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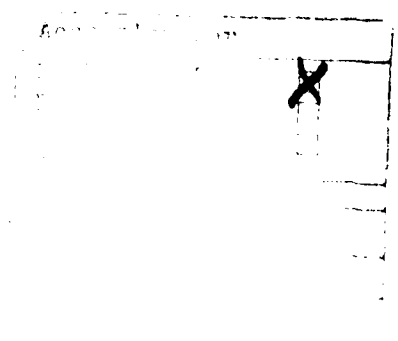
NUMERICAL FACILITY: CONVERGENCE OF COGNITIVE AND FACTOR ANALYTIC MODELS

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Summary

The importance of general as opposed to domain specific processes for the explanation of individual differences in intelligence is an unresolved and divisive issue in human abilities research. The present study was designed to clarify this issue by examining the patterns of convergent and discriminant validity in relating variables which represent basic numerical operations, an index of facility of performing these operations within working memory, and pencil-and-paper measures which require the processing of numbers but which define distinct ability factors. To achieve this end, 112 Air Force recruits responded to 400 arithmetic problems in a true-false reaction-time (RT) verification paradigm, solved a series of computer-administered tasks of the attentional allocation aspect of working memory capacity, and were administered ability measures spanning Numerical Facility, Perceptual Speed, General Reasoning, and Memory Span factors. The 400 cognitive arithmetic problems comprised 80 problems of each of five types; simple addition, multi-column complex addition, multi-digit complex addition, simple multiplication, and complex multiplication. All individual-differences measures of numerical facility, general reasoning, and memory span require processing of numbers; however, the memory span tests, unlike the tests of numerical facility and general reasoning, do not require arithmetic.

Information-processing results indicated processing strategies consistent with those employed by college students for the mental solution of each of the five arithmetic problem types, and verified the importance of the elementary operations of retrieving facts from a long-term memory network and carrying to the next column for complex problems as the substantive processes required for

the mental solution of arithmetic equations. The relation between process components, working memory capacity, and the ability measures was examined by means of structural equation modeling. A series of structural equation models indicated a convergence between process variables and theoretically similar ability measures. A direct relation between a factor subsuming efficiency of fact retrieval and the carry operation and the Numerical Facility and General Reasoning factors was found, as was a direct relation between a separate factor subsuming speed of encoding digits and decision and response times and the Perceptual Speed factor. A working memory capacity latent variable showed a direct relation to the General Reasoning and Memory Span factors. The discriminant validity of these results was demonstrated by no relation between variables representing elementary arithmetical operations and the memory span measures, and by no direct relation between the latent variable subsuming speed of encoding digits and decision and response times and the General Reasoning factor. Furthermore, it was demonstrated that the correlation within sets of ability factors was related to shared underlying operations, or similar working memory demands, but that different sets of ability factors was correlated due to different underlying processes, and not because of a pervasive general ability. Implications for future studies of the relation between variables representing elementary information processes and individual-differences measures were discussed.

PREFACE

This report presents a description of all work completed on Air Force grant, AFOSR-88-0239. The experiment described herein was conducted at the Learning Abilities Measurement Project laboratory of the Air Force Human Resources Laboratory and provided a study of the feasibility of a process-based assessment measure of basic quantitative skills. Performance measures derived from the process-model showed predicted patterns of convergent and discriminant validity in their relation to a battery of psychometric ability measures. A strong convergence between scores derived from the process-model and traditional tests of numerical abilities was demonstrated. Moreover, it was shown that the level-of-mastery of basic numerical skills, as measured by the process-model, was related to performance on rather more complex reasoning tasks which required basic number knowledge. As such, this study indicated the process model presented within this document should, with further refinement, provide a valid and useful technique for the assessment of basic quantitative skills.

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NUMERICAL FACILITY: CONVERGENCE OF COGNITIVE AND FACTOR ANALYTIC MODELS

Early studies of human intelligence employed correlational methods to examine the covariance among traditional ability tests. Such studies reliably produced what has been termed positive manifold; that is, ability measures were, nearly always, positively correlated with other ability measures (Cattell, 1963; French, 1951; Guilford, 1972; Horn, 1968; Horn & Cattell, 1966; Spearman, 1927; Thomson, 1951; Thurstone, 1938; Thurstone & Thurstone, 1941; Vernon, 1965). Spearman (1927) argued this finding indicated performance differences across all cognitive tasks were primarily due to individual differences in a global biologically mediated ability; that is, general intelligence, or *g*. Thurstone (1938) found, by means of factor analytic procedures, that tests tended to cluster in groups and argued for the existence of a finite number of relatively independent cognitive domains. In fact, both Spearman and Thurstone acknowledged the likely existence of global and domain specific processes (Carroll, 1988; Eysenck, 1988; Spearman & Jones, 1950; Thurstone & Thurstone, 1941). Nevertheless, the importance of global relative to specific abilities in explaining individual differences in intelligence remains unresolved and in fact continues to be a divisive issue in human abilities research (Eysenck, 1988; Horn, 1984; Keating & MacLean, 1987; Larson & Saccuzzo, 1989).

Thus, numerous studies of human intelligence continue to be conducted and vigorously debated (Braden, 1989; Carlson & C. M. Jensen, 1982; Carlson, C. M. Jensen, & Widaman, 1983; Eysenck, 1988; Geary & Burlingham-Dubree, 1989; Geary & Widaman, 1987; Gustafsson, 1984; Hunt, 1983; Hunt, Davidson, & Lansman, 1981; Hunt, Lunneborg, & Lewis, 1975; A. R. Jensen, 1982; A. R. Jensen,

Larson, & Paul, 1988; Keating & Bobbitt, 1978; Keating, List, & Merriman, 1985; Lansman, 1981; Lansman, Donaldson, Hunt, & Yantis, 1982; Palmer, MacLeod, Hunt, & Davidson, 1985; Pellegrino & Glaser, 1979; Sternberg, 1977; Sternberg & Gardner, 1983; Vernon, 1983; Vernon, Nador, & Kantor, 1985a). These more contemporary experiments, however, have sought to identify and isolate the basic information processes which define human abilities (e.g., Hunt et al., 1975; Jensen et al., 1988; Lansman et al., 1982; Sternberg, 1977; Sternberg & Gardner, 1983). Typically, such studies, pioneered by Hunt and his colleagues, involve examining the relationship between traditional individual-differences measures and scores on information-processing parameters derived from experimental tasks; although these studies have followed two rather different paradigms.

Studies which follow the first model attempt to isolate the processes, or processing characteristics, common to all cognitive tasks; that is, those operations defining g (e.g., Jensen, 1982; Larson & Saccuzzo, 1989). Experiments which follow the second model attempt to isolate the elementary operations defining specific ability domains, such as inductive reasoning or verbal skills (e.g., Hunt et al., 1975; Sternberg & Gardner, 1983). Theoretical and empirical research following each of these paradigms has produced rather different perspectives on how to best represent and explain individual differences in human intelligence. So, a brief overview of each of these paradigms will be presented. The overview will be followed by a description of the present study which was designed to determine if the covariance among related ability factors is due to a common feature of the

information-processing system, or to more restricted domain specific processes.

Information-processing studies of general intelligence. Experimental tasks included in studies of general intelligence require subjects to make noncomplex decisions or execute relatively simple cognitive operations (Carlson & C. M. Jensen, 1982; A. R. Jensen et al., 1988; Vernon, 1983). To illustrate, in a recent study subjects were administered a series of information-processing tasks and a battery of psychometric ability measures (Larson & Saccuzzo, 1989). One of the experimental tasks, the Inspection Time Test, required subjects to choose which of two briefly presented horizontal lines was longer. The duration of stimulus presentation varied and was the primary determinant of task difficulty. The score on this test was the number of correct responses. Three composite variables, based on the variability (i.e., the standard deviation) in accuracy or response times across experimental tasks, were constructed and correlated with an indicator of *g*. These analyses produced reliable correlations between the intelligence measure and parameters representing variability in the rate of information processing and working memory capacity. This experiment, as well as other studies which have employed a similar methodology, have found a modest inverse relationship between rate and stability of information processing and efficiency of information manipulation in short-term memory and traditional intelligence measures. This pattern of results, combined with other findings, led Larson and Saccuzzo to argue that "information process/intelligence correlations are not task specific, rather, they are primarily based on *g*" (p. 5).

Moreover, such findings are often interpreted as indicating a biological basis to individual differences in g , because the content of the information-processing tasks and the psychometric measures typically do not overlap (Eysenck, 1988; Jensen, 1982; Larson & Saccuzzo, 1989). Thus, the empirical relationship between the process variables and the intelligence tests, theoretically, could not be explained by individual differences in learning histories or familiarity with overlapping contents (but see Carlson & Widaman, 1987; Keating & MacLean, 1987). The basic implication of these studies is that the primary determinants of individual differences in human intelligence are the biological factors which are responsible for the rate and stability of information processing and efficiency of information manipulation in working memory, factors which span all cognitive domains (Eysenck, 1988).

Information-processing studies of specific abilities. Scientists who have sought to isolate the processes defining more specific ability domains have followed a somewhat different paradigm. Here, parameters derived from componential models of the same ability, or from experimental tasks in the same cognitive domain, are related to performance on theoretically similar ability tests (Geary & Burlingham-Dubree, 1989; Geary & Widaman, 1987; Hunt, 1978; Hunt et al., 1975; Kover et al., 1985; Lansman, 1981; Lansman et al., 1982; Pellegrino & Glaser, 1979; Sternberg, 1977; Sternberg & Gardner, 1983). In fact, many of these studies have followed a convergent and discriminant validation format (Campbell & Fiske, 1959); that is, these studies have sought to demonstrate that the rate or efficiency of executing elementary component processes are related to individual differences on theoretically similar ability measures (convergent validity) and not related to individual

differences on theoretically different ability measures (discriminant validity).

In one such study, Sternberg and Gardner (1983) examined the pattern of relations between variables derived from a componential model of inductive reasoning and individual differences measures of reasoning and perceptual speed. This analysis indicated that the rate of executing several substantive operations defining the process of inductive reasoning were significantly correlated with performance on pencil-and-paper measures of reasoning ability, but were not correlated with the measures of perceptual speed. In a similar study, Geary and Widaman (1987) examined the pattern of relations between rate of executing the processes underlying the mental solution of arithmetic problems and performance on traditional measures of numerical facility (i.e., addition, subtraction, multiplication, and division tests), perceptual speed, and spatial relations (i.e., tests requiring the rotation of images in two- and three-dimensional space). The rate of executing the elementary operations of retrieving arithmetic facts from a long-term memory network and of carrying to the next column for complex arithmetic problems was strongly related to performance on the numerical facility tests, but was unrelated to performance on measures of spatial ability. Not all studies, however, have been successful in demonstrating patterns of convergent and discriminant relationships between experimentally derived process variables and performance on the theoretically similar and dissimilar ability measures (e.g., Keating et al., 1985).

Nevertheless, it follows from the few studies which have demonstrated a pattern of convergent and discriminant validity in relating process variables

to ability measures that the theoretical importance of g , or a set of global processing characteristics, as the primary determinant of individual differences in human intelligence across all cognitive domains is in doubt. Rather, these studies suggest that human intelligence is best understood in terms of systems of interrelated processes (Allen, 1983; Luria, 1980). These functional systems might in fact define the separable "lower-order" ability domains identified by means of factor analytic studies of traditional measures (e.g., Carroll, 1976; Carroll, 1988; Coombs, 1941; French, 1951; Horn & Cattell, 1966; Thomson, 1951; Thurstone, 1938). An important implication of such componential studies is that the positive correlation between a set of ability measures is likely related to common underlying domain specific processes or to similar task demands (e.g., a common demand would be to maintain attention on relevant task features; Carlson & C. M. Jensen, 1982; Carlson & Widaman, 1987), and not necessarily due to a fundamental and pervasive biological parameter, such as "efficient neural circuits" (Larson & Saccuzzo, 1989, p. 23), or the "number of neural elements activated by a stimulus and . . . rate of oscillation of the excitatory-refractory phases of the activated elements" (Jensen, 1982, p. 123).

The Present Study

The present study was designed to provide further evidence for the specificity of the relationship between variables which represent basic processes and traditional ability measures of the same cognitive skill. Moreover, rather than assess the discriminant validity of these relationships with the use of obviously different ability measures, such as numerical facility/spatial relations or reasoning/perceptual speed, we sought to assess

the pattern of convergent and discriminant validity for relating the elementary operations underlying the processing of numerical information to a battery of ability tests which also require processing of numbers but which define separate ability factors. In this way, a more rigorous test of the specificity of the relationship between process variables and similar and subtly dissimilar ability measures was obtained. More importantly, this method allowed us to empirically assess whether the positive correlations among ability factors were all related to a single common process or whether the correlation between different sets of ability factors was due to different underlying processes. The former result would provide support for the importance of *g* in explaining individual ability differences, whereas the latter would argue for the specificity of process/ability relationships.

More precisely, we examined the pattern of relations comparing process variables derived from our componential model for mental arithmetic (Widaman, Geary, Cormier, & Little, 1989), a variable which indexed the facility of executing arithmetical operations in working memory (Christal, 1988), with performance on traditional ability measures which define the Numerical Facility, Perceptual Speed, General Reasoning, and Memory Span factors. Both of the general reasoning measures require knowledge of and/or execution of arithmetic operations and procedures. The Memory Span factor was indexed by the Auditory Number Span Test and the Visual Number Span Test (Ekstrom, French, & Harman, 1976). Thus, all measures defining the Numerical Facility, General Reasoning, and Memory Span factors require the processing of numerical information.

If the relationship between information-processing variables and ability tests is domain specific, then parameters which estimate the rate of executing the substantive processes required for the mental solution of arithmetic problems, i.e., the operations of fact retrieval and carrying, should be significantly related to measures which require arithmetic; that is, the tests of numerical facility and general reasoning. Moreover, these variables should not be directly related to measures which require the processing of numbers but do not specifically require arithmetic; that is, the memory span tests. Evidence for the specificity of the process/ability relationship would be further bolstered with the demonstration that variables which represent the rate of executing theoretically important arithmetic processes (e.g., fact retrieval), not variables which index more basic processes (e.g., decision and response times), contribute to individual differences on measures which require arithmetic.

Furthermore, successful performance on the tests of general reasoning and memory span demands working memory resources (Carroll, 1976). We therefore expected to find a direct relationship between the above noted index of the attentional allocation aspect of working memory capacity (Christal, 1980; Woltz, 1988) and performance on measures which define both the General Reasoning and Memory Span factors. Thus, performance on the general reasoning tests should, in theory, be related to both the rate of executing substantive arithmetical operations and to working memory capacity, whereas performance on the memory span tests should only be related to working memory capacity. Empirical support for this theoretically defensible pattern of results would provide strong evidence for both the convergent and discriminant validity of

the relationship between process variables and theoretically similar and dissimilar ability measures, and concurrently would argue against the importance of g as the primary determinant of individual differences in human intelligence. Finally, as noted above, derivation of the variables which represent each of the various arithmetical processes (e.g., encoding of numbers, retrieving facts, carrying) was based on a componential model. Statistical and conceptual details of the model have been presented elsewhere (Geary, Widaman, Little, 1986; Widaman et al., 1989). So, here we present only a brief description of this model.

Componential model for arithmetic. The experimental procedure involves presenting an arithmetic problem along with a stated answer on a video screen controlled by a microcomputer. The subject must then solve the problem and determine whether the stated answer is "true" (correct) or "false" (incorrect); solution times and accuracy are recorded for each problem. Our conceptual model represents each of the processing stages required for the solution of arithmetic problems of varying complexity and operation, and the accompanying statistical model enables the derivation of variables which estimate the duration of each of the requisite processes.

Problem solving begins with the determination of the number of digits to be processed in the units column. If two columnar digits are presented, then each of the numbers is encoded, and the process of retrieving the columnar answer from a long-term memory network of arithmetic facts is executed. Greater than two columnar digits requires first scanning the array and encoding the two largest value integers. Following the encoding process, the associated answer is retrieved from long-term memory. This provisional answer

is held in working memory, while the values of the remaining digits are incremented, one at a time, in a unit-by-unit fashion onto the provisional answer until a final columnar answer is obtained. The next stage of processing requires a decision as to the correctness of the columnar answer. If the obtained and stated answers are unequal, then problem solving is self-terminated and the response "false" executed. If the obtained and stated answers are identical and there are no further columns of digits to be processed, then the response "true" is executed. If there are further columns of digits to be processed, then the just described stages are recycled until all columns are processed or until a columnar error is encountered. The only modification of these recycling loops occurs if a columnar answer exceeds nine, in which case a carrying operation is performed.

This componential model has been shown to easily accommodate each of the processing stages required for the mental solution of simple and various forms of complex addition problems, as well as both simple and complex forms of multiplication problems (Geary et al., 1986; Widaman et al., 1989). As such, the model provides a theoretically justifiable framework for the establishment of convergent and discrimination patterns of relationship between elementary numerical operations and traditional ability measures which require the processing of numbers (Carroll, 1988; Keating et al., 1985; Keating & MacLean, 1987).

Method

Subjects

Subjects were U.S. Air Force recruits in their eleventh day of basic training at Lackland Air Force Base, Texas. In all, 112 subjects completed

the ability test battery and the experimental measures; however, ten subjects were eliminated due to high (greater than 20 percent) error rates on one or more of the experimental tasks. Of the remaining 102 subjects, 54 were male (mean age = 20.0 years, SD = 2.8) and 48 were female (mean age = 20.4 years, SD = 3.2). Subject racial and educational characteristics are presented in Table 1.

Insert Table 1 about here

Experimental Tasks

Arithmetic problem sets. A total of 400 arithmetic problems served as stimuli. The global set consisted of 80 of each of five types of problems: simple addition; multi-column complex addition; multi-digit complex addition; simple multiplication; and complex multiplication.

Simple addition. Each of the 80 simple addition problems consisted of two vertically placed single-digit integers with a stated sum. Forty of the problems were selected from the 56 possible nontie (a tie problem is, e.g., $2 + 2$, $4 + 4$) pair-wise combinations of the integers 2 through 9 as the augend and the same integers as the addend; the 40 problems were presented with the correct sum. The frequency and placement of all integers were counterbalanced. That is, each integer (2 through 9) appeared ten times across the 40 problems; five times as the augend and five times as the addend. The remaining simple addition problems were the same 40 pairs of integers, but these were presented with a stated sum incorrect by ± 1 or ± 2 . The magnitude of the error was counterbalanced across the 40 false stimuli. No repetition of either the augend, the addend, or of the stated sum was allowed across

consecutive trials, and no more than four consecutive presentations of true or false problems were allowed.

Multi-column complex addition. Each of the 80 multi-column complex addition problems consisted of two vertically placed double-digit integers with a stated sum. The 40 correct problems were constructed with the constraint that each problem consisted of four unique digits with the values 2 through 9. The frequency of each individual digit 2 through 9 was counterbalanced for position. The remaining 40 problems consisted of the same 40 pairs of integers, but these were presented with a stated sum incorrect by ± 1 , ± 2 , ± 10 , or ± 20 . The placement of the error was counterbalanced for position, as was the presence versus absence of the carry operation. No repetition of columnar digits or of the stated sum was allowed across consecutive trials, and no more than four consecutive presentations of true or false problems were allowed.

Multi-digit complex addition. Each of the 80 multi-digit complex addition problems consisted of three vertically placed single-digit integers with a stated sum. The 40 correct problems were constructed with the constraint that each problem consisted of three unique digits with the values 2 through 9. The frequency of each individual digit 2 through 9 was counterbalanced for position. The remaining 40 problems consisted the same 40 triplets, but these were presented with a stated sum incorrect by ± 1 , or ± 2 . The magnitude of the error was counterbalanced across the 40 false stimuli. No repetition of any integer occupying the same position or of the stated sum was allowed across consecutive trials, and no more than four consecutive presentations of true or false problems were allowed.

Simple multiplication. Each of the 80 simple multiplication problems consisted of two vertically placed single-digit integers with a stated product. Forty of the problems were selected from the 56 possible nontie pair-wise combinations of the integers 2 through 9 as the multiplicand and the same integers as the multiplier; the 40 problems were presented with the correct product. The frequency and placement of all integers was counterbalanced; that is; each integer appeared ten times across the 40 problems; five times as the multiplicand and five times as the multiplier. The remaining simple multiplication problems were the same 40 pairs of integers, but these were presented with a stated product incorrect by ± 1 , ± 2 , or ± 10 . Sixteen problems were incorrect by ± 10 ; and six problems were incorrect for each of the four remaining values of difference (e.g. $+1$, -2). No repetition of either the multiplicand, the multiplier, or of the stated product was allowed across consecutive trials, and no more than four consecutive presentations of true or false problems were allowed.

Complex multiplication. Each of the 80 complex multiplication problems consisted of a double-digit multiplicand placed vertically over a single-digit multiplier and presented with a stated product. The 40 correct problems were constructed with the constraint that each problem consisted of three unique digits with the values 2 through 9. The frequency of each individual digit 2 through 9 was counterbalanced for position. The remaining 40 problems consisted of the same 40 pairs of integers, but these were presented with a stated product incorrect by ± 1 , ± 2 , ± 10 , ± 20 , or ± 100 . The placement of the error was counterbalanced for position. No repetition of columnar digits or

of the stated product was allowed across consecutive trials, and no more than four consecutive presentations of true or false problems were allowed.

Working memory task. The 21 item ABC-assignment task, developed at the Air Force Human Resources Laboratory, was used as the measure of the attentional capacity aspect of working memory (Christal, 1988; Woltz, 1988). For each item, numerical values or simple equations are assigned to the letters A, B, and C. For each of the 21 items, requisite information is presented on three successive screens. To illustrate, consider the following item, which is of intermediate difficulty: "A = 69"; "B = 8 x 7"; "C = B/4." Here, each equivalence is presented on a successive screen, with the constraint that subjects are not allowed to re-examine previously presented information. Following the presentation of the third screen, three response probes, such as "C = ?", are presented in a randomly determined order. Subjects then answer each response probe by depressing appropriate number keys at the top of the keyboard and then pressing ENTER. Accuracy rather than speed is emphasized, and study time for each screen is self-paced with subjects hitting the space bar to move to the next screen. Accuracy feedback for the entire task is provided following the presentation of the last item. The score for the ABC-assignment task is the percentage of items answered correctly.

Apparatus. The arithmetic problems and the working memory items were presented at the center of an EGA color video monitor controlled by a Zenith Z-248 microcomputer. A software program ensured the collection of RTs with ± 1 -ms accuracy. Subjects were seated approximately 70 cm from the video screen and, for the arithmetic problems, responded "true" by depressing a

response key with their right index finger and "false" by depressing a response key using their left index finger.

For each arithmetic problem, a READY prompt appeared at the center of the video screen for a 1000-ms duration, followed by a 1000-ms period during which the screen was blank. Then, an arithmetic problem appeared on the screen and remained until the subject responded, at which time the problem was removed. If the subject responded correctly, the screen was blank for a 1000-ms duration, and then the READY prompt for the next problem appeared. If the subject responded incorrectly, a WRONG prompt with a 1000-ms duration followed the removal of the stimulus and preceded the 1000-ms interproblem blank period.

Procedure. Subjects were tested in groups of up to 31 subjects, with each subject in an individual partitioned carrel. Following a brief orientation to the experimental session, subjects were told by means of computer administered instructions that they were going to be presented with five individual sets of arithmetic problems and a memory task. Also by means of computer administered instructions, subjects were told that their task, for the arithmetic problems, was to respond "true" or "false" to each presented problem by pressing the appropriate key. Equal emphasis was placed on speed and accuracy. Subjects were told the type of problem (e.g., simple addition) to be presented before each set. Sixteen practice problems were presented at the beginning of the first set and eight practice problems preceded the administration of each of the four remaining problem sets. A short rest period followed each of the sets. Finally, the experimental tasks were independently administered in the following order: simple addition, multi-column complex addition, multi-digit

complex addition, simple multiplication, complex multiplication, and the ABC-assignment task. The entire testing session lasted approximately 90 min.

Ability Test Battery

Four sets of ability tests were used in the study: tests spanning the Numerical Facility, Perceptual Speed, General Reasoning, and Memory Span factors. Two or three measures of each of these mental abilities were administered, and, where appropriate, alternate forms of each individual measure were administered.

Numerical Facility. The three measures of Numerical Facility were taken from the Educational Testing Service (ETS) test battery (Ekstrom, French, & Harman, 1976). The three measures were the Addition Test (N-1), the Division Test (N-2), and the Subtraction and Multiplication Test (N-3). Both forms of all three measures were administered. The score for each form was the total number of items answered correctly. The total score for each measure was the sum of both forms.

Perceptual Speed. The three measures of Perceptual Speed were taken from the ETS test battery (Ekstrom et al., 1976). The three measures were the Finding As Test (P-1), the Number Comparison Test (P-2), and the Identical Pictures Test (P-3). Both forms of all three measures were administered. The score for each form of the Finding As Test was the total number of words marked correctly. The score for each form of the Number Comparison Test was the number of items correct minus the number of items incorrect. The score for each form of the Identical Pictures Test was the number of items correct minus a fraction of the number of items incorrect. The total score for each measure was the sum of both forms.

General Reasoning. The two measures of General Reasoning were taken from the ETS test battery (Ekstrom et al., 1976). The two measures were the Arithmetic Aptitude Test (RG-1), and the Necessary Arithmetic Operations Test (RG-3). The score for each form of both measures was the number of items correct minus a fraction of the number of items incorrect. The total score for each measure was the sum of both forms.

Memory Span. The two measures of Memory Span were taken from the ETS test battery (Ekstrom et al., 1976). The two measures were the Auditory Number Span Test (MS-1), and the Visual Number Span Test (MS-2). Due to time constraints the number of items on the Visual Number Span Test was reduced from 24 items to 20 items. The score for each measure was the number of items recalled correctly.

Procedure. The ten ability tests were administered in a classroom to subject groups, with group size being not more than 31 subjects. Each group completed the ability tests within a single testing session that lasted approximately 90 min. The ten tests were timed according to instruction in the manual (Ekstrom et al., 1976) and were administered in the following order: N-1, N-2, N-3, P-1, P-2, P-3, RG-1, RG-3, MS-1, and MS-2. All subjects completed the test battery before the reaction-time measures were administered.

Analytic Procedures

Analyses of the ability test battery and the combined data, information-processing (IP) and test battery, were based on covariance structures and followed the LISREL VI program (Jöreskog & Sörbom, 1984). These analyses employed covariance matrixes (Cudeck, 1989) and the ρ value was

adopted as the practical goodness-of-fit index for the various structural equation models (Bentler & Bonett, 1980; Marsh, Balla, & McDonald, 1988; Tucker & Lewis, 1973). A structural equation model which produced a ρ value of at least .90 was considered acceptable (Bentler & Bonett, 1980).

Results and Discussion

For clarity of presentation, the results from the current study will be presented in three major sections, followed by a general discussion of the results and their implications. In the first major section, results for the IP tasks, which included the arithmetic problem sets and the ABC-assignment task, will be presented. In the second major section, results for the ability test battery will be presented. The final section will present analyses of the relationship between performance on the IP tasks and the ability measures.

Information-Processing Tasks

Analyses of the arithmetic problem sets were based on the previously described componential model for mental arithmetic (Geary et al., 1986; Widaman, et al., 1989). Here, hierarchical regression equations, embodying variables representing the processes identified in the componential model, were fit to average RT data. The product (Prod) of columnar digits, or of a combination of two digits for multi-digit complex addition problems, was used to represent the memory retrieval process (Geary et al., 1986; Miller, Perlmutter, & Keating, 1984; Stazyk, Ashcraft, & Hamann, 1982; Widaman et al., 1989). Conceptually, the product variable represents a memory network with two orthogonal axes representing nodes for the integers to be added or multiplied. The distance between nodal values is assumed to be constant. Activation of the network begins at the origin and proceeds at a constant rate

and as a linear function of the area of the network that must be traversed. The area of network activation is defined by the rectangle formed by the origin, the values of the nodes representing the two integers, and the point of intersection of orthogonal projections from these two nodal values. Thus, the product of the integers represents the area of the network activated and is therefore linearly related to search time required to arrive at the correct answer (Widaman et al., 1989).

Additional processes were represented by variables estimating (a) intercept differences between correct and incorrect problems for verification tasks (Truth: coded 0 for correct and 1 for incorrect problems), (b) rate of encoding digits (Encode: coded the total number of digits in the problem including the stated sum, except when the stated units-column answer was incorrect, for complex problems, in which case Encode was coded 3), and (c) rate of carrying to the next column for complex problems (Carry: coded 0 for the absence and 1 for the presence of a carry). Moreover, variables were coded so as to represent self-terminating, as opposed to exhaustive, processing of complex problems (Geary et al., 1986). Self-termination of a complex problem occurs when a units column error is encountered. At this point, the processing of the problem stops, and the response "false" is executed; therefore, variables representing any process following a units column error (e.g., carry) were coded 0.

Finally, the solution of both multi-digit complex addition problems and complex multiplication problems requires the execution of one additional process; incrementing a number onto a provisional sum or product. Widaman et al. (1989) determined multi-digit complex addition problems were processed two

integers at a time. First, the sum of the two largest value integers is retrieved from long-term memory and then the smallest value digit is incremented in a unit-by-unit fashion onto the provisional sum. Thus, the modeling of solution times to multi-digit complex addition problems included a variable representing this incrementing process (Min: coded the value of the smallest integer). For complex multiplication problems, an additional parameter representing the incrementation of the value of the carry onto the provisional tens column product was included in associated the regression equation (Carrem: coded the value of the remainder following the units column multiplication). For an illustration of this coding scheme see Geary and Widaman (1987, Table 1).

Addition

Simple addition. A total of 80 simple addition problems were included in the study, resulting in a total of 8160 RTs across the 102 subjects. Overall error rate was 3.32 percent and is consistent with studies that have required samples of undergraduate students to solve comparable problem sets (Ashcraft & Stazyk, 1981; Geary et al., 1986; Widaman et al., 1989). An additional 1.27 percent of responses were eliminated as outliers. All subsequently described analyses excluded error and outlier RTs.

Statistical summaries of regression analyses for addition problems are presented in Table 2; the first two equations represent process models for simple addition. The first of these equations provides estimates for rate of encoding single digits (Encode), memory retrieval rate (Prod), and intercept differences comparing correct with incorrect problems (Truth); however, the partial F ratio for the Encode parameter was not significant, $p > .10$. So, the

Encode variable was dropped and the equation was recomputed, providing the second equation presented in Table 2. Here, rate of encoding digits would theoretically be estimated within the intercept term, along with decision and response times (Ashcraft & Battaglia, 1978; Widaman et al., 1989). This second equation provided a highly significant overall level of fit, $R^2 = .705$, $p < .0001$. This overall level of fit, as well as the significance of the Prod variable, is comparable to previous studies of simple addition (Geary et al., 1986; Widaman et al., 1989). This equation also provided a better representation of simple addition RTs than did a model representing an implicit counting strategy (here, the Min parameter was fit to RTs in place of Prod; see Groen & Parkman, 1972), $R^2 = .638$. For the second equation, the Prod by Truth interaction was not significant. $F(1,76) = 0.15$, $p > .50$.

Insert Table 2 about here

Finally, inspection of Table 2 indicates a mean solution time of 1600-ms. This solution time ranges from about 350-ms to 500-ms longer than for samples of undergraduate students solving comparable problem sets (Ashcraft & Battaglia, 1978; Ashcraft & Stazyk, 1981; Geary et al., 1986; Widaman et al., 1989). This finding combined with the above described results suggests that the Air Force recruits were using the same processes as do college students to solve simple addition problems, but with longer overall solution times.

Multi-column complex addition. Across the 102 subjects, 8160 RTs to multi-column complex addition problems were obtained. Overall error rate was 4.22 percent and is comparable to previous studies (Geary et al., 1986;

Widaman et al., 1989). An additional 1.18 percent of responses were eliminated as outliers.

The third equation presented in Table 2 enabled the representation of each of the processing components (e.g., Encode, Carry) proposed in our componential model (Widaman et al., 1989) for the solution of multi-column complex addition problems. Inspection of Table 2 reveals that the level of fit of the overall equation was adequate, $R^2 = .853$, and comparable, in terms of R^2 and significance of individual parameters, to previous studies (Geary et al., 1986; Widaman et al., 1989). Moreover, variables which represented each individual component process showed highly significant partial F ratios, $ps < .01$. For this equation, the memory retrieval parameters (i.e., unitprod and tenprod) were initially estimated separately for each column. Inspection of these results revealed highly similar columnwise slope estimates. Accordingly, the columnwise slope estimates for the units and tens columns were constrained to be equal. Constraining columnwise slope estimates to be equal resulted in a nonsignificant decrease in the full-model R^2 , $F(1,74) = 0.50$, $p > .50$. Identical slope estimates are therefore presented in Table 2 for the units and tens columns.

The interactions between the Truth variable and the Encode, columnar product, and Carry parameters were not significant, $ps > .05$. Finally, the mean solution time of 3610-ms was significantly longer than for samples of college students solving comparable problem sets. Here, mean solution times for multi-column complex addition problems have ranged between 2254-ms (Widaman et al., 1989) and 2272-ms (Geary et al., 1986). Thus, the Air Force recruits appear to have employed the same processes as do college students to solve

multi-column complex addition problems, but with longer overall solution times.

Multi-digit complex addition. Across the 102 subjects, 8160 RTs to multi-digit complex addition problems were obtained. Overall error rate was 3.57 percent which was not significantly higher than the 3.63 percent error rate for a comparable problem set administered to a sample of college students (Widaman et al., 1989). An additional 1.81 percent of responses were eliminated as outliers. The mean solution time of 3478-ms was longer than the mean solution time of 2027-ms for a comparable problem set administered to the just noted college sample (Widaman et al., 1989).

The final section of Table 2 presents the two equations which were used to model the component processes invoked for the solution of multi-digit complex addition problems. The first of these equations was specified based on earlier findings (Widaman et al., 1989), but excluded the Encode parameter due to a nonsignificant partial F ratio. Here, the sum of the two largest value digits (Largprod) is retrieval from long-term memory and held in working memory while the smallest value digit (Min) is incremented in a unit-by-unit fashion onto this provisional sum. The equation representing these processes provided an adequate overall level of fit, $R^2 = .723$.

However, because the solution of multi-digit complex addition problems is limited by the processing of two digits at any given step (Widaman et al., 1989), the possibility exists for a variety of initial digit-combination strategies. For example, subject's might first process the two largest value digits, as was found by Widaman et al. (1989), or first chunk digits that summed to ten. To assess the goodness-of-fit of this alternative chunking

strategy, a second equation was used to model solution times to multi-digit complex addition problems. Here, we assumed the modal strategy first involved chunking any two digits that summed to ten. If no such combination was presented in the problem, then the first combination involved processing the two largest value digits.

To accommodate this strategy, a Scan variable was incorporated into the regression equation. This parameter represented the number of digits that had to be scanned before a chunk (two digits with a sum of ten) was found. If a chunk was presented in the problem and involved the first two digits, then Scan was coded 2; otherwise Scan was coded 3. If no chunk was presented in the problem and the two largest value digits were in the first and second positions, then Scan was coded 5; otherwise Scan was coded 6. For the initial modeling of this strategy two retrieval parameters were required; one variable for chunk problems (coded the product of the two chunked digits and coded 0 for problems without a chunk), and a second variable for problems without a chunk (coded the product of the two largest value digits and coded 0 for chunk problems). Accompanying the retrieval parameters were two variables representing the value of the remaining digit; termed Min2 for problems without a chunk and Remainder for problems with a chunk. Of course, for problems with a chunk Min2 was coded 0 and for problems without a chunk Remainder was coded 0.

The initial equation included the Scan parameter, the two retrieval variables, Min2, Remainder, and the Truth variables. The resulting regression equation provided an improved level of fit, $R^2 = .898$, relative to our first model for this problem type. Inspection of this equation revealed similar

slope estimates for the two retrieval parameters. So, the slope estimates for the two retrieval variables were forced to be equal. Enforcing this equality constraint resulted in a small, $\Delta R^2 = .0004$, and nonsignificant decrease in the overall level of model fit, $F(1,73) = 0.29$, $p > .50$. Forcing the slope estimates for the Min2 and Remainder variables to equality, however, resulted in a significant decrease in the level of model fit, $F(1,74) = 40.07$, $p < .001$. So, the final model, presented as the fifth equation in Table 2, included the Scan parameter, a single retrieval variable (Prod2), and the Min2, Remainder and Truth parameters. For this equation, the Truth variable did not significantly interact with any of the four remaining parameters ($p > .50$).

Briefly, then, the modal strategy for the solution of multi-digit complex addition problems involved first scanning the presented integers until two of these digits could be chunked (i.e., retrieving "ten" from long-term memory) or, if no chunk was possible, retrieving the sum of the two largest value digits. The rate with which provisional answers for chunk and non-chunk problems were retrieved from long-term memory did not differ significantly. For non-chunk problems, the Min2 variable should theoretically represent the rate by which subjects increment the smallest value digit onto the provisional sum in a unit-by-unit fashion by means of an implicit speech process (Widaman et al., 1989). Assuming a relatively slow implicit speech rate for Air Force recruits, as compared to college students, the estimated regression weight, $b = 337$ -ms (compared to 239-ms for college students) is in accord with this interpretation. For chunk problems, however, the estimated regression weight for the Remainder variable, $b = 123$ -ms, appears to be too low to represent an

implicit counting strategy. The psychological processes represented by this variable are, at this point, unclear.

Finally, to determine if all subjects were using the chunking strategy, three independent regression equations were fit to individual RT data for each of the 102 subjects in the study. The first two equations were identical to those presented, respectively, as the fourth equation and the fifth equation in Table 2. The third model represented a strategy whereby the subject first retrieved the sum of the digits presented in the first and second positions and then incremented in a unit-by-unit fashion the value of the digit in the third position onto the provisional sum. Based on the goodness-of-fit of competing models and the values of the associated regression weights, we determined that the modal strategy for ten subjects was best represented by this third model; 61 subjects followed the chunking strategy; and, the remaining 31 subjects followed the strategy represented by the fourth equation in Table 2. Thus, the final equation presented in Table 2 provides the best representation of the modal strategy employed by the Air Force recruits for the solution of multi-digit complex addition problems; although several alternative strategy approaches to this problem type were evident.

Multiplication

Simple multiplication. Across the 102 subjects, 8160 RTs were obtained. Overall error rate was 3.10 percent and is comparable to previous studies which have required samples of college students to solve comparable problem sets (Geary et al., 1986; Parkman, 1972; Stazyk et al., 1982). An additional 1.91 percent of responses were deleted as outliers. The mean solution time of 1738-ms was longer than the mean RT of 1232-ms (Geary et al., 1986) and

1129-ms (Stazyk et al., 1982 for correct nontie problems) reported for samples of college students.

Statistical summaries of regression analyses for multiplication problems are presented in Table 3. The first equation presented in Table 3 fits a model to simple multiplication RTs identical to the model fit to simple addition RTs and identical to the model which best fitted solution times for a comparable problem set administered to a sample of undergraduate students (Geary et al., 1986). The equation showed an adequate level of fit, $R^2 = .651$, which was better than the level of fit for a model representing a set-wise counting strategy (represented by the Min variable; e.g. $3 \times 4 = "4"$, $"8"$, $"12"$), $R^2 = .608$. For the first equation presented in Table 3, the Prod by Truth interaction was not significant, $p > .10$.

Insert Table 3 about here

Complex multiplication. Across the 102 subjects, 8160 RTs were obtained. Overall error rate was 7.29 percent and is comparable to previous research (Geary et al., 1986). An additional 1.84 percent of responses were deleted as outliers. The mean solution time of 4542-ms is higher than the mean solution time of 2840-ms found for a comparable problem set administered to a sample of undergraduate students (Geary et al., 1986).

The second equation presented in Table 3 fitted a model identical, except for the inclusion of the Carrem variable, to the model fit to multi-column complex addition problems. The equation is also identical to the equation found to best represent solution times to a comparable set of complex multiplication problems administered to the just noted sample of undergraduate

students (Geary et al., 1986). Inspection of Table 2 reveals that the level of fit of the overall equation was adequate, $R^2 = .873$, and highly comparable to our previous finding ($R^2 = .878$; Geary et al., 1986). For this equation, the memory retrieval parameters (i.e., unitprod and tenprod) were initially estimated separately; however, constraining columnwise slope estimates to be equal resulted in a nonsignificant decrease in the full model R^2 , $F(1,73) = 0.71$, $p > .50$. Identical slope estimates are therefore presented in Table 3 for the units and tens columns. Finally, the interactions between the Truth variable and the Encode, columnar product, Carry and Carrem parameters were not significant, $ps > .10$.

Summary of IP Models for Addition and Multiplication

An important finding of the above described modeling of solution times for each of the five sets of arithmetic problems was that the processing strategies invoked by the Air Force recruits for solving all five problem types were easily accommodated by the Widaman et al. (1989) model. Moreover, except for individual differences in initial digit-combination strategies for multi-digit complex addition problems, process models representing RTs for the current sample did not differ substantively from the best fitting regression equations used to model solution times to comparable problem sets administered to several samples of undergraduate students (Geary et al., 1986; Widaman et al., 1989). The only substantive difference, comparing the current sample with previous college samples, was in terms of mean solution times for each of the five problem sets. Here, the Air Force recruits required significantly longer to solve comparable arithmetic problems. The finding of highly similar error

rates indicates that these longer solution times were not due to a speed/accuracy trade-off.

Briefly, then, as was found for samples of college students, for this primarily non-college sample, addition and multiplication problems were processed in a column-wise fashion. Columnar answers were retrieved from a long-term memory network of arithmetic facts, and complex problems were self-terminated when an error in the units column of the stated sum or product was encountered. Additional component processes required for the mental solution of addition and multiplication problems included encoding single integers, carrying to the next column for complex problems, and incrementing in a unit-by-unit fashion a digit onto a provisional sum or product. This latter process is invoked when greater than two single integers need to be processed to obtain a columnar answer, as was described for multi-digit complex addition problems. For a more detailed discussion of the psychological processes modeled by the regression equations presented in Table 2 and in Table 3 see Geary et al. (1986) and Widaman et al. (1989).

Working Memory Task

The mean percent correct for the ABC-assignment task was 47.40 (SD = 21.47). The reliability of the task, derived by means of the Spearman-Brown prophecy formula (based on the correlation between odd and even items), was .883. Both the mean percent correct and the reliability estimate were very similar to the respective values of 49.39 (SD = 25.72) and .90 obtained with an independent sample of Air Force recruits for a similar working memory task (Woltz, 1988).

Structural Model for the Ability Test Battery

Table 4 presents descriptive statistics and reliability estimates for the four sets of ability measures. Total score (form 1 + form 2) reliability estimates, obtained with the Spearman-Brown prophecy formula, ranged in value from .580 to .937, with a median value of .812.

The reliability estimates, for the numerical facility and perceptual speed measures, were highly comparable to those reported by Geary and Widaman (1987) for a sample of undergraduate students. The present sample, however, did differ in mean performance relative to the sample assessed in our earlier study (Geary & Widaman, 1987). Relative to the college sample, the Air Force recruits correctly solved between 15.4 percent (Addition) and 36.7 percent (Division) fewer arithmetic problems and between 3.6 percent (Finding As) and 8.4 percent (Identical Pictures) fewer items on the measures of perceptual speed.

Insert Table 4 about here

Covariances among the ability tests were computed and the dimensional structure of the tests was assessed by fitting a confirmatory factor analytic model to the data (Jöreskog, 1969; Jöreskog & Sörbom, 1984). First, a null model fitting only unique variances was estimated, $\chi^2_{(45)} = 400.72$, $p < .001$. Next, a four-common-factor model was formulated. The four hypothesized factors were Numerical Facility, Perceptual Speed, General Reasoning, and Memory Span; the indicators for these factors were as noted in Table 4. The loading of each ability test on its respective common factor was estimated, as were interfactor correlations, in the first nested model, $\chi_{(31)} = 46.52$, $p =$

.036. This model produced an adequate value for the practical goodness-of-fit index, $\rho = .937$, and was therefore accepted as providing an adequate representation of the covariance among the ability tests.

The maximum likelihood estimates for the just noted model were in a covariance metric. To make the estimates more readily interpretable, these values were converted to a standardized metric by means of the following equation:

$$S_i = (\Lambda_i^2 / \Lambda_i^2 + \theta_i)^{1/2}, \quad (1)$$

where S_i = the standardized factor loading for variable i , Λ_i = the factor loading in the covariance metric, and θ_i = the unique variance in the covariance metric. The standardized unique variances were calculated by means of the following equation:

$$U_i = 1 - S_i^2, \quad (2)$$

where U_i = the standardized unique factor variance. The resulting standardized common- and unique-factor loadings, as well as factor intercorrelations, are presented in Table 5. Inspection of the bottom portion of Table 5 reveals a matrix of positive correlations among all ability factors; although the correlation between the Memory Span and Numerical Facility factors, and between the General Reasoning and Perceptual Speed factors, was not significant, $p < .05$. Nevertheless, this matrix of positive correlations might be interpreted as indicating the existence of g , or a set of processing characteristics influencing performance on all of these individual-differences measures.

Insert Table 5 about here

Structural Models for the Combined Data

Component scores (raw regression weights) for the intercept term, memory retrieval variable (i.e., product), and the carry parameter for all 102 subjects across the five arithmetic problem types were obtained (component scores were taken from the appropriate equation for multi-digit complex addition problems). Variables for which the psychological processes modeled by the parameter are unknown (e.g., Truth) or had no counterpart from other types of problems (e.g., Scan) were not included in these analyses. Because the Encode parameter was not significant for several of the problem types, the regression equations for complex addition and complex multiplication problems were re-estimated with no independent variable for encoding speed. As a result, speed of encoding digits was incorporated into the intercept value for all equations.

In all, component scores for 12 variables across the five arithmetic problem types were used from the IP analyses. In the resulting matrix of 1224 (12 X 102) component scores, 39 values were negative and therefore not interpretable. These 39 scores were replaced by the appropriate variable mean. Finally, due to a large variance for several of the IP variables (e.g., carry for complex addition and complex multiplication) a square-root transformation of all variables was performed. The zero-order correlation between the transformed scores and raw scores ranged in value from .97 to 1.00 ($M = .98$, $SD = .009$), with a modal value of .99. Thus, the transformation did not alter the pattern of individual differences.

Covariances among the component scores for the 12 IP variables, the ABC-assignment task, and the ten ability tests were computed. The resulting

covariance matrix was analyzed by means of the LISREL VI program (Jöreskog & Sörbom, 1984). First, a null model estimating only unique variances was estimated, $\chi^2_{(253)} = 1105.9$, $p < .001$. Next, the initial measurement model, termed Model 1, was estimated. Model 1 included the four common factors for the measures in the test battery, factors described earlier, and three trait factors for the IP variables. The IP factors consisted of (a) an Arithmetic Processes latent variable for which the memory retrieval (i.e., the product) variable from each of the five problem types and the carry parameter from the two complex multi-column problem types served as indicators, (b) a combined Intercept: Encode-Decode-Respond latent variable with loadings estimated for each of the five intercept terms, and (c) a Working Memory Capacity latent variable defined by the ABC-assignment task variable.

Furthermore, Model 1 included the estimation of 17 covariances among uniqueness terms. Each of these involved either the estimation of (a) the covariance between variables derived from the same regression equation (e.g., the covariance between the unique variances for the intercept term and the product variable were estimated for all five problem types) or (b) the covariance between IP variables defining the same factor (e.g., the covariance between the unique variances for the product variable for simple addition and the product variable for multi-column complex addition was estimated). The net result of allowing for the estimation of these 17 covariances was a better definition of the IP latent variables and the removal of method variance from substantive aspects of the structural model. Finally, the covariances among the four ability test factors and among the three IP latent variables were

estimated. All other latent variable covariances were fixed at zero, and all nondefining factor loadings were fixed at zero.

Table 6 presents overall goodness-of-fit indexes for all of the structural equation models and Table 7 presents indexes of differences in fit between nested structural equation models. Inspection of Table 7 reveals that estimation of Model 1 resulted in a highly significant improvement in model fit, $\chi^2_{(46)} = 692.3$, $p < .001$. The overall level of fit, however, was not acceptable, $\rho = .703$, as noted in Table 6. A graphical representation of the latent variables represented in Model 1 is presented in Figure 1. Here, only significant ($p < .05$) or marginally significant ($p < .10$) correlations among the latent variables are presented.

Insert Table 6 and Table 7 about here

Insert Figure 1 about here

The next modification of the structural equation model involved the estimation of five directed paths from the IP latent variables to the ability test factors. Each of the five directed paths was theoretically justified (Keating et al., 1985; Keating & MacLean, 1987). First, based on our earlier findings (Geary & Widaman, 1987) a directed path from the Arithmetic Processes IP factor to the Numerical Facility ability factor was estimated, as was a second directed path from the Intercept: Encode-Decide-Respond latent variable to the Perceptual Speed factor (Geary & Widaman, 1987; Hunt et al., 1975; Lansman et al., 1982). Directed paths from the Working Memory Capacity factor to both the General Reasoning factor and the Memory Span factor were

estimated, based on Carroll's (1976) task analysis of an earlier version of these measures. The fifth directed path was from the Arithmetic Processes latent variable to the General Reasoning ability factor. Here, as previously mentioned, we reasoned since both of the general reasoning measures required knowledge of and/or the execution of arithmetic operations and procedures, the rate of executing arithmetical processes (e.g., carrying to the next column) should be significantly and inversely related to performance on these measures of general reasoning.

Inspection of Table 7 reveals estimation of Model 2 provided a significant improvement in model fit, $\chi^2_{(5)} = 101.4$, $p < .001$, as well as an improvement in the level of practical fit, $\Delta\rho = .134$. Moreover, each of the resulting path coefficients differed significantly from zero ($ps < .05$) and each was in the predicted direction. However, inspection of Table 6 reveals an unacceptable ρ value (.837) for the overall model.

Substantive considerations as well as modification indexes were used to improve the level of fit for the overall model, and involved the estimation of 14 post hoc covariances between uniqueness terms. The estimation of these 14 covariances, which produced Model 3, provided a significant improvement in the statistical fit of the model, $\chi^2_{(14)} = 86.5$, $p < .001$, as well as an acceptable overall index of practical fit, $\rho = .941$, as noted in Table 6. The values of the standardized path coefficients for the five directed paths did not change significantly ($M\Delta = .015$) with the addition of these 14 covariances and all remained significantly different from zero ($ps < .05$).

To further insure that the addition of these 14 covariances did not influence our substantive results, we fixed each of the five previously

described directed paths at zero, which produced Model 4. Inspection of Table 7 reveals that fixing these paths at zero resulted in a significant worsening of model fit, $\chi^2_{(5)} = 114.9$, $p < .001$, and an unacceptable ρ value (.774), as noted in Table 6. This result indicates that estimation of the relationship between the IP factors and the ability test factors represented by the five directed paths was required by the data.

Finally, to assess the discriminant validity of the five directed paths, two additional paths were estimated, which yielded Model 5. Here, a directed path from the Arithmetic Processes IP factor to the Memory Span factor was estimated, as was a directed path from the combined Intercept:

Encode-Decide-Respond factor to the General Reasoning ability factor.

Inspection of Table 7 reveals that the estimation of these two paths produced a nonsignificant change in the overall level of statistical fit, $\chi^2_{(2)} = 3.3$, $p > .15$, and no change in the level of practical fit ($\Delta\rho = .000$). Moreover, neither of the two path coefficients approached statistical significance ($ps > .10$).

Based on this result and on the overall level of practical fit ($\rho = .941$), we therefore accepted Model 3 as providing an adequate representation of these data. Moreover, examination of the modification indexes for the directed path matrix for Model 3 indicated that any respecifications of the model would not have led to substantial improvements in model fit. Trait- and unique-factor loadings were standardized by means of Eq 1 and Eq 2 and are presented in Table 8.

Insert Table 8 about here

In Figure 2, the final structural relations among the seven trait factors from Model 3 are presented. The important estimates of structural relations, embodied in the coefficients for the directed paths from the IP factors to the ability test common factors, all differed significantly from zero and were in the predicted direction. The estimation of the directed path from the Arithmetic Processes IP factor to the Numerical Facility common factor was based on our previous findings for a sample of undergraduate students (Geary & Widaman, 1987).

Insert Figure 2 about here

In this previous study, a path coefficient, estimating the relationship between the elementary operations subsumed by the Arithmetic Processes factor and performance on the measures of numerical facility, of $-.879$ was found. For the current sample, estimation of the identical path produced a highly comparable estimate of $-.885$. This result provides a strong replication, with a rather different sample, of this earlier finding of a convergence in the processes identified in our componential model for arithmetic (Geary et al., 1986; Widaman et al., 1989) and performance on pencil-and-paper measures that traditionally span the Numerical Facility factor (Coombs, 1941; Spearman, 1927; Thurstone, 1938; Thurstone & Thurstone, 1941). The component processes which underlie numerical facility appear to be the elementary operations of information retrieval from a stored network of arithmetic facts (Ashcraft & Battaglia, 1978) and carrying to the next column for multi-column complex addition and multiplication problems (Ashcraft & Stazyk, 1981) combined with an adequate understanding of arithmetical procedures (Baroody, 1983; Coombs,

1941). Finally, for adults, individual differences in basic numerical abilities appear to be related to the rate with which these two elementary operations are executed.

The directed path from the Intercept: Encode-Decide-Respond IP factor to the Perceptual Speed ability factor was also estimated in an attempt to replicate previous results (Geary & Widaman, 1987; Hunt et al., 1975; Lansman et al., 1982). Here, the value of the resulting path coefficient, $-.538$, was lower than was found for a sample of undergraduate students ($-.701$; Geary & Widaman, 1987), but the same basic relationship was replicated. The rate of encoding single digits, along with decision and response times associated with the verification task paradigm, was inversely related to performance on the measures of perceptual speed. This result suggests the perceptual speed tests index the rate of encoding overlearned information and the rate of making noncomplex decisions. Moreover, this finding is consistent with the traditional interpretation of the Perceptual Speed factor (e.g., Thurstone & Thurstone, 1941).

Each of the three remaining path coefficients presented in Figure 2 represent an extension of our earlier study (Geary & Widaman, 1987) and more importantly enabled an empirical test of the pattern of convergent and discriminant validity in relating variables which represent basic numerical operations to ability tests which require number processing. Here, it was hypothesized that performance on the working memory capacity task should theoretically show a positive relationship to performance on measures of both general reasoning and memory span (e.g., Carroll, 1976; Horn & McArdle, 1980). Indeed, both of these hypothesized relationships were supported empirically by

the current study. Here, the better the attentional allocation aspect of working memory capacity (Woltz, 1988) the better the performance on measures which defined both the General Reasoning factor and the Memory Span factor.

The two measures of general reasoning included in this study required the execution of basic arithmetic operations and the knowledge of arithmetical procedures. So, it was hypothesized that the rate of executing the component processes of fact retrieval and the carry operation should contribute, in addition to working memory capacity, to general reasoning ability. This hypothesis was empirically supported by a significant directed path from the Arithmetic Processes IP factor to the General Reasoning common factor. The set of directed paths to this factor indicates that general reasoning abilities are related to both the ability to allocate attentional resources within working memory and to the rate of executing basic content relevant operations (e.g., retrieval of facts from long-term memory) and procedures; although, it is likely that individual differences in reasoning ability are also related to rate of executing additional component processes, such as those required for inferring relationships between important problem variables (Sternberg, 1977; Sternberg & Gardner, 1983).

Each of the five path coefficients presented in Figure 2 represent empirically a convergence between elementary information processes and theoretically related traditional ability measures. The two directed paths described for Model 5 were estimated to assess the discriminant validity of the above described relationships. The first of these involved the estimation of a directed path from the Arithmetic Processes IP factor to the Memory Span ability factor. Performance on both the measures of general reasoning and the

memory span measures was directly related to working memory capacity and all of these ability tests require the processing of numbers. The memory span tests however, unlike the general reasoning measures, do not require arithmetic. Thus, support for discriminant validity of the relationship between the measures subsumed by the Arithmetic Processes factor and the General Reasoning ability factor would be found if no direct relationship between the Arithmetic Processes IP factor and the Memory Span factor was found. This discriminant relationship was supported by the finding that the estimated path coefficient, from the Arithmetic Processes factor to the Memory Span factor, did not differ significantly from zero. In all, the pattern of results described thus far indicate that the rate of executing basic arithmetical operations is related to performance on traditional ability measures which require arithmetic and not directly related to similar measures which do not require arithmetic.

Finally, Model 5 also included the estimation of a directed path from the Intercept: Encode-Decode-Respond factor to the General Reasoning common factor and again the resulting path coefficient did not differ significantly from zero. This result suggests a discriminant relationship between the rate of executing the processes underlying the mental solution of arithmetic problems and performance on general reasoning measures which require arithmetic. Specifically, these data indicate that individual differences on these measures of general reasoning were more strongly influenced by the rate of executing content relevant operations and procedures (e.g., retrieval of arithmetic facts from long-term memory) than by the rate of executing related but more fundamental (e.g., decision and response times) processes.

General Discussion

This study examined the pattern of convergent and discriminant relationships between variables which represented the rate of executing elementary numerical operations, facility of performing these operations within working memory, and a battery of individual differences measures which required processing of numbers, but which defined distinct ability factors. The experimental design was therefore biased against finding discriminant relationships between the process factors and the ability factors, due to a similar content across all measures. Indeed, in the initial analysis of the psychometric tests, the matrix of positive correlations among the ability factors suggested a common source of variance spanning all of these individual-differences measures.

The pattern of structural relations, however, indicated the existence of rather specific process/ability relationships. The directed paths between the IP latent variables and the ability factors, represented in Figure 2, suggested that individual differences in the rate of executing the elementary operations of fact retrieval and carrying contributes to individual differences on the measures defining both the Numerical Facility and General Reasoning factors. The estimation of these two paths in effect partialled rate of executing the arithmetical operations from the covariance between the numerical facility and general reasoning measures, and the simultaneously estimated correlation between these two ability factors dropped to nonsignificance (the actual value was $-.008$). This result suggests that the original correlation between the Numerical Facility and General Reasoning factors was due to the fact that both sets of measures share a distinct set of

underlying elementary processes; that is, those operations subsumed by the Arithmetic Processes factor (Carroll, 1976).

A similar argument could be advanced in explanation of the original correlation between the General Reasoning and Memory Span factors, but here the covariance was due, in part, to these measures having similar working memory demands. Within this scheme, the lack of significant correlation between the Numerical Facility and Memory Span factors would be related to the lack of common underlying operations required for successful performance on these measures; although the tests of numerical facility do require working memory resources (Carroll, 1976; Hitch, 1978). The relationship between working memory capacity and numerical facility, however, would appear to be indirect. As was shown in Figure 1, working memory capacity was inversely related to rate of executing the substantive arithmetical processes. Thus, facility of information manipulation in working memory was associated with a shorter duration of basic numerical operations, such as carrying, and the rate of executing these operations, in turn, appeared to be the primary determinant of individual differences on the tests of numerical facility (Geary & Widaman, 1987). Thus, working memory capacity indirectly influences performance on the numerical facility measures through its impact on the rate of executing the processes defining these basic numerical abilities; that is, the operations of fact retrieval and carrying.

Nevertheless, it could be argued that the lack of covariance between the numerical facility and memory span tests was due to the fact that the former are timed measures, whereas the latter are not. Vernon and his colleagues (Vernon, 1986; Vernon & Kantor, 1986; Vernon, Nador, & Kantor, 1985b),

however, have reported that the correlation between speed-of-processing variables and timed and untimed indicators of g did not differ significantly. In theory, then, a variable which represented the processes underlying g should show a correlation of the same magnitude with timed and untimed ability measures. So, variables which indexed basic speed of information processing, such as rate of fact retrieval, should have been directly related to all of the ability measures used in this study.

These results suggest that the covariance between ability measures which define distinct factors is due to a common underlying set of elementary operations, or to similar working memory demands, but that different operations might underlie the covariance between different sets of ability factors. Indeed, the two above described sets of ability factors (numerical facility/general reasoning, memory span/general reasoning) were correlated for different reasons, not because of a processing characteristic which spanned all of the measures used in this study. This result argues against the existence of a pervasive biologically based factor as the primary determinant of individual differences in human abilities across all cognitive domains.

The reliable correlations between basic processing parameters and measures of g are, then, not readily interpretable. The key to resolving these contradictory sets of findings might be found with careful examination of indicators of g . These measures are typically composed of rather complex cognitive tasks which often span many ability domains. For example, one study defined g as the weighted composite of four tests; the Surface Development Test (a spatial visualization measure, Ekstrom et al., 1976); the advanced form of the Raven Progressive Matrices Test (Raven, 1962); and both the Verbal

and Mathematics section of the Scholastic Aptitude Test (Larson & Saccuzzo, 1989, Experiment 1). Thus, this and many other indicators of g likely reflect individual differences in an assortment of skills, such as reasoning, efficiency in representing and mentally manipulating visual-spatial information, and work memory resources. The modest but reliable correlations between IP tasks and measures of g might represent a shared source of variance, but it is not clear, in the absence of componential studies of these g measures, whether the various IP/ g correlations all reflect the same single common source of variance.

To illustrate, Keating and Bobbitt (1978) examined the relationship between age, ability (defined by the Raven Progressive Matrices Test), and performance on three information-processing tasks. These tasks provided variables which represented choice reaction time, rate of retrieving name codes from long-term memory (Hunt et al., 1975; Posner, Boies, Eichelman, & Taylor, 1969), and rate of scanning information in short-term memory (Sternberg, 1966). In one multiple regression analysis these three variables were found to be highly related to performance on the intelligence test, $R = .72$. Keating and Bobbitt reported, however, that "it is also important to note that the central cognitive-processing variables contribute different sources of covariance to the prediction (of intelligence)" (p. 165). In other words, these three basic IP parameters were all correlated with an excellent measure of g (see Jensen, 1982), but each of these correlations, to a large extent, represented a unique, not common, relationship between the IP variable and the intelligence test. Thus, an array of basic cognitive skills likely influence individual differences on measures of g . The relationship between

an IP task variable and a test of *g* might be due to a common source of variance (e.g., reasoning skills), but it is possible that a different IP task is related to *g* due to a relatively independent source of variance (e.g., spatial skills).

In fact, the results of the current study suggests that the mind might be demarcated into relatively discrete and basic cognitive functions. Complex cognitive skills, such as those assessed by traditional psychometric tests, would involve the coordination and integration of systems of more basic operations (Allen, 1983; Luria, 1980; McCloskey, Caramazza, & Basili, 1985; Milberg, Alexander, Charness, McGlinchey-Berroth, & Barrett, 1988). In this view, distinct primary ability factors might index well developed functional systems of basic cognitive operations. Different ability factors would be correlated the extent to which they share some of the same basic operations, or require similar working memory resources. Superordinate clustering of factors, such as those subsumed by Crystallized and Fluid Intelligence (Horn & Cattell, 1966), might reflect even more fundamental features of the human mind. To illustrate, the evolution of human language preceded the development of what are not considered basic cognitive abilities (e.g., numerical facility). The neural substrate underlying human language appears to support many culture-mediated skills, e.g., reading, writing, and arithmetic (Boller & Grafman, 1983; Dahmen, Hartje, Büssing, & Sturm, 1982; Deloche & Seron, 1982; Luria, 1980), and perhaps the representation of declarative information in long-term memory. The neural substrate which enabled the evolution of language might then serve as the basis for a broad range of human skills, such as those subsumed by Crystallized Intelligence. Thus, ability measures which

require culture-mediated skills would cluster together, but would define separable ability factors due to individual differences in learning history for, e.g., arithmetic as contrasted with verbal knowledge, or vocabulary.

In summary, the present study demonstrated empirically a pattern of convergent and discriminant relationships between rate of executing basic numerical operations, facility of performing these operations within working memory, and a battery of tests which required number processing but which defined distinct ability factors. These results suggested different sets of ability factors might be correlated for different reasons, and not because of a pervasive biologically mediated processing characteristic which spans all cognitive domains. This result, nevertheless, does not militate against the clustering of groups of factors, such as those subsumed by Crystallized and Fluid Intelligence, as the results of the present study and numerous other studies clearly supports the existence of factor clusters (e.g., Carroll, 1988; Horn & Cattell, 1966, Spearman & Jones, 1950). Finally, the results of this study strongly suggest that to avoid confusion as to the source of the correlation between intelligence tests and IP parameters, future studies of the relationship between individual differences in cognitive abilities and elementary processes should be based on careful task analysis of both the psychometric tests and the IP tasks (Carroll, 1976; Keating et al., 1985).

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Table 1

Subject Racial and Educational Characteristics

Variable	n	percentage
Race		
Caucasian	74	72.5
Black	21	20.6
Other	7	6.9
Education		
12 years, no diploma	1	1.0
High school diploma	54	52.9
GED	2	2.0
Some College	37	36.3
Associate degree	3	2.9
Bachelor degree	5	4.9

Table 2

Statistical Summaries of Regression Analyses: Addition

Equation	R^2	F	df	MS_e
Simple				
RT = 784 + 69(Encode) + 15.8(Prod) + 208(Truth)	.710	62.00	3,76	196
Partial F_s = 1.31, 84.13, 22.58				
RT = 1,001 + 17.0(Prod) + 208(Truth)	.705	91.98	2,77	197
Partial F_s = 161.47, 22.49				
\overline{RT} = 1,600				
Complex: Multi-column				
RT = 1341 + 209(Encode) + 9.7(Unitprod) + 647(Carry) + 9.7(Tenprod) + 406(Truth)	.853	108.95	4,75	303
Partial F_s = 49.34, 22.45, 43.05, 22.45, 23.71				
\overline{RT} = 3,610				
Complex: Multi-digit				
RT = 1654 + 12.3(Largeprod) + 344(Min) + 305(Truth)	.723	66.03	3,76	404
Partial F_s = 17.00, 70.82, 11.41				
RT = 1391 + 105(Scan) + 9.1(Prod2) + 337(Min2) + 123(Remainder) + 305(Truth)	.898	130.23	5,74	249
Partial F_s = 7.99, 18.15, 155.88, 48.49, 30.18				
\overline{RT} = 3,478				

Note. All models are significant, $p < .0001$; all partial F ratios are significant $p < .01$, except for the Encode parameter in the first equation, $p > .10$. Encode = number of digits encoded; Prod = product of augend and addend; Truth = correct (0) or incorrect (1) stated sum; Unitprod and Tenprod = product of digits in units and tens columns, respectively; Carry = presence (1) or absence (0) of a carry from the units to tens column;

Largeprod = product of the two largest value digits; Min = value of the smallest digit; Scan = number of digits scanned before executing the memory retrieval process; Prod2 = product of the two largest value digits, unless two of the problem's digits sum to ten, then Prod2 = the product of these digits; Min2 = value of smallest digit when no two digits sum to ten; Remainder = value of remaining digit when two digits sum to ten.

Table 3

Statistical Summaries of Regression Analyses: Multiplication

Equation	R^2	F	df	MS _e
Simple				
RT = 1349 + 10.0(Prod) + 188 (Truth) Partial F_s = 110.64, 33.04	.651	71.84	2,77	146
\overline{RT} = 1,738				
Complex				
RT = 1727 + 190(Encode) + 13.7(Unitprod) + 707(Carry) + 13.7(Tenprod) + 164(Carrem) + 377(Truth) Partial F_s = 4.30, 23.74, 7.89, 23.74, 14.55, 13.49	.873	102.00	5,74	398
\overline{RT} = 4,542				

Note. All models are significant, $p < .0001$; all partial F ratios are significant, $p < .05$. Prod = product of multiplicand and multiplier; Truth = correct (1) or incorrect (1) stated sum; Encode = number of digits encoded; Unitprod and Tenprod = product of digits in the units and tens columns, respectively; Carry = presence (1) or absence (0) of a carry from units to tens column; Carrem = value of the remainder following the units column multiplication.

Table 4

Descriptive Statistics for Measures in the Ability Test Battery

Test	<u>M</u>	<u>SD</u>	Spearman-Brown reliability estimates
<hr/>			
Numerical Facility			
Addition	36.07	10.26	.892
Division	23.66	11.82	.911
Subtraction/multiplication	48.91	17.25	.937
Perceptual Speed			
Finding A's	63.36	14.79	.840
Number comparison	24.17	5.86	.692
Identical pictures	69.61	12.98	.884
General Reasoning			
Arithmetic aptitude	10.41	6.42	.704
Necessary arithmetic operations	10.46	5.51	.791
Memory Span			
Auditory number span	8.96	3.11	.580
Visual number span	9.34	3.06	.683

Note. Reliability estimates for the Auditory Number Span and the Visual Number Span measures were based on the zero-order correlation between odd and even items.

Table 5

Results of Confirmatory Factor Analysis of Measures in the Ability Test Battery

Variable	Factor				Unique Variance
	Numerical Facility	Perceptual Speed	General Reasoning	Memory Span	
Factor pattern					
Addition	.872(.083)				.240(.057)
Division	.725(.089)				.474(.075)
Subtraction/ multiplication	.922(.080)				.150(.055)
Finding A's		.358(.116)			.872(.131)
Number comparison		.737(.127)			.457(.154)
Identical pictures		.433(.116)			.813(.128)
Arithmetic aptitude			.807(.067)		.349(.073)
Necessary arithmetic operations			.921(.076)		.152(.073)
Auditory number span				.725(.076)	.474(.101)
Visual number span				.747(.078)	.442(.102)

Factor intercorrelations

Factor

Numerical Facility			
Perceptual Speed	.650(.109)		
General Reasoning	.310(.102)	.210(.133)	
Memory Span	.083(.124)	.298(.146)	.376(.114)

Note. The latent variable variances were fixed at unity in order to identify the model. Tabled values are loading estimates; associated standard errors are in parentheses. Empty cells signify parameters fixed at zero. The loading values defining the General Reasoning factor and the Memory Span factor were constrained to equality, in the covariance metric, to empirically identify the estimates. All Tabled values are significant, $p < .05$; except for the correlation between the Memory Span and Numerical Facility factors, and between the General Reasoning and Perceptual Speed factors, $p > .05$.

Table 6

Goodness-of-Fit Indexes for Structural Equation Models Relating Information Processing Parameters to Ability Test Measures

Model	<u>df</u>	χ^2		χ^2 / df	
Overall fit of alternative models					
Null	253	1,105.9	.001	4.37	---
1:Seven trait factors and 17 covariances a between uniqueness terms	207	413.6	.001	2.00	.703
2:Model 1 plus five directed paths	202	312.2	.001	1.55	.837
3:Model 2 plus 14 covariances between uniqueness terms	188	225.7	.031	1.20	.941
4:Model 4 minus five directed paths	193	340.6	.001	1.76	.774
5:Model 3 plus two directed paths	186	222.4	.035	1.20	.941

a

A table of covariances between uniqueness terms is available from the first author upon request.

Table 7

Indexes of Difference Between Nested Structural Equation Models
Relating Information-Processing Parameters to Ability Test
Measures

Comparison	Differences		p	Differences	
	<u>df</u>	χ^2		χ^2 / df	f^2
Null vs. Model 1	46	692.3	.001	2.37	---
Model 1 vs. Model 2	5	101.4	.001	0.45	.134
Model 2 vs. Model 3	14	86.5	.001	0.35	.104
Model 3 vs. Model 4	5	114.9	.001	0.56	.167
Model 3 vs. Model 5	2	3.3	.150	0.00	.000

Table 8

Estimates from Structural Equation Model 3

Observed measures	Trait factor		Unique Factor	
	Loading	SE	Loading	SE
Information-processing parameters				
Arithmetic processes				
Simple addition: product	.685	.092	.531	.095
Multi-column complex addition: product	.578	.096	.666	.102
Multi-digit complex addition: product	.267	.100	.929	.131
Simple multiplication: product	.571	.096	.674	.102
Complex multiplication: product	.176	.103	.969	.136
Multi-column complex addition: carry	.317	.101	.900	.128
Complex multiplication: carry	.500	.095	.750	.108
Intercept: encode, decide, respond				
Simple addition	.435	.121	.811	.128
Multi-column complex addition	.700	.097	.510	.097
Multi-digit complex addition	.405	.101	.836	.121
Simple multiplication	.408	.126	.834	.130
Complex multiplication	.736	.095	.458	.093
Working memory capacity				
ABC -assignment	.894	.079	.201 ^a	---
Ability tests				
Numerical facility				
Addition	.845	.076	.286	.057
Division	.753	.069	.433	.068
Subtraction/multiplication	.889	.077	.210	.045
Perceptual speed				
Finding <u>As</u>	.423	.094	.821	.126
Number comparison	.754	.126	.431	.147
Identical pictures	.377	.097	.858	.128
General reasoning				
Arithmetic aptitude	.840	.065	.294	.067
Necessary arithmetic operations	.848	.066	.281	.070
Memory span				
Auditory number span	.715	.077	.489	.103
Visual number span	.733	.079	.463	.105

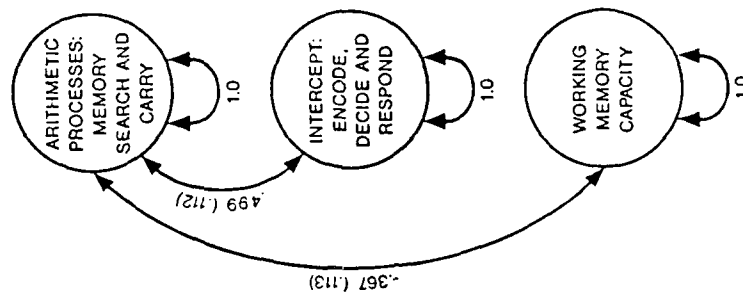
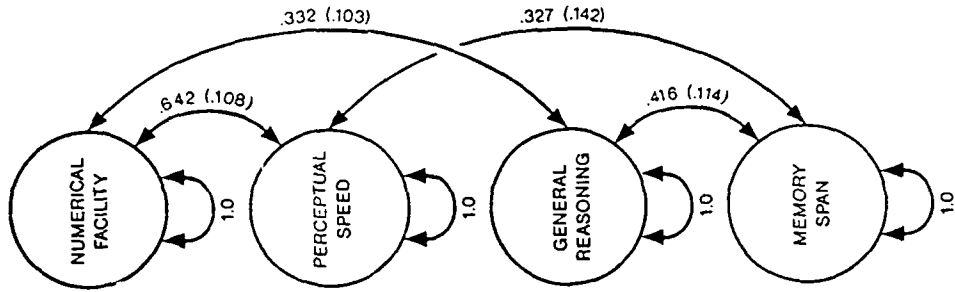
Note: All reported loadings are significant, $p < .05$, except for the product variable for complex multiplication, which was marginally significant, $p < .10$.

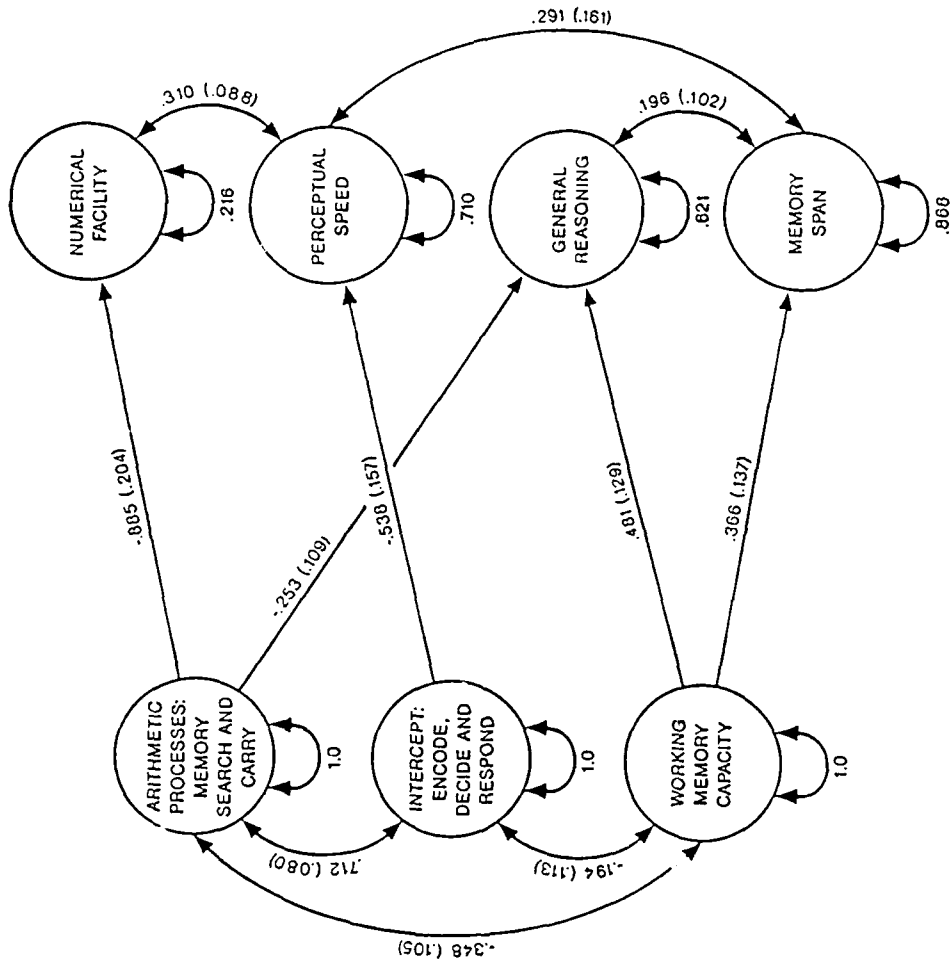
The factor loadings for the two General Reasoning tests and for the two Memory

Span tests were constrained to equality, in the covariance metric, to empirically identify the estimates. All remaining, nonreported loadings, were fixed at zero.

^a

Parameter fixed at this value, in the covariance metric, based on the parameter's reliability estimate.





Appendix A
Experimental Stimuli
Arithmetic Problem Sets

Experimental stimuli: Simple addition (Set 1)

STIMULI	TRUE/FALSE	STIMULI	TRUE/FALSE
5+4 = 9	0	4+5 = 9	0
2+8 = 10	0	2+9 = 11	0
5+9 = 16	1	6+3 = 9	0
3+8 = 11	0	9+4 = 14	1
2+7 = 9	0	4+3 = 5	1
6+4 = 10	0	8+5 = 11	1
3+5 = 9	1	7+2 = 9	0
5+6 = 12	1	5+9 = 14	0
6+7 = 11	1	2+3 = 4	1
3+6 = 9	0	7+4 = 11	0
5+2 = 6	1	8+6 = 15	1
4+6 = 11	1	9+5 = 13	1
2+9 = 12	1	4+9 = 15	1
7+3 = 10	0	9+3 = 12	0
6+7 = 13	0	6+8 = 13	1
8+2 = 12	1	3+7 = 8	1
9+8 = 17	0	7+8 = 15	0
6+2 = 9	1	4+3 = 7	0
3+7 = 10	0	7+9 = 14	1
2+8 = 11	1	3+8 = 10	1
4+9 = 13	0	2+7 = 8	1
5+4 = 11	1	5+6 = 11	0
9+6 = 15	0	8+4 = 14	1
6+3 = 7	1	2+5 = 7	0
9+8 = 15	1	7+4 = 10	1
8+7 = 17	1	9+5 = 14	0
6+2 = 8	0	8+7 = 15	0
9+3 = 13	1	4+2 = 4	1
2+5 = 9	1	9+4 = 13	0
8+6 = 14	0	8+2 = 10	0
5+2 = 7	0	5+7 = 12	0
7+3 = 8	1	2+3 = 5	0
4+2 = 6	0	3+9 = 10	1
3+5 = 8	0	7+8 = 14	1
6+4 = 9	1	4+6 = 10	0
7+9 = 16	0	3+9 = 12	0
5+7 = 13	1	9+6 = 17	1
4+5 = 8	1	6+8 = 14	0
8+4 = 12	0	7+2 = 11	1
3+6 = 11	1	8+5 = 13	0

Experimental stimuli: Multi-column complex addition (Set 2)

STIMULI	TRUE/FALSE	STIMULI	TRUE/FALSE
93+67 = 159	1	28+47 = 95	1
59+36 = 95	0	92+56 = 148	0
75+68 = 143	0	76+48 = 124	0
28+47 = 75	0	27+86 = 113	0
63+98 = 161	0	82+43 = 135	1
54+69 = 125	1	93+67 = 160	0
48+26 = 74	0	37+25 = 72	1
86+39 = 105	1	25+34 = 49	1
97+24 = 121	0	56+72 = 128	0
49+78 = 137	1	35+46 = 81	0
92+56 = 149	1	83+52 = 145	1
37+25 = 62	0	94+37 = 131	0
69+57 = 126	0	79+85 = 164	0
97+24 = 111	1	95+42 = 136	1
76+48 = 122	1	57+83 = 140	0
65+73 = 158	1	48+26 = 76	1
46+97 = 143	0	62+35 = 87	1
64+82 = 136	1	38+59 = 95	1
82+43 = 125	0	49+78 = 127	0
27+86 = 133	1	58+93 = 151	0
84+23 = 107	0	64+82 = 146	0
35+46 = 82	1	23+79 = 102	0
23+79 = 104	1	57+83 = 120	1
87+92 = 180	1	95+42 = 137	0
78+54 = 122	1	74+28 = 100	1
43+65 = 108	0	26+95 = 122	1
38+59 = 97	0	42+89 = 151	1
65+73 = 138	0	78+54 = 132	0
43+65 = 110	1	59+36 = 105	1
69+57 = 124	1	26+95 = 121	0
32+74 = 108	1	83+52 = 135	0
75+68 = 123	1	79+85 = 163	1
62+35 = 97	0	84+23 = 87	1
39+64 = 103	0	46+97 = 163	1
86+39 = 125	0	25+34 = 59	0
63+98 = 159	1	54+69 = 123	0
94+37 = 130	1	87+92 = 179	0
56+72 = 129	1	42+89 = 131	0
39+64 = 83	1	58+93 = 150	1
74+28 = 102	0	32+74 = 106	0

Experimental stimuli: Multi-digit complex addition (Set 3)

STIMULI	TRUE/FALSE	STIMULI	TRUE/FALSE
5+8+6 = 19	0	9+5+2 = 14	1
4+6+7 = 17	0	3+4+6 = 13	0
9+7+2 = 18	0	5+9+8 = 22	0
4+6+7 = 16	1	9+5+2 = 16	0
9+3+5 = 19	1	2+6+4 = 14	1
6+2+7 = 15	0	9+8+5 = 23	1
4+3+8 = 17	1	2+3+4 = 9	0
6+9+4 = 19	0	8+7+5 = 21	1
7+3+2 = 12	0	7+2+4 = 12	1
8+9+3 = 20	0	3+9+5 = 19	1
3+7+4 = 14	0	6+5+3 = 14	0
5+2+3 = 11	1	9+4+8 = 23	1
6+7+8 = 19	1	6+5+3 = 15	1
7+4+9 = 20	0	2+4+5 = 11	0
6+2+7 = 16	1	8+3+6 = 16	1
8+9+3 = 18	1	5+4+2 = 9	1
6+7+8 = 21	0	3+6+9 = 17	1
4+5+7 = 17	1	5+2+3 = 10	0
8+2+9 = 19	0	8+5+9 = 24	1
6+8+7 = 21	0	5+9+8 = 23	1
7+6+2 = 14	1	7+3+2 = 11	1
5+8+6 = 21	1	2+5+8 = 15	0
7+2+4 = 13	0	4+7+6 = 18	1
5+4+2 = 11	0	2+4+5 = 13	1
2+5+8 = 14	1	6+9+4 = 21	1
3+7+4 = 13	1	4+3+8 = 15	0
2+9+7 = 16	1	7+8+9 = 24	0
3+2+6 = 11	0	4+7+6 = 17	0
4+5+7 = 16	0	6+8+7 = 19	1
9+8+5 = 22	0	8+7+5 = 20	0
7+4+9 = 19	1	3+6+9 = 18	0
2+6+4 = 12	0	9+7+2 = 16	1
4+8+3 = 15	0	8+5+9 = 22	0
8+2+9 = 21	1	5+6+3 = 15	1
7+6+2 = 15	0	8+3+6 = 17	0
3+2+6 = 9	1	5+6+3 = 14	0
9+3+5 = 17	0	9+4+8 = 21	0
7+8+9 = 22	1	2+3+4 = 8	1
3+4+6 = 11	1	4+8+3 = 16	1
2+9+7 = 18	0	3+9+5 = 17	0

Experimental stimuli: Simple multiplication (Set 4)

STIMULI	TRUE/FALSE	STIMULI	TRUE/FALSE
6x7 = 42	0	2x7 = 24	1
8x4 = 34	1	3x5 = 14	1
3x6 = 18	0	6x8 = 38	1
9x8 = 82	1	4x3 = 12	0
2x5 = 10	0	2x4 = 6	1
8x3 = 22	1	4x8 = 31	1
4x5 = 19	1	5x9 = 45	0
6x3 = 18	0	9x2 = 18	0
5x6 = 32	1	4x8 = 32	0
7x3 = 20	1	9x4 = 26	1
5x8 = 40	0	7x8 = 56	0
7x5 = 35	0	2x9 = 18	0
9x7 = 63	0	9x4 = 36	0
4x3 = 14	1	6x9 = 44	1
3x2 = 6	0	2x7 = 14	0
2x9 = 28	1	6x9 = 54	0
6x7 = 32	1	9x2 = 19	1
7x4 = 27	1	6x4 = 14	1
8x3 = 24	0	3x6 = 19	1
7x5 = 25	1	7x3 = 21	0
5x3 = 15	0	2x6 = 12	0
2x5 = 20	1	8x4 = 32	0
7x9 = 53	1	5x3 = 13	1
8x5 = 40	0	9x7 = 73	1
9x6 = 53	1	5x2 = 10	0
4x2 = 8	0	4x5 = 20	0
8x6 = 48	0	3x7 = 19	1
7x4 = 28	0	6x4 = 24	0
3x9 = 37	1	9x6 = 54	0
7x8 = 66	1	5x2 = 11	1
5x6 = 30	0	3x9 = 27	0
6x8 = 48	0	2x6 = 14	1
3x5 = 15	0	5x8 = 50	1
8x2 = 17	1	4x2 = 10	1
4x7 = 26	1	9x8 = 72	0
2x4 = 8	0	8x5 = 41	1
8x6 = 46	1	3x7 = 21	0
7x9 = 63	0	8x2 = 16	0
4x7 = 28	0	5x9 = 35	1
6x3 = 20	1	3x2 = 7	1

Experimental stimuli: Complex multiplication (Set 5)

STIMULI	TRUE/FALSE	STIMULI	TRUE/FALSE
78x2 = 156	0	39x5 = 197	1
86x7 = 622	1	97x4 = 488	1
32x8 = 276	1	72x3 = 216	0
73x6 = 438	0	38x9 = 342	0
38x9 = 322	1	57x3 = 171	0
59x4 = 236	0	63x4 = 262	1
78x2 = 155	1	54x7 = 378	0
82x6 = 482	1	26x4 = 104	0
57x3 = 173	1	83x5 = 415	0
65x9 = 485	1	96x8 = 767	1
58x6 = 348	0	54x7 = 388	1
45x3 = 135	0	37x6 = 122	1
27x9 = 243	0	85x4 = 350	1
83x5 = 515	1	74x8 = 592	0
29x3 = 87	0	47x2 = 84	1
73x6 = 436	1	69x8 = 572	1
64x5 = 310	1	45x3 = 155	1
47x2 = 94	0	97x4 = 388	0
96x8 = 768	0	56x2 = 112	0
62x7 = 434	0	27x9 = 223	1
58x6 = 328	1	95x6 = 570	0
32x8 = 256	0	34x2 = 67	1
29x3 = 89	1	23x8 = 184	0
42x9 = 278	1	59x4 = 216	1
76x5 = 380	0	48x5 = 140	1
25x7 = 173	1	84x9 = 757	1
69x8 = 552	0	93x2 = 186	1
48x5 = 240	0	64x5 = 320	0
95x6 = 571	1	93x2 = 187	0
56x2 = 212	1	37x6 = 222	0
42x9 = 378	0	74x8 = 591	1
86x7 = 602	0	49x7 = 343	0
98x3 = 295	1	72x3 = 214	1
84x9 = 756	0	25x7 = 175	0
39x5 = 195	0	82x6 = 492	0
62x7 = 534	1	26x4 = 114	1
85x4 = 340	0	49x7 = 345	1
23x8 = 182	1	34x2 = 68	0
98x3 = 294	0	76x5 = 370	1
65x9 = 585	0	63x4 = 252	0

Appendix B

Computer-administered Instructions

Arithmetic Problem Sets

Instructions: Subject Instructions

<u>Screen</u>	<u>Instructions</u>	<u>Notes</u>
1	Please type in your 3-digit ID Please hit Space Bar to continue.	Between presentation of screens, which don't include the arithmetic stimuli, have a 500-ms pause.
1b	Please type in your age. Please hit Space Bar to continue.	
2	Today, you will solve 5 sets of Arithmetic problems. Three sets of addition problems and two sets of multiplication problems. Please hit Space Bar to continue.	
3	Each Problem will be presented at the center of your screen. Please hit Space Bar to continue.	
4	You are to decide whether the problem is CORRECT or NOT CORRECT. Please hit Space Bar to continue.	
5	For example, look at this problem: $\begin{array}{r} 1 \\ +2 \\ \hline 3 \end{array}$ <p>The answer is CORRECT. So, PRESS the L (for LIKE) on the key board with your right index finger.</p>	If they press "L" move to screen 6. If they press any other key move to screen 5b.
5b	You pressed the wrong key. Remember, if the answer is CORRECT press L. Now, press L with your right index finger.	When they press L move to screen 6.

<u>Screen</u>	<u>Instructions</u>	<u>Notes</u>
6	<p>Now, look at this problem:</p> $\begin{array}{r} 2 \\ +2 \\ \hline 5 \end{array}$ <p>The answer is NOT CORRECT. So, PRESS D (for DIFFERENT) on the keyboard with your left index finger.</p>	If they press D move to screen 7; otherwise, to screen 6b.
6b	<p>You pressed the wrong key.</p> <p>Remember, if the answer is NOT CORRECT PRESS D.</p> <p>Now, press D with the index finger of your left hand.</p>	When they press D move to screen 7.
7	<p>Remember, if the answer is CORRECT press L.</p> <p>Please press L.</p>	When they press L move to screen 8.
8	<p>And, if the answer is NOT CORRECT PRESS D.</p> <p>Please press D.</p>	When they press D move to screen 9.
9	<p>OK, now let's try a few practice problems.</p> <p>Are these problems CORRECT (L) or NOT CORRECT (D).</p> <p>Press Space Bar to Begin.</p>	
10	<p>1</p> $\begin{array}{r} +3 \\ 4 \end{array}$	Present these three practice problems one at a time, as per instructions for experimental stimuli. If they press the appropriate key (L, or D) then remove the problem and present a "GOOD" prompt for a 1,000-ms duration then go to next screen.
11	<p>2</p> $\begin{array}{r} +1 \\ 2 \end{array}$	

<u>Screen</u>	<u>Instructions</u>	<u>Notes</u>
12	3 +3 6	If the wrong key is pressed remove the problem and present a "WRONG" prompt for a 1,000-ms duration. Then present screen 10b.
10b	Remember, CORRECT PRESS L NOT CORRECT PRESS D. Press Space Bar to continue.	
13	When you solve the airthmetic problems try to solve them as quickly as you can. Press Space Bar to continue.	
14	BUT, try not to make any mistakes. Press Space Bar to continue.	
14	If you make a mistake you will see WRONG following that problem. Press Space Bar to continue.	
15	So, if you see several WRONGs - SLOW DOWN Press Space Bar to continue.	
16	If you have questions ask the experimenter. Press Space Bar to continue.	
17	Let's get started. Press Space Bar to continue.	
18	You'll begin by solving simple addition problems. Press Space Bar to continue.	
19	This set will include 8 practice problems and 80 more problems. Press Space Bar to continue.	
20	Remember, go as quickly as you can without making alot of mistakes.	

<u>Screen</u>	<u>Instructions</u>	<u>Notes</u>
	Press Space Bar to continue.	
21	Here come the 8 practice problems.	After this screen present the 8 practice problems for SET = 1 (according to enclosed instructions).
	Press Space Bar to Begin.	
22	Now, you'll solve 80 more problems.	Present this screen after they finish the 8th practice problem. Then, after this screen present the SET = 1 experimental stimuli as per instructions.
	Press Space Bar to Begin.	
23	Good, you've finished the first set of problems.	Present this screen after they have finished problem number 80 in SET = 1.
	Press Space Bar to continue.	
24	Now, you'll solve complex addition problems.	
	Press Space Bar to continue.	
25	Beginning with 8 practice problems.	After this screen present the 8 practice stimuli, according to instructions, for SET = 2.
	Press Space Bar to Begin.	
26	Now, you'll solve 80 more problems.	Present this screen after they have solved the 8th practice problem. Then, after this screen present the SET = 2 experimental stimuli as per instructions.
	Press Space Bar to Begin.	
27	Good, you've finished the second set of problems.	Present this screen after they have completed the 80th problem in SET = 2.
	Press Space Bar to continue.	

<u>Screen</u>	<u>Instructions</u>	<u>Notes</u>
28	Now, you'll solve a different type of addition problem. Press Space Bar to continue.	
29	Beginning with 8 practice problems. Press Space Bar to Begin.	After this screen present the 8 practice problems for SET = 3.
30	Now, you'll solve 80 more problems. Press Space Bar to Begin.	Present this screen after they have solved the 8th practice problem (SET = 3). After this screen present the 80 experimental stimuli for SET = 3.
31	Good, you've finished the third set of problems. Press Space Bar to continue.	Present this screen after they've solved the 80th problem in SET = 3.
32	Now, you'll solve simple multiplication problems. Press Space Bar to continue.	
33	Beginning with 8 practice problems. Press Space Bar to Begin.	After this screen present the 8 practice problems for SET = 4.
34	Now, you'll solve 80 more problems. Press Space Bar to Begin.	Present this screen after they have solved the 8th practice problems (SET = 4). Then, present the experimental stimuli for SET = 4.
35	Good, one more set to go. Press Space Bar to continue.	Present this screen after they have finished the 80th problem in SET = 4.

<u>Screen</u>	<u>Instructions</u>	<u>Notes</u>
36	In this last set, you'll solve complex multiplication problems. Press Space Bar to continue.	
37	Beginning with 8 practice problems.	After this screen present the 8 practice problems for SET - 5.
38	Press Space Bar to Begin. Now, you'll solve 80 more problems.	Present this screen after they have solved the 8th practice problem in SET - 5. Present the 80 stimuli for SET - 5 after this screen.
	Press Space Bar to Begin.	
39	Good, you've now completed all of the arithmetic problem sets.	

Appendix C

Statistical Analysis System Program for Mathematical Modeling
of Solution Times to Arithmetic Problem Sets

```
CMS FILEDEF GEARY DISK DUMMY DUMMY T;
DATA ALL; SET GEARY.AFPRT;
```

```
*** GENERAL SETUP;
IF SET = 1 OR SET = 2 THEN TRUSUM = (NUM1 + NUM2);
IF SET = 3 THEN TRUSUM = (NUM1 + NUM2 + NUM3);
IF SET = 4 OR SET = 5 THEN TRUPROD = (NUM1*NUM2);
IF SET = 1 OR SET = 2 OR SET = 3 THEN ERRORSUM = (STSUM - TRUSUM);
IF SET = 4 OR SET = 5 THEN ERRORPRD = (STSUM - TRUPROD);
*** END GENERAL SETUP;
```

```
*** SETUP PROCESS CODES: SET = 1 AND SET = 4;
IF SET = 1 OR SET = 4 THEN DO;
SUM = NUM1 + NUM2;
SUM2 = SUM**2;
PROD = NUM1*NUM2;
MIN = MIN(NUM1,NUM2);
MAX = MAX(NUM1,NUM2);
END;
*** END SETUP PROCESS CODES: SET = 1 AND SET = 4;
```

```
*** SETUP ACT VARIABLE: SET = 1 AND SET = 4;
IF SET = 1 OR SET = 4 THEN DO;
ACT = (NUM1 + 1)*(NUM2 + 1);
END;
*** END SETUP ACT VARIABLE: SET = 1;
```

```
*** SETUP ENCODING PARAMETER: SET = 1;
IF SET = 1 THEN DO;
  IF STSUM > 9 THEN NI = 4;
  IF STSUM < 10 THEN NI = 3;
END;
*** END SETUP ENCODING PARAMETER: SET = 1;
```

```
*** SETUP RE-ENCODING PARAMETER: SET = 1;
IF SET = 1 AND TRUTH = 1 THEN DO;
  IF NI = 4 THEN REEN = 2;
  IF NI = 3 THEN REEN = 1;
END;
IF SET = 1 AND TRUTH = 0 THEN REEN = 0;
IF SET = 1 THEN COMEN = NI + REEN;
*** END SETUP RE-ENCODING PARAMETER: SET = 1;
```

```
*** SETUP ENCODING PARAMETER: SET = 4;
IF SET = 4 THEN DO;
  F STSUM > 9 THEN NI = 4;
  F STSUM < 10 THEN NI = 3;
END;
*** END SETUP ENCODING PARAMETER: SET = 4;
```

```
*** SETUP RE-ENCODING PARAMETER: SET = 4;
IF SET = 4 AND TRUTH = 1 THEN DO;
```

```

    IF NI = 4 THEN REEN = 2;
    IF NI = 3 THEN REEN = 1;
END;
IF SET = 4 AND TRUTH = 0 THEN REEN = 0;
IF SET = 4 THEN COMEN = NI + REEN;
*** END SETUP RE-ENCODING: SET = 4;

*** SETUP COLUMNAR PROCESS CODES: SET = 2;
IF SET = 2 THEN DO;
UNIT1 = MOD(NUM1,10);
UNIT2 = MOD(NUM2,10);
UNITSUM = UNIT1 + UNIT2;
UNITMIN = MIN(UNIT1,UNIT2);
UNITPROD = (UNIT1*UNIT2);
UNITSUM2 = UNITSUM**2;
UNITACT = (UNIT1 + 1)*(UNIT2 + 1);
IF UNITSUM > 9 THEN CARRY = 1;
        ELSE CARRY = 0;

TEN1 = NUM1/10;
TEN2 = NUM2/10;
TENS1 = INT(TEN1);
TENS2 = INT(TEN2);
TENSUM = TENS1 + TENS2 + CARRY;
TENSUM2 = TENSUM**2;
TENMIN = MIN(TENS1,TENS2);
TENPROD = ((TENS1 + CARRY)*TENS2);
TENACT = ((TENS1 + CARRY + 1))*(TENS2 + 1);
COLMN = UNITMIN + TENMIN;
END;
*** END SETUP PROCESS CODES: SET = 2;

*** SETUP SERIAL SELF-TERMINATING CODES: SET = 2;
IF SET = 2 THEN DO;
STSUNIT = MOD(STSUM,10);
    IF UNITSUM > 9 THEN UNITST = MOD(UNITSUM,10);
        ELSE UNITST = UNITSUM;
IF UNITST = STSUNIT THEN UNITCORR = 1;
IF UNITST NE STSUNIT THEN UNITCORR = 0;
IF UNITSUM > 9 THEN CARRYST = 1;
        ELSE CARRYST = 0;
IF UNITCORR = 0 THEN CARRYST = 0;
IF CARRY = 0 THEN CARRYST = 0;

    IF UNITCORR = 0 THEN TENSUM = 0;
    IF UNITCORR = 0 THEN TENPROD = 0;
    IF UNITCORR = 0 THEN TENACT = 0;
    IF UNITCORR = 0 THEN TENMIN = 0;
    IF UNITCORR = 0 THEN TENSUM2 = 0;
** COLUMNAR CONSTRAINTS: SET = 2;
COLPROD = UNITPROD + TENPROD;
COLMIN = UNITMIN + TENMIN;
COLSUM2 = UNITSUM2 + TENSUM2;

```

```

** END COLUMNAR CONSTRAINTS: SET = 2;
END;
*** END SETUP SELF-TERMINATING CODES: SET = 2;

*** SETUP ENCODING PARAMETER: SET = 2;
IF SET = 2 THEN DO;
  IF TRUTH = 0 AND STSUM < 100 THEN NI = 6;
  IF TRUTH = 0 AND STSUM > 99 THEN NI = 7;
  IF TRUTH = 1 AND UNITCORR = 0 THEN NI = 3;
  IF TRUTH = 1 AND UNITCORR = 1 AND STSUM < 100 THEN NI = 6;
  IF TRUTH = 1 AND UNITCORR = 1 AND STSUM > 99 THEN NI = 7;
END;
*** END SETUP FOR ENCODING: SET = 2;

*** SETUP RE-ENCODING PARAMETER: SET = 2;
IF SET = 2 AND TRUTH = 1 THEN DO;
  IF NI = 3 THEN REEN = 1;
  IF NI = 6 THEN REEN = 2;
  IF NI = 7 THEN REEN = 3;
END;
IF SET = 2 AND TRUTH = 0 THEN REEN = 0;
IF SET = 2 AND TRUTH = 0 THEN COMEN = (NI + REEN);
IF SET = 2 AND TRUTH = 1 THEN COMEN = (NI + REEN);
*** END SETUP RE-ENCODING PARAMETER: SET = 2;

*** SETUP PROCESS CODES: SET = 3;
IF SET = 3 THEN DO;
M1 = MIN(NUM1, NUM2);
MIN = MIN(NUM3, M1);
MIN2 = MIN;
MX1 = MAX(NUM1, NUM2);
MAX = MAX(NUM3, MX1);
MD1 = (MIN + MAX);
MID = TRUSUM - MD1;
FPROD = (NUM1*NUM2);
LARGPROD = MID*MAX;
LARGPRD2 = LARGPROD;
** CHUNK;
IF NUM1 + NUM2 = 10 THEN CHUNK = 1;
IF NUM1 + NUM3 = 10 THEN CHUNK = 1;
IF NUM2 + NUM3 = 10 THEN CHUNK = 1;
IF CHUNK NE 1 THEN CHUNK = 0;
IF CHUNK = 1 THEN LARGPRD2 = 0;
IF CHUNK = 1 THEN MIN2 = 0;
IF NUM1 + NUM2 = 10 THEN REM = NUM3;
IF NUM1 + NUM3 = 10 THEN REM = NUM2;
IF NUM2 + NUM3 = 10 THEN REM = NUM1;
IF REM = . THEN REM = 0;

IF NUM1 + NUM2 = 10 THEN POS = 0;
IF NUM1 + NUM3 = 10 THEN POS = 1;
IF NUM2 + NUM3 = 10 THEN POS = 0;
IF POS = . THEN POS = 0;

```

```
IF NUM1 + NUM2 = 10 THEN CPROD = (NUM1*NUM2);
IF NUM1 + NUM3 = 10 THEN CPROD = (NUM1*NUM3);
IF NUM2 + NUM3 = 10 THEN CPROD = (NUM2*NUM3);
IF CHUNK = 0 THEN CPROD = 0;
IF CPROD NE 0 THEN CPROD2 = 1;
      ELSE CPROD2 = 0;
*** END CHUNK;
```

```
*** FINAL MODEL CODES: SET = 3;
      IF CPROD NE 0 THEN LARGPRD2 = 0;
      IF REM NE 0 THEN MIN2 = 0;
      TOTPROD = LARGPRD2 + CPROD;
      TOTMIN = MIN2 + REM;
END;
* END FINAL CODES;
*** END SETUP PROCESS CODES: SET = 3;
```

```
*** SCANNING CODES: SET = 3;
IF SET = 3 THEN DO;
  IF NUM1 + NUM2 = 10 THEN SCAN = 2;
  IF NUM1 + NUM3 = 10 THEN SCAN = 3;
  IF NUM2 + NUM3 = 10 THEN SCAN = 3;
  IF (MAX + MID) = (NUM1 + NUM2) THEN SCAN = 5;
  IF SCAN = . THEN SCAN = 6;
END;
IF SET = 3 AND SCAN = . THEN SCAN = 3;
*** END SCANNING CODES: SET = 3;
```

```
*** COMPARISON CODES: SET = 3;
IF SET = 3 THEN DO;
  IF NUM1 + NUM2 = 10 THEN COMP = 1;
  IF NUM2 + NUM3 = 10 THEN COMP = 2;
END;
IF SET = 3 AND COMP = . THEN COMP = 0;
*** END COMPARISON CODES: SET = 3;
```

```
*** SETUP POSITION2 VARIABLE: SET = 3;
  IF SET = 3 AND CHUNK = 0 THEN DO;
    IF (MAX + MID) = (NUM1 + NUM3) THEN POS2 = 1;
    ELSE POS2 = 0;
  END;
  IF SET = 3 AND CHUNK = 1 THEN POS2 = 0;
  IF SET = 3 THEN COMPOS = (POS + POS2);
*** END SETUP POSITION2 VARIABLE: SET = 3;
```

```
*** SETUP ENCODING PARAMETER: SET = 3;
IF SET = 3 THEN DO;
  IF STSUM > 9 THEN NI = 5;
  IF STSUM < 10 THEN NI = 4;
END;
*** END SETUP ENCODING PARAMETER: SET = 3;
```

```
*** SETUP ENCODING MODIFICATION FOR SET = 3;
IF SET = 3 THEN DO;
  STUN3 = MOD(STSUM,10);
  TRUN3 = MOD(TRUSUM,10);
  IF STUN3 = TRUN3 THEN UN3CORR = 1;
  IF STUN3 NE TRUN3 THEN UN3CORR = 0;
  IF (NI = 5) AND (UN3CORR = 0) THEN NIT = 4;
  ELSE NIT = NI;
END;
*** END ENCODING MODIFICATION FOR SET = 3;

*** SETUP RE-ENCODING PARAMETER: SET = 3;
IF SET = 3 AND TRUTH = 1 THEN DO;
  IF NI = 4 THEN REEN = 1;
  IF NI = 5 THEN REEN = 2;
END;
IF SET = 3 AND TRUTH = 0 THEN REEN = 0;
IF SET = 3 THEN COMEN = (NI + REEN);
*** END RE-ENCODING SETUP: SET = 3;

*** SETUP PROCESS CODES: SET = 5;
IF SET = 5 THEN DO;
  B = MOD(NUM1,10);
  C = (NUM1/10);
  A = INT(C);

UNITPROD = (B*NUM2);
UNITMIN = MIN(B,NUM2);
  IF UNITPROD >= 10 THEN CARRY = 1;
  ELSE CARRY = 0;

C = (UNITPROD/10);
CAREM = INT(C);

TENPROD = (A*NUM2);
TENMIN = MIN(A,NUM2);
END;
*** END SETUP PROCESS CODES: SET = 5;

*** SETUP SERIAL SELF-TERMINATING CODES: SET = 5;
IF SET = 5 THEN DO;
STSUNIT = MOD(STSUM,10);
  IF UNITPROD > 9 THEN UNITST = MOD(UNITPROD,10);
  ELSE UNITST = UNITPROD;
IF UNITST = STSUNIT THEN UNITCORR = 1;
IF UNITST NE STSUNIT THEN UNITCORR = 0;
IF UNITPROD > 9 THEN CARRYST = 1;
  ELSE CARRYST = 0;
IF UNITCORR = 0 THEN CARRYST = 0;
IF CARRY = 0 THEN CARRYST = 0;
```

```
    IF UNITCORR = 0 THEN TENPROD = 0;
    IF UNITCORR = 0 THEN CAREM = 0;
    COLPROD = UNITPROD + TENPROD;
    COLMIN = UNITMIN + TENMIN;
END;
*** END SETUP SELF-TERMINATING CODES: SET = 5;

*** SETUP ENCODING PARAMETER: SET = 5;
IF SET = 5 THEN DO;
    IF TRUTH = 0 AND STSUM > 99 THEN NI = 6;
    IF TRUTH = 0 AND STSUM < 100 THEN NI = 5;
    IF TRUTH = 1 AND UNITCORR = 0 THEN NI = 3;
    IF TRUTH = 1 AND UNITCORR = 1 AND STSUM > 99 THEN NI = 6;
    IF TRUTH = 1 AND UNITCORR = 1 AND STSUM < 100 THEN NI = 5;
END;
*** END SETUP ENCODING PARAMETER: SET = 5;

*** SETUP RE-ENCODING PARAMETER: SET = 5;
IF SET = 5 AND TRUTH = 1 THEN DO;
    IF NI = 3 THEN REEN = 1;
    IF NI = 5 THEN REEN = 2;
    IF NI = 6 THEN REEN = 3;
END;
IF SET = 5 AND TRUTH = 0 THEN REEN = 0;
IF SET = 5 THEN COMEN = (NI + REEN);
*** END SETUP RE-ENCODING: SET = 5;

*** SETUP AVERAGE REACTION TIME;
RTSUM = 0;
NC = 0;

ARRAY RT
RT002 RT004 RT006 RT007 RT011 RT013-RT023 RT028 RT030-RT033 RT037-RT052
RT060-RT061 RT063-RT065 RT067 RT069 RT071 RT072-RT075 RT078 RT080 RT081
RT083-RT087
RT090 RT092 RT094-RT097 RT106-RT108 RT113 RT116
RT119-RT122
RT148 RT151 RT155 RT169 RT170 RT171 RT172 RT173 RT174
RT176 RT177 RT178 RT179 RT180 RT181 RT184 RT186 RT187
RT188 RT189 RT190 RT191 RT192 RT193 RT194 RT195 RT198 RT200
RT201 RT203;

ARRAY CT
C002 C004 C006 C007 C011 C013-C023 C028 C030-C033 C037-C052
C060-C061 C063-C065 C067 C069 C071 C072-C075 C078 C080 C081
C083-C087
C090 C092 C094-C097 C106-C108 C113 C116
C119-C122
C148 C151 C155 C169 C170 C171 C172 C173 C174
C176 C177 C178 C179 C180 C181 C184 C186 C187
```

```
C188 C189 C190 C191 C192 C193 C194 C195 C198 C200
C201 C203;

DO OVER RT;
  IF CT = 0 THEN RT = .;
  IF CT = 1 AND RT NE . THEN RTSUM = RTSUM + RT;
  IF CT = 1 AND RT NE . THEN NC = NC + 1;
END;

IF RTSUM = . THEN RTSUM = 0;
IF NC = . OR NC = 0 THEN NC = 1;
AVERT = RTSUM/NC;
*** END SETUP REACTION TIME;

*** SETUP COMPONENT SCORES: SET = 3

DATA ONE; SET ALL; IF SET = 3;

PROC STANDARD DATA=ONE OUT=ONEB REPLACE;
VAR
RT002 RT004 RT006 RT007 RT011 RT013-RT023 RT028 RT030-RT033 RT037-RT052
RT060-RT061 RT063-RT065 RT067 RT069 RT071 RT072-RT075 RT078 RT080 RT081
RT083-RT087
RT090 RT092 RT094-RT097 RT106-RT108 RT113 RT116
RT119-RT122
RT148 RT151 RT155 RT169 RT170 RT171 RT172 RT173 RT174
RT176 RT177 RT178 RT179 RT180 RT181 RT184 RT186 RT187
RT188 RT189 RT190 RT191 RT192 RT193 RT194 RT195 RT198 RT200
RT201 RT203;

DATA ONEC; SET ONEB;

PROC REG DATA=ONEB OUTEST=ONEP NOPRINT;
MODEL
RT002 RT004 RT006 RT007 RT011 RT013-RT023 RT028 RT030-RT033 RT037-RT052
RT060-RT061 RT063-RT065 RT067 RT069 RT071 RT072-RT075 RT078 RT080 RT081
RT083-RT087
RT090 RT092 RT094-RT097 RT106-RT108 RT113 RT116
RT119-RT122
RT148 RT151 RT155 RT169 RT170 RT171 RT172 RT173 RT174
RT176 RT177 RT178 RT179 RT180 RT181 RT184 RT186 RT187
RT188 RT189 RT190 RT191 RT192 RT193 RT194 RT195 RT198 RT200
RT201 RT203 = FPROD NUM3 TRUTH;

DATA COMPON; SET ONEP;
KEEP INT3C SIG3C PROD3C MIN3C TRUTH3C N;
INT3C = INTERCEP;
SIG3C = _SIGMA_;
PROD3C = FPROD;
```

```
MIN3C = NUM3;  
TRUTH3C = TRUTH;  
N = _N_;
```

```
PROC SORT; BY N;
```

```
DATA A; SET GEARY.AFABIL2;  
N = _N_;
```

```
PROC SORT; BY N;
```

```
DATA GEARY.AFABIL2; MERGE A COMON; BY N;
```