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FORTTRAN SUBROUTINES FOR THE EVALUATION OF THE
CONFLUENT HYPERGEOMETRIC FUNCTIONS

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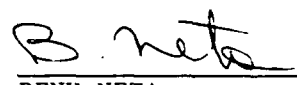
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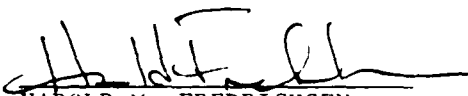
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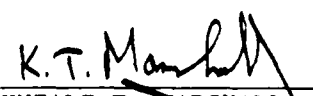

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Abstract

In this report we list the Fortran subroutines for evaluating the confluent hypergeometric functions $M(a,b;x)$ and $U(a,b;x)$. These subroutines use the stable recurrence relations given e.g. in Wimp.

Key words:
confluent hypergeometric functions
stable algorithm
Fortran subroutine
recurrence relation

Introduction

It is well known that the ordinary differential equation

$$x \frac{d^2 y}{dx^2} + (1-x) \frac{dy}{dx} - ay = 0$$

has a solution

$$y(x) = AM(a, 1; x) + BU(a, 1; x)$$

if a is not a negative integer.

This problem arises e.g. when solving the linearized shallow water equations with the full linear variation in depth included (see Williams, Staniforth and Neta, [1]).

The computation of the confluent hypergeometric functions is based on the Miller algorithm (see e.g. Wimp, [2]). In general, one has a second order difference equation

$$z(n) + a(n)z(n+1) + b(n)z(n+2) = 0, \quad n \geq 0, \quad b(n) \neq 0.$$

If $b(n) = 0$ for some n , in some cases one can make a change of variable $Y(n) = \lambda(n)z(n)$ which will produce an equation of the desired type. Let $w(n)$ be a nontrivial solution and the sum of the normalizing series

$$S = \sum_{k=0}^{\infty} c(k)w(k) \neq 0$$

is known. Let N be a large integer and define $z_N(n)$, $0 \leq n \leq N+1$, by

$$z_N(n) = \begin{cases} 0 & n = N+1 \\ 1 & n = N \end{cases}$$

$$z_N(n) + a(n)z_N(n+1) + b(n)z_N(n+2) = 0, \quad n = N-1, \dots, 1, 0.$$

One can approximate $w(n)$ by $w_N(n)$

$$w_N(n) = Sz_N(n)/S_N$$

where

$$S_N = \sum_{k=0}^N c(k)z_N(k).$$

The algorithm is said to converge if

$$w_N(n) \rightarrow w(n) \quad \text{as } N \rightarrow \infty.$$

The function $M(a, b; x)$ satisfies the recurrence relation

$$\begin{aligned} (2n+b+2)(n+a)z(n) - (2n+b+1)\left\{(2a-b) + \frac{(2n+b)(2n+b+2)}{x}\right\}z(n+1) \\ - (2n+b)(n+b+1-a)z(n+2) = 0. \end{aligned}$$

The minimal solution is

$$w(n) = \frac{x^n (a)_n}{(b)_{2n}} M(a+n, 2n+b; x)$$

where

$$(c)_n = \frac{\Gamma(n+c)}{\Gamma(c)} .$$

The normalization relationship used in our subroutine is

$$S = b-1 = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} (b-1)_k (b+2k-1) w(k) .$$

An obvious modification must be made if $b = 1$. The algorithm is not defined if b , $b+1-a$, a are negative integers or zero.

The function $U(a,b;x)$ satisfies the relationship

$$\begin{aligned} (n+a)(n+a+1-b)z(n) - (n+1)[2(n+a+1)+x-b]z(n+1) \\ + (n+1)(n+2)z(n+2) = 0 . \end{aligned}$$

The minimal solution is

$$w(n) = \frac{x^n (a)_n (a+1-b)_n}{n!} U(a+n, b; x)$$

for $|\arg x| < \pi$. A normalization relation is

$$1 = \sum_{k=0}^{\infty} w(k) .$$

In the next section we give a listing of the Fortran subroutines.

Subroutine Miller

```
SUBROUTINE MILLER(N,ALPHA,BETA,X,S,SS,COEFF)
INTEGER N
REAL*8 ALPHA,BETA,X,SS
REAL*8 S(0:1000)
EXTERNAL COEFF
C   USES THE J.C.P. MILLER ALGORITHM TO COMPUTE
C   S(0:N).
C   BEGIN
      INTEGER NN,K
      REAL*8 T,D,EPS,A,B,C
      REAL*8 OLDS(0:1000)
      EPS = 0.000000001
C   INITIALIZE OLDS.
      DO 1 K = 0, 1000
        OLDS(K) = 0
1    CONTINUE
C   CHOOSE INITIAL NN.
      NN = N + 2
C   INITIALIZE K, S AND T.
2    K = NN
      S(K+1) = 0
      S(K) = 1
      CALL COEFF(K,ALPHA,BETA,X,A,B,C)
      T = 2*C*S(K)
C   TAKE A BACKWARD RECURRENCE STEP AND UPDATE IT.
3    K = K - 1
      CALL COEFF(K,ALPHA,BETA,X,A,B,C)
      S(K) = A*S(K+1) + B*S(K+2)
C   CHECK FOR OVERFLOW AND RESCALE IF NECESSARY.
      D = DABS(S(K))
      IF (D .GT. 1.D30) THEN
C   BEGIN
        CALL SCALE(K,NN,S,T,D)
      END IF
      IF (K .GT. 0) THEN
C   BEGIN
        T = T + 2*C*S(K)
        GO TO 3
      END IF
      T = T + C*S(0)
      DO 4 K = 0, N
        S(K) = S(K)/T
4    CONTINUE
C   TEMPORARY PRINT STATEMENT.
C   PRINT*, S(0)
C   TEST FOR CONVERGENCE.
      D = 0
      DO 5 K = 0, N
        D = D + S(K)**2
5    CONTINUE
      D = DSQRT(D)
      T = 0
```

```

        DO 6 K = 0, N
          T = T + (S(K) - OLDS(K))**2
6      CONTINUE
        T = DSQRT(T)
C      TAKE ANOTHER STEP IF NO CONVERGENCE.
        IF (T .GT. EPS*D) THEN
C      BEGIN
          NN = 2*NN
          DO 7 K = 0, N
            OLDS(K) = S(K)
7          CONTINUE
          IF(NN .LE. 1000) GO TO 2
          PRINT 999, NN, ALPHA, BETA, X, T
999      FORMAT(' ** NO CONVERGENCE ** NN AP CP X T ', I5.4E14.7)
        END IF
        SS=S(0)
        RETURN
      END

```

```

SUBROUTINE COEFF(N,ALPHA,BETA,X,A,B,C)
INTEGER N
REAL*8 ALPHA,BETA,X,A,B,C
C   COMPUTES COEFFICIENTS USED BY J.C.P. MILLER ALGORITHM FOR
C   A CONFLUENT HYPERGEOMETRIC FUNCTION  $M(a,b;x)$ 
C   SEE JET WIMP, COMPUTATION WITH RECURRENCE RELATIONS.
C   PITMAN 1984 PP. 61-62
C   BEGIN
      INTEGER M,K
      REAL*8 T,U,V,W
      S = 2*ALPHA - BETA
      T = N + ALPHA
      M = 2*N
      U = M + BETA
      V = U + 1
      W = V + 1
      A = (S/W + U/X)*V/T
      B = (N + BETA - ALPHA + 1)*U/T/W
      T = 1
      IF (N .GT. 0) THEN
C   BEGIN
          S = BETA - 1
          DO 1 K = 1, N-1
              T = -T*(1+S/K)
1          CONTINUE
              T = -T*(1+S/M)
          END IF
          C = T
          RETURN
      END

```

```

SUBROUTINE SCALE(K,N,S,T,D)
INTEGER N,K
REAL*8 T,D
REAL*8 S(0:1000)
C   BEGIN
      INTEGER J
      T = T/D
      DO 1 J = K, N
          S(J) = S(J)/D
1      CONTINUE
      RETURN
      END

```

```

SUBROUTINE COEFU(N,ALPHA,BETA,X,A,B,C)
INTEGER N
REAL*8 ALPHA, BETA,X,A,B,C
C   COMPUTES COEFFICIENTS USED BY J.C.P. MILLER ALGORITHM FOR
C   A CONFLUENT HYPERGEOMETRIC FUNCTION  $U(a,b;x)$ 
C   SEE JET WIMP, COMPUTATION WITH RECURRENCE RELATIONS.
C   PITMAN 1984 PP. 63-64
C   BEGIN
      INTEGER M,K
      REAL*8 S,T,U,V,W
      S = ALPHA + QFLOAT(N)
      T = S + 1.D0
      U = S*(T - BETA)
      V = QFLOAT(N + 1)
      W = V + 1.D0
      A = (2*T + X - BETA)*V/U
      B = - V*W/U
      C = 1
      RETURN
END

```

Remark: The program that calls Miller must supply as a last parameter either COEFF (for M) or COEFU (for U).

The subroutines are available on a diskette from either author upon request. These subroutines were tested extensively for various values of a , b and x .

Remark: If the parameter is a negative integer, the solution of the differential equation is

$$y = AL_n(x) + B\{\ln|x|L_n(x) + \sum_{m=0}^{\infty} \beta_m x^m\}$$

where $n = -a$.

$L_n(x)$ are Laguerre polynomials whose coefficients a_i satisfy

$$a_i = \frac{i-n-1}{i^2} a_{i-1} \quad , \quad i = 2, \dots, n \quad ,$$

$$a_1 = -n \quad ,$$

The coefficients β_m satisfy

$$\beta_{m+1} = \frac{(m-n)\beta_m + \left(1 - \frac{2(m-n)}{m+1} a_m\right)}{(m+1)^2} \quad m = 1, \dots, n-1$$

$$\beta_m = \frac{1}{(n+1)^2} a_n \quad m = n$$

$$\beta_m = \frac{m-n-1}{m^2} \beta_{m-1} \quad m = n+1, n+2, \dots$$

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1. R.T. Williams, A.N. Staniforth and B. Neta, Solution of a generalized Sturm-Liouville Problem, IMA Conference on Computational Ordinary Differential Equations. Imperial College, London, July 3-7, 1989.
2. J. Wimp, Computation with Recurrence Relations. Pitman Advanced Pub. Program, Boston, 1984.

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