

Naval Research Laboratory

Washington, DC 20375-5000

2



NRL Memorandum Report 6549

DTIC FILE COPY

AD-A213 137

# A Solvable Self-Similar Model of the Sausage Instability in a Resistive Z-Pinch

MARTIN LAMPE

*Beam Physics Branch  
Plasma Physics Division*

DTIC  
ELECTE  
OCT 05 1989  
S D & D

September 20, 1989

Approved for public release; distribution unlimited.

89 10 4 054

REPORT DOCUMENTATION PAGE				Form Approved OMB No 0704-0188	
1a REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>		1b RESTRICTIVE MARKINGS			
2a SECURITY CLASSIFICATION AUTHORITY		3 DISTRIBUTION AVAILABILITY OF REPORT			
2b DECLASSIFICATION/DOWNGRADING SCHEDULE		Approved for public release; distribution unlimited.			
4 PERFORMING ORGANIZATION REPORT NUMBER(S)  NRL Memorandum Report 6549		5 MONITORING ORGANIZATION REPORT NUMBER(S)			
6a NAME OF PERFORMING ORGANIZATION  Naval Research Laboratory		6b OFFICE SYMBOL (If applicable)  Code 4790	7a NAME OF MONITORING ORGANIZATION		
6c ADDRESS (City, State, and ZIP Code)  Washington, DC 20375-5000		7b ADDRESS (City, State, and ZIP Code)			
8a NAME OF FUNDING/SPONSORING ORGANIZATION  Office of Naval Research		8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8c ADDRESS (City, State, and ZIP Code)  Arlington, VA 22217		10 SOURCE OF FUNDING NUMBERS	PROGRAM ELEMENT NO	PROJECT NO	TASK NO
			ONR		WORK UNIT ACCESSION NO
11 TITLE (Include Security Classification)  A Solvable Self-Similar Model of the Sausage Instability in a Resistive Z-Pinch					
12 PERSONAL AUTHOR(S)  Lampe, Martin					
13a TYPE OF REPORT  Interim		13b TIME COVERED FROM _____ TO _____		14 DATE OF REPORT (Year, Month, Day)  1989 September 20	15 PAGE COUNT  38
16 SUPPLEMENTARY NOTATION					
17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Z-pinch	Sausage instability	
			Pinch	MHD instability	
19 ABSTRACT (Continue on reverse if necessary and identify by block number)  A solvable model is developed for the linearized sausage mode within the context of resistive MHD. The model is based on the assumption that the fluid motion of the plasma is self-similar, as well as several assumptions pertinent to the long-wavelength limit. The perturbations to the magnetic field are not assumed to be self-similar, but rather are calculated. Effects arising from time dependences of the equilibrium, e.g., current rising as $t^\alpha$ , ohmic heating, and time variation of the pinch radius, are included in the analysis. The formalism appears to provide a good representation of those modes that involve coherent sausage distortion of the entire cross section of the pinch, but excludes modes that are localized radially, and higher radial eigenmodes. For this and other reasons, it is expected that the model underestimates the maximum  <span style="float: right;">(Continues)</span>					
20 DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21 ABSTRACT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>		
22a NAME OF RESPONSIBLE INDIVIDUAL  Martin Lampe			22b TELEPHONE (Include Area Code)  (202) 767-4041	22c OFFICE SYMBOL  Code 4792	

19. ABSTRACTS (Continued)

instability growth rates, but is reasonable for global sausage modes. The net effect of resistivity and time variation of the equilibrium is to decrease the growth rate if  $\alpha < 1$ , but never by more than a factor of about two. The effect is to increase the growth rate if  $\alpha > 1$ .

CONTENTS

I. INTRODUCTION ..... 1

II. SELF-SIMILAR EQUILIBRIUM ..... 4

III. LINEARIZED PERTURBATION ANALYSIS ..... 8

IV. SOLUTION OF THE PERTURBATION EQUATIONS ..... 16

CONCLUSIONS ..... 22

ACKNOWLEDGMENTS ..... 23

REFERENCES ..... 24

DISTRIBUTION LIST ..... 29



# A SOLVABLE SELF-SIMILAR MODEL OF THE SAUSAGE INSTABILITY IN A RESISTIVE Z-PINCH

## I. Introduction

Since the early days of controlled fusion research, it has been generally accepted that simple Z-pinchs are violently unstable to both the sausage and kink modes. Specifically, ideal MHD unequivocally predicts instability for a time-independent pinch surrounded by vacuum, with growth rate of the order of  $kv_A$  for  $ka_0 \leq 1$ , where  $k$  is the axial wave number,  $a_0$  is the equilibrium pinch radius, and  $v_A$  is the Alfvén speed. The early experiments were indeed observed to be strongly unstable, in general agreement with this picture. These conclusions have, over the years, suppressed interest in the Z-pinch as a quiescent plasma confinement geometry, and apparently have also discouraged theoretical investigation of the stability problem from a more sophisticated point of view, e.g., with the inclusion of dissipative and kinetic effects, as well as those effects arising from time dependence of the equilibrium state.

However, recent experiments with deuterium-fiber-initiated Z-pinchs at NRL<sup>1</sup> and Los Alamos<sup>2</sup> exhibit apparent stability over a very large number of Alfvén growth times  $a_0/v_A$ . For example, in the NRL experiment, the pinch was apparently stable over the entire duration of the current rise, which is typically 130 ns but has been varied down to 110 ns to test sensitivity to this parameter. The pinch then goes sausage unstable after peak current is reached. Robson<sup>1</sup> has suggested that the stability is a direct consequence of rising current - the "I hypothesis". These observations have prompted a number of theoretical efforts to re-examine stability, with the emphasis being on the sausage mode, since the kink mode is not seen at all in the recent experiments. These recent and ongoing efforts include linearized one-dimensional (radial) treatments which assume a mode structure  $\exp(ikz)$  and model the plasma either with resistive MHD,<sup>3</sup> or MHD with the full Braginskii transport coefficients,<sup>4</sup> or Chew-

Goldberger-Low,<sup>5,6</sup> or Vlasov ions.<sup>7</sup> In addition, there have been two-dimensional nonlinear fluid simulations that include resistivity<sup>8</sup> or full Braginskii transport,<sup>4</sup> or even the ongoing ablation of the deuterium fiber.<sup>9</sup> Preliminary results from some of this work has suggested that resistivity could be a significant stabilizing factor.<sup>3,6</sup> All of these treatments are essentially numerical, and do not provide analytic scaling laws, nor direct insights into the precise nature of stabilizing mechanisms. Also, these stability calculations have in most cases assumed time dependence  $(t/t_0)^{1/3}$  for the current  $I(t)$ , which simplifies and restricts the problem by leading to a time-independent pinch radius.<sup>10,11</sup>

In this work we investigate the resistive MHD sausage linear stability problem, using an alternative approach which has the virtues of simplicity and generality, and directly provides analytic scaling laws as well as insight into physical mechanisms. We develop a model that admittedly includes some features that are not strictly correct, but which we expect to be at least reasonable. The model is motivated by those features appropriate to long-wavelength modes, but one hopes that it applies qualitatively for  $ka_0$  up to order unity. The principal assumptions are as follows: (i) The plasma temperature  $T(r,z,t)$  is assumed to be independent of  $r$ , in both the equilibrium and the perturbation. (ii) Self-similar equilibria are investigated with  $I(t) \propto (t/t_0)^\alpha$ , where  $\alpha$  can be any number. (iii) No heat transport along  $z$  is included in the investigations to date (but this could easily be included, and would be destabilizing). (iv) Radial pressure balance, i.e., the Bennett pinch condition,<sup>12,13</sup> is assumed in both the equilibrium and the perturbation. (v) The current  $I(z,t)$  is assumed to be unchanged by the perturbation, and thus independent of  $z$ . (vi) Some terms of order  $k^2 a_0^2$  are dropped for convenience. (vii) The fluid motion of the sausageing is assumed to be self-similar. However,

the magnetic field  $B(r,z,t)$  is not assumed to be self-similar, but rather is calculated from resistive MHD. With these assumptions, we are able to obtain an analytic solution for the growth of sausage perturbations.

In undertaking this calculation, we had in mind two particular resistive effects which, coupled to increasing  $I(t)$ , could be stabilizing. The first is that, according to the Bennett relation, the necks of the sausages must be hotter, and therefore less resistive, than the bulges; thus, if  $\dot{I} > 0$ , the skin effect is enhanced in the necks and (under some circumstances) the current in the necks is concentrated in a smaller fraction of the pinch area at the outside. This would reduce, at the necks, the magnetic pressure in the interior which pushes plasma axially out of the necks and thus drives the instability. The second effect is that (under some circumstances) ohmic energy dissipation is enhanced at the necks, where the plasma is compressed, and reduced at the bulges. This can be interpreted mathematically as an effective decrease in the plasma compressibility, i.e., an increase in the adiabatic index from its actual value  $5/3$ , as regards the perturbation. We have calculated these and other resistive effects fully self-consistently, within the model described above. We find that if  $\alpha \leq 1$ , the growth rate is reduced, but not by more than about a factor of two, from the well-known value from ideal MHD with an assumed time-independent equilibrium.<sup>14</sup> In fact, there are other resistive terms which are destabilizing (as was recognized by R. J. Tayler<sup>15</sup> many years ago), and also some geometric terms associated with the time variation of the equilibrium radius if  $\alpha \neq 1/3$ . If  $\alpha \geq 1$ , the net result is that the growth rate is larger than the value from ideal MHD with a time-dependent equilibrium. Our model also can be interpreted as representing a plasma with large shear viscosity. Thus, it appears that within the context of a linearized fluid treatment neither resistivity nor

shear viscosity can account for the long period of stability observed in the deuterium-fiber experiments.

## II. Self-Similar Equilibrium

The "equilibrium" state of the pinch is, in general, time-dependent, since  $I(t)$  will have a (specified) time dependence and the pinch will heat up resistively and expand or contract to maintain radial pressure balance. However, by definition, the equilibrium is  $z$ -independent. We assume radial pressure balance is maintained, and that the equilibrium is isothermal and has equal electron and ion temperatures. (However, all the conclusions also follow if  $T_e \gg T_i$  or if  $T_e/T_i$  is a fixed ratio.) We also assume that the conductivity

$$\sigma \propto T^{3/2}, \quad (1)$$

in accordance with the Spitzer formula<sup>13</sup> with the variation of the Coulomb logarithm neglected, so that  $\sigma_0$  is also uniform within the plasma. The continuity, radial momentum, and magnetic diffusion equations are then

$$\frac{\partial \rho_0}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} r v_{r0} \rho_0 = 0, \quad (2)$$

$$\frac{\partial P_0}{\partial r} + \frac{\partial}{\partial r} \frac{B_0^2}{8\pi} = - \frac{B_0^2}{4\pi r}, \quad (3)$$

$$\frac{\partial B_0}{\partial t} = - \frac{\partial}{\partial r} v_{r0} B_0 - \frac{c^2}{4\pi\sigma_0} \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} r B_0, \quad (4)$$

where  $\rho$  is the density,  $P$  the pressure,  $v_r$  the radial velocity, and subscript-zero denotes equilibrium. There is also an energy equation which we will write down later in radially-integrated form.

It is well known<sup>10,11,16</sup> that these equations admit a self-similar solution, defined by the relations

$$x = \frac{r}{a_0(t)}, \quad (5)$$

$$v_{r0} = \dot{a}_0(t)x, \quad (6)$$

$$\rho_0(r,t) = \frac{N_0 m_i}{\pi a_0^2(t)} \hat{\rho}_0(x), \quad (7)$$

$$B_0(r,t) = \frac{2I(t)}{ca_0(t)} \hat{B}_0(x), \quad (8)$$

$$J_0(r,t) = \frac{I(t)}{\pi a_0^2(t)} \hat{J}_0(x) = \frac{c}{4\pi} \frac{1}{r} \frac{\partial}{\partial r} r B_0(r,t), \quad (9)$$

where  $a_0(t)$  is the pinch outer radius,  $N_0$  is the number of ions per unit length, and  $m_i$  is the ion mass.

Equation (2) is automatically satisfied by (5) - (7). Equation (3) implies the Bennett relation,

$$T_0 = \frac{I^2}{4N_0 c^2}, \quad (10)$$

where the temperature  $T$  is in ergs. Beyond that, we shall not need to solve (3) for the pressure  $P_0$ . Equation (4) reduces to a Bessel equation,

$$\frac{\partial}{\partial x} \frac{1}{x} \frac{\partial}{\partial x} x \hat{B}_0 + \frac{4\pi\sigma_0 a_0^2}{c^2} \frac{\dot{I}}{I} \hat{B}_0 = 0, \quad (11)$$

to which the solution [normalized to unity at  $x = 1$  as required by (8)] is

$$\hat{B}_0(x) = \begin{cases} \frac{I_1(qx)}{I_1(q)} & , 0 \leq x \leq 1 \\ x^{-1} & , 1 \leq x \end{cases} \quad (12a)$$

$$(12b)$$

where  $I_1$  is a modified Bessel function and

$$q^2 = \frac{4\pi\sigma_0(t)a_0(t)}{c^2} \frac{\dot{I}(t)}{I(t)}. \quad (13)$$

However, we have assumed  $\hat{B}_0(x)$ , and therefore  $q$ , to be time-independent.

This can be satisfied if

$$I(t) = I_0(t/t_0)^\alpha, \quad (14)$$

in which case (1) and (10) imply

$$\sigma_0(t) \propto (t/t_0)^{3\alpha}, \quad (15a)$$

and thus it is required that

$$a_0(t) \propto (t/t_0)^{(1-3\alpha)/2} \quad (15b)$$

so that

$$q^2 = \frac{\alpha}{t} \frac{4\pi\sigma_0 a_0^2}{c^2} \quad (16)$$

is indeed time-independent.

The energy equation, including both ohmic and compressional heating and assuming an ideal gas with adiabatic index 5/3, is

$$3 N_0 \dot{T}_0 + \frac{4\dot{a}_0}{a_0} N_0 T_0 = \frac{1}{\sigma_0} \int_0^{a_0} dr 2\pi r J_0^2. \quad (17)$$

Using the Bennett relation (10) to eliminate  $T_0$ , Eqs. (9), (12) and (13) to determine  $J_0(r,t)$ , and Eqs. (14) and (15b) to specify  $\dot{I}/I$  and  $\dot{a}_0/a_0$ , Eq.

(17) reduces to an equation that determines  $q$  as a function of  $\alpha$ :

$$4\alpha = \frac{I_1^2(q)}{\int_0^1 dx \times I_0^2(qx)} \quad (18)$$

Equation (18) is plotted in Fig. 1. In the limiting cases of small and large  $\alpha$ , the Bessel functions can be expanded and we find

$$q = \begin{cases} \sqrt{8\alpha} & , \quad \alpha \leq 1 \\ 2\alpha & , \quad \alpha \gg 1 \end{cases} \quad (19a)$$

$$(19b)$$

Also, in the case  $\alpha = 1/3$  which corresponds to a fixed radius  $a_0$ ,

$$q(\alpha = 1/3) = 1.65 \quad (19c)$$

Recently, Rosenau, Nebel and Lewis<sup>17</sup> have shown numerically that the self-similar solutions are stable, and are attractors for a broad class of initial conditions, as long as the pinch is legislated mathematically to be uniform in  $z$ .

In this paper, we shall apply our perturbation model to this particular equilibrium. However, the perturbation analysis is more general, and could be applied to other assumed equilibria. We have in fact, as a check on the perturbation model, used it to calculate ideal MHD growth rates for various assumed time-independent equilibria, but these calculations will not be presented here.

### III. Linearized Perturbation Analysis

In the present section, we allow all quantities to depend on  $z$  as well as  $r$  and  $t$ , except that we specify that  $I(t)$  is  $z$ -independent so as to prevent charge accumulation anywhere in the pinch. We shall soon specialize to the case of small departures from a  $z$ -independent equilibrium, but we begin by writing down the nonlinear equations for continuity, momentum, energy, and magnetic field diffusion. With long wavelength perturbations in mind, we continue to assume that radial pressure balance is satisfied, and that  $T(r,z,t)$  is independent of  $r$  but that there is no heat conduction along  $z$ . The basic equations then are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z} (\rho v_z) + \frac{1}{r} \frac{\partial}{\partial r} r \rho v_r = 0, \quad (20)$$

$$\frac{\partial}{\partial r} \left( P + \frac{B^2}{8\pi} \right) = - \frac{B^2}{4\pi r}, \quad (21)$$

$$\rho \frac{\partial v_z}{\partial t} = - \frac{\partial}{\partial z} \left( P + \frac{B^2}{8\pi} \right), \quad (22)$$

$$\frac{\partial B}{\partial t} + \frac{\partial}{\partial r} (v_r B) + \frac{\partial}{\partial z} (v_z B) = \frac{c^2}{4\pi\sigma} \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} r B + \frac{c^2}{4\pi} \frac{\partial}{\partial z} \frac{1}{\sigma} \frac{\partial B}{\partial z}. \quad (23)$$

Now we make the key simplification that allows us to develop a solvable model: we require the fluid motion to be radially self-similar, i.e., that  $\rho(r,z,t)$  be of the form

$$\rho(r,z,t) = \frac{N(z,t)m_i}{\pi a^2(z,t)} \tilde{\rho}(x,z), \quad (24a)$$

where  $N(z,t)$  is the perturbed number of ions per unit length,  $a(z,t)$  is the perturbed outer radius of the pinch, and now

$$x \equiv r/a(z,t). \quad (25)$$

This is equivalent to specifying that

$$v_r(r, z, t) = x \frac{\partial}{\partial t} a(z, t), \quad (24b)$$

$$v_z(r, z, t) = v_z(z, t), \text{ independent of } r. \quad (24c)$$

We have seen that self-similar forms were exact solutions for the equilibrium, but Eqs. (24) are not in general exact solutions of the z-dependent perturbation equations; they are imposed externally to specify our model. However, we regard them as a qualitatively reasonable representation, particularly in the long-wavelength limit, which is the focus of our study. For the class of sausage perturbations that are readily observed experimentally and that are expected to be most disruptive - i.e., those that consist of a more or less coherent pinching of the entire plasma cross-section at the "necks", and expansion of the entire cross-section at the bulges. Our model excludes modes which are strongly localized to a limited shell in r, and higher radial modes which may have parts of the plasma cross-section moving inward at the same time when other parts are moving outward. In some cases, particularly within ideal MHD, these types of modes may have the largest linear growth rates, but one does not expect them to have the same type of disruptive effect, when they reach large amplitude, as the global sausage modes. Since we are excluding certain classes of modes, in general, we expect our model to yield linear growth rates which are somewhat less unstable than the exact results of resistive MHD, but we yield qualitatively accurate growth rates for any global modes that are present. Indeed, we have tested the model by applying it with zero resistivity to some simple time-independent equilibria, and found this to be the case.

It is also worth noting that Eq. (24c), which is equivalent in itself to the self-similar assumption, becomes exact in the limit of long wavelength and large shear viscosity. Thus, the present model becomes a true representation of the physics in that limit.

The inexactness of the model becomes clear when Eq. (22) is solved; in general, the solution will be of a value of  $v_z$  that varies with  $r$ , contradicting Eq. (24c). Thus, to make our model self-consistent, we set  $v_z$  equal to the average value given by (22), i.e., we replace (22) with

$$Nm_i \frac{\partial v_z}{\partial t} = - \frac{\partial}{\partial z} \int_0^a dr 2\pi r \left( P + \frac{B^2}{8\pi} \right). \quad (26)$$

We now add an energy equation to complete our set of nonlinear fluid equations:

$$\frac{\partial}{\partial t} (3 NT) + \frac{\partial}{\partial z} (3 NT v_z) + \left( \frac{2\dot{a}}{a} + \frac{\partial v_z}{\partial z} \right) 2 NT = \frac{1}{\sigma(z,t)} \int_0^a dr 2\pi r J^2(r,z,t). \quad (27)$$

However, we can simplify (27) by using the Bennett relation,

$$N(z,t) T(z,t) = \frac{I^2(t)}{4c^2}, \quad (28)$$

which follows from (21). We have assumed

$$I(z,t) = I(t) \quad (29)$$

to be independent of  $z$ . Then (27) becomes

$$\frac{3\dot{I}}{I} + \left( \frac{2\dot{a}}{a} + \frac{5}{2} \frac{\partial v_z}{\partial z} \right) = \frac{2c^2}{I^2 \sigma} \int_0^a dr 2\pi r J^2(r,z,t). \quad (30)$$

Now we linearize our basic set of Eqs. (20), (21), (23), (28), and (30), by defining

$$\frac{a(z,t)}{a_0(t)} = 1 + \hat{a}, \quad (31a)$$

$$\frac{\sigma(z,t)}{\sigma_0(t)} = 1 + \hat{\sigma}, \quad (31b)$$

$$\frac{N(z,t)}{N_0(t)} = 1 + \hat{N}, \quad (31c)$$

$$I(t) = I_0(t), \quad (31d)$$

$$B(r,z,t) = \frac{a_0}{a} B_0\left(\frac{a_0}{a} r, t\right) + \frac{2I}{ca} \hat{B}(x,z,t), \quad (32a)$$

$$J(r,z,t) = \left(\frac{a_0}{a}\right)^2 J_0\left(\frac{a_0}{a} r, t\right) + \frac{I}{\pi a^2} \hat{J}(x,z,t). \quad (32b)$$

Equations (32) define the first-order quantities  $\hat{B}$  and  $\hat{J}$  in terms of the difference between the exact values  $B(r,z,t)$ ,  $J(r,z,t)$  and self-similar forms that carry the same current. This somewhat unusual approach is very convenient for handling the boundary conditions at  $r = a$ , where both  $J(r,z,t)$  and  $\sigma(r,z,t)$  jump discontinuously to zero. With the definitions (32) and using  $x$  rather than  $r$  as our independent variable, all discontinuities occur exactly at  $x = 1$ , it is not necessary to add any surface terms to the first-order equations, and the boundary conditions are

$$\hat{B}(0,t) = 0, \quad (33a)$$

$$\hat{B}(x,t) = 0, \text{ for } x \geq 1, \quad (33b)$$

$$\hat{J}(x,t) = 0, \text{ for } x > 1. \quad (33c)$$

Note that  $\hat{J}$  is, in general, discontinuous at  $x = 1$ , but if  $\sigma$  is finite  $\hat{B}$  is continuous.

We treat all quantities with carets as first-order small quantities with z-dependence  $\exp(ikz)$ , neglect second order, and also define

$$\hat{v} = ikv_z. \quad (34)$$

Then Eq. (20) reduces to

$$\frac{d\hat{N}}{dt} = -\hat{v}. \quad (35)$$

Equations (28), (31d), and (35) give

$$\frac{d\hat{T}}{dt} = \hat{v}, \quad (36)$$

and continuing to use  $\sigma \propto T^{3/2}$ , we have

$$\frac{d\hat{\sigma}}{dt} = \frac{3}{2} \hat{v}. \quad (37)$$

It is useful to explicitly expand Eqs. (32) to first order, using (8) and (9)

$$B(r, t) = \frac{2I}{ca_0} \left[ (1 - \hat{a}) \hat{B}_0(x) + \hat{B}(x, t) \right], \quad (38a)$$

$$J(r, t) = \frac{I}{\pi a_0^2} \left[ (1 - 2\hat{a}) \hat{J}_0(x) + \hat{J}(x, t) \right]. \quad (38b)$$

We integrate (21), using (38), (12b), and (33b), to obtain

$$\begin{aligned} P + \frac{B^2}{8\pi} &= \int_r^\infty \frac{dr' B^2(r')}{4\pi r'} \\ &= \frac{I^2}{\pi c^2 a_0^2} \left[ (1 - 2\hat{a}) \left( \frac{1}{2} + \int_x^1 \frac{dx'}{x'} \hat{B}_0^2(x') \right) + 2 \int_x^1 \frac{dx'}{x'} \hat{B}_0(x') \hat{B}(x') \right]. \end{aligned} \quad (39)$$

We now use this in the equation of motion (26). Reversing the order of integrations and explicitly using (12) for the equilibrium field, we obtain

$$\frac{\hat{d}\hat{v}}{dt} = \frac{2k^2 I_1^2}{N_0 m_i c^2} \left[ -\hat{a} \left( \frac{1}{2} + \int_0^1 \frac{dxx I_1^2(qx)}{I_1^2(q)} \right) + \int_0^1 \frac{dxx I_1(qx) \hat{B}(x)}{I_1(q)} \right]. \quad (40)$$

After some algebra but no further approximations, the magnetic field Eq. (23) can be put in the form

$$\begin{aligned} & \left( \frac{\partial}{\partial t} + \frac{\alpha}{t} - \frac{c^2}{4\pi\sigma_0 a_0^2} \frac{\partial}{\partial x} \frac{1}{x} \frac{\partial}{\partial x} x + \frac{k^2 c^2}{4\pi\sigma_0} \right) \hat{B} \\ & = \left[ \frac{\alpha}{t} \hat{a} - \hat{v} - \frac{\alpha}{t} (\hat{\sigma} + 3\hat{a}) + \frac{k^2 c^2}{4\pi\sigma_0} \hat{a} \right] \frac{I_1(qx)}{I_1(q)}. \end{aligned} \quad (41)$$

Finally, the ohmic heating term in (30) can be calculated in linearized form and the energy equation put in the form

$$\frac{\hat{d}\hat{a}}{dt} = -\frac{5}{4} \hat{v} - \frac{1-3\alpha}{2t} \hat{A} - \frac{c^2}{4\pi\sigma_0 a_0^2} \left[ \frac{q^2}{2\alpha} (\hat{\sigma} - 2\hat{a}) + 4q^2 \int_0^1 \frac{dxx I_1(qx) \hat{B}(x)}{I_1(q)} \right], \quad (42)$$

where we have explicitly used the time dependence (15b) of  $a_0(t)$ .

Equations (37), (40), (41), and (42) with the boundary conditions (33a,b) constitute a complete set of homogeneous equations for the time evolution of  $\hat{\sigma}(t)$ ,  $\hat{v}(t)$ ,  $\hat{a}(t)$ , and  $\hat{B}(x,t)$ . We clean up the equations a bit by dropping the  $k^2 c^2$  terms on each side of (41); these terms correspond to magnetic diffusion along  $z$ , and are stabilizing but small in the long wavelength limit. We also use (13) and (14) to eliminate  $\sigma_0$  in terms of  $q$ , and define a characteristic Alfvén velocity  $v_A$  by

$$v_A^2 = \frac{B_0^2(a_0)}{4\pi(N_0 m_i / \pi a_0^2)} = \frac{I^2}{N_0 m_i c^2}. \quad (43)$$

Then Eqs. (40) - (42) become

$$\frac{\dot{v}}{dt} = k^2 v_A^2 \left\{ - [1 + 2I_{11}(q)] \hat{a} + 2 \int_0^1 \frac{dxx I_1(qx) \hat{B}(x, t)}{I_1(q)} \right\}, \quad (44)$$

$$\frac{\dot{a}}{dt} = -\frac{5}{4} v - \frac{1-3\alpha}{2t} \hat{a} - \frac{\hat{a}}{t} - \frac{\hat{\sigma}}{2t} - \frac{4\alpha}{t} \int_0^1 \frac{dxx I_1(qx) \hat{B}(x, t)}{I_1(q)}, \quad (45)$$

$$\left( \frac{\partial}{\partial t} + \frac{\alpha}{t} - \frac{\alpha}{q^2 t} \frac{\partial}{\partial x} \frac{1}{x} \frac{\partial}{\partial x} x \right) \hat{B}(x, t) = \left[ -\hat{v} - \frac{\alpha}{t} (\hat{\sigma} + 2\hat{a}) \right] \frac{I_1(qx)}{I_1(q)}, \quad (46)$$

where

$$I_{11}(q) \equiv \frac{\int_0^1 dxx I_1(qx)}{I_1^2(q)}. \quad (47a)$$

The quantity  $I_{11}$ , which can be regarded as a function of  $\alpha$ , is shown in Fig. 1. Using (18) and doing some integrations by parts, we can write

$$I_{11} = \frac{I_0(q)}{q I_1(q)} - \frac{1}{4\alpha}, \quad (47b)$$

in which form it is clear that

$$I_{11}(0) = \frac{1}{4}, \quad \text{if } \alpha \ll 1, \quad (48a)$$

$$I_{11}(\infty) = \frac{1}{4\alpha}, \quad \text{if } \alpha \gg 1. \quad (48b)$$

$I_{11}$  is a monotonic decreasing function of  $\alpha$  (i.e., of  $q$ ) which is well approximated by

$$I_{11}(\alpha) \approx \frac{1}{4(1+\alpha)}. \quad (49)$$

The expression (49) is always smaller than the exact value of  $I_{11}$ , with the inaccuracy peaking at  $\sim 12\%$  over a wide range of  $\alpha$  from  $\sim 0.5$  to  $\sim 3$ .

Before proceeding with the solution of Eqs. (37), (44) - (46), and (33), we shall pause to discuss the general nature of these equations and the origin and significance of their terms. If instability is present, in general, all time derivatives will be positive at times of primary interest (instability is non-oscillatory), as will be discussed. At a sausage neck, where  $\dot{a} < 0$ , we will find  $\dot{v} > 0$  (plasma flowing out axially), and thus,  $\dot{N} < 0$ , and thus  $\dot{\sigma} > 0$  (the neck must be hotter, according to the Bennett relation, and so has higher conductivity). The basic instability, which is familiar from ideal MHD analysis of a time-invariant equilibrium, is given by the first terms on the right-hand side (RHS) of (44) and (45); these yield a growth rate on the scale of  $kv_A$ , with some dependence on the equilibrium current profile through  $I_{11}$ . The last term of (44) represents the effect of non-self-similar evolution of B: typically  $J(r,z,t)$  will be more peaked at the outside of the pinch at a neck, i.e.,  $\hat{B}(x)$  for  $0 < x < 1$  will be negative where  $\dot{a} < 0$ , which weakens the pressure pushing plasma out of the neck. Thus, this term is stabilizing. In Eq. (45), the term  $(1-3\alpha)\dot{a}/2t$  arises simply from the fact that perturbations to  $a(z,t)$  must be scaled to  $a_0$ , which is itself a time-varying quantity; this geometric effect does have a significant effect on instability growth rates, as we shall see. The last three terms of (45) arise from the differences in ohmic heating between sausage necks and bulges: if there is extra heating at the necks, this is stabilizing by, in effect, increasing the adiabatic index (stiffness) of the plasma and opposing further shrinkage of the neck. The  $\dot{a}/t$  term in (45) represents increase of ohmic heating at a neck due to self-similar compression of the current density; the  $\dot{\sigma}/2t$  term represents decrease of ohmic heating due to increased conductivity; and the last term represents increased ohmic heating due to current concentration at the

outside resulting from departures from self-similarity. It remains to be determined under what circumstances the stabilizing terms win out.

Equation (46) shows the ways in which  $B(r)$  departs from self-similarity. The first term on the RHS represents decrease of  $B$  at a neck due to axial convection of  $B$  along with the outflowing plasma. The  $\alpha(\hat{\sigma} + 2\hat{a})/t$  term represents the first-order contribution to the skin effect in a situation where  $\dot{I} > 0$ . As in the discussion of Eq. (45), the strength of the skin effect scales as the ratio of the magnetic diffusion time  $4\pi\sigma_0 a_0^2/c^2$  to the current rise time  $I/\dot{I}$ ; hence, the first-order term scales as  $\alpha(\hat{\sigma} + 2\hat{a})/t$ . Clearly, these terms, which arise from the combined effect of resistivity and rising current, have the potential to be stabilizing, but must be evaluated to determine their effect.

#### IV. Solution of the Perturbation Equations

Equations (37), (44)-(46), and (33a,b) could be solved numerically, e.g., by using a Fourier-Bessel expansion of (46). However, as our objective here is to obtain analytic scaling laws in simple form, we choose to perform a simpler approximate analysis. We note that  $\hat{B}(x,t)$  appears in Eqs. (44) and (45) only in the integral form

$$b(t) \equiv \int_0^1 \frac{dx x I_1(qx) \hat{B}(x,t)}{I_1(q)}. \quad (50)$$

Equation (46) for  $\hat{B}$  has the particular solution

$$\hat{B}_1(x,t) = F(t) \frac{I_1(qx)}{I_1(q)}, \quad (51)$$

$$F(t) \equiv \int_{-\infty}^t dt' \left[ \hat{v} + \frac{\alpha}{t'} (\hat{\sigma} + 2\hat{a}) \right]. \quad (52)$$

However, (51) fails to satisfy the boundary condition (33b), and thus, we must add in a homogeneous solution  $B_2(x,t)$  with the correct boundary condition:

$$\hat{B}(x,t) = \hat{B}_1(x,t) + \hat{B}_2(x,t), \quad (53)$$

$$\left(\frac{\partial}{\partial t} + \frac{\alpha}{t} - \frac{\alpha}{q^2 t} \frac{\partial}{\partial x} \frac{1}{x} \frac{\partial}{\partial x} x\right) \hat{B}_2 = 0, \quad (54a)$$

$$\hat{B}_2(0,t) = 0, \quad (54b)$$

$$\hat{B}_2(1,t) = -F(t). \quad (54c)$$

We shall argue that over the time scale of primary interest to us, it is reasonable qualitatively to neglect  $B_2$ .

In general, we cannot produce a closed-form solution for  $\hat{B}_2(x,t)$ . But for the special case where

$$F(t) = t^\eta, \quad (55)$$

the solution to Eq. (54) is

$$\hat{B}_2(x,t) = -F(t) \frac{I_1(\sqrt{1+\eta/\alpha} qx)}{I_1(\sqrt{1+\eta/\alpha} q)}. \quad (56)$$

In this case, the integral  $b(t)$  of Eq. (50) can be written as a sum of contributions from  $B_1(x,t)$  and  $B_2(x,t)$ :

$$b(t) = b_1(t) + b_2(t), \quad (57)$$

$$b_1(t) = F(t) \int_0^1 dx x \frac{I_1^2(qx)}{I_1^2(q)} \quad (58a)$$

$$b_2(t) = -F(t) \int_0^1 dx x \frac{I_1(qx) I_1(\sqrt{1+\eta/\alpha} qx)}{I_1(q) I_1(\sqrt{1+\eta/\alpha} q)}. \quad (58b)$$

If

$$\frac{\eta + \alpha}{\alpha} q^2 \gg 1, \quad (59a)$$

and also

$$\frac{\eta + \alpha}{\alpha} q^2 \gg q^2, \quad (59b)$$

then  $\hat{B}_2(x, t)$  is localized near  $x = 1$  and as a result

$$b_2 \gg b_1. \quad (60)$$

Conditions (59) are both satisfied if and only if

$$\eta q^2 / \alpha \gg 1. \quad (61)$$

In fact,  $b_2/b_1$  is of order  $(\alpha/\eta q^2)^{1/2}$ . The interpretation of these equations is that if the perturbation grows rapidly enough compared to the equilibrium current-rise time scale, then  $\hat{B}_2(x, t)$  is essentially a surface current perturbation, which does not have time to diffuse substantially into the pinch; this current contributes little to the integrated quantity  $b(t)$  that occurs in (44) and (45).

If the time dependence of  $F(t)$  is not a power law, we cannot solve for  $\hat{B}_2(x, t)$  in closed form, but the same physical argument applies with  $\eta$  defined by

$$\eta = \frac{tF'(t)}{F(t)}. \quad (62)$$

If, for example,  $F(t) = e^{\gamma t}$ , the condition for neglecting  $B_2$  is

$$\gamma t \gg \alpha/q^2. \quad (63)$$

For the limiting cases of small and large  $\alpha$ , we see from Eq. (19) that this reduces to

$$\gamma t \gg \frac{1}{8}, \text{ if } \alpha \ll 1, \quad (64a)$$

or

$$\gamma t \gg 1/4\alpha, \text{ if } \alpha \gg 1. \quad (64b)$$

In either case, the criterion is well satisfied at a time when no more than one e-fold of unstable growth has occurred. We are generally interested in the longer time scale over which an instability e-folds several times, so it is reasonable to approximate  $\hat{B}(x,t)$  by the particular solution  $\hat{F}_1(x,t)$  whose x-dependence is known. We note from (51), (56) and (58) that this approximation increases  $|b(t)|$ , and this is true quite generally if  $\dot{I} > 0$ . The quantity  $b(t)$ , i.e., the effect of the deviation of  $B(r,t)$  from self-similarity, is unambiguously stabilizing, as will be discussed below. Thus, the approximation of  $\hat{B}(x,t)$  by  $\hat{F}_1(x,t)$  reduces the growth rates which we shall calculate.

Referring to Eqs. (50) - (52), (57) and (58), we see that the partial differential equation (46) can now be replaced by an ordinary differential equation for  $b(t)$ , so that (37), (44) - (46), and (33a,b) reduce to a set of four homogeneous first-order linear ordinary differential equations, which can be written in the matrix form

$$\frac{d}{dt} \begin{bmatrix} \hat{v} \\ \hat{a} \\ b \\ \hat{\sigma} \end{bmatrix} = - \begin{bmatrix} 0 & (1+2I_{11})k_{vA}^2 & -2k_{vA}^2 & 0 \\ 5/4 & 3(1-\alpha)/2t & 4\alpha/t & 1/2t \\ I_{11} & 2I_{11}\alpha/t & 0 & I_{11}\alpha/t \\ -3/2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{v} \\ \hat{a} \\ b \\ \hat{\sigma} \end{bmatrix} \quad (65)$$

The eigenvalues of this matrix give the instantaneous growth rate of perturbations; this growth rate will generally be time-varying, since the equilibrium varies with time, i.e., growth will not be purely exponential.

It is convenient to define  $\Gamma$  as the eigenvalue scaled to  $kv_A(t)$ . We then find the following quartic equation to determine the four eigenvalues:

$$0 = 4(kv_A t)^2 \Gamma^4 + 6(1-\alpha)kv_A t \Gamma^3 - [(5+2I_{11})(kv_A t)^2 + 32\alpha^2 I_{11}] \Gamma^2 + [-3+(6-4\alpha)I_{11} + 32\alpha I_{11}^2] kv_A t \Gamma + 6(1+\alpha+8\alpha I_{11})\alpha I_{11}. \quad (66)$$

The ideal MHD limit corresponds to  $kv_A t \rightarrow \infty$ . (The temperature increases steadily as  $T \propto I^2$ , so resistivity becomes negligible at late times.) It is also possible to express  $kv_A t$  as

$$kv_A t = \frac{\alpha}{q} (ka_0)S, \quad (67)$$

$$S = \frac{4\pi\sigma_0(t)a_0^2(t)v_A(t)}{c^2 a_0(t)}, \quad (68)$$

is the Lundquist number,<sup>18</sup> which has been used in the literature<sup>3,8</sup> as an indication of the strength of resistive effects. It is apparent from Eqs. (65) - (67) that the growth rate and other characteristics of the instability, for a given value of  $\alpha$ , can depend on  $S$  and  $ka_0$  only as the combination  $(ka_0)S$ .  $S$  is a monotonically increasing function of time, and large  $S$  corresponds to the ideal MHD limit. In this limit, (66) gives

$$\Gamma = \left( \frac{5+2I_{11}}{4} \right)^{1/2}, \quad (69)$$

i.e., growth is on the Alfvén time scale, as expected. We may regard the four eigenvalues  $\Gamma_1, \Gamma_2, \Gamma_3$  and  $\Gamma_4$  as functions of  $kv_A t$ . The eigenvalue that appears in Eq. (69), which we shall denote  $\Gamma_1(kv_A t)$ , is always the largest and thus of primary interest. We have solved Eq. (66) for the value of  $\Gamma$  as a function of  $kv_A t$  or equivalently of  $(ka_0)S$ . It is convenient to plot  $\Gamma_1$  as a function of  $(kv_A t)^{-1}$ , which is an indication of

the strength of non-ideal effects. The results are shown in Fig. 2. It is apparent that for  $\alpha \ll 1$ , the value of  $\Gamma_1$  is reduced by non-ideal effects, but only to  $\Gamma_1 = 1/2$  even for  $\alpha \rightarrow 0$  in the limit  $kv_A t \rightarrow 0$ . For small finite values of  $\alpha$ , e.g. the case  $\alpha = 1/3$  shown in Fig. 2,  $\Gamma_1$  decreases as a function of  $(kv_A t)^{-1}$ , but goes complex for times earlier than a finite value  $kv_A t^*$ . Thus, the instability is predicted to exhibit oscillatory growth for  $t < t^*$ . However,  $t < t^*$  corresponds to less than one e-fold of growth, and our neglect of  $\hat{B}_2(x,t)$  [Eqs. (53) - (58)] becomes questionable at these early times, so we do not plot the complex values of  $\Gamma_1$ . In no case do we find stability, or even reduction of the ideal MHD growth rate by more than about a factor of two, for time scales of interest.

For  $\alpha \geq 1$ , we find  $\Gamma_1$  to be a rapidly increasing function of  $(kv_A t)^{-1}$ , i.e., the growth rate is much larger than the ideal MHD growth rate for a static equilibrium. Tracing back through the mathematics, we can see that this rapid growth is primarily due to the 2-2 matrix element of Eq. (65), i.e., to the term  $(1-3\alpha)\hat{a}/2t$  on the RHS of Eq. (45) for  $\hat{d}a/dt$ . This is not a resistive effect, but simply a geometric one arising from the increasing  $I(t)$ , which is not balanced by enough ohmic heating to maintain pressure balance at constant pinch radius  $a_0$ . If  $\alpha > 1/3$ , the equilibrium radius  $a_0(t)$  of the pinch steadily decreases as  $t^{(1-3\alpha)/2}$ , but any perturbations to the radius do not collapse as quickly. This effect tends to increase  $\hat{a}(t) \equiv (a - a_0)/a_0$ , which is the quantity that drives the growth of the perturbations. Additionally, because of this effect perturbations to  $\sigma a^2$  are dominated by the  $2\hat{a}$  perturbation rather than the  $\hat{\sigma}$  perturbation: hence the skin effect is weaker at a sausage neck, which is destabilizing, as discussed in Sec. III.

### Concluding Remarks

We have used a self-similar model to calculate sausage mode growth rates within the framework of resistive MHD. Although our model is a simplification of the perturbation dynamics, it does treat quite a general class of time-dependent exact equilibria, it is fully self-consistent in its treatment of equilibrium and perturbations, and it is a fully resistive treatment from the outset rather than being a perturbative treatment of departures from ideal MHD. Furthermore, we expect the self-similar assumption to be at least qualitatively reasonable in calculating growth rates for global modes, and in any event to yield growth rates that are smaller than exact solutions of resistive MHD. Thus, it is disappointing that our results fail to explain the long-lived apparent stability of the recent deuterium-fiber Z-pinch experiments.<sup>1,2</sup> We do find reductions in the instability growth rate for the case  $I \propto t^{1/3}$  where the equilibrium pinch radius is time-independent, and for other cases  $I \propto t^\alpha$  with  $\alpha$  small, but the reductions are by no more than a factor of two, which is insufficient to explain the experiments.

Assuming the experimental observations to be a valid indication of stability over a significant parameter regime, we may well ask what types of physical effects could account for this stability. Several types of effects come to mind, which have been excluded by the assumptions of the present model, and also by assumptions made in other recent calculations. First, we (and others) have assumed the current  $I(z,t)$  to be z-independent and equal to the equilibrium current. If, in fact,  $I(z,t)$  is reduced at the sausage necks - which would amount to some capacitive loading with charge accumulating on the shoulders of the sausages - that would have a stabilizing effect on the mode. Secondly, we (and others) have assumed the temperature within a given slice (i.e., at a given value of  $z$ ) to be

independent of  $r$ . This is in agreement with the numerical equilibrium calculations of Lindemuth, et al<sup>19</sup> provided there is no remaining residual solid fiber. However, these authors argue that many tens of nanoseconds pass before the fiber is fully ablated, and find that the presence of the fiber at  $r = 0$ , which cools the plasma there, results in a ramped profile for  $T(r)$ . It is possible that such a profile is more stable to sausage perturbations. We have not calculated either of these effects and merely suggest them as possible avenues for future investigation.

More generally, it should be noted that diagnostics used in the present generation of fiber z-pinch experiments would detect sausage instability only when it has reached large amplitude. Thus, it is possible that nonlinear effects excluded from the present linearized treatment, and possibly coupled to the effects of increasing current, resistivity, and other dissipative mechanisms may play a role in explaining the stability observed in the experiments.

#### Acknowledgments

The author is grateful to F. L. Cochran, A. E. Robson, M. Coppins, and I. D. Culverwell for discussions of work in progress. Some of the present work was inspired by previous calculations performed by these investigators. The author also has profited from conversations with D. Mosher. This work was performed in part while the author was a guest at the Laboratory for Plasma Research at the University of Maryland, whose hospitality is appreciated.

This work was supported by the Office of Naval Research.

### References

1. J. D. Sethian, A. E. Robson, K. A. Gerber, and A. W. DeSilva, Phys. Rev. Lett. 59, 892 (1987); 59, 1790 (1987).
2. J. E. Hammel and D. W. Scudder, Proc. 14th European Conference on Controlled Fusion and Plasma Physics, Madrid, 1987, Contributed papers, Part 2, p. 450.
3. I. D. Culverwell, M. Coppins, and M. G. Haines, Second International Conference on High Density Pinches, Laguna Beach, CA, 1989, in press.
4. A. H. Glasser and R. A. Nebel, *ibid.*
5. M. Coppins, Phys. Fluids, in press.
6. M. Coppins and J. Scheffel, *op. cit.* Ref. 3.
7. T. D. Arber and M. Coppins, *ibid.*
8. F. L. Cochran and A. E. Robson, *ibid.*
9. I. R. Lindemuth, *ibid.*
10. S. I. Braginskii, Sov. Phys. JETP 6, 494 (1958).
11. M. G. Haines, Proc. Phys. Soc. London 76, 250 (1960).
12. W. H. Bennett, Phys. Rev. 45, 890 (1934).
13. L. Spitzer, Physics of Fully Ionized Gases, Interscience, New York, 1962.
14. To be precise, it has been well known for decades that the growth rate is of the order of  $kv_A$ , but exact growth rates from ideal MHD for diffuse profiles have recently been calculated by M. Coppins, Plasma Phys. and Controlled Fusion 30, 201 (1988).
15. R. J. Tayler, Rev. Mod. Phys. 32, 907 (1960).
16. M. Coppins, I. D. Culverwell, and M. G. Haines, Phys. Fluids 31, 2688 (1988).
17. P. Rosenau, R. A. Nebel, and H. R. Lewis, Phys. Fluids (in press).

18. S. Lundquist, Ark. Fys. 5, 297 (1952).
19. I. R. Lindemuth, G. H. McCall, and R. A. Nebel, Phys. Rev. Lett. 62, 264 (1989).

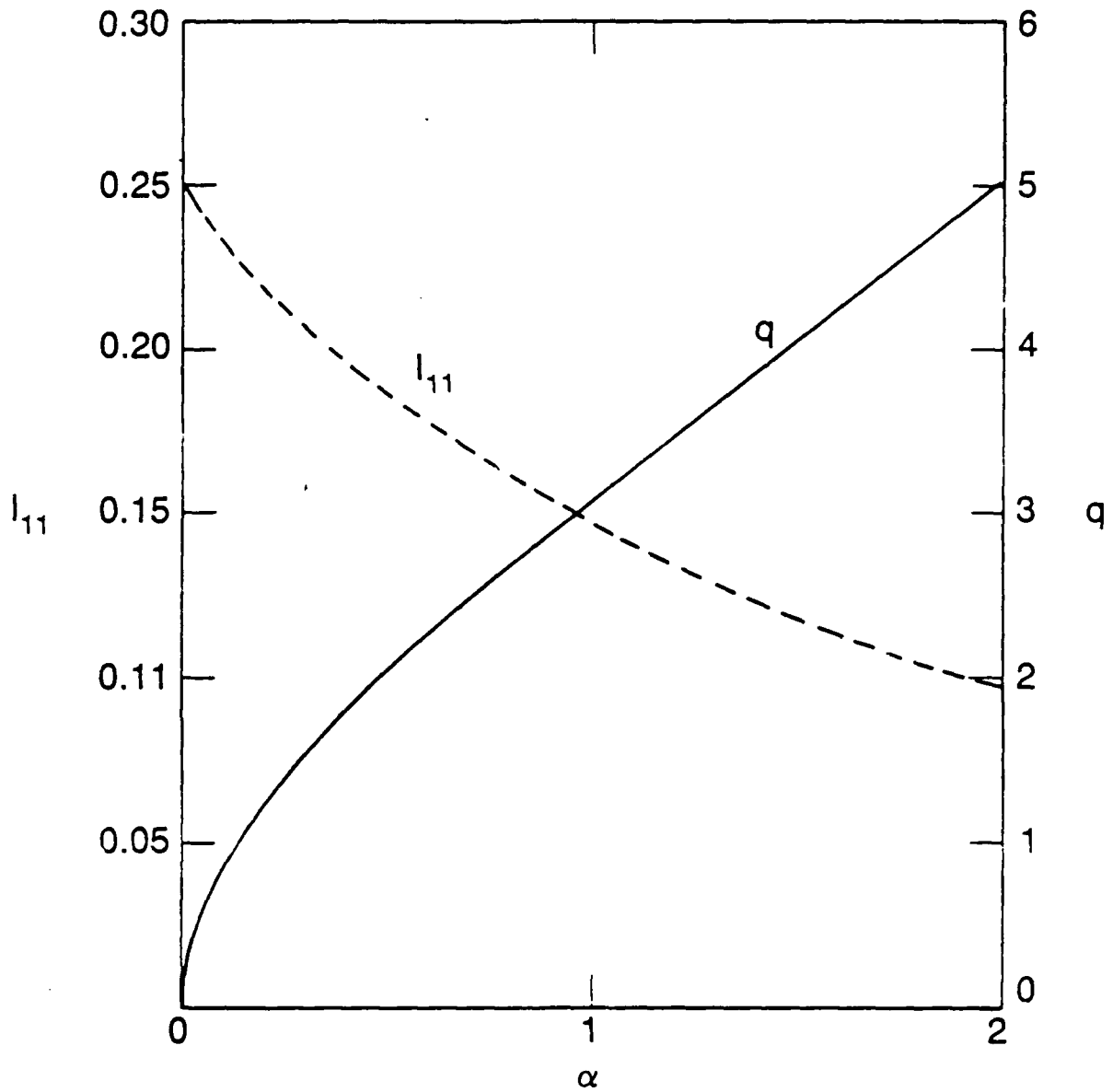


Fig. 1. The function  $I_{11}(\alpha)$  defined in Eqs. (47) is shown as the dashed curve, using the scale at left. The function  $q(\alpha)$ , from Eq. (18) is shown as the solid curve, using the scale at right.

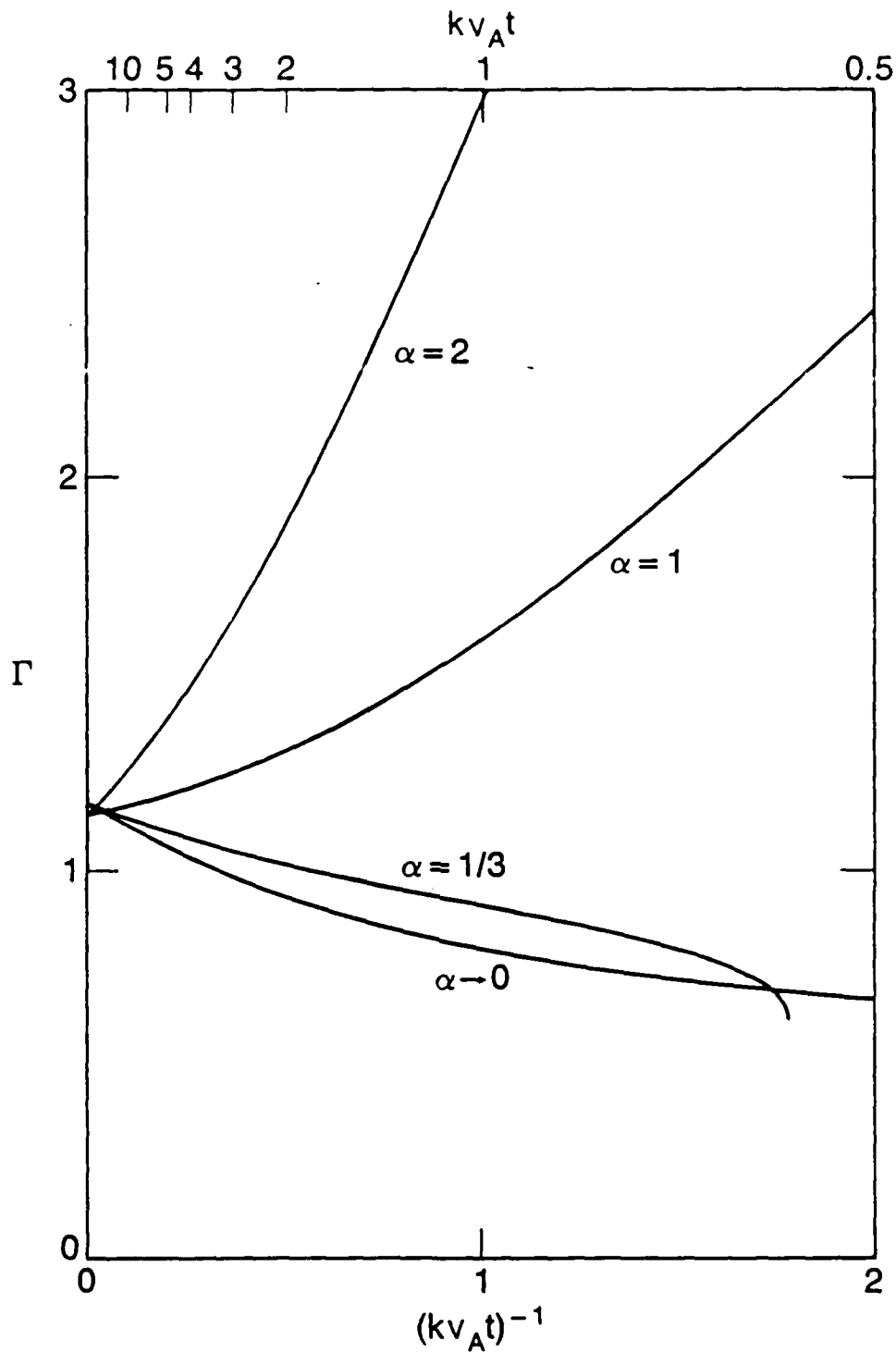


Fig. 2. The instability growth rate  $\Gamma$  [normalized to  $kv_A$ , where the Alfvén speed  $v_A$  is defined in Eq. (43)], for four different values of the parameter  $\alpha$ , which specifies the time dependence of the current. Note that the value of  $\Gamma$  for  $\alpha = 1/3$  becomes complex for  $(kv_A t)^{-1} > (kv_A t^*)^{-1} \approx 1.8$ .

Distribution List\*

Naval Research Laboratory  
4555 Overlook Avenue, S.W.

Attn: CAPT J. J. Donegan, Jr. - Code 1000  
Dr. M. Lampe - Code 4792 (20 copies)  
Dr. T. Coffey - Code 1001  
Head, Office of Management & Admin - Code 1005  
Deputy Head, Office of Management & Admin - Code 1005.1  
Directives Staff, Office of Management & Admin - Code 1005.6  
Director of Technical Services - Code 2000  
ONR - Code 0124  
NRL Historian - Code 2604  
Dr. W. Ellis - Code 4000  
Dr. S. Ossakow - Code 4700 (26 copies)  
B. Pitcher - Code 4790A  
Library - Code 2628 (22 copies)  
D. Wilbanks - Code 2634

\* Every name listed on distribution gets one copy except for those where extra copies are noted.

Dr. John P. Apruzese  
Naval Research Laboratory  
Plasma Radiation Branch  
Code 4720  
Washington, DC 20375

Dr. John F. Benage, Jr.  
Los Alamos National Laboratory  
MS-E526  
Los Alamos, NM 87545

Dr. Ekkehard P. Boggasch  
University of Maryland  
Laboratory for Plasma Research  
College Park, MD 20742

Dr. Thomas J. Burgess  
Sandia National Laboratories  
Division 1261  
P.O. Box 5800  
Albuquerque, NM 87185

Dr. Frederick L. Cochran  
Berkeley Research Associates, Inc.  
P.O. Box 852  
Springfield, VA 22150

Dr. Denis G. Colombant  
Naval Research Laboratory  
Code 4790  
Washington, DC 20375

Dr. Jack Davis  
Naval Research Laboratory  
Plasma Radiation Branch  
Code 4720  
Washington, DC 20375-5000

Dr. Christoph Deeney  
Physics International  
2700 Merced Street  
San Leandro, CA 94577

Dr. James H. Degnan  
Air Force Weapons Laboratory  
Kirtland AFB, NM 87117

Dr. William F. Dove  
Advanced Fusion Concepts Brnch  
Div. of Applied Plasma Physics  
Office Fusion Energy/ER-543-GTN  
U.S. Department of Energy  
Washington, DC 20545

Dr. Shimon Eckhouse  
Maxwell Laboratories, Inc.  
9244 Balboa Avenue  
San Diego, CA 92123

Dr. Carl Ekdahl  
Los Alamos National Laboratory  
P.O. Box 1663  
Mail Stop D410  
Los Alamos, NM 87545

Dr. Franklin S. Felber  
JAYCOR  
11011 Torreyana Road  
P.O. Box 85154  
San Diego, CA 92138

Dr. Michael Finkenthal  
Johns Hopkins University  
Dept. of Physics & Astronomy  
Rowland Hall  
Baltimore, MD 21218

Dr. Amnon Fisher  
University of California, Irvine  
Physics Department  
Irvine, CA 92717

Mr. Kent A. Gerber  
Naval Research Laboratory  
Code 4760  
Washington, DC 20375-5000

Ms. Miriam Gersten  
Maxwell Laboratories, Inc.  
8888 Balboa Avenue  
San Diego, CA 92123

Dr. John L. Giuliani  
Naval Research Laboratory  
Plasma Radiation Branch  
Plasma Physics Division  
Code 4720  
Washington, DC 20375

Dr. Alan H. Glasser  
Los Alamos National Laboratory  
CTR-6 Group  
Mail Stop F-642  
P.O. Box 1663  
Los Alamos, NM 87545

Dr. Arthur E. Greene  
Los Alamos National Laboratory  
Xx-10, MS B259  
Los Alamos, NM 87545

Dr. Hans R. Griem  
University of Maryland  
Lab for Plasma Research  
Energy Research Building  
College Park, MD 20742

Dr. Jay E. Hammel  
Los Alamos National Laboratory  
P.O. Box 1663  
CTR-8, MS K639  
Los Alamos, NM 87545

Dr. Charles W. Hartman  
Lawrence Livermore National Lab.  
P.O. Box 808  
L-637  
Livermore, CA 94550

Dr. David D. Hinshelwood  
Naval Research Laboratory  
Code 4773  
Washington, DC 20375-5000

Dr. Warren W. Hsing  
Sandia National Laboratories  
P.O. Box 5800  
Albuquerque, NM 87185

Dr. Thomas W. Hussey  
Sandia National Laboratories  
P.O. Box 5800  
Albuquerque, NM 87185

Dr. Franz Jahoda  
Los Alamos National Laboratory  
P.O. Box 1663  
CTR-8, MS K639  
Los Alamos, NM 87545

Dr. Robert A. Krakowski  
Los Alamos National Laboratory  
CTR-12, MSF641  
P.O. Box 1663  
Los Alamos, NM 87545

Dr. Mahadevan Krishnan  
Physics International  
2700 Merced Street  
San Leandro, CA 94577-0599

Dr. M. Kristiansen  
Texas Tech University  
Dept. of Electrical Engineering  
Lubbock, TX 79409-4439

Dr. Martin Lampe  
Naval Research Laboratory  
Code 4792  
Washington, DC 20375

Mr. P. David LePell  
Physics International Company  
2700 Merced Street  
San Leandro, CA 94577

Dr. Jae K. Lee  
General Atomics (13-318)  
P.O. Box 85608  
San Diego, CA 92138

Dr. Irvin R. Lindemuth  
Los Alamos National Laboratory  
Applied Theoretical Physics Div.  
MS-E531  
P.O. Box 1663  
Los Alamos, NM 87545

Dr. Nicholas G. Loter  
Maxwell Laboratories, Inc.  
8888 Balboa Avenue  
San Diego, CA 92123

Dr. Ralph H. Lovberg  
Los Alamos National Laboratory  
P.O. Box 1663  
CTR-8, MS K639  
Los Alamos, NM 87545

Dr. Rodney J. Mason  
Los Alamos National Laboratory  
X-1, MS-E531  
Los Alamos, NM 87545

Stephen M. Matthews  
Lawrence Livermore National Lab.  
L-389, Box 808  
Livermore, CA 94550

Dr. Keith. Matzen  
Sandia National Laboratories  
Code 1273  
P.O. Box 5800  
Albuquerque, NM 87185

George H. Miley  
University of Illinois  
214 Nuclear Engr Lab  
103 S. Goodwin Avenue  
Urbana, IL 61801

Cornelius A. Morgan  
University of Maryland  
Plasma Spectroscopy Group  
Building #223  
College Park, MD 20742

Dr. David Mosher  
Naval Research Laboratory  
Code 4770.1M  
Washington, DC 20375

Dr. V. Nardi  
Stevens Institute of Technology  
Hoboken, NJ 07803

Dr. Thomas Nash  
Physics International Co.  
2700 Merced Street  
San Leandro, CA 94577

Dr. Richard A. Nebel  
Los Alamos National Laboratory  
MS F642  
Los Alamos, NM 87545

Dr. Nino R. Pereira  
Berkeley Research Associates, Inc.  
P.O. Box 852  
Springfield, VA 22150

Dr. Joseph D. Perez  
Auburn University  
Physics Department  
206 Allison Lab  
Auburn University, AL 36849-5311

Dr. Darrell L. Peterson  
Los Alamos National Laboratory  
MS B259  
Los Alamos, NM 87545

Dr. John L. Porter  
Sandia National Laboratories  
P.O. Box 5800  
Albuquerque, NM 87185

Dr. Charles Powell  
Stevens Institute of Technology  
Dept. of Physics  
Hoboken, NJ 07030

Mr. Rahul Prasad  
Yale University  
Mason Lab.  
9 Hillhouse Avenue  
New Haven, CT 06520

Dr. Peter E. Pulsifer  
Berkeley Research Associates, Inc.  
P.O. Box 852  
Springfield, VA 22150-0852

Dr. Niansheng Qi  
Laboratory of Plasma Studies  
Cornell University  
369 Upson Hall  
Ithaca, NY 14853

Dr. John C. Riordan  
Physics International Co.  
2700 Merced Street  
San Leandro, CA 94577

Dr. William Rix  
Maxwell Laboratories, Inc.  
8888 Balboa Avenue  
San Diego, CA 92123

Charles W. Roberson  
Office of Naval Research  
Physics Division, Code 1112  
800 North Quincy St.  
Arlington, VA 22308

Dr. Anthony E. Robson  
Naval Research Laboratory  
Code 4760  
4555 Overlook Ave., SW  
Washington, DC 20375

Professor Norman F. Roderick  
University of New Mexico  
Dept. Chemical & Nuclear Eng  
Albuquerque, NM 87131

Dr. Norman Rostoker  
University of California, Irvine  
Physics Department  
Irvine, CA 92717

Dr. Gary A. Saenz  
Rockwell International  
Rocketdyne Division  
6633 Canoga Ave.  
M/S: FA03  
Canoga Park, CA 91303

Dr. David W. Scudder  
Los Alamos National Laboratory  
P.O. Box 1663  
MS K639  
Los Alamos, NM 87545

Dr. John D. Sethian  
Naval Research Laboratory  
Code 4762  
4555 Overlook Ave., SW  
Washington, DC 20375

Dr. Jack S. Shlachter  
Los Alamos National Laboratory  
CTR-8, MS K639  
P.O. Box 1663  
Los Alamos, NM 87545

Dr. Rick B. Spielman  
Sandia National Laboratories  
P.O. Box 5800  
Albuquerque, NM 87115

Dr. Stavros Stephanakis  
Naval Research Laboratory  
Code 4773  
4555 Overlook Avenue, S.W.  
Washington, DC 20375-5000

Dr. Leaf Turner  
Los Alamos National Laboratory  
CTR-6, MS F642  
P.O. Box 1663  
Los Alamos, NM 87545

Dr. Han S. Uhm  
Naval Surface Warfare Center  
R-41  
White Oak  
Silver Spring, MD 20903-5000

Dr. Kenneth D. Ware  
Maxwell Laboratories  
8888 Balboa Avenue  
San Diego, CA 92123

Dr. Frank J. Wessel  
University of California, Irvine  
Physics Department  
Irvine, CA 92717

Dr. Kenneth G. Whitney  
Naval Research Laboratory  
Plasma Radiation Branch  
Code 4720  
4555 Overlook Avenue, SW  
Washington, DC 20375

Dr. Andrew R. Wilson  
S-Cubed  
A Division of Maxwell Laboratories  
PO Box 1620  
La Jolla, CA 92038

Dr. Gerold Yonas  
Titan Technologies  
9191 Towne Centre Dr.  
Suite 600  
San Diego, CA 92122

Dr. Frank C. Young  
Naval Research Laboratory  
Code 4770.1  
Washington, DC 20375-5000

Naval Research Laboratory  
Washington, DC 20375-5000  
Code 1220

Do NOT make labels  
for these two-below:  
Records---(1 copy)

Director of Research  
U. S. Naval Academy  
Annapolis, MD 21402  
(2 copies)

Naval Research Laboratory  
Washington, DC 20375-5000  
Code 2630  
Timothy Calderwood