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## LOCALIZATION OF ULTRASOUND IN THICK COMPOSITES

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### INTRODUCTION

The propagation of ultrasound in fiber-reinforced composites is controlled by the relative magnitudes of characteristic length variables. These length variables are the wavelength ( $\Lambda$ ), the fiber diameter ( $a$ ), the thicknesses of plies ( $h$ ), and the overall thickness of the component ( $H$ ). It may generally be assumed that  $\Lambda$  is much larger than the fiber diameter ( $a \sim 8 \mu\text{m}$ ). On the other hand wavelengths of the order of thicknesses of plies ( $h \sim 100 \mu\text{m}$ ), but smaller than the overall thickness of the component may well occur. In that range of wavelengths the propagation of ultrasonic waves gives rise to interesting multiple reflection phenomena which are the topic of this paper.

In this paper we consider the propagation of longitudinal waves in a cross-ply fiber-reinforced composite. Since  $\Lambda \gg a$ , we will use an effective modulus representation for the mechanical behavior of the plies. This means that in the direction normal to the plies longitudinal waves will propagate with a wave speed which is independent of the fiber direction. Hence the problem considered here is a one dimensional problem of wave propagation through layers with identical mechanical impedances which are separated by interfaces.

Since the interfaces of the plies and/or laminates may reflect sound, a composite may display some of the features that are characteristic of the propagation of ultrasound in periodic media, such as passing and stopping bands in the frequency spectrum. Strict structural periodicity is, however, an idealization. In reality there will be random deviations from periodicity, which will in fact destroy the passing bands, and will give rise to attenuation at all frequencies. This attenuation is often referred to as localization. The present paper analyzes such localization.

### INTERFACES

Reflection and transmission of ultrasonic wave motion by an interface or an interphase between two solid materials has been considered for a number of special cases. The insert of Fig. 1 shows an incident wave:  $u = \exp(ikx)$ , a reflected wave:  $u = R \exp(-ikx)$ , and a transmitted wave:  $u = T \exp(ikx)$ . Here we will consider the case that the materials on both sides of the interface or interphase are identical. Typical examples of interphases for such a configuration are a distribution of cracks, as considered by Sotiropoulos and Achenbach [1], or a distribution of spherical cavities as discussed by Achenbach and Kitahara [2]. Another

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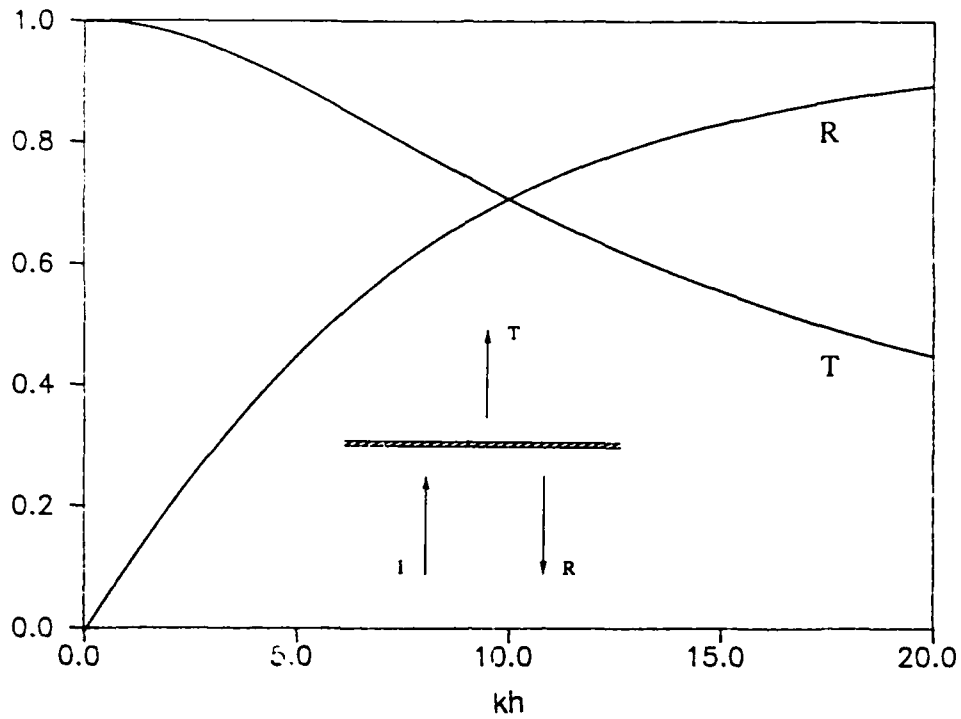


Fig. 1. Magnitudes of reflection and transmission coefficients versus  $kh$  according to Eqn. (1a,b)

typical example of an interphase is a thin layer of slightly different material properties which may develop at a plane of juncture. An interphase is often represented by a layer of springs. For the cases of cracks, cavities and springs, the reflection and transmission coefficients ( $R$  and  $T$ , respectively) depend on the frequency. The general form of the dependence of absolute values on the frequency is illustrated in Fig. 1.

The curves shown in Fig. 1 can conveniently be represented by functional forms that depend on a single dimensionless parameter  $\alpha$  :

$$R = -\frac{i\alpha kh}{1 - i\alpha kh}, \quad T = \frac{1}{1 - i\alpha kh} \quad (1a,b)$$

### PROPAGATOR MATRIX FORMULATION

Figure 2 shows a layer of thickness  $H$ , containing  $n$  identical interfaces separated by distances  $h$ , thus

$$H = (n+1)h \quad (2)$$

The layer is coupled on both sides to another medium, in this case water. Waves propagating in the positive and negative  $x$ -directions have amplitudes  $A$  and  $B$ , respectively.

First we consider the interface  $j$ . Amplitudes of waves to the left of this interface are denoted by  $A_{j-1}$  and  $B_{j-1}$ , while amplitudes to the right are denoted by  $A_j$  and  $B_j$ . Let  $A_{j-1} \exp(ikx)$  and  $B_j \exp(-ikx)$  be considered as incident waves. Then if  $R$  and  $T$  are the reflection and transmission coefficients, respectively, we have

$$B_{j-1} = RA_{j-1} + TB_j, \quad A_j = TA_{j-1} + RB_j \quad (3a,b)$$

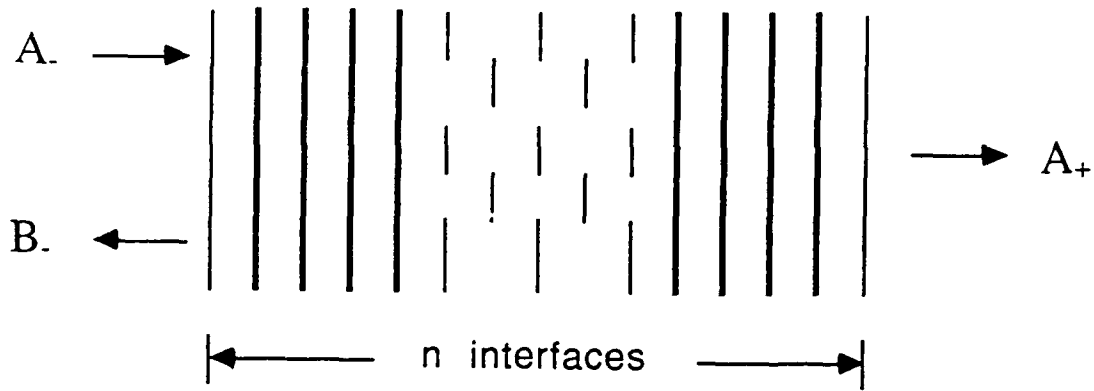


Fig. 2. A layer of thickness  $H$ , containing  $n$  identical interfaces separated by distances  $h$ , and coupled on both sides to water

This system may be expressed in the form

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = \underline{\underline{S}} \begin{pmatrix} A_{j-1}' \\ B_{j-1}' \end{pmatrix}, \quad \text{with} \quad \underline{\underline{S}} = \frac{1}{T} \begin{pmatrix} T^2 - R^2 & R \\ -R & 1 \end{pmatrix}, \quad (4a,b)$$

where  $\underline{\underline{S}}$  is the scattering matrix. Since

$$\begin{pmatrix} A_{j-1}' \\ B_{j-1}' \end{pmatrix} = \underline{\underline{P}} \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix}, \quad \text{with} \quad \underline{\underline{P}} = \begin{pmatrix} e^{ikh} & 0 \\ 0 & e^{-ikh} \end{pmatrix}, \quad (5a,b)$$

Eqns. (4) and (5) may also be expressed as

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = \underline{\underline{W}} \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix} \quad (6)$$

where  $\underline{\underline{W}} = \underline{\underline{S}} \underline{\underline{P}}$  is the propagator matrix.

$$\underline{\underline{W}} = \frac{1}{T} \begin{pmatrix} (T^2 - R^2)e^{ikh} & Re^{-ikh} \\ -Re^{ikh} & e^{-ikh} \end{pmatrix} \quad (7)$$

Now let a wave in the water of amplitude  $A_-$  be incident from the left on the layer of thickness  $H$ . The amplitude of the reflected wave is denoted by  $B_-$ , and the amplitude of the transmitted wave to the right by  $A_+$ . It is then not difficult to see that  $A_+$ ,  $A_-$  and  $B_-$  are related by

$$\begin{pmatrix} A_+ \\ 0 \end{pmatrix} = \underline{\underline{S}}_0 \underline{\underline{P}} \underline{\underline{W}}_n \underline{\underline{W}}_{n-1} \cdots \underline{\underline{W}}_1 \underline{\underline{S}}_0^{-1} \begin{pmatrix} A_- \\ B_- \end{pmatrix} \quad (8a)$$

where

$$\underline{\underline{S}}_0 = \frac{1}{2k_w} \begin{pmatrix} k_w + k & k_w - k \\ k_w - k & k_w + k \end{pmatrix} \quad (8b)$$

Here  $\underline{S}_0$  is the scattering matrix of the solid-water interface,  $k_w$  is the wavenumber in water,  $k_w = \omega/c_w$ .

Equation (8a) can be solved for  $A_+$  and  $B_-$  in terms of  $A_-$ , the amplitude of the incident wave. The reflection and transmission coefficients of the layer may then be defined as

$$R_l = \frac{B_-}{A_-}, \quad T_l = \frac{A_+}{A_-} \quad (9a,b)$$

For the case that all interfaces are the same and of the form shown in Fig. 1, results are displayed in Fig. 3 for the case  $\alpha = 0.1$  and  $n = 21$ . The peaks at  $k_w h = 0.11$  correspond to resonances for the overall thickness of the layer. The drop of the transmission coefficient to zero and the corresponding increase of the reflection coefficient to one occurs at  $k_w h = 4.4$ , which is just the frequency at which the stopping bands start for wave propagation in the unbounded periodic medium.

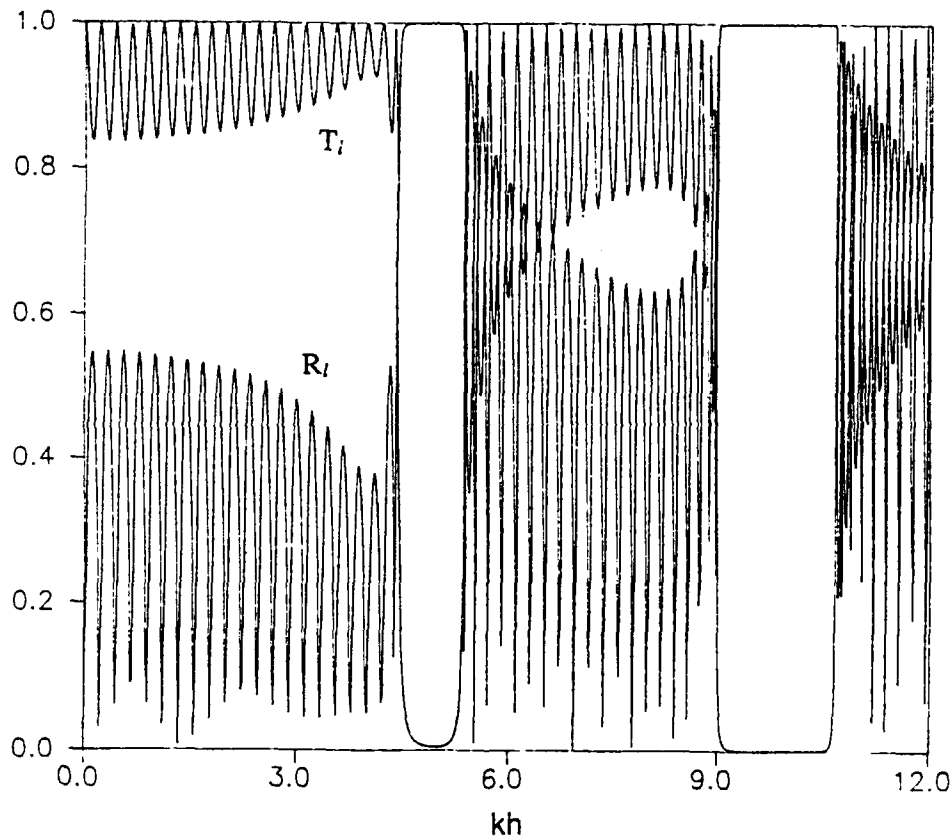


Fig. 3. Magnitudes of reflection and transmission coefficients of the layer versus  $kh$ , for  $\alpha = 0.1$ ,  $n=21$

### INTERFACES WITH STATISTICAL PROPERTIES

For composite materials periodicity is an idealization, since the real ply-structure will always have some random features. Within the context of this paper's approach, statistical properties of an interphase can be taken into account by considering the parameter  $\alpha$  in Eqn. (1) as a random variable. The propagation in a basic cell can still be defined by a propagator matrix, but now this matrix has random coefficients. Following the approach developed for the deterministic system, see Eqn. (8a,b), we have to consider the product of a finite number of random matrices. Unfortunately it is very difficult to make rigorously valid statements about such a product. It is, however, possible to draw rigorously valid conclusions when the number of matrices becomes very large. This can be done by the use of a theorem due to Fürstenberg [3].

When the number of interfaces is large, the asymptotic behavior of the total transmission coefficient  $T_l$  has a close relationship with the magnitude of eigenvalues of the propagator matrix  $\underline{W}$ . Let  $\exp(\lambda)$  be the eigenvalue of  $\underline{W}$ , then  $|\exp(\lambda)| = \exp[\text{Re}(\lambda)]$ . If  $\text{Re}(\lambda) = 0$ , as in passing bands of the periodic structure,  $|\exp(\lambda)| = 1$ , and

$$T_l = \frac{A_+}{A_-} \sim O(1) \quad (10)$$

If, on the other hand,  $\text{Re}(\lambda) \neq 0$ , as in stopping bands of the periodic structure,  $|\exp(\lambda)| \neq 1$ , and

$$T_l = \frac{A_+}{A_-} \sim o(1) \quad (11)$$

Fürstenberg's theorem states that for the mean value of the eigenvalues,  $\exp(\lambda)$ , of the random matrix  $\underline{W}$ ,  $\text{Re}(\lambda)$  will never be equal to zero, which implies that for a solid with random interfaces, at all frequencies we will have

$$T_l \sim o(1) \quad (12)$$

Since in our problem, conservation of rates of energies holds, the information on  $T_l$  subsequently supplies the information on  $R_l$ .

Fürstenberg's theorem may be stated as follows: For random matrices  $\underline{W}_i$ ,  $i = 1, 2, \dots, n$ , there exists a  $\gamma > 0$ , such that for each  $z_0 \neq 0$ ,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \ln |\underline{W}_n \cdots \underline{W}_2 \underline{W}_1 z_0| = \gamma \quad (13)$$

with probability of unity, and

$$z_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta} \\ e^{-i\theta} \end{pmatrix} \quad (14)$$

$$\gamma = \iint \ln |\underline{W} z_0| p(\alpha) d\alpha v(\theta) d\theta \quad (15)$$

Here  $\alpha$  and  $\theta$  are random variables in  $\underline{W}$  and  $z_0$ , respectively, and  $p(\alpha)$  and  $v(\theta)$  are corresponding probability density functions. The constant  $\gamma$  is usually called the localization constant, while  $l = 1/\gamma$  is called the localization length [4].

Suppose that the phase of  $z_0$  is evenly distributed over the interval  $[0, \pi]$ , and the interface parameter  $\alpha$  obeys a Gaussian distribution. Then, assuming that  $\alpha$  deviates slightly from its mean value, it can be shown by evaluating Eqn. (15) that

$$\gamma \propto \sigma_\alpha^2 \omega^2 \quad (16)$$

$$l = \frac{1}{\gamma} \propto \frac{1}{\sigma_{\alpha}^2 \omega^2} \quad (17)$$

where  $\sigma_{\alpha}$  is the deviation of the interface parameter  $\alpha$ . Let

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \underline{S}_0 \underline{P} \underline{W}_n \underline{W}_{n-1} \cdots \underline{W}_1 \underline{S}_0^{-1} \quad (18)$$

then according to Fürstenberg theorem,  $a_{ij} \sim \exp(n\gamma)$ . By solving Eqn. (8) for  $T_l$  and  $R_l$ , and using Eqn. (18), we obtain

$$T_l = \frac{A_+}{A_-} = \frac{1}{a_{22}} \sim \exp(-n\gamma) \quad (19)$$

(It can be proven that  $\det(a_{ij}) = 1$ ).

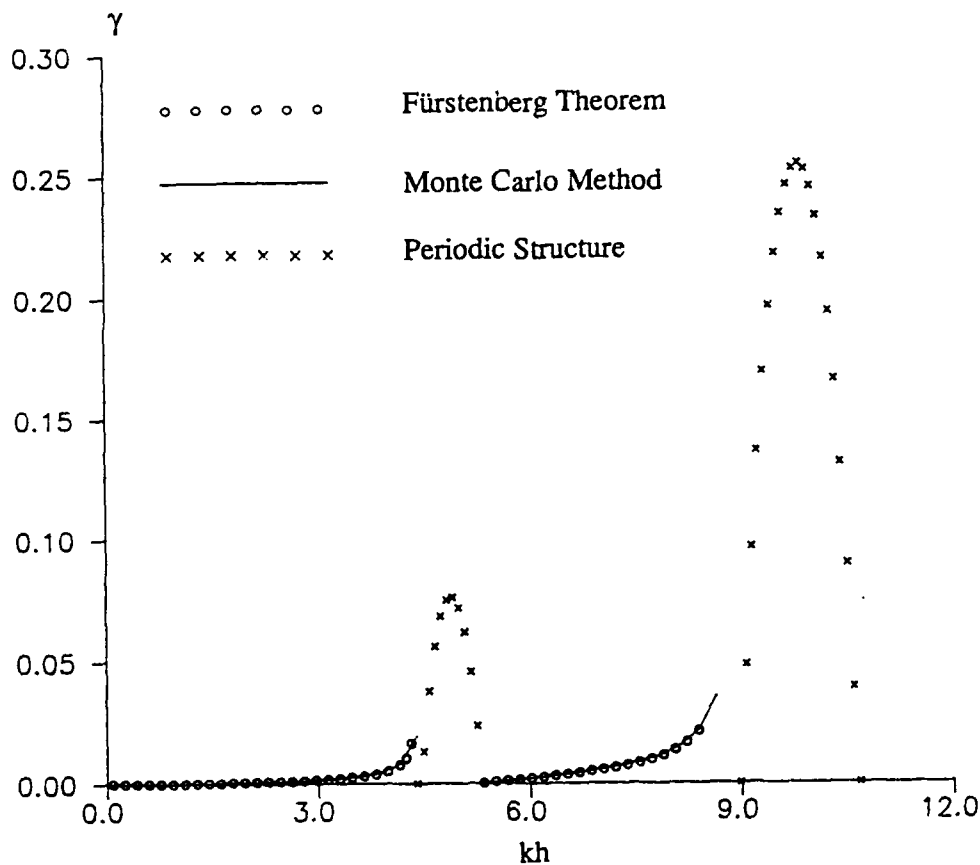


Fig. 4. Localization constant  $\gamma$  versus  $kh$  for  $\bar{\alpha} = 0.1$ ,  $\sigma_{\alpha} = 0.3$

Figures 4 and 5 show the localization constant  $\gamma$  and the localization length  $l$  as functions of the dimensionless wave number  $kh$ . Results based on Fürstenberg theorem are compared with those obtained by the use of Monte Carlo simulation and for the strictly periodic structure. These results have been computed for

$$\bar{\alpha} = 0.1 \text{ and } \sigma_{\alpha} = 0.3 \quad (20)$$

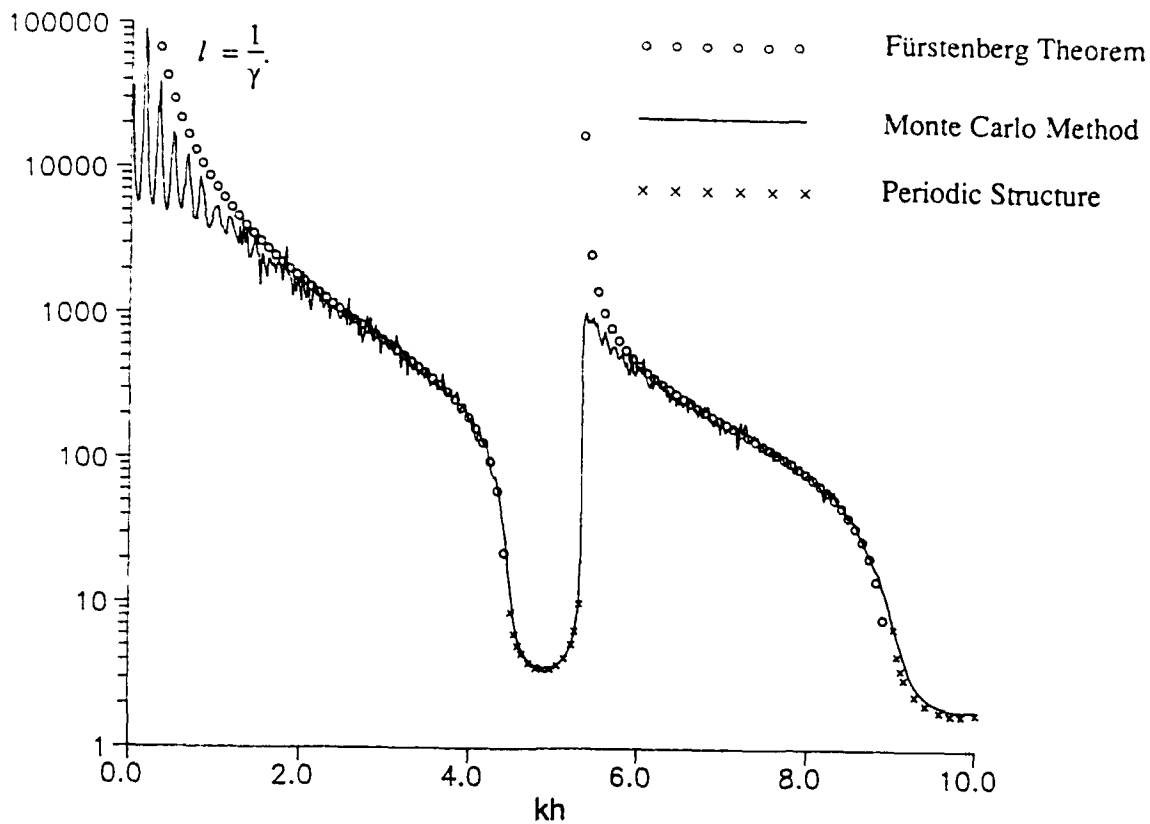


Fig. 5. Localization length  $l$  versus  $kh$  for  $\bar{\alpha} = 0.1$ ,  $\sigma_{\alpha} = 0.3$

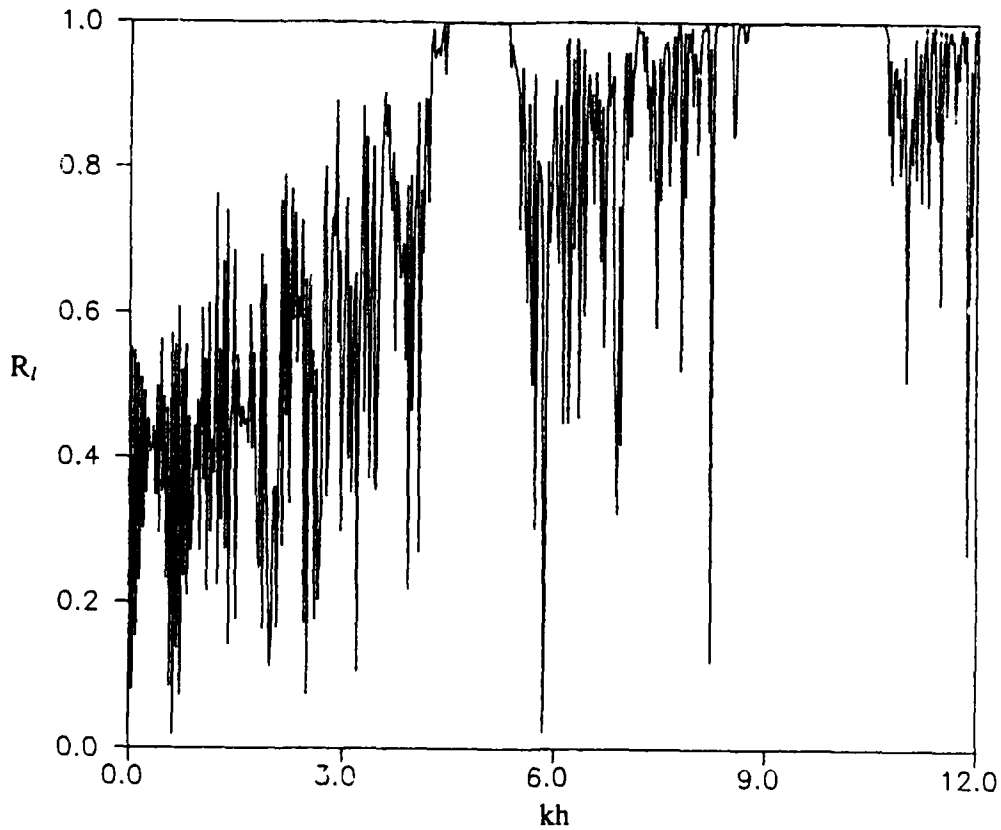


Fig. 6. Magnitude of reflection coefficient versus  $kh$  for a layer of 200 interfaces,  $\bar{\alpha} = 0.1$ ,  $\sigma_{\alpha} = 0.3$

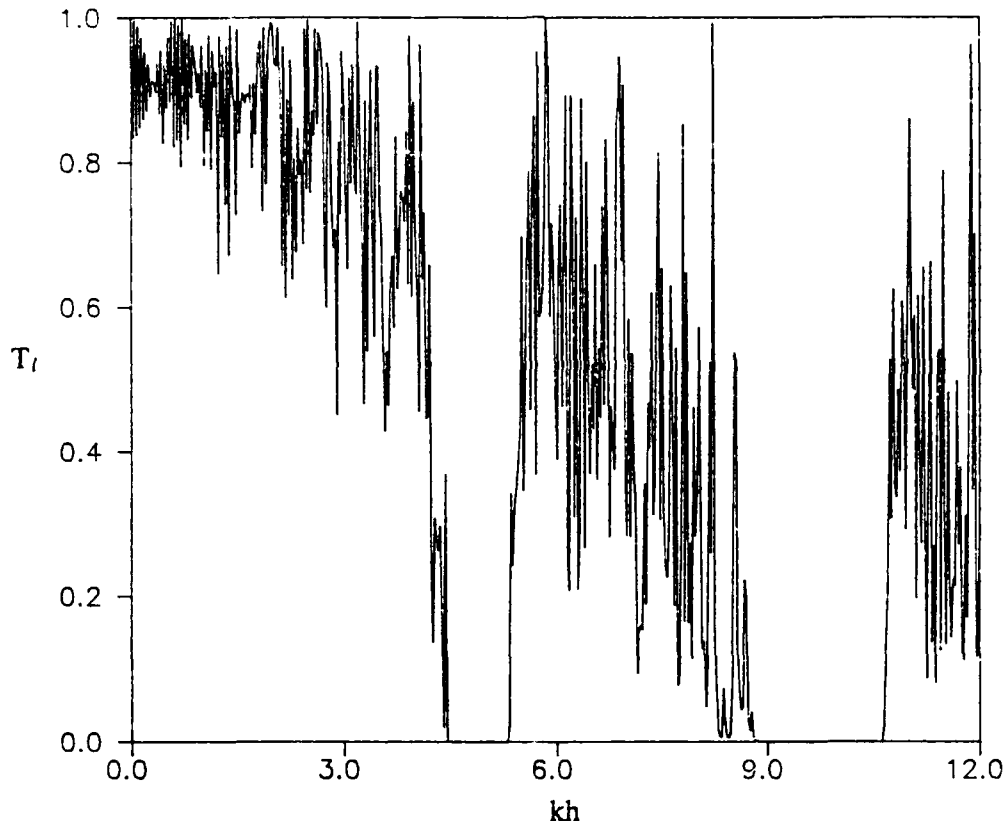


Fig. 7. Magnitude of transmission coefficient versus  $kh$  for a layer of 200 interfaces,  $\bar{\alpha} = 0.1$ ,  $\sigma_{\alpha} = 0.3$

where the bar denotes the mean. It is important to note that  $\gamma$  is not equal to zero in the range of  $kh$  which would correspond to the passing bands of the strictly periodic composite. The results presented in Fig. 6 and Fig. 7 show the direct matrix multiplication for  $T_t$  and  $R_t$ , where the number of interfaces is 200. It can be easily seen that the transmission coefficient  $T_t$  decreases with increasing frequency and correspondingly, the reflection coefficient  $R_t$  increases with increasing frequency.

## CONCLUSION

It has been shown in this paper that the propagator matrix approach can be used to study one dimensional propagation of ultrasound in composites. Random variations of the interface properties give rise to the localization phenomenon. As a consequence, ultrasound will be confined to a local region near the insonified area. It should be expected that localization will adversely affect ultrasonic testing of thick composites.

## ACKNOWLEDGMENT

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