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TECHNICAL MEMORANDUM 89/219
August 1989

CHORDWISE LOADING AND CAMBER FOR TWO-DIMENSIONAL THIN SECTIONS

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Defence Research Establishment Atlantic

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Canada

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Approved by L.J. Leggat
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Abstract

Derivations are given for the calculation of section camberline from a given segmented chordwise load distribution, and for the inverse problem. In both cases, the algorithms are exact within the limits of thin-wing theory.

From these derivations, codes were developed which are included in the section design procedure at DREA. The results from example cases show very good agreement with analytical solutions, even for the extreme situation of infinite leading or trailing edge slopes.

Résumé

Présentation de dérivations pour le calcul d'une ligne de courbure à partir de segments formés par des cordes correspondant à une répartition de charges, ainsi que pour le problème inverse. Dans les deux cas les algorithmes sont exacts dans les limites de la théorie des ailes minces.

À partir de ces dérivations, on a développé des codes qui sont inclus dans la méthode de calcul des sections du CRDA. Les résultats des cas donnés en exemple montrent que la concordance avec les solutions analytiques est bonne, même pour la situation extrême des pentes infinies des bords d'attaque et des bords de fuite.

Table of Contents

Nomenclature

c_l	section lift coefficient
c_{li}	ideal, or design lift coefficient
x	chordwise coordinate/chord (abscissa)
y	camberline offset, or normal coordinate/chord (ordinate)
α	angle of attack
α_i	ideal angle of attack
α_0	zero-lift angle of attack
γ	nondimensional loading, or circulation per unit span
θ	polar coordinate
ξ	chordwise coordinate/chord

1 Introduction

This document is concerned with two-dimensional, thin, lifting sections and gives the derivation of the camberline from a segmented chordwise loading (the design problem) and the calculation of loading from a discretely-defined camberline (the analysis problem). To the author's knowledge, these derivations have not been published for the general case, although the results for some specific cases are well known[1,2]. Good examples are the NACA a-series camberlines[1], obtained from simple two-segment load distributions, which have been widely used for many years.

Modern section design methods, such as the Eppler code[3], tend to generate complete sections rather than treat thickness and camber separately. DREA employs the AADP9 code[4] which can be used in either fashion, or in a hybrid of the two; Reference 5 illustrates its use in a typical design exercise. The algorithms presented here were required to extend that previous work to the development of sections for which the load distribution is controlled by defining the camberline.

Based on the present derivations, two utility programs, CAMBER (design) and CLOAD (analysis) were written for use with AADP9. Examples using these programs are presented and compared with analytical results.

2 Deriving Camber from Chordwise Loading

This memorandum uses the standard conventions for section definition. Thus, the camberline runs from the leading edge at the origin of the axis system to the trailing edge at x, or ξ, equal to 1. Camberline offsets are denoted by y and lengths are nondimensionalized by the section chord.

In thin-wing theory[6], the nondimensional loading, $\gamma(x)$, on a thin section is identified with the difference in local velocity between the upper and lower surfaces. From this definition it follows that the lift coefficient is:

$$c_l = 2\int_0^1 \gamma(x)dx$$

If $\gamma(x)$ is given for an arbitrary angle of attack, α, the camberline slope is found from the following relationship[6]:

$$y'(\xi) - \alpha = \frac{1}{2\pi}\int_0^1 \frac{\gamma(x)dx}{x-\xi}$$

In practice, $\gamma(x)$ is invariably defined for the ideal angle of attack, α_i, otherwise the leading edge loading, $\gamma(0)$, becomes infinite. At the ideal angle of attack, the lift coefficient is called the section design lift coefficient, c_{l_i}.

For this analysis, the chordwise loading, $\gamma(x)$, is defined in N segments, so that the n^{th} segment lies between ordinates x_{n-1} and x_n. Therefore, the first segment, starting at the leading edge, lies between $x_0 = 0$ and x_1, and the last segment, ending on the trailing edge, lies between x_{N-1} and $x_N = 1$, Figure 1. The camberline slope is then:

$$y'(\xi) - \alpha = \frac{1}{2\pi}\sum_{n=1}^{N}\int_{x_{n-1}}^{x_n} \frac{\gamma(x)dx}{x-\xi}$$

We proceed by obtaining the general result for a single one of these integrals so that:

$$y'(\xi) - \alpha = \frac{1}{2\pi}\sum_{n=1}^{N} I_n(\xi) \tag{1}$$

where

$$I_n(\xi) = \int_{x_{n-1}}^{x_n} \frac{\gamma(x)dx}{x - \xi}$$

The loading is chosen to be a linear function of x on each segment, so that using the notation γ_n for the loading at ordinate x_n, the loading on the n^{th} segment is:

$$\gamma(x) = \gamma_n - \frac{(\gamma_n - \gamma_{n-1})}{(x_n - x_{n-1})} \cdot (x_n - x)$$

That is,

$$\gamma(x) = a_n + b_n x \qquad\qquad (x_{n-1} \le x \le x_n)$$

where a_n and b_n are respectively the n^{th} loading segment offset and slope given by:

$$a_n = \gamma_n - \gamma_n' x_n \qquad \text{and} \qquad b_n = \gamma_n' \tag{2}$$

To obtain the camberline slope, we substitute for $\gamma(x)$ in the integrals I_n:

$$I_n(\xi) = a_n \int_{x_{n-1}}^{x_n} \frac{dx}{x - \xi} + b_n \int_{x_{n-1}}^{x_n} \frac{x dx}{x - \xi}$$

These are standard integrals and lead to the same result whether ξ is inside or outside the n^{th} segment, that is:

$$I_n(\xi) = b_n(x_n - x_{n-1}) + (a_n + b_n\xi)\left\{\ln|x_n - \xi| - \ln|\xi - x_{n-1}|\right\} \tag{3}$$

The camberline is obtained by integrating the equation for camberline slope. Reversing the order of summation and integration this gives:

$$y(\xi) - \alpha\xi = \frac{1}{2\pi}\sum_{n=1}^{N}\int_0^{\xi} I_n(x)dx$$

or,

$$y(\xi) - \alpha\xi = \frac{1}{2\pi}\sum_{n=1}^{N} J_n(\xi) \tag{4}$$

where

$$J_n(\xi) = \int_0^{\xi} I_n(x)dx$$

Therefore, substituting for $I_n(x)$,

$$\begin{aligned} J_n(\xi) \;=\; & b_n\xi(x_n - x_{n-1}) \\ & + a_n \int_0^{\xi} \left\{\ln|x_n - x| - \ln|x - x_{n-1}|\right\} dx \\ & + b_n \int_0^{\xi} \left\{x\ln|x_n - x| - x\ln|x - x_{n-1}|\right\} dx \end{aligned}$$

Evaluating the integrals, we finally obtain:

$$\begin{aligned} J_n(\xi) &= b_n\xi(x_n - x_{n-1}) \\ &\quad -(a_n + b_n x_n)(x_n - \xi)\{\ln|x_n - \xi| - 1\} \\ &\quad +(a_n + b_n x_n)x_n\{\ln|x_n| - 1\} \\ &\quad -(a_n + b_n x_{n-1})(\xi - x_{n-1})\{\ln|\xi - x_{n-1}| - 1\} \\ &\quad -(a_n + b_n x_{n-1})x_{n-1}\{\ln|x_{n-1}| - 1\} \\ &\quad +b_n\frac{(x_n - \xi)^2}{2}\left\{\ln|x_n - \xi| - \tfrac{1}{2}\right\} \\ &\quad -b_n\frac{x_n^2}{2}\left\{\ln|x_n| - \tfrac{1}{2}\right\} \\ &\quad -b_n\frac{(\xi - x_{n-1})^2}{2}\left\{\ln|\xi - x_{n-1}| - \tfrac{1}{2}\right\} \\ &\quad +b_n\frac{x_{n-1}^2}{2}\left\{\ln|x_{n-1}| - \tfrac{1}{2}\right\} \end{aligned} \tag{5}$$

For $\alpha = \alpha_i$, the section design lift coefficient, c_{li}, is obtained directly. Since, for the segmented loading (using the superscript i to denote that γ is defined at the angle of attack),

$$c_{li} = 2\sum_{n=1}^{N}\int_{x_{n-1}}^{x_n}\gamma^i(x)dx$$

simple substitution gives:

$$c_{li} = \sum_{n=1}^{N}\left\{2a_n^i(x_n - x_{n-1}) + b_n^i(x_n^2 - x_{n-1}^2)\right\} \tag{6}$$

In summary, the camberline slope is given by equations (1), (2) and (3), the camberline by equations (4) and (5), and the design lift by equations (2) and (6).

It is easy to verify that these results are correct for simple cases. For example, the NACA $(a = 1)$ mean line is defined as corresponding to a uniform load, γ_*, at the ideal angle of attack which, since the loading is symmetrical about midchord, is zero. Therefore,

$$a_1^i = \gamma_* \quad , \quad b_1^i = 0 \quad , \quad x_0 = 0 \quad \text{and} \quad x_1 = 1$$

so that

$$c_{li} = 2\gamma_*$$

$$y'(\xi) = \frac{c_{li}}{4\pi}\{\ln|1 - \xi| - \ln|\xi|\}$$

and

$$y(\xi) = -\frac{c_{li}}{4\pi}\{(1 - \xi)\ln|1 - \xi| + \xi\ln|\xi|\}$$

which is the result given by Abbott and von Doenhoff[1], and also by Riegels[2], who calls this the S_5 camberline.

Finally, in the general case, thin-wing theory gives the lift at zero incidence:

$$c_{l(\alpha=0)} = c_l - 2\pi\alpha$$

and the zero lift angle:

$$\alpha_0 = \alpha - \frac{c_l}{2\pi}$$

It should be noted that if γ is non-zero at the leading or trailing edges, then the camberline slope at these points is infinite. Furthermore, the Kutta condition requires that the loading go to zero at the trailing edge. In practice, these strictures are ignored: for example the NACA a-series mean lines are all loaded at the leading edge, and the $(a = 1)$ mean line is also loaded at the trailing edge. Difficulties do not arise because, invariably, the last few percent of chord at each edge are modified for manufacturing convenience and, in addition, the effective camber at the trailing edge is modified by the boundary layer.

3 Deriving Chordwise Loading from Camber

Suppose the loading is distributed in segments as before. The end-points of these segments may have the same abscissae as the points defining the camberline, Figure 2. In the previous section, we obtained the following result for the camberline:

$$y(\xi) - \alpha\xi = \frac{1}{2\pi}\sum_{n=1}^{N} J_n(\xi)$$

$J_n(\xi)$ is a continuous function of ξ, but considering only the discrete values of ξ at which the camberline is defined, we can replace the function $J_n(\xi)$ by discrete values $J_{m,n}$. Then the equation for the m^{th} point on the camberline is:

$$y_m - \alpha x_m = \frac{1}{2\pi}\sum_{n=1}^{N} J_{m,n}$$

Since $J_{m,n}$ can be expanded in γ_n this can be written as:

$$y_m - \alpha x_m = \frac{1}{2\pi}\sum_{n=0}^{N-1} C_{m,n}\gamma_n$$

where $C_{m,n}$ are coefficients of the expansion and the limits of the summation have been shifted to accomodate the leading edge $(n = 0)$ and to exclude the trailing edge (since the Kutta condition requires that $\gamma_N = 0$).

It is straightforward algebraic manipulation to obtain $C_{m,n}$ by evaluating the previously-given expression for $J_n(\xi)$, substituting for a_n and b_n, and setting $x_m = \xi$. The final result is:

$$C_{m,n} = \begin{cases} D_{m,1} & (n = 0) \\ D_{m,n+1} + E_{m,n} & (\text{otherwise}) \end{cases} \tag{7}$$

where

$$\begin{aligned}
D_{m,n} \;=\; & -x_m \\
& -(x_m - x_{n-1})\left\{\ln|x_m - x_{n-1}| - 1\right\} \\
& -x_{n-1}\left\{\ln|x_{n-1}| - 1\right\} \\
& -\frac{(x_n - x_m)^2}{2(x_n - x_{n-1})}\left\{\ln|x_n - x_m| - \tfrac{1}{2}\right\} \\
& +\frac{x_n^2}{2(x_n - x_{n-1})}\left\{\ln|x_n| - \tfrac{1}{2}\right\} \\
& +\frac{(x_m - x_{n-1})^2}{2(x_n - x_{n-1})}\left\{\ln|x_m - x_{n-1}| - \tfrac{1}{2}\right\} \\
& -\frac{x_{n-1}^2}{2(x_n - x_{n-1})}\left\{\ln|x_{n-1}| - \tfrac{1}{2}\right\}
\end{aligned} \tag{8}$$

and

$$\begin{aligned}
E_{m,n} \;=\; & x_m \\
& -(x_n - x_m)\left\{\ln|x_n - x_m| - 1\right\} \\
& +x_n\left\{\ln|x_n| - 1\right\} \\
& +\frac{(x_n - x_m)^2}{2(x_n - x_{n-1})}\left\{\ln|x_n - x_m| - \tfrac{1}{2}\right\} \\
& -\frac{x_n^2}{2(x_n - x_{n-1})}\left\{\ln|x_n| - \tfrac{1}{2}\right\} \\
& -\frac{(x_m - x_{n-1})^2}{2(x_n - x_{n-1})}\left\{\ln|x_m - x_{n-1}| - \tfrac{1}{2}\right\} \\
& +\frac{x_{n-1}^2}{2(x_n - x_{n-1})}\left\{\ln|x_{n-1}| - \tfrac{1}{2}\right\}
\end{aligned} \tag{9}$$

Thus, since $y_0 = 0$, the loading $\gamma_{n,(0 \le n \le N-1)}$ is given by the solution to the set of simultaneous equations:

$$y_m - \alpha x_m = \frac{1}{2\pi}\sum_{n=0}^{N-1} C_{m,n}\gamma_n \qquad (1 \le m \le N) \tag{10}$$

where the coefficients $C_{m,n}$ are defined in equations (7), (8) and (9).

The loadin[illegible] the leading edge is finite, according to thin-wing theory, only at the ideal angle of attack, [illegible] the camberline is invariably defined for $\alpha = 0$. If it is required, an approximation for α_i can be obtained from the definition[6]:

$$\alpha_i = \frac{1}{\pi}\int_0^{\pi} y'(\theta)\,d\theta$$

where

$$\cos\theta = 1 - 2x$$

We replace the integral by a summation and discretize the terms:

$$\alpha_i = \frac{1}{\pi}\sum_{n=1}^{N}\frac{(y_n - y_{n-1})}{(x_n - x_{n-1})}\left\{\arccos(1-2x_n) - \arccos(1-2x_{n-1})\right\}$$

This approximation may not be sufficiently accurate for cases with only a few points defining the camberline in the leading or trailing edge regions.

4 Examples

The first example has already been discussed in Section 2: the constant load NACA ($a = 1$) mean line. The camberline and slope for a loading of $\gamma_* = 0.5$ ($c_{li} = 1$) obtained with the CAMBER code were identical with the analytical solutions to within computer precision except at the leading and trailing edges, where theoretically infinite slopes were encountered. This situation results in an attempted evaluation of the natural logarithm of zero. In CAMBER, the problem is avoided by setting a lower limit of 10^{-20} to the argument of the natural logarithm.

The next example is for a loading function defined so that:

n	x_n	γ_n	γ_n/γ_{MAX}
0	0.00	0.080247	0.2
1	0.10	0.320988	0.8
2	0.25	0.401235	1.0
3	0.70	0.401235	1.0
4	0.90	0.200617	0.5
5	1.00	0.000000	0.0

The load distribution and design lift were arbitrarily chosen to represent a typical mean line with reduced leading edge loading. CAMBER allows the input load distribution to be normalized in order to obtain a specific lift coefficient. In this example, the values in the last column of the above table were input and scaled by CAMBER for $c_l = 0.65$; the values tabulated in the third column are the results. Since $\gamma(0)$ is finite, these represent the loading at the ideal angle of attack. The predicted camberline was subsequently input to CLOAD to determine how well the input load distribution was recovered. Table I and Figure 3 show the results of these calculations.

There was a small deficit in loading concentrated at the leading edge. Other examples show that the effect only occurs to this extent when the loading does not go to zero at the leading (as in this case) or trailing edge. This is a numerical artifact, occurring in the matrix inversion required to solve the system of simultaneous equations (Equation 10). It is beyond the scope of this document to discuss numerical problems in detail, but it is pertinent to note that a standard partial pivoting routine is used in CLOAD, and that better methods are available.

Generally, in this example, the predicted loading from CLOAD agreed with the input to CAMBER to within one percent of the maximum value.

The last example illustrates the evaluation of a Riegels S_1 (Birnbaum-Glauert) camberline with CLOAD. This is a class of camberlines synthesised from circular arcs and reflex profiles with the general form:

$$y(\xi) = 4f\xi(1-\xi)\cdot\left(1+\ell_1\xi+\ell_2\xi^2\right)$$

For this case, the parameters were chosen to be: $f = 0.1$, $\ell_1 = -2.0$ and $\ell_2 = 0.5$, giving a moderately cambered, highly reflexed camberline. The predictions of CLOAD are compared with Riegels' analytical results[2] in Table II and on Figure 4.

This time, since the loading was zero at both leading and trailing edges, the accuracy achieved by CLOAD was improved. Agreement between predicted and analytical loading was generally within much less than one percent of the maximum value.

5 Concluding Remarks

This document has presented derivations of the relationships between camber and chordwise loading for the general case in which these quantities are expressed at a finite number of discrete points. For segmented load distributions, typified by the NACA a-series meanlines, the derivations are exact in terms of thin wing theory. For most other cases, they give very good approximations, provided the discretization is done over small enough intervals.

Worked examples using computer codes based on these derivations showed very satisfactory results for both the design and analysis problems. In the former case, the numerical implementation is straightforward, and only limited by computer precision. For analysis, the present implementation is perfectly adequate for, for example, designing meanlines for propeller blade sections, but a robust, high precision linear solver may be required for extremely accurate results.

TABLE I. ARBITRARY LOADING EXAMPLE

Input		CAMBER		CLOAD
x	γ	y	y'	γ
0.0000	0.080247	0.000000	0.737107	-0.041351
0.0050	0.092284	0.001168	0.224251	0.087432
0.0075	0.098303	0.001725	0.221690	0.078498
0.0125	0.110340	0.002827	0.219440	0.095316
0.0250	0.140432	0.005558	0.217869	0.123398
0.0500	0.200618	0.010973	0.214717	0.192775
0.0750	0.260803	0.016247	0.206002	0.254378
0.1000	0.320988	0.021184	0.184839	0.315498
0.1500	0.347737	0.029323	0.146886	0.343371
0.2000	0.374486	0.036035	0.122036	0.370805
0.2500	0.401235	0.041499	0.094693	0.398039
0.3000	0.401235	0.045526	0.068639	0.398409
0.3500	0.401235	0.048451	0.048879	0.398719
0.4000	0.401235	0.050448	0.031222	0.398968
0.4500	0.401235	0.051589	0.014502	0.399192
0.5000	0.401235	0.051901	-0.002023	0.399381
0.5500	0.401235	0.051378	-0.019033	0.399568
0.6000	0.401235	0.049976	-0.037387	0.399719
0.6500	0.401235	0.047592	-0.058661	0.399883
0.7000	0.401235	0.043975	-0.089828	0.400022
0.7500	0.351081	0.038627	-0.119926	0.350013
0.8000	0.300926	0.032151	-0.138104	0.300004
0.8500	0.250772	0.024889	-0.151944	0.249995
0.9000	0.200617	0.016942	-0.168446	0.200005
0.9500	0.100309	0.008218	-0.174343	0.099892
0.9750	0.050154	0.003946	-0.166203	0.049851
1.0000	0.000000	0.000000	-0.145228	0.000000

Input γ is interpolated from the data on page 6; input $c_l = 0.65$

CAMBER output:

$c_l = 0.650000$, $\alpha_i = 0.4897$ deg. and $\alpha_0 = -5.4348$ deg.

CLOAD output:

$c_{l_i} = 0.644172$, $\alpha_i = 0.4393$ deg. and $\alpha_0 = -5.4349$ deg.

TABLE II. TYPICAL BIRNBAUM-GLAUERT CAMBERLINE

Input		Analytical	CLOAD
x	y	γ	γ
0.0000	0.000000	0.000000	0.028792
0.0050	0.001970	0.167450	0.173618
0.0075	0.002933	0.203708	0.202769
0.0125	0.004814	0.259452	0.260255
0.0250	0.009266	0.354560	0.358425
0.0500	0.017124	0.467274	0.470707
0.0750	0.023666	0.531787	0.533524
0.1000	0.028980	0.568800	0.572114
0.1500	0.036274	0.591310	0.595202
0.2000	0.039680	0.569600	0.572151
0.2500	0.039844	0.519615	0.521716
0.3000	0.037380	0.450925	0.452642
0.3500	0.032874	0.370128	0.371667
0.4000	0.026880	0.282181	0.283540
0.4500	0.019924	0.191038	0.192347
0.5000	0.012500	0.100000	0.101233
0.5500	0.005074	0.011940	0.013207
0.6000	-0.001920	-0.070545	-0.069288
0.6500	-0.008076	-0.144999	-0.143645
0.7000	-0.013020	-0.208965	-0.207563
0.7500	-0.016406	-0.259808	-0.258252
0.8000	-0.017920	-0.294400	-0.292766
0.8500	-0.017276	-0.308510	-0.306632
0.9000	-0.014220	-0.295200	-0.294158
0.9500	-0.008526	-0.238868	-0.234843
0.9750	-0.004628	-0.179388	-0.184229
1.0000	0.000000	0.000000	0.000000

Analytical results:
$c_{l_i} = 0.235619$, $\alpha_i = 4.2972$ deg. and $\alpha_0 = 2.1486$ deg.

CLOAD output:
$c_{l_i} = 0.240397$, $\alpha_i = 4.2379$ deg. and $\alpha_0 = 2.0458$ deg.

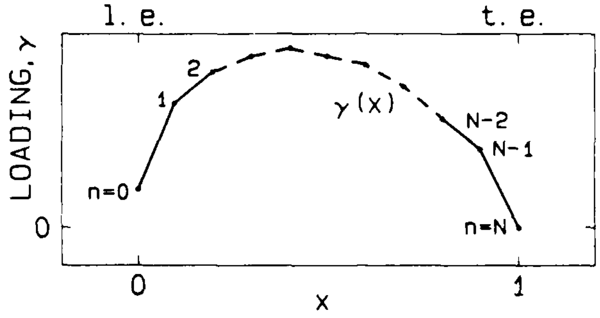

FIG.1 SEGMENTED LOAD DISTRIBUTION

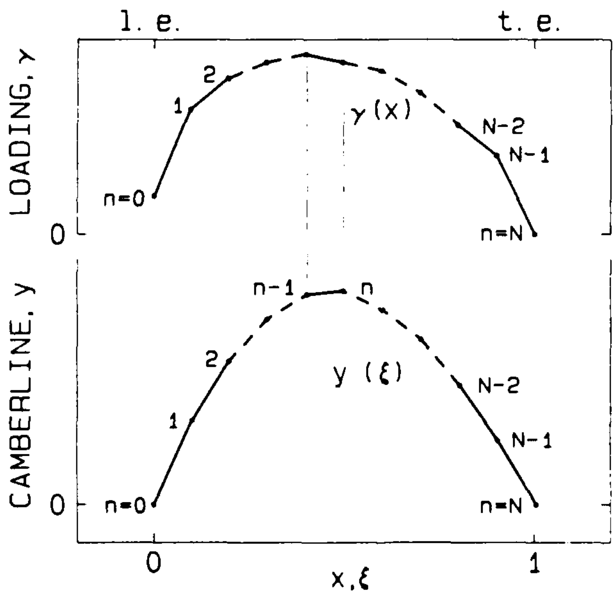

FIG.2 CORRESPONDING LOADING AND CAMBERLINE SEGMENTS

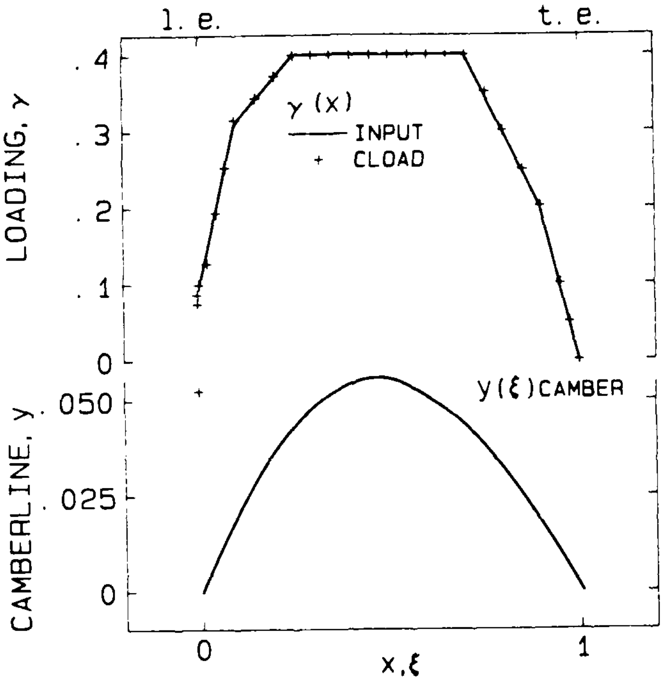

FIG.3 ARBITRARY LOADING EXAMPLE

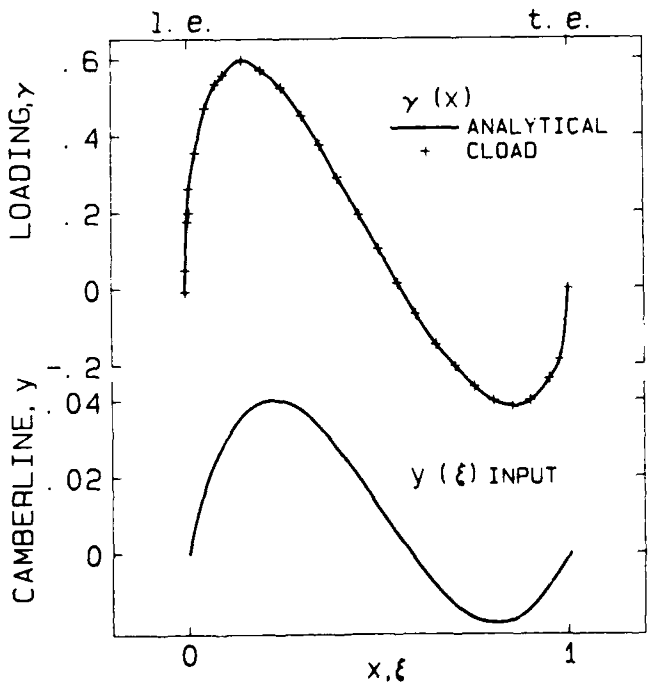

FIG.4 TYPICAL BIRNBAUM-GLAUERT CAMBERLINE

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Derivations are given for the calculation of camberline from a given chordwise load distribution, and for the inverse problem. In both bases, the algorithms are exact within the limits of thin-wing theory.

From these derivations, codes were developed which are included in the section design procedures at DREA. The results from example cases show very good agreement with analytical solutions, even for the extreme situation of infinite leading or trailing edge slopes.

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Airfoils
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