considered here, the reflection matrix elements are given by the surface integrals

$$R_{nq}^{mp} = \iint_S d\theta_s dr_s r_s \frac{u_{mp}(r_s, \theta_s) u_{nq}(r_s, \theta_s)}{\left[1 + \frac{r_o^2}{b_o^2}\right]^{1/2}} \left[\frac{1 + \frac{z_s^2(r_s)}{b_o^2}}{1 + \frac{z_s^2(r_s)}{b_i^2}} \right]^{1/2} e^{i\Delta(r_s, \theta_s)} \times e^{i\zeta_{nq}^i(z_s) - i\zeta_{mp}^o(z_s)} . \quad (9)$$

The mirror surface S is spherical, expressed in the coordinate system (r_s, θ_s, z_s) with origin located at the mirror center, by

$$(z_s - R_m)^2 + r_s^2 = R_m^2 , \quad (10)$$

where R_m is the mirror radius of curvature. Equation (10) is used to express z_s on the surface S in terms of r_s . The mirror boundary is given by

$$r_s^2 = \rho^2 , \quad (11)$$

where ρ is the radius of the mirror cross-section.

The phase factor $\Delta(r_s, \theta_s)$, related to the optical path along the various rays connecting the incoming wavefront with its mirror image (reflected), must be approximately constant. Therefore, the curvature of

the outgoing wavefront is related to the incoming and the mirror curvatures through

$$\frac{1}{R_o} = \frac{2}{R_m} - \frac{1}{R_i} . \quad (12)$$

The Rayleigh length b_o and the waist location l_o of the outgoing modes are yet to be determined.

It has been argued^{5,6} that the amount of radiation scattered into other than the incoming modes, as well as that escaping behind the mirror, depends on three factors:

(a) Finite mirror size effects, of the order of

$$\exp \left(- \frac{\rho^2}{W_i^2} \right) , \quad (13)$$

where ρ is the mirror cross-section radius and W_i the incoming radiation spot size.

(b) Spherical aberration effects, of the order of

$$kW_i \left(\frac{W_i^2}{R_m} \right) , \quad (14)$$

coming from the phase term $\Delta(r_s, \theta_s)$ inside (9). Spherical aberration exists even when the mirror curvature matches the radiation curvature. It is caused by the fact that rays ending on a given spherical wavefront are not exactly perpendicular to it, since they originated from a finite size waist and not from a point at the center of curvature.

(c) Surface imperfections, for example, when the reflecting surface is not perfectly spherical.

Spherical aberration effects are usually less important. They will be addressed in future work, together with the potentially more important consequences caused by mirror deformations (buckling) due to heating.

Ignoring (b) and (c) amounts to setting $\Delta(r_s, \theta_s) = 0$ in (9). After substituting expression (2) for the eigenmodes u_{mp} , (9) becomes,

$$R_{nq}^{mp}(\mu, \alpha) = \delta_{pq} C_{nq}^{mp} \int_0^{2\mu} d\xi (\alpha^2 \xi)^{\frac{q}{2}} \xi^{\frac{p}{2}} L_n^q(\alpha^2 \xi) L_m^p(\xi) e^{-\frac{\alpha^2+1}{2} \xi}, \quad (15)$$

where $C_{nq}^{mp} = \alpha [m!n!/(m+p)!(n+q)!]^{1/2}$ and $\xi = 2r^2/W_i^2$. Since the surface S has rotational symmetry about the z-axis, it couples modes with the same poloidal θ dependence, $p = q$. The radial integration is carried out in Appendix A.

In general, the matrix R involves two independent parameters, the ratio of the mirror radius ρ to the radiation spot size squared, $\mu = (\rho/W_i)^2$, and the ratio of the incoming to outgoing spot sizes $\alpha = (W_i/W_o)$. Only the curvature of the reflected mode is set by the mirror, while the outgoing spot size is still a free parameter. This can be exploited by choosing the value W_o that maximizes the coefficient for the fundamental mode in the reflected radiation, i.e.,

$$\frac{\partial R_{00}^{00}}{\partial \alpha} = 0. \quad (16)$$

Once W_o is selected, the exact location and size of the waist(s) for the reflected modes is determined by solving the system of equations

$$W_o = w_o \left[1 + \frac{l_o^2}{b_o^2} \right]^{1/2}, \quad \frac{1}{R_o} = \frac{l_o}{l_o^2 + b_o^2}. \quad (17)$$

The transmission matrix T through an aperture is given by

$$T = e^{i\pi} R, \quad (18)$$

where R is the reflection matrix for a plane mirror ($R_m = \infty$) of the same cross section ρ , while the curvature transformation is

$$\frac{1}{R_0} = \frac{1}{R_i} . \quad (19)$$

Aberration and surface imperfections do not affect transmission through apertures.

III. CAVITY EIGENMODES

We seek a class of cavity eigenmodes with the waist located in the middle of the vacuum chamber. Since the mode spot must remain unchanged $W_i = W_0$ during each transmission or reflection we elect $\alpha = 1$ inside the transfer matrix (5) for every optical element. The curvature R is not changed during transmission through an aperture. The mirror curvature, however, must match the incoming and outgoing radiation curvature, $R_i = R_0 = R_m$ in (12). Therefore, for eigenmodes to exist, the equations

$$R_1 = \frac{L_1}{L_1^2 + b^2}, \quad R_2 = \frac{L_2}{L_2^2 + b^2}, \quad (20)$$

must admit a positive solution for b . This is possible when

$$\left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right) > 0, \quad L = L_1 + L_2. \quad (21)$$

Equation (21) is the optical stability condition for the cavity. Even with (21) satisfied the cavity modes will still decay slowly in time because of the finite size of the mirrors. According to (20), the Rayleigh length b for various wavelengths remains fixed for a given mirror configuration, while the waist w varies as $w = (b\lambda/\pi)^{1/2}$. The cross section of the electron beam is adjustable to give a good filling factor for the particular wavelength. The cross section of the chamber is oval and is approximated with a circular one of effective radius $\rho = (\rho_1 \rho_2)^{1/2}$, where ρ_1 and ρ_2 are the major and minor aperture radii.

Because of the imposed axisymmetry, we look for cavity eigenmodes involving combinations of axisymmetric vacuum modes $p = q = 0$. The coefficients $c_j^{(n)}$ of the modes u_{n0} inside the j -th cavity mode C_j

$$C_j = [c_j^{(0)}, c_j^{(1)}, \dots, c_j^{(n)}, \dots], \quad (22)$$

and the corresponding eigenvalues v_j are found by numerical diagonalization of the transfer matrix M , utilizing expressions (A3) with $\alpha = 1$ for each individual optical element inside (5). Eventually, the cavity matrix M depends on the spot to aperture size ratios for the four optical elements,

$$M[\mu_1(\lambda), \mu_2(\lambda), \mu_3(\lambda), \mu_4(\lambda)] , \quad (23)$$

where in turn μ_i depends on the wavelength through

$$\mu_i(\lambda) = \left(\frac{\lambda}{\pi b} \right)^{1/2} \frac{\rho_i}{\left(1 + L_i^2/b^2 \right)^{1/2}} . \quad (24)$$

Two general conclusions are made. First, it is found that, for radiation spot sizes smaller than about one third the aperture sizes, the dominant contribution in each cavity eigenmode comes from a single coefficient $c_j^{(n)}$. Cavity eigenmodes, in this case, approach pure vacuum modes, as expected from the smallness of the off-diagonal matrix elements in M . Second, if the eigenvalues are arranged according to magnitude, $|v_0| > |v_1| > \dots |v_j| > |v_{j+1}| \dots$, the largest eigenvalue corresponds to the eigenmode closest to the fundamental vacuum mode u_{00} , i.e., the eigenmode with $|c_0^{(0)}| \approx 1$. The next largest eigenvalue corresponds to the eigenmode closest to the first radial vacuum mode, i.e., with $|c_1^{(1)}| \approx 1$, and so on. This is expected as the rms spot size for the n -th radial mode $u_{n0}(r;W)$ increases with n as $(n+1)^{1/2}W$.

The NIST/NRL oscillator has been designed for eigenmodes of Rayleigh length equal to half the vacuum chamber length. Two different arrangements, one with full wiggler and one with half length wiggler will be used for the wavelength regimes of $0.2\mu\text{m}$ to $2\mu\text{m}$ and $2\mu\text{m}$ to $10\mu\text{m}$ respectively. The design parameters are $b = L_w/2 = 107.5\text{cm}$, $L_1 = 521\text{cm}$,

$L_2 = 386\text{cm}$, $R_1 = 543\text{cm}$, $R_2 = 417\text{cm}$, $\rho_1 = \rho_2 = 2.54\text{cm}$ for the half wiggler and $b = L_w/2 = 198\text{cm}$, $L_1 = 431\text{cm}$, $L_2 = 477\text{cm}$, $R_1 = 521\text{cm}$, $R_2 = 559\text{cm}$ for the full wiggler.

The fractional power loss $\eta_j = 1 - |v_j|^2$ for the first 5 cavity eigenmodes of the half wiggler arrangement is plotted in Fig. 2 as a function of the wavelength λ . Through the planned regime of operation the loss for the fundamental cavity mode never exceeds 1%. The loss factor for the next two modes is also very small, so that the mode selection is going to be determined by the differences in the radiation gain for each mode. In Fig. 3(a) we plot the expansion coefficients $c_n^{(0)}$ of the fundamental cavity mode C_0 into the vacuum modes u_{n0} . It shows that the cavity mode profile is very close to a pure u_{00} vacuum mode, with other modes contributing less than 1%. The expansion coefficients for the second and third cavity modes are shown in Figs. 3(b) and 3(c) respectively. If the complete expansion coefficients are written as

$$c_j^{(n)} = |c_j^{(n)}| \exp(i\chi_{jn}) ,$$

then the complex amplitude for the j -th eigenmode C_j is given by

$$A_j(r) = |A_j(r)| e^{i\phi_j(r)} ,$$

where

$$|A_j(r)| = \left[Ar_j^2 + Ai_j^2 \right]^{1/2} , \quad \phi_j(r) = \tan^{-1} \left(Ar_j / Ai_j \right) ,$$

$$Ar_j = \sum_n u_{n0}(r) |c_j^{(n)}| \cos \chi_{jn} , \quad Ai_j = \sum_n u_{n0}(r) |c_j^{(n)}| \sin \chi_{jn} .$$

The resulting amplitude profiles for the first three modes at $z = 0$ are shown in Fig. 4 for $\lambda = 2\mu\text{m}$ and $\lambda = 10\mu\text{m}$. In Fig. 5(a) we show the power fraction η_j for the first four cavity modes of Rayleigh length $b = L_w/6$. The vacuum expansion coefficients $c_0^{(n)}$ for the fundamental cavity mode are shown in Fig. 5(b) against the wavelength λ . The transverse amplitude profiles for the first three modes at $\lambda = 2.2\mu\text{m}$ and $\lambda = 10\mu\text{m}$ respectively in Figs. 5(c) and 5(d), show considerable departure from vacuum modes.

The cavity eigenmodes for the full wiggler arrangement are extremely close to vacuum modes and the fractional power losses for the first five of them are below 10^{-3} over the frequency regime from $0.5\mu\text{m}$ to $2\mu\text{m}$. This is caused by the combination of a longer Rayleigh length with a spot size that gets smaller with shorter wavelength.

In case of small gain we can include the effect of the electron beam on the cavity eigenmodes by introducing the amplitude gain matrix G . In the linear regime the cross-coupling among various transverse modes is unimportant and G is diagonal, given by

$$G_{mn} = g(\lambda) f_n \delta_{mn}, \quad (25a)$$

where

$$g(\lambda) = 0.5 F_1^2 \frac{\pi^2}{\sigma_R} \frac{I}{I_A} \frac{\lambda_w^2}{\gamma_0^2} K^2 N^3 \quad (25b)$$

is the amplitude gain for the fundamental vacuum mode. In Eqs. (25) N is the number of wiggler periods, γ_0 is the initial relativistic factor, $\sigma_R = \pi w^2$ is the radiation cross section at the waist, $I_A = 17 \times 10^3$ A, I is the current in amperes, $K = |e| B_w \lambda_w / 2\pi m c^2$ is the wiggler parameter, B_w is the rms magnetic field of the wiggler, λ_w is the wiggler wavelength, related

to the radiation wavelength λ by $\lambda_w = 2\gamma^2\lambda/(1+K^2)$, $F_1 = J_0(b) - J_1(b)$ with $b = K^2/2(1+K^2)$ and f_n is the normalized (to the fundamental) filling factor

$$f_n = \int_0^\infty dr \, r \, j(r) \, u_{n0}(r) / \int_0^\infty dr \, r \, j(r) \, u_{00}(r) , \quad (26)$$

with the parabolic current profile given by $j(r) = j_0 (1 - r^2/\rho_b^2)$ for $r \leq \rho_b$, $j(r) = 0$ for $r > \rho_b$, ρ_b being the beam radius. The round trip gain for the j -th beam-cavity eigenmode is given by

$$g_j = [1 + g(\lambda)] \tilde{v}_j, \quad (27)$$

where $\tilde{v}_j$ is the eigenvalue for the eigenmode $\tilde{C}_j$ of the combined gain-transfer matrix

$$M_g = G M . \quad (28)$$

The fractional power loss $\tilde{\eta}_j = 1 - |\tilde{v}_j|^2$ for the first five eigenmodes is shown in Fig. 6 as a function of the wavelength, while the expansion coefficients of the fundamental eigenmode in terms of vacuum modes are shown in Figs. 7(a)-7(c). The losses for the higher eigenmodes $j > 0$ are now considerably higher than the fundamental mode (compare Figs. 2 and 6), thusly, mode selection among transverse modes occurs through amplification of the radiation, because of the differences in the filling factor f_n . The transverse amplitude profiles at $\lambda = 2.2 \, \mu\text{m}$ and $\lambda = 10 \, \mu\text{m}$ appear in Figs. 8(a) and 8(b).

ACKNOWLEDGMENT

This work supported by ONR Contract No. N00014-87-f-0066 through National Institute of Standards and Technology (NIST).

REFERENCES

1. S. Penner et al., IEEE Trans. Nucl. Sci. NS-32 (5), 2669 (1985).
2. C. M. Tang, P. Sprangle, S. Penner and X. Maruyama, J. Appl. Phys. 63, 5233 (1988).
3. S. Penner et al., Nucl. Instr. and Methods Phys. Res., A272, 73 (1988).
4. See, for example, H. A. Haus in Waves and Fields in Optoelectronics, (Prentice Hall, New Jersey, 1984), p. 180, and references therein.
5. S. Riyopoulos, C. M. Tang and P. Sprangle, IEEE J. Quant. Electronics 25, 803 (1989).
6. S. Riyopoulos, P. Sprangle, C. M. Tang and A. Ting, Nucl. Instr. and Methods in Phys. Res. A272, 543 (1988).

APPENDIX A. Computation of the Reflection Matrix Element

The associated Laguerre polynomials are given by

$$L_m^p(\xi) = \sum_{k=0}^m (-1)^k \frac{(m+p)!}{k!(m-k)!(p+k)!} \xi^k. \quad (A1)$$

Substituting (A1) inside (15) and integrating by factors, using

$$\int d\xi \xi^n e^{-\xi} = -e^{-\xi} \left[\sum_{k=0}^n \frac{n!}{(n-k)!} \xi^{n-k} \right], \quad (A2)$$

one obtains

$$R_{nq}^{mp}(\mu, \alpha) = \delta_{pq} \left[m!n!(m+p)!(n+q)! \right]^{1/2} \frac{\alpha^{p+1}}{\left(\frac{1+\alpha^2}{2} \right)^{p+1}} \sum_{k=0}^m \sum_{l=0}^n \alpha^{2k} \left(\frac{2}{\alpha^2+1} \right)^{k+1} \\ \frac{(-1)^{k+1}(k+l+p)!}{k!l!(m-k)!(n-l)!(p+k)!(p+l)!} \left(1 - e^{-(\alpha^2+1)\mu} \sum_{i=0}^{k+l+p} (2\mu\alpha)^{k+l+p-i} \frac{\left(\frac{\alpha^2+1}{2} \right)^{p-i}}{(k+l+p-1)!} \right). \quad (A3)$$

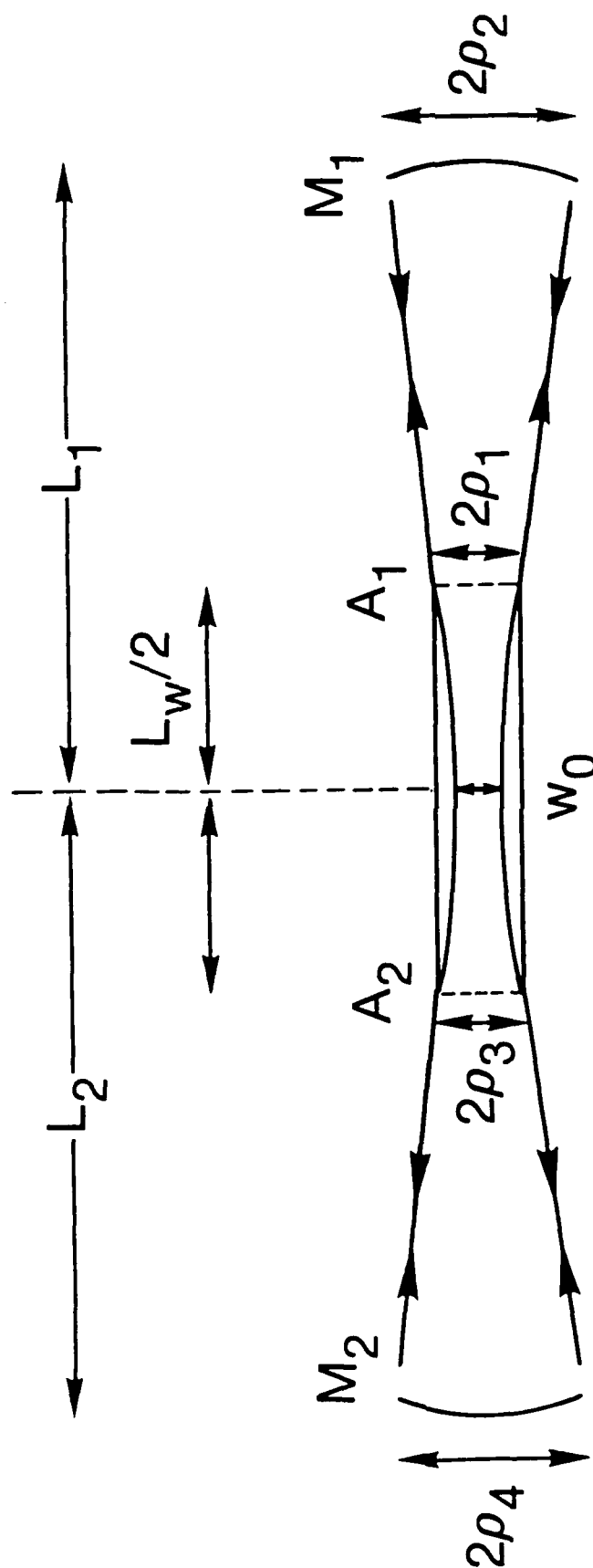


Fig. 1 — Schematic illustration of the NIST/NRL optical cavity.

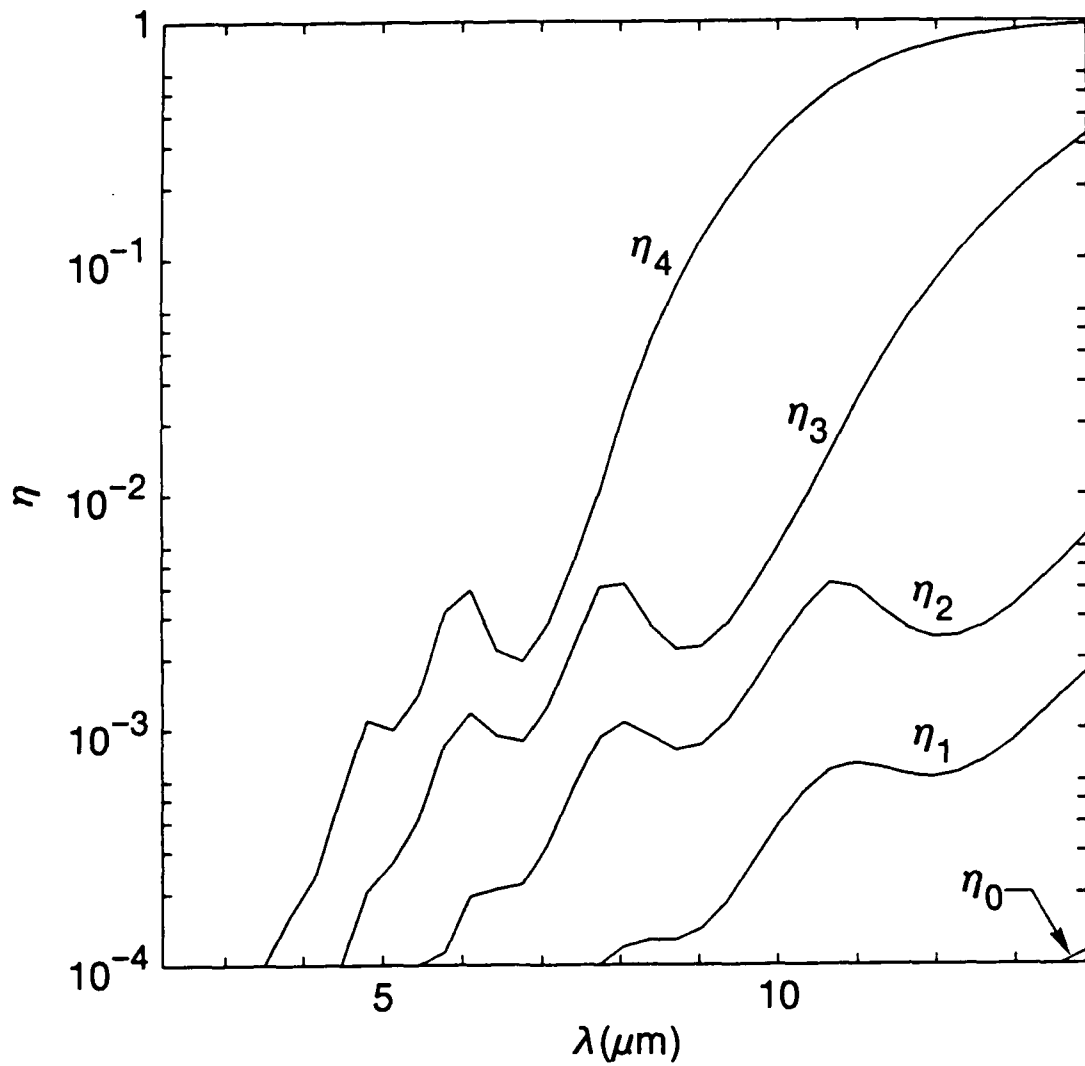


Fig. 2 — Fractional per round trip power loss versus wavelength for the first five resonator eigenmodes in vacuum. The Rayleigh length b is half the wiggler length. The cavity parameters for the half wiggler arrangement are $L_1 = 521\text{cm}$, $L_2 = 386\text{cm}$, $R_1 = 543\text{cm}$, $R_2 = 417\text{cm}$ and $\rho_1 = \rho_2 = 2.54\text{cm}$.

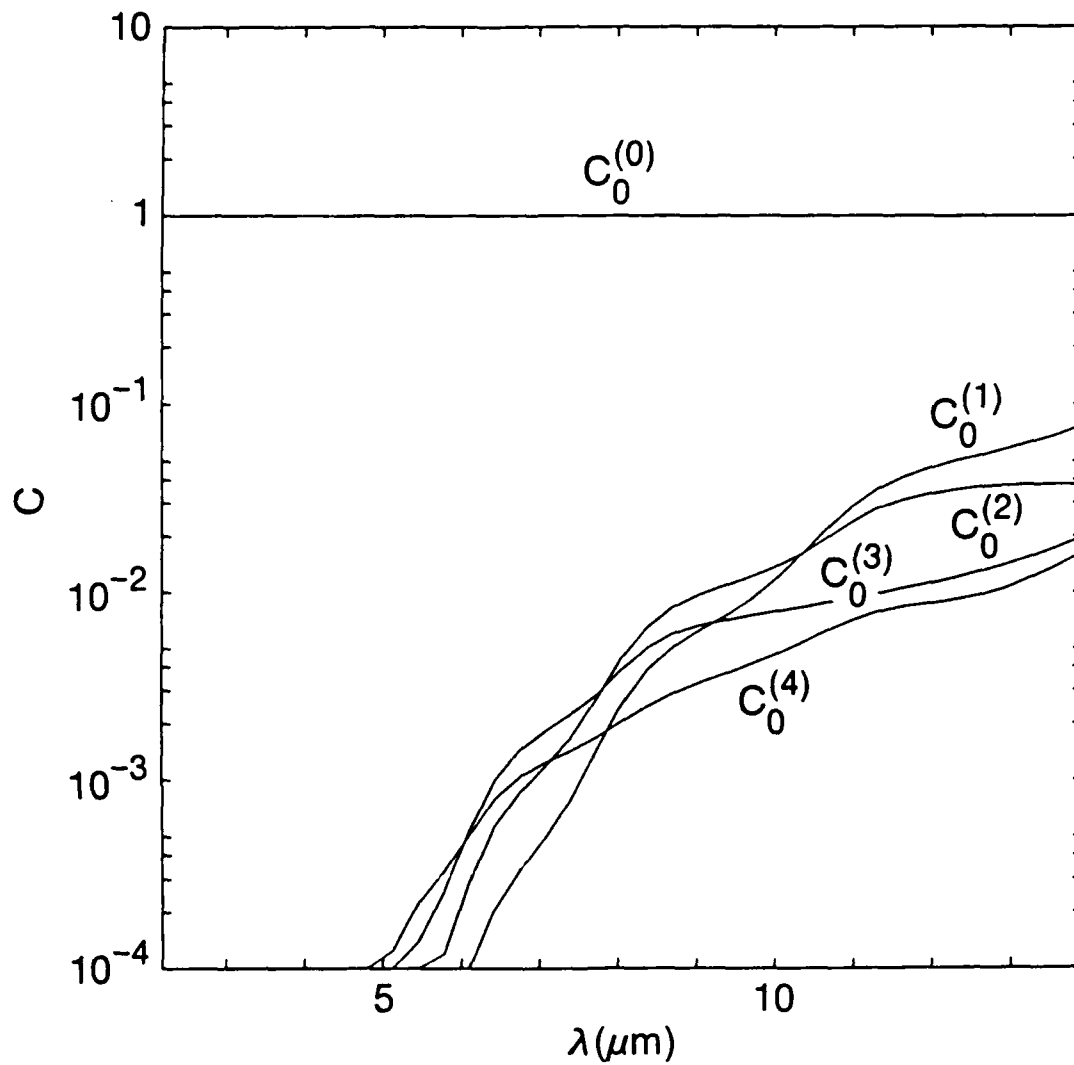


Fig. 3a — Expansion coefficients into vacuum modes for the fundamental cavity eigenmode versus wavelength.

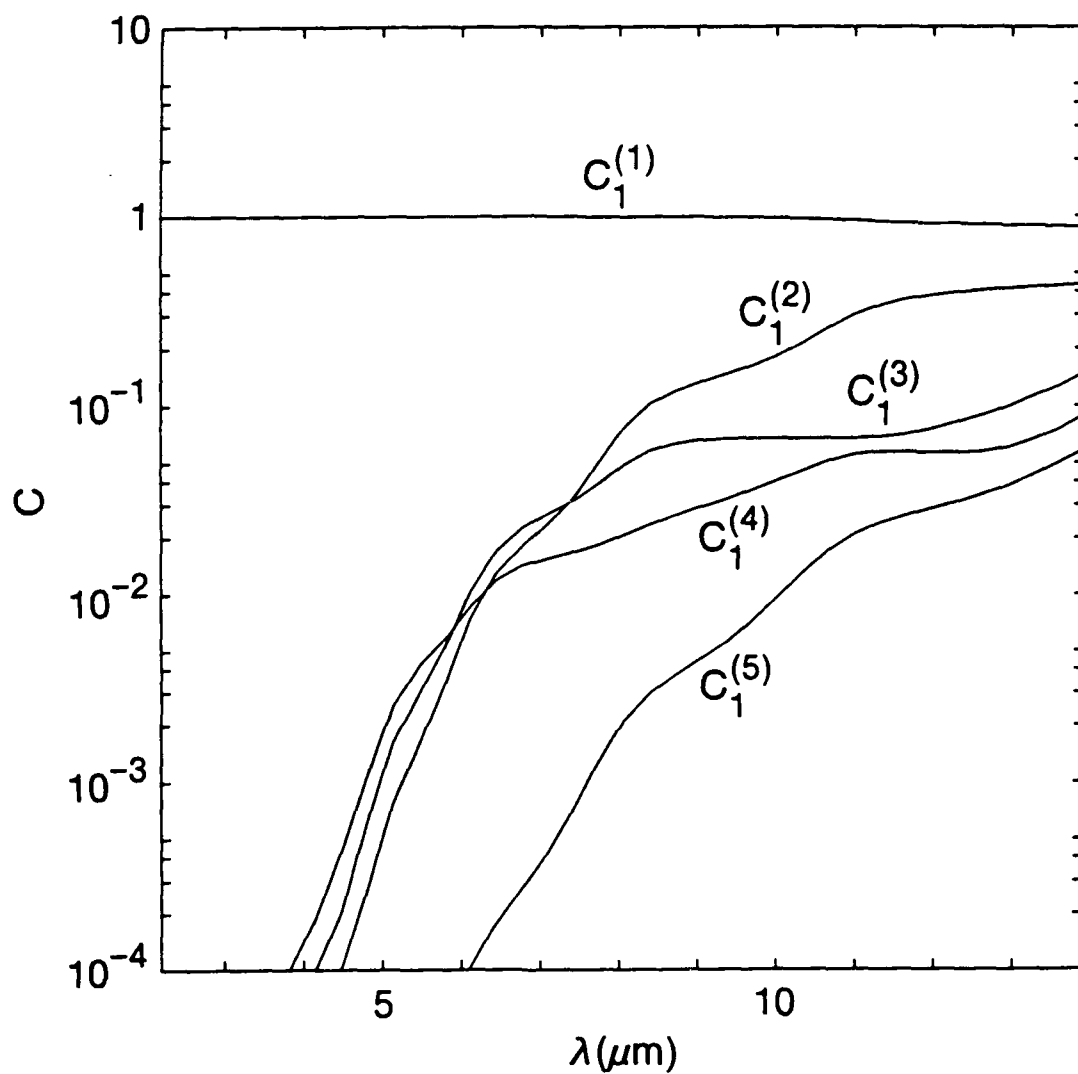


Fig. 3b — Expansion coefficients into vacuum modes for the first cavity eigenmode versus wavelength.

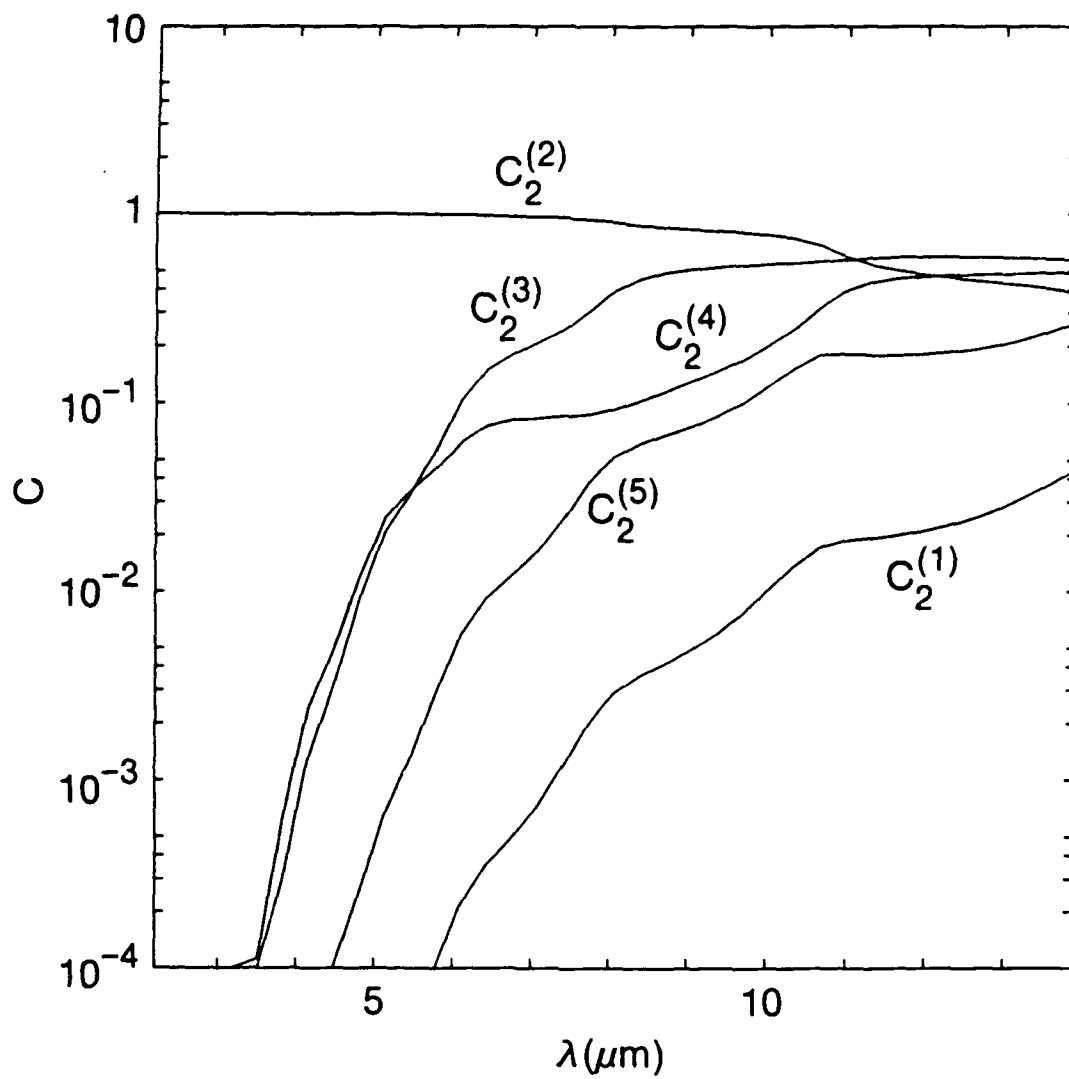


Fig. 3c — Expansion coefficients into vacuum modes for the second cavity eigenmode versus wavelength.

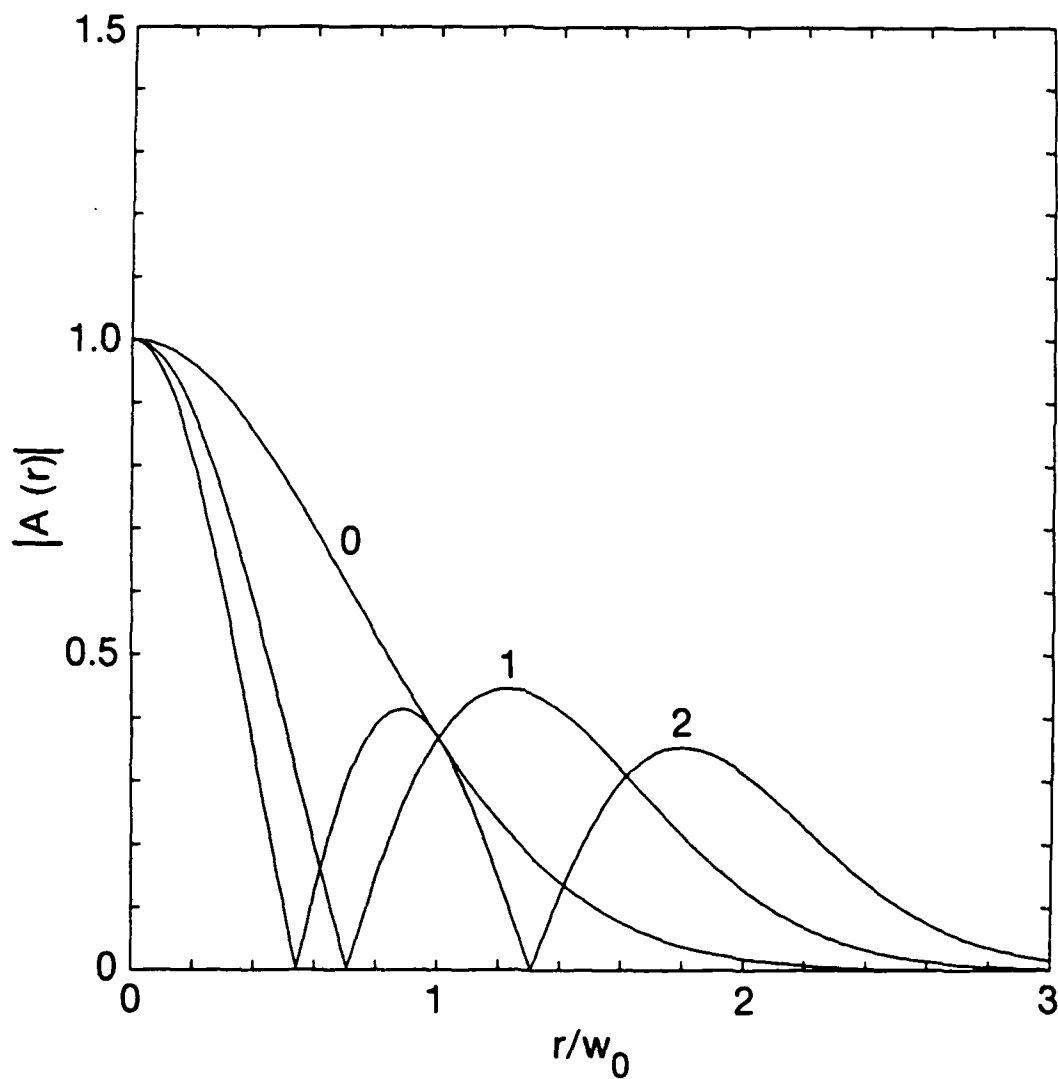


Fig. 4a — The transverse amplitude profiles $|A(r)|$ for the first three axisymmetric cavity modes for wavelength $\lambda = 2.2\mu\text{m}$.

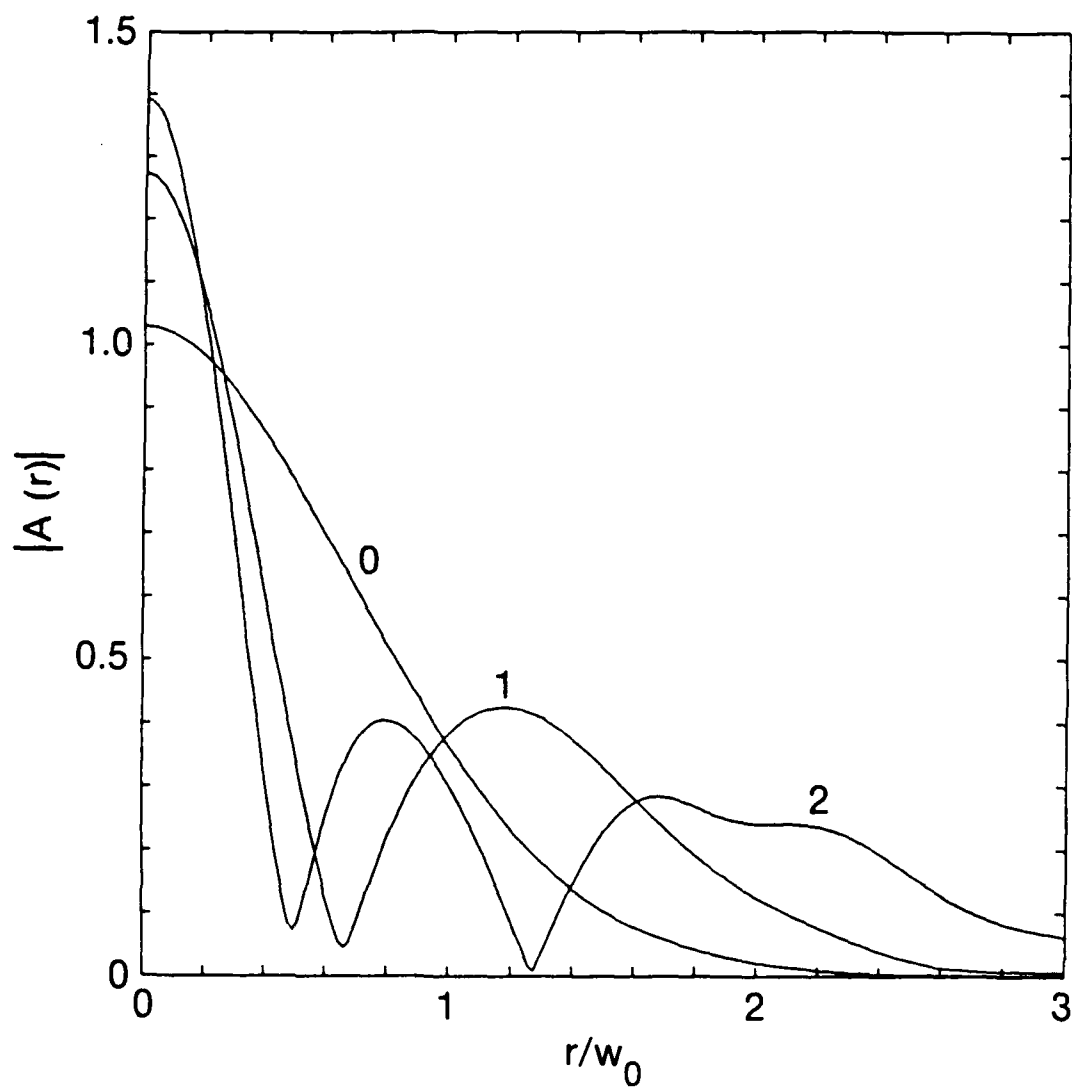


Fig. 4b — The transverse amplitude profiles $|A(r)|$ for the first three axisymmetric cavity modes for wavelength $\lambda = 10.0\mu$.

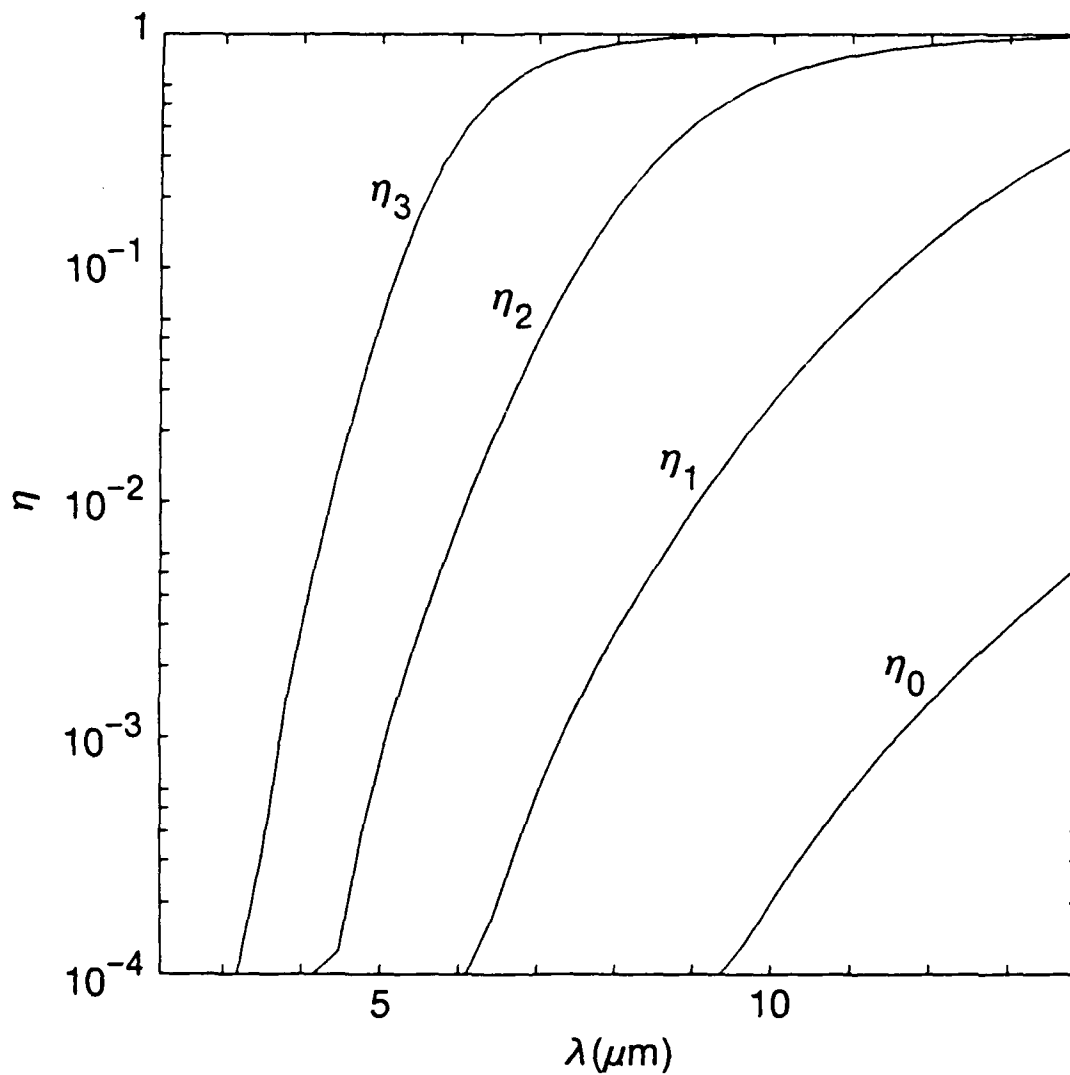


Fig. 5a — Plots of fractional round trip losses η for the first four eigenmodes versus wavelength where the fundamental cavity mode structure has Rayleigh length of $b = L_w/6$ and corresponding mirror curvatures of $R_1 = 523\text{cm}$ and $R_2 = 389\text{cm}$.

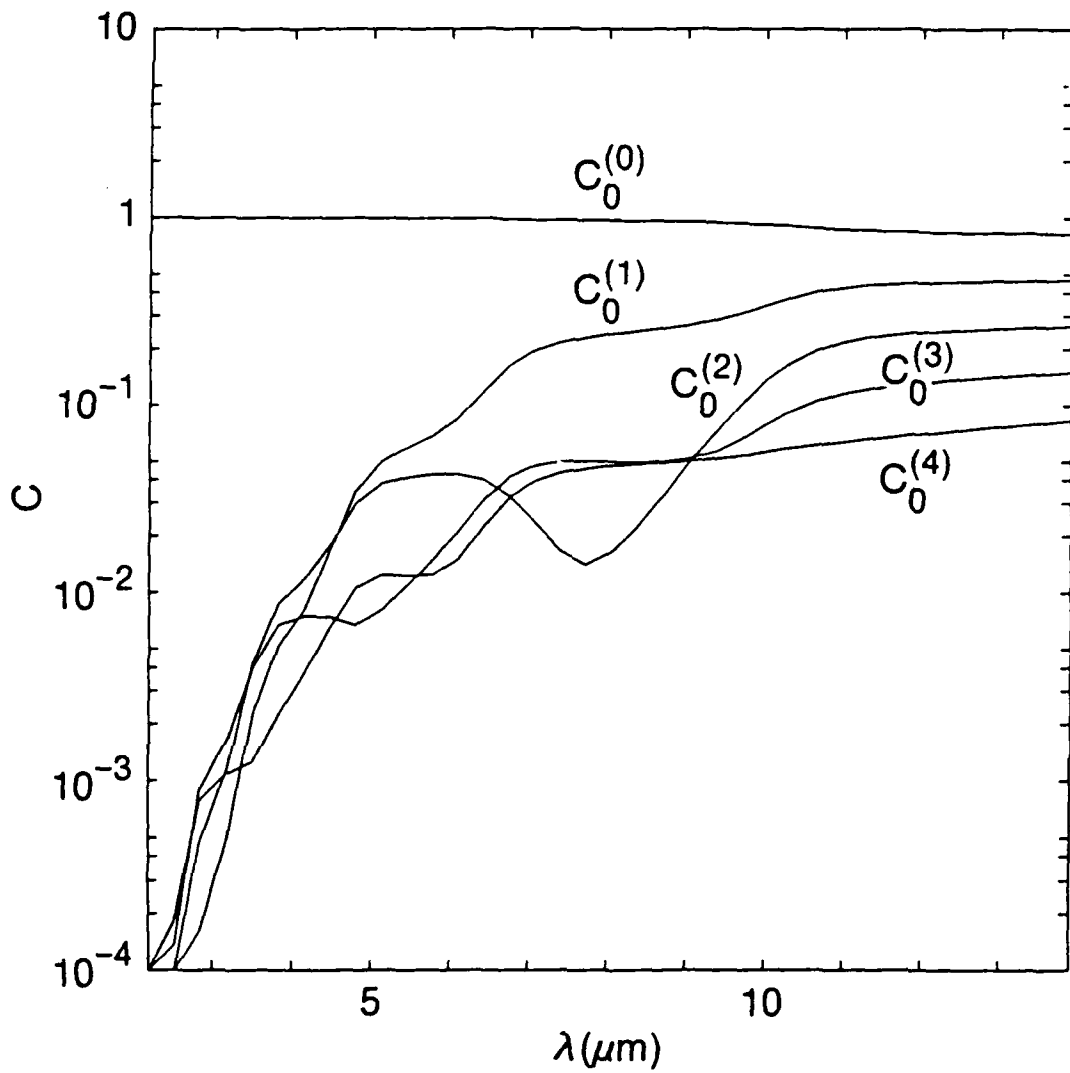


Fig. 5b — Plots of expansion coefficients of the fundamental cavity mode in vacuum modes versus wavelength where the fundamental cavity mode structure has Rayleigh length of $b = L_w/6$ and corresponding mirror curvatures of $R_1 = 523\text{cm}$ and $R_2 = 389\text{cm}$.

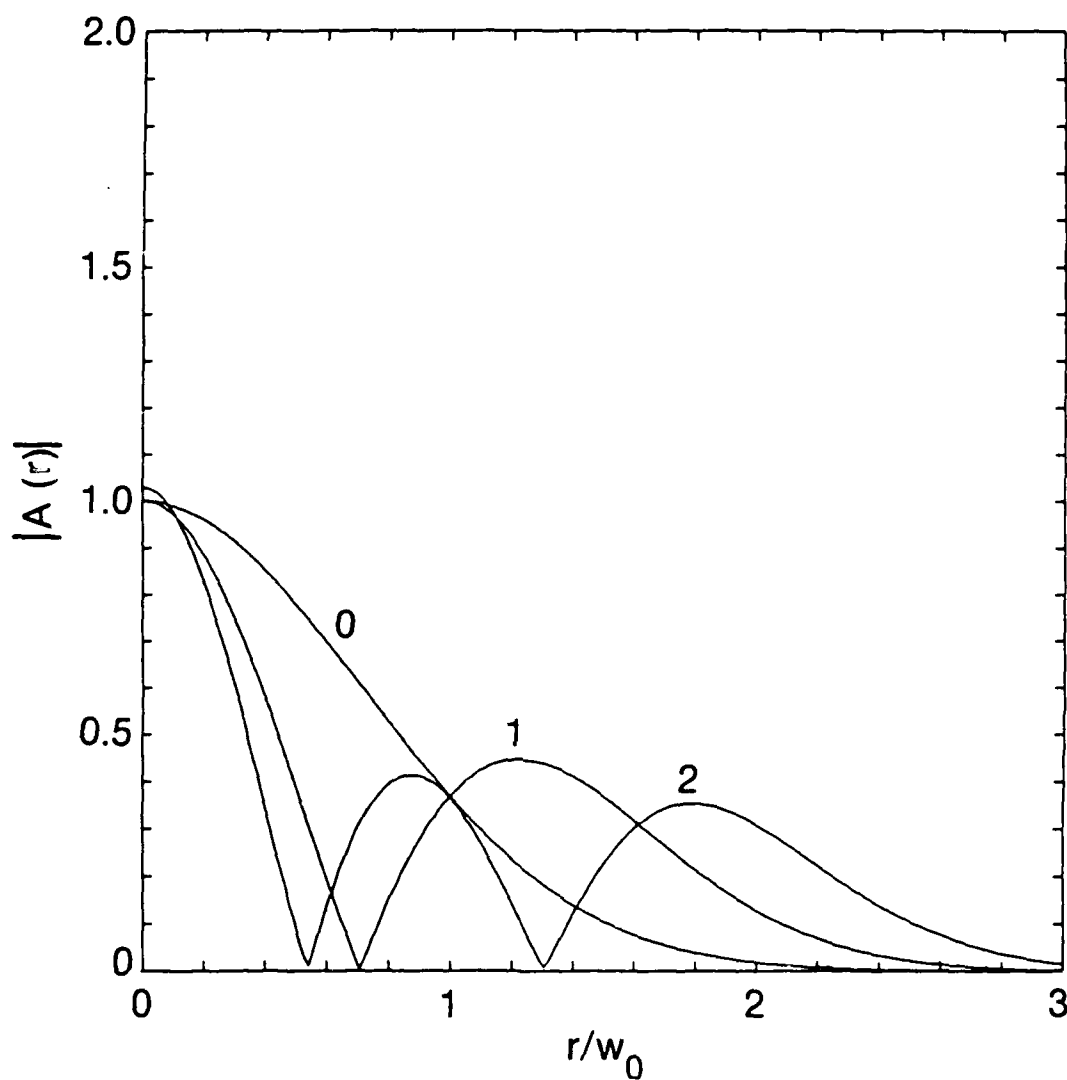


Fig. 5c — Plots of transverse profiles for the first three cavity modes at wavelength $2.2 \mu\text{m}$ where the fundamental cavity mode structure has Rayleigh length of $b = L_w/6$ and corresponding mirror curvatures of $R_1 = 523\text{cm}$ and $R_2 = 389\text{cm}$.

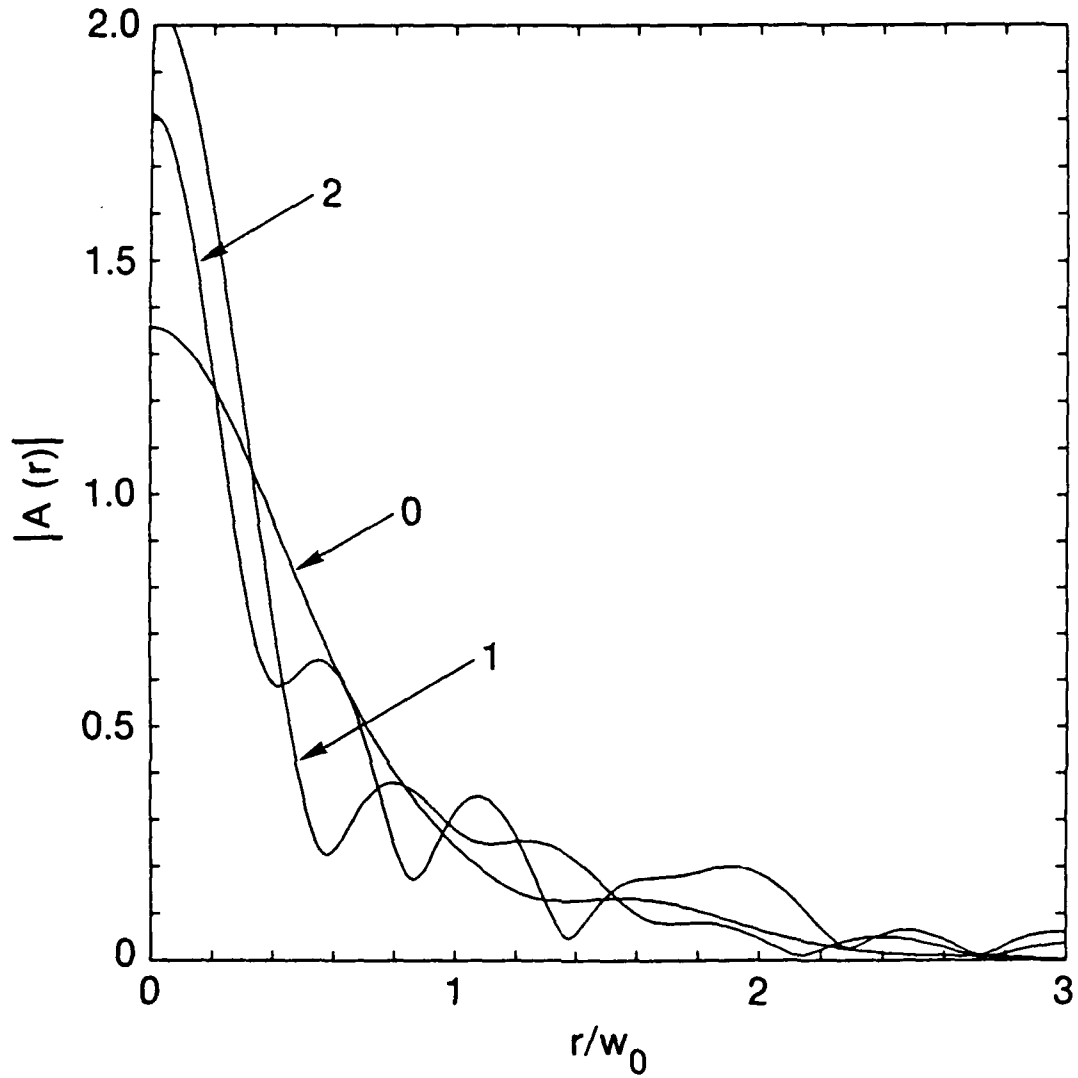


Fig. 5d — Plots of transverse profiles for the first three cavity modes at wavelength $10\text{ }\mu\text{m}$ where the fundamental cavity mode structure has Rayleigh length of $b = L_w/6$ and corresponding mirror curvatures of $R_1 = 523\text{cm}$ and $R_2 = 389\text{cm}$.

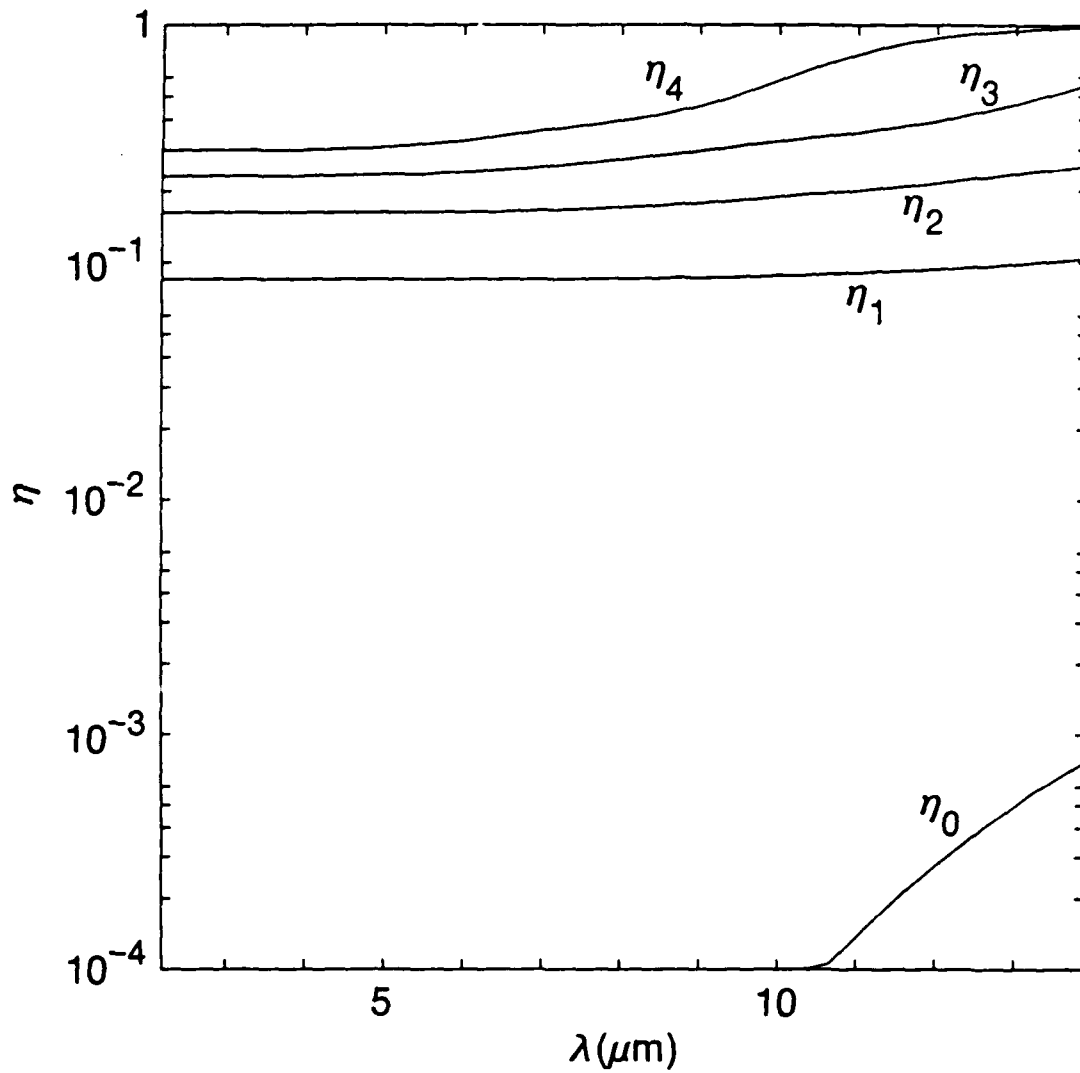


Fig. 6 — Same as in Fig. 2, including the effects of the beam filling factor for small gain.

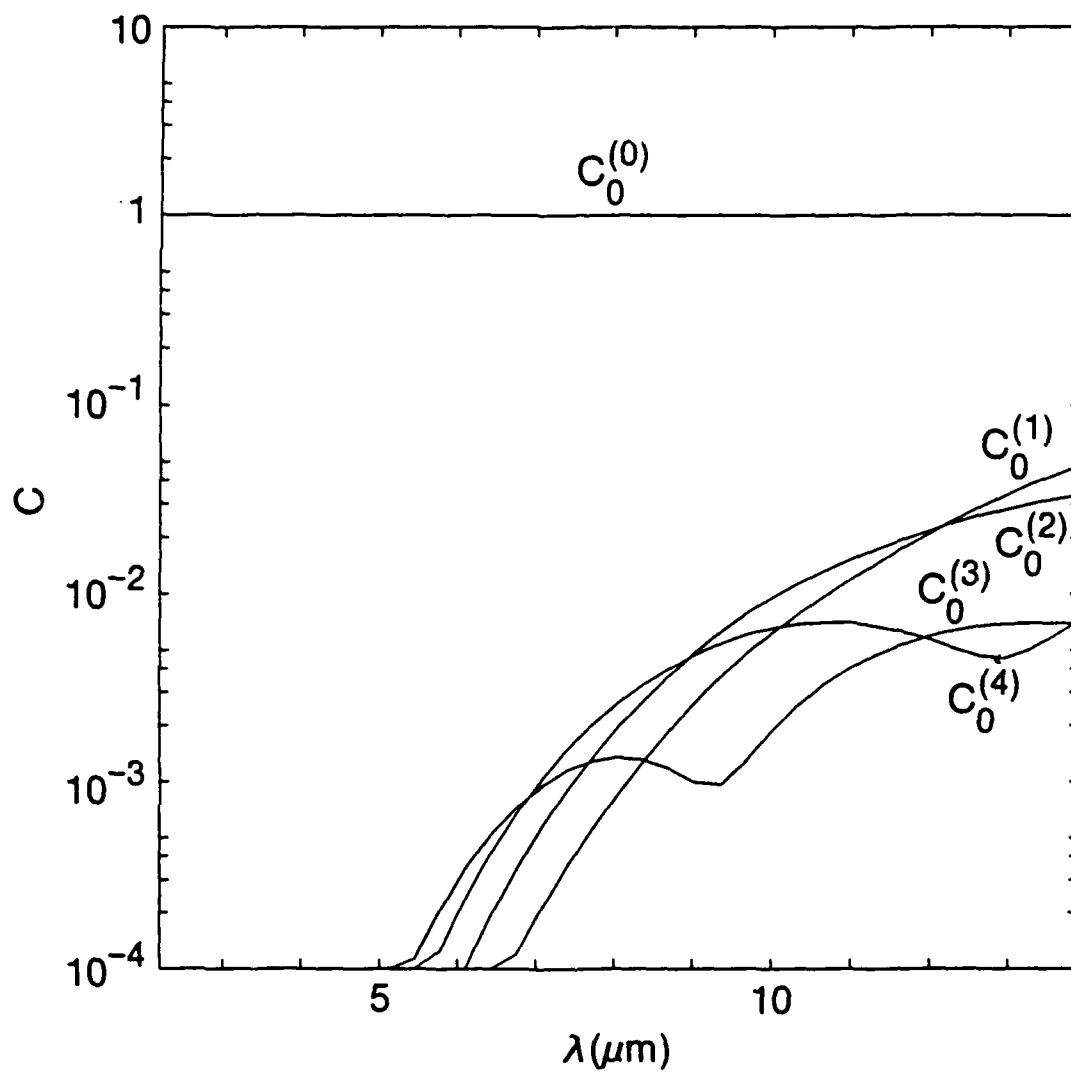


Fig. 7a — Structure of the combined beam-cavity eigenmodes. Same notation as in Fig. 3.

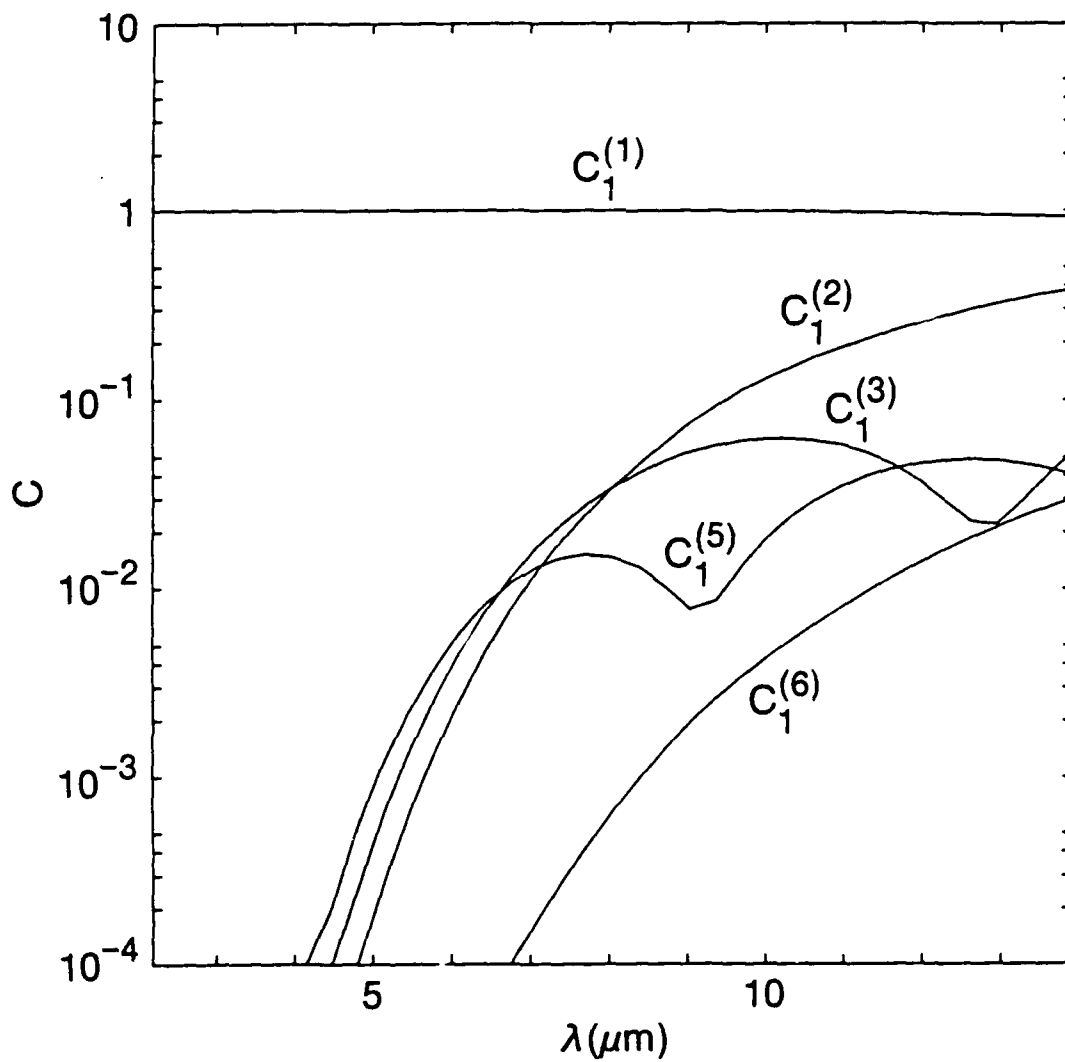


Fig. 7b — Structure of the combined beam-cavity eigenmodes. Same notation as in Fig. 3.

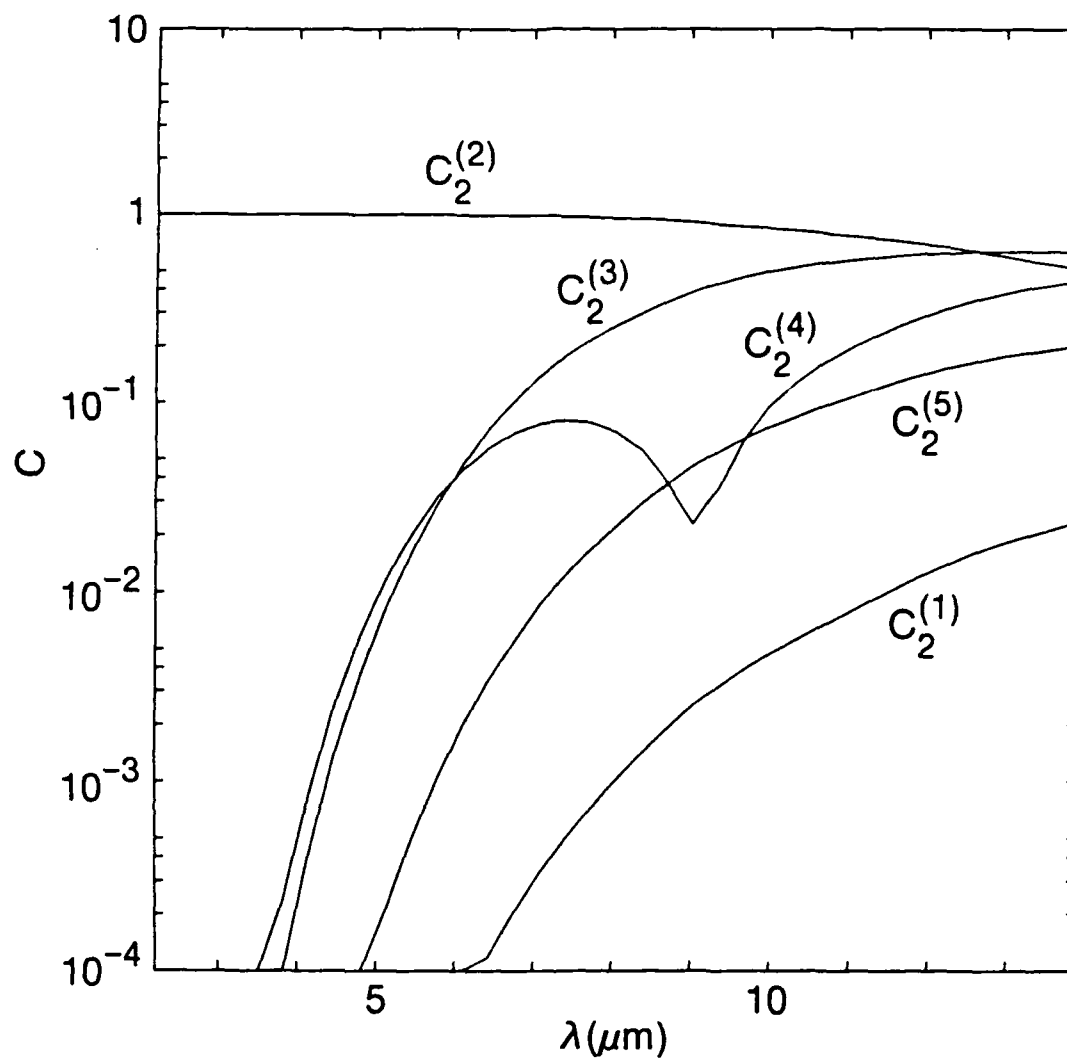


Fig. 7c — Structure of the combined beam-cavity eigenmodes. Same notation as in Fig. 3.

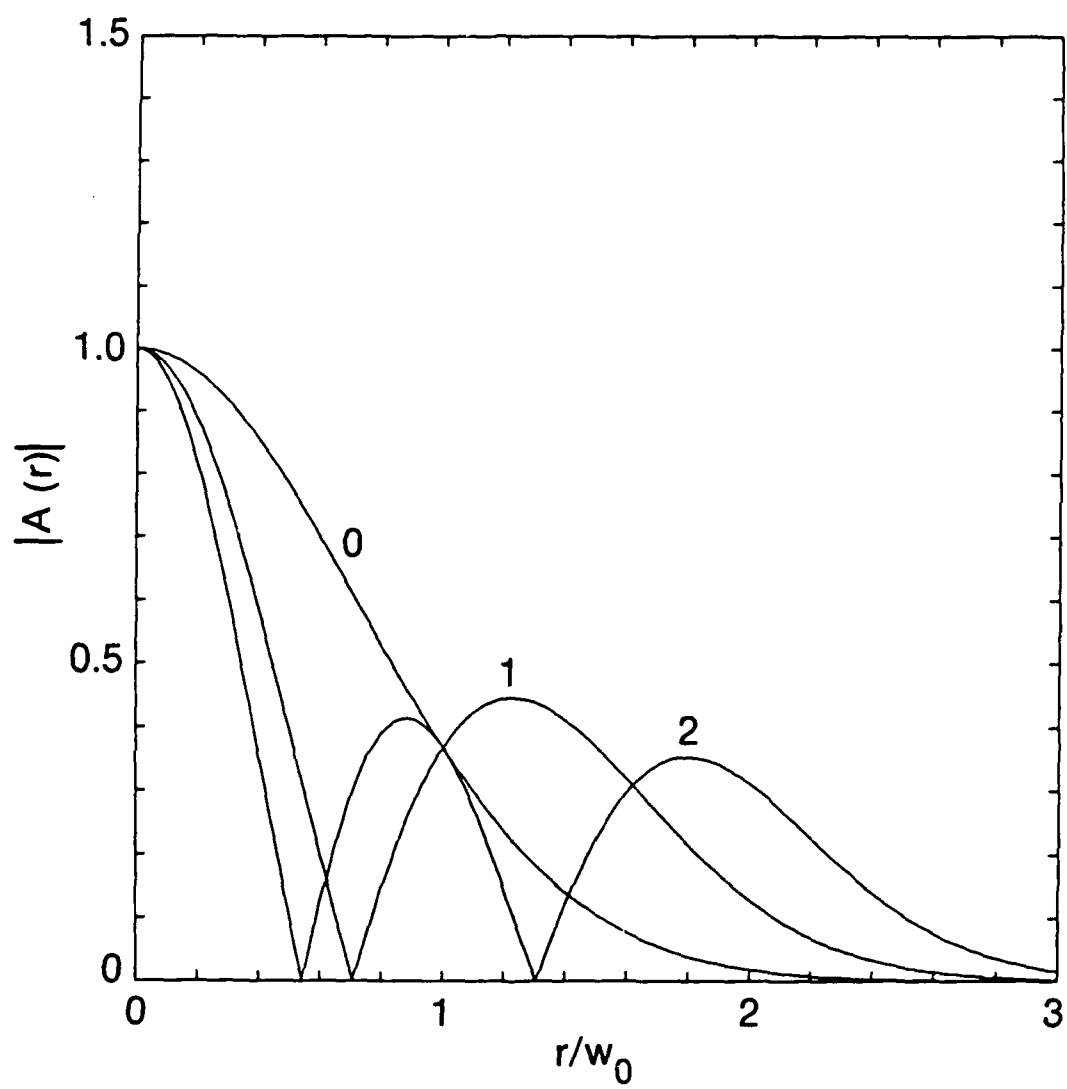


Fig. 8a — Profiles of the first three beam-cavity eigenmodes. Same notation as in Fig. 4.

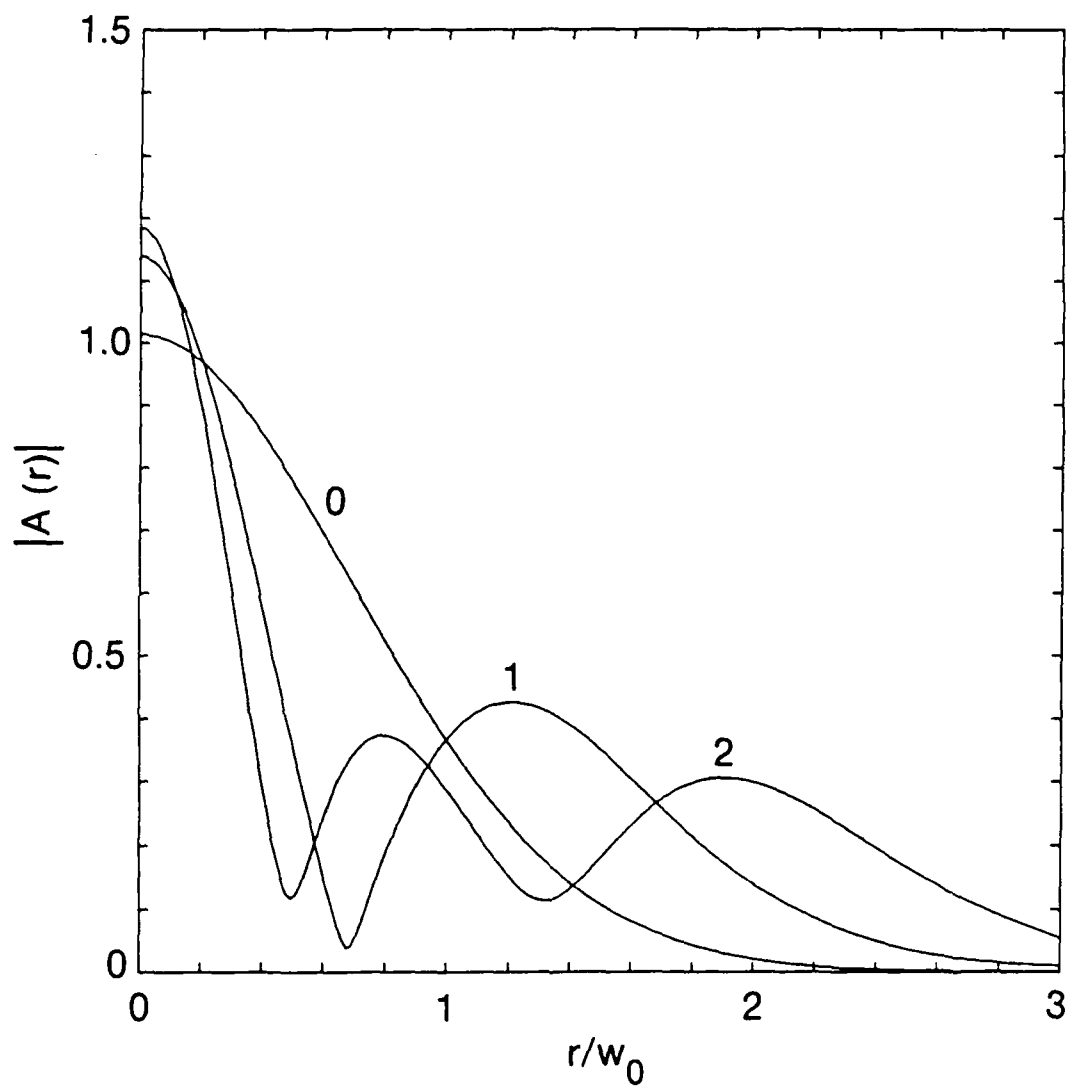


Fig. 8b — Profiles of the first three beam-cavity eigenmodes. Same notation as in Fig. 4.

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