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AERONAUTICAL RESEARCH LABORATORY

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Aircraft Structures Report 438

A PENALTY ELEMENT FORMULATION FOR CALCULATING BULK STRESS

by

W. Waldman, T.E. Tay and R. Jones

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SUMMARY

This report describes the formulation of a three-dimensional penalty element that uses the bulk stress at each node as a degree of freedom, in addition to the usual u, v, and w displacements. The penalty function method is used to define a modified energy functional that is constrained by the equation for bulk stress, and a solution is obtained by specifying stationarity of this modified functional. Edge-cracked and centre-cracked panels are modelled in order to investigate the practical implementation of the penalty element formulation and to compare the results with those obtained using standard methods.

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1.0 INTRODUCTION

In recent years the thermoelastic effect has been greatly exploited as a non-contact method of experimental stress analysis. A device known as SPATE (Stress Pattern Analysis by measurement of Thermal Emission) has been used extensively at the Aeronautical Research Laboratory to analyze many different components to gain data on the surface stress distributions. The SPATE apparatus accomplishes its analysis by measuring the thermal emission of the surface of the component which is being sinusoidally loaded about some mean load. The thermal emission is proportional to the surface temperature and, because the thermal fluctuations are proportional to the changes in bulk stress $(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$, a strain gauge can be used to calibrate the output from SPATE in order to determine the actual bulk stress.

Although the use of SPATE for the analysis of components is a powerful and versatile technique, the tensorial nature of stress is ignored since temperature and bulk stress are scalar quantities. This places a significant limitation on the ability of SPATE data to provide a quantitative measure of the stress field, except in circumstances where it is known that the stress field is primarily uni-directional. Another disadvantage is that no direct information concerning the nature of the internal stress field of a three-dimensional structure can be obtained from the simple surface analysis performed by SPATE.

It is well known that the stress field existing in a component must satisfy equilibrium and any imposed boundary conditions. For two-dimensional problems, Ryall and Wong [1] have shown that when additional stress-related information is available, such as experimentally obtained bulk stress data from a SPATE analysis, it is no longer necessary to have a complete knowledge of the boundary conditions governing the problem in order to obtain a useful solution. Their method involved the use of an assumed stress potential Φ, with unknown coefficients, from which the bulk stress could be calculated analytically. The values of the unknown coefficients were determined by formulating the problem using a best-fit criterion where the difference between the analytical expression for the bulk stress and the experimentally observed bulk stresses was minimized in the least squares sense.

This report describes the formulation of an alternative finite element approach using a three-dimensional penalty element that uses the bulk stress at each node as a degree of freedom, in addition to the usual u, v, and w displacements. The penalty function method, as described in [2], will be used to define a modified energy functional that is constrained by the equation for bulk stress, a solution being obtained by specifying stationarity of this functional.

This approach leads to an interesting possible application where a finite element model of a component is created, and the bulk stress, as obtained by an experimental technique such as SPATE, is prescribed at various points. The resulting system can then be solved, making use of any other available data concerning the applied loads and boundary conditions, thus determining the full three-dimensional stress field in the component. In problems where ill-conditioning is encountered, such as adhesively bonded repairs, it may also be possible to use the measured bulk stress field as a means of improving the conditioning, thus driving the solution to a more reliable answer.

2.0 MATHEMATICAL FORMULATION

In the finite element method the displacements **u** at any point within an element may be written as

$$\mathbf{u} = \mathbf{N}\mathbf{a}, \tag{2.1}$$

where the components of **N** are prescribed functions of position and **a** represents the vector of nodal displacements for a particular element. The vector **a** may be written in terms of the n nodal points as

$$\mathbf{a} = \left\{ \begin{array}{c} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_n \end{array} \right\}, \tag{2.2}$$

and if each $\mathbf{a}_i$ is allowed horizontal, lateral, and vertical displacement degrees of freedom then we can write

$$\mathbf{a}_i = \left\{ \begin{array}{c} u_i \\ v_i \\ w_i \end{array} \right\}. \tag{2.3}$$

With the displacements known at all nodal points, then by using the linear theory of elasticity the strains can be written as

$$\boldsymbol{\epsilon} = \left\{ \begin{array}{c} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{array} \right\} = \left\{ \begin{array}{c} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{array} \right\} = \mathbf{B}\mathbf{u}, \tag{2.4}$$

where

$$\mathbf{B} = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{pmatrix} \tag{2.5}$$

and the relationship between stress and strain is of the form

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \mathbf{D}\boldsymbol{\varepsilon}, \tag{2.6}$$

where $\mathbf{D}$ is an elasticity matrix containing the appropriate material properties.

We will now introduce the definitions of the bulk stress σ_b, the bulk strain e, and the bulk modulus of the material κ:

$$\sigma_b = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \tag{2.7}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z \tag{2.8}$$

$$\kappa = \frac{E}{1 - 2\nu} \tag{2.9}$$

where σ_{xx} σ_{yy} σ_{zz} are the principal stresses, and ε_x ε_y ε_z are the principal strains, E is Young's modulus, and ν is Poisson's ratio.

The equation relating the bulk stress σ_b to the bulk strain e is

$$\sigma_b = \kappa e \tag{2.10}$$
$$= \kappa(\varepsilon_x + \varepsilon_y + \varepsilon_z) \tag{2.11}$$
$$= \boldsymbol{\kappa}^{\mathrm{T}}\boldsymbol{\varepsilon}. \tag{2.12}$$

and the bulk modulus vector $\boldsymbol{\kappa}$ is defined as

$$\boldsymbol{\kappa} = \begin{Bmatrix} \kappa \\ \kappa \\ \kappa \\ 0 \\ 0 \\ 0 \end{Bmatrix}. \tag{2.13}$$

Equation (2.12) may be written in the form

$$\sigma_b - \boldsymbol{\kappa}^{\mathrm{T}} \boldsymbol{\varepsilon} = 0 \tag{2.14}$$

and we note that the bulk stress σ_b is a scalar quantity.

We now proceed to define an energy functional Π which, in the absence of body forces, is a combination of the standard energy functional Π_a due to elastic deformations, and a penalty function term Π_σ which involves the bulk stress σ_b, such that

$$\Pi(\boldsymbol{\varepsilon}, \sigma_b) = \Pi_a + \Pi_\sigma \tag{2.15}$$

where

$$\Pi_a = \frac{1}{2} \int \boldsymbol{\varepsilon}^{\mathrm{T}} \mathbf{D} \boldsymbol{\varepsilon} \, dV - \int \mathbf{u}^{\mathrm{T}} \mathbf{T} \, dS \tag{2.16}$$

$$\Pi_\sigma = \frac{P}{2} \int \left(\sigma_b - \boldsymbol{\kappa}^{\mathrm{T}} \boldsymbol{\varepsilon} \right)^2 dV. \tag{2.17}$$

and $\mathbf{T}$ is the surface traction vector $\{T_x, T_y, T_z\}^{\mathrm{T}}$.

Now, by substituting for $\mathbf{u}$, the strains $\boldsymbol{\varepsilon}$ can be written in terms of the nodal displacements as follows

$$\boldsymbol{\varepsilon} = \mathbf{BNa}. \tag{2.18}$$

Hence, the standard energy functional Π_a may be expressed as

$$\Pi_a = \frac{1}{2} \mathbf{a}^{\mathrm{T}} \mathbf{K} \mathbf{a} - \mathbf{a}^{\mathrm{T}} \mathbf{f}_{\mathrm{T}} \tag{2.19}$$

where the stiffness matrix $\mathbf{K}$ and the force vector $\mathbf{f}_{\mathrm{T}}$ are given by

$$\mathbf{K} = \int \mathbf{N}^{\mathrm{T}} \mathbf{B}^{\mathrm{T}} \mathbf{D} \mathbf{B} \mathbf{N} \, dV \tag{2.20}$$

$$\mathbf{f}_{\mathrm{T}} = \int \mathbf{N}^{\mathrm{T}} \mathbf{T} \, dS \tag{2.21}$$

and the penalty functional Π_σ becomes

$$\Pi_\sigma = \frac{P}{2}\int(\sigma_b^2 - 2\sigma_b\boldsymbol{\kappa}^{\mathrm{T}}\boldsymbol{\epsilon} + \boldsymbol{\kappa}^{\mathrm{T}}\boldsymbol{\epsilon}\boldsymbol{\kappa}^{\mathrm{T}}\boldsymbol{\epsilon})\,dV \tag{2.22}$$

$$= \frac{P}{2}\int(\sigma_b^2 - 2\sigma_b\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{BNa} + \mathbf{a}^{\mathrm{T}}\mathbf{N}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\boldsymbol{\kappa}\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{BNa})\,dV \tag{2.23}$$

$$= \frac{P}{2}\int\sigma_b^2\,dV - P\int\sigma_b\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{BN}\,dV\,\mathbf{a} + \frac{P}{2}\mathbf{a}^{\mathrm{T}}\mathbf{K}_{aa}\mathbf{a} \tag{2.24}$$

where

$$\mathbf{K}_{aa} = \int\mathbf{N}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\boldsymbol{\kappa}\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{BN}\,dV \tag{2.25}$$

$$= \kappa^2\int\mathbf{N}^{\mathrm{T}}\mathbf{JN}\,dV \tag{2.26}$$

and

$$\mathbf{J} = \begin{pmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x\partial y} & \frac{\partial^2}{\partial x\partial z} \\ \frac{\partial^2}{\partial y\partial x} & \frac{\partial^2}{\partial y^2} & \frac{\partial^2}{\partial y\partial z} \\ \frac{\partial^2}{\partial z\partial x} & \frac{\partial^2}{\partial z\partial y} & \frac{\partial^2}{\partial z^2} \end{pmatrix} \tag{2.27}$$

and P is a penalty parameter whose nominal value is assigned such that the order of magnitude of the two functionals Π_a and Π_σ is approximately the same. In our particular case, it will be shown later that the penalty parameter can be specified to be equal to either $1/E$ or $1/\kappa$ without significant change in the solution.

Let us now define σ_b in terms of a shape function $\mathbf{M}$ and the values of the bulk stress at the nodal points, the latter denoted by the bulk stress vector $\mathbf{b}$, such that

$$\sigma_b = \mathbf{M}^{\mathrm{T}}\mathbf{b}, \tag{2.28}$$

and by noting that $\mathbf{M}^{\mathrm{T}}\mathbf{b} = \mathbf{b}^{\mathrm{T}}\mathbf{M}$ we obtain a new expression for the penalty functional

$$\Pi_\sigma = \frac{P}{2}\int\mathbf{b}^{\mathrm{T}}\mathbf{M}\mathbf{M}^{\mathrm{T}}\mathbf{b}\,dV - P\int\mathbf{b}^{\mathrm{T}}\mathbf{M}\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{BN}\,dV\,\mathbf{a} + \frac{P}{2}\mathbf{a}^{\mathrm{T}}\mathbf{K}_{aa}\mathbf{a} \tag{2.29}$$

$$= \frac{P}{2}\mathbf{b}^{\mathrm{T}}\mathbf{K}_{bb}\mathbf{b} - P\mathbf{b}^{\mathrm{T}}\mathbf{K}_{ba}\mathbf{a} + \frac{P}{2}\mathbf{a}^{\mathrm{T}}\mathbf{K}_{aa}\mathbf{a} \tag{2.30}$$

where

$$\mathbf{K}_{bb} = \int\mathbf{M}\mathbf{M}^{\mathrm{T}}\,dV \tag{2.31}$$

$$\mathbf{K}_{ba} = \int\mathbf{M}\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{BN}\,dV. \tag{2.32}$$

Hence the complete expression for the energy functional Π is

$$\Pi(\mathbf{a},\mathbf{b}) = \frac{1}{2}\mathbf{a}^{\mathrm{T}}\mathbf{K}\mathbf{a} - \mathbf{a}^{\mathrm{T}}\mathbf{f}_{\mathrm{T}} + \frac{P}{2}\mathbf{b}^{\mathrm{T}}\mathbf{K}_{bb}\mathbf{b} - P\mathbf{b}^{\mathrm{T}}\mathbf{K}_{ba}\mathbf{a} + \frac{P}{2}\mathbf{a}^{\mathrm{T}}\mathbf{K}_{aa}\mathbf{a} \qquad (2.33)$$

where Π is now a function of the displacement degrees of freedom **a** and the bulk stress degrees of freedom **b** specified at each node.

In order to obtain a solution, we apply the principle of stationarity to the energy functional Π, namely

$$\frac{\partial\Pi}{\partial\mathbf{a}} = \mathbf{0} \qquad (2.34)$$

$$\frac{\partial\Pi}{\partial\mathbf{b}} = \mathbf{0} \qquad (2.35)$$

and we obtain the following expressions

$$\frac{\partial\Pi}{\partial\mathbf{a}} = \mathbf{K}\mathbf{a} - \mathbf{f}_{\mathrm{T}} + P\mathbf{K}_{aa}\mathbf{a} - P\mathbf{K}_{ba}\mathbf{b} \qquad (2.36)$$

$$\frac{\partial\Pi}{\partial\mathbf{b}} = -P\mathbf{K}_{ba}\mathbf{a} + P\mathbf{K}_{bb}\mathbf{b}. \qquad (2.37)$$

Hence, the stationarity condition leads to a set of linear equations, which can be expressed in matrix form as

$$\left(\begin{bmatrix}\mathbf{K} & 0\\ 0 & 0\end{bmatrix} + P\begin{bmatrix}\mathbf{K}_{aa} & -\mathbf{K}_{ba}\\ -\mathbf{K}_{ba} & \mathbf{K}_{bb}\end{bmatrix}\right)\begin{Bmatrix}\mathbf{a}\\ \mathbf{b}\end{Bmatrix} = \begin{Bmatrix}\mathbf{f}_{\mathrm{T}}\\ \mathbf{0}\end{Bmatrix}. \qquad (2.38)$$

We note that the term involving **K** is just the standard finite element formulation of the stiffness matrix, while the matrix term multiplied by the penalty parameter P originates from the penalty element formulation for the bulk stress degrees of freedom.

3.0 VALIDATION OF PENALTY ELEMENT FORMULATION

In order to check the theoretical formulation of the penalty element approach, as well as its practical implementation into a finite element package, two demonstration problems will be analyzed. These consist of three-dimensional finite element models of an edge-cracked panel and a centre-cracked panel, from which it is possible to determine bulk stresses and displacements at various nodes in the finite element mesh. Figures 1 and 2 show the general dimensions of the edge-cracked and centre-cracked panels that were studied.

The test problems were analyzed using the PAFEC finite element package running on the ELXSI 6400 computer at ARL. The penalty element formulation was incorporated into the PAFEC program by re-writing the code that deals with 20-noded orthotropic brick elements in order to incorporate the penalty element.

The penalty element was assigned an element-type number of 37150, and this element type overwrites the PAFEC isotropic element of the same name. The orthotropic element type was chosen for this modification because the required changes were easier to code than for an isotropic element.

3.1 Edge-cracked and centre-cracked panels

Figure 3 shows the finite element mesh used for modelling the edge-cracked panel, and it details the major dimensions, including those of the simulated crack. Due to symmetry only 1/4 of the structure is shown, and the full model represented by this mesh can be obtained by reflection in the X-Z and X-Y planes. Therefore, the nodes along the x-axis correspond to interior points in the plate. The loading applied to the finite element model consists of a uniform prescribed v-displacement of 0.01 mm along its top surface (see Figures 1 and 2), and it should be noted that restraints are applied on that face to prevent any strains in the x-direction.

The centre-cracked panel was modelled by applying additional restraints to the finite element model of the edge-cracked panel, making use of the existing mesh. Hence, with the appropriate restraints, the finite element mesh shown in Figure 3 also corresponds to 1/8 of the full centre-cracked panel, and the full model can be obtained by reflecting the mesh in the X-Y, X-Z and Y-Z planes. The loading for the centre-cracked panel was identical to that used for the edge-cracked panel.

The material chosen for the two different types of panel corresponds to an epoxy adhesive, whose material properties are presented in Table 1. Although these properties are input as for an orthotropic material, they are chosen so as to make the material behave in an isotropic manner. The data are given for two different values of Poisson's ratio, where $\nu = 0.30$ corresponds to the values used for the centre-cracked panel, and $\nu = 0.35$ corresponds to the edge-cracked panel. Initially some of the finite element calculations were carried out using $\nu = 0.35$, but in order to match the Poisson's ratio used for obtaining the stress intensity factor for a centre-cracked panel [3], it was also necessary to use a Poisson's ratio of $\nu = 0.30$.

Figure 4 shows a typical deformed finite element mesh that was obtained for the edge-cracked panel when it was loaded by a uniform displacement. The deformation of the mesh in the vicinity of the crack tip can be seen clearly.

3.2 Bulk stress calculations

The edge-cracked and centre-cracked panels were analyzed using both the standard and penalty element formulations. For the standard element case, bulk stresses were calculated at a number of nodes ahead of the crack tip by simply summing the principal stresses according to Equation (2.7). Since the nodes chosen for this comparison were placed at element boundaries, the bulk stresses were calculated as the average of the results obtained for adjacent elements.

Tables 2 and 3 present the bulk stresses calculated for the edge-cracked and centre-cracked panels. When using the penalty element formulation, two different values of the penalty parameter were tried, the first corresponding to $P = 1/E$ and the second to $P = 1/\kappa$. The data presented in these two tables indicates that there is very good agreement between the standard and the penalty element formulations. In areas where there is a rapid stress variation, the answers generally agree to within 7%, while the average difference is about 4%. Note that the results obtained for the two different values of the penalty parameter are also in very good agreement, indicating that the solution is not very sensitive to changes in the value of the penalty parameter when it is of the order of magnitude $O(1/E)$. Also note that there is a stress singularity at node 12, since this is right at the crack tip, and consequently the finite element results are inaccurate at that location; the results for node 12 were included only to enable a qualitative comparison to be made between the standard and penalty element formulations.

3.3 Stress intensity factor calculations

In order to give the required $\sqrt{r}$ displacement behaviour for near crack tip points in the vicinity of the crack front, special elements were used that had their midside nodes shifted to the quarter point position. The value of the stress intensity factor K_I near the crack front can be determined from the opening displacement of the crack near the tip using the plane strain equation

$$K_I = \frac{\delta E}{4(1-\nu^2)}\sqrt{\frac{2\pi}{r\left(1-\frac{r}{2l}\right)}} \tag{3.1}$$

where

δ is the displacement in the y-direction of a point lying in the X-Z plane near the crack front

r is the perpendicular distance of the point from the crack front

l is the crack length for an edge-cracked panel, and is the crack half-length for a centre-cracked panel

E is Young's modulus

ν is Poisson's ratio.

Table 4 presents values of the stress intensity factor K_I as obtained using both the standard and penalty element formulations, and compares them to the values of K_I given by Rooke & Cartwright [3] for both an edge-cracked and centre-cracked panel. The calculations were performed for an applied v-displacement of 0.01 mm, and it is seen that the two different finite element formulations give results that are in good agreement with those obtained from [3]. Note that the results obtained using standard three-dimensional elements are closer to those given in [3], but in any case the maximum difference is only about 4%, which is not very significant when the accuracy of the results from [3] are quoted as being within an error bound of a similar magnitude. However, the difference in values of K_I calculated using standard and penalty elements is only about 1.5%, which is very small.

Tables 5 and 6 present values of crack opening displacement δ, stress intensity factor K_I, and reaction force F_y obtained for the edge-cracked and centre-cracked panels. In order to study the sensitivity of the penalty element solution to different values of the penalty parameter P, a wide range of values were used, with $P = 100$ being the largest value that could be used without numerical problems. The maximum variation in crack opening displacement and stress intensity factor experienced for the edge-cracked panel was 1.5%, while for the centre-cracked panel it was 2.0%.

4.0 CONCLUSION

By using a penalty element formulation, the standard finite element method has been modified to allow the calculation of bulk stresses as an additional degree of freedom in the finite element model of a structure. The bulk stress penalty elements have been incorporated into the PAFEC finite element package.

In order to check the practical implementation of the penalty element formulation, edge-cracked and centre-cracked panels were modelled, and the solutions obtained were compared with those calculated using the standard finite element method. Also, both the standard and penalty element formulations have been used to obtain crack opening displacements, from which the values of K_I were calculated and compared with those from [3].

The results of these comparisons indicate that bulk stresses can be accurately predicted using the penalty element method, and that there is no significant effect on the quality of the results obtained for the other degrees of freedom. It has also been found that the present application of the penalty element formulation is very insensitive to the choice of penalty parameter P, and a suitable value for P can be defined by using either Young's modulus or the bulk modulus, the appropriate values being given by $P = 1/E$ or $P = 1/\kappa$.

It is envisaged that this methodology will subsequently be used, in conjunction with experimentally obtained values for the bulk stress, to develop a hybrid experimental/numerical procedure for determining individual stress components based on the work presented in [1].

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[2] O.C. Zienkiewicz, The Finite Element Method, Third Edition, McGraw-Hill Book Company (UK) Limited, 1977.

[3] D.P. Rooke and D.J. Cartwright, Compendium of Stress Intensity Factors, Her Majesty's Stationery Office, London, 1976.

TABLE 1

Mechanical Properties of Adhesive For
Two Values of Poisson's Ratio

MECHANICAL PROPERTY (MPa^{-1})	CENTRE-CRACKED PANEL $\nu = 0.30$	EDGE-CRACKED PANEL $\nu = 0.35$
S_{XX}	5.2910×10^{-4}	5.2910×10^{-4}
S_{YY}	5.2910×10^{-4}	5.2910×10^{-4}
S_{ZZ}	5.2910×10^{-4}	5.2910×10^{-4}
S_{XY}	-1.5873×10^{-4}	-1.8519×10^{-4}
S_{YZ}	-1.5873×10^{-4}	-1.8519×10^{-4}
S_{ZX}	-1.5873×10^{-4}	-1.8519×10^{-4}
SH_{XY}	1.3757×10^{-3}	1.4286×10^{-3}
SH_{YZ}	1.3757×10^{-3}	1.4286×10^{-3}
SH_{ZX}	1.3757×10^{-3}	1.4286×10^{-3}

TABLE 2

Bulk Stress at Different Nodes Calculated Using Standard and Penalty Elements with Different Values of Penalty Parameter *P*

EDGE-CRACKED PANEL				
NODE NO.	DISTANCE FROM CRACK FACE (mm)	BULK STRESS (MPa)		
		Standard Element	Penalty Element	
			$P = 1/E$	$P = 1/\kappa$
12	0.0000	26.31	27.28	28.09
13	0.1667	12.29	11.47	11.48
14	0.5000	5.19	5.11	5.08
15	1.6667	2.65	2.75	2.74
16	5.0000	2.02	2.03	2.01

TABLE 3

Bulk Stress at Different Nodes Calculated Using Standard and Penalty Elements with Different Values of Penalty Parameter *P*

CENTRE-CRACKED PANEL				
NODE NO.	DISTANCE FROM CRACK FACE (mm)	BULK STRESS (MPa)		
		Standard Element	Penalty Element	
			$P = 1/E$	$P = 1/\kappa$
12	0.0000	21.70	22.72	23.09
13	0.1667	9.98	9.48	9.43
14	0.5000	4.65	4.38	4.37
15	1.6667	2.48	2.57	2.55
16	5.0000	2.00	2.02	2.01

TABLE 4

Values of Stress Intensity Factor K_I as Obtained Using Different Methods

SOLUTION METHOD	STRESS INTENSITY FACTOR K_I (MPa$\sqrt{\text{m}}$)	
	EDGE-CRACKED PANEL	CENTRE-CRACKED PANEL
Rooke & Cartwright [3]	0.155	0.140
Standard Elements	0.151	0.137
Penalty Elements ($P = 1/E$)	0.149	0.135

TABLE 5

Finite Element Results For Crack Opening, Stress Intensity Factor, and Reaction Force Obtained With Different Values of Penalty Parameter P

EDGE-CRACKED PANEL			
PENALTY PARAMETER P	CRACK OPENING δ AT NODE 11 (m)	STRESS INTENSITY FACTOR K_I (MPa$\sqrt{m}$)	REACTION F_y AT NODE 3 (N)
$0.000\times10^{+0}$*	1.4062×10^{-6}	0.1508	18.299
1.000×10^{-6}	1.4061×10^{-6}	0.1508	18.299
1.000×10^{-5}	1.4051×10^{-6}	0.1507	18.300
1.000×10^{-4}	1.3984×10^{-6}	0.1500	18.304
1.587×10^{-4}†	1.3956×10^{-6}	0.1497	18.306
5.291×10^{-4}‡	1.3874×10^{-6}	0.1488	18.314
1.000×10^{-2}	1.3778×10^{-6}	0.1478	18.338
$1.000\times10^{+0}$	1.3858×10^{-6}	0.1486	18.362
$1.000\times10^{+1}$	1.3909×10^{-6}	0.1492	18.413
$1.000\times10^{+2}$	1.4280×10^{-6}	0.1532	18.827

* standard finite element formulation
† corresponds to $P = 1/\kappa$
‡ corresponds to $P = 1/E$

TABLE 6

Finite Element Results For Crack Opening, Stress Intensity Factor, and Reaction Force Obtained With Different Values of Penalty Parameter P

CENTRE-CRACKED PANEL			
PENALTY PARAMETER P	CRACK OPENING δ AT NODE 11 (m)	STRESS INTENSITY FACTOR K_I (MPa$\sqrt{m}$)	REACTION F_y AT NODE 3 (N)
$0.000\times10^{+0}$*	1.3211×10^{-6}	0.1367	18.549
1.000×10^{-6}	1.3211×10^{-6}	0.1367	18.549
1.000×10^{-5}	1.3205×10^{-6}	0.1366	18.549
1.000×10^{-4}	1.3160×10^{-6}	0.1361	18.552
2.116×10^{-4}†	1.3124×10^{-6}	0.1358	18.554
5.291×10^{-4}‡	1.3067×10^{-6}	0.1352	18.558
1.000×10^{-2}	1.2959×10^{-6}	0.1340	18.580
$1.000\times10^{+0}$	1.2986×10^{-6}	0.1343	18.602
$1.000\times10^{+1}$	1.2976×10^{-6}	0.1342	18.604
$1.000\times10^{+2}$	1.2966×10^{-6}	0.1341	18.542

* standard finite element formulation
† corresponds to $P = 1/\kappa$
‡ corresponds to $P = 1/E$

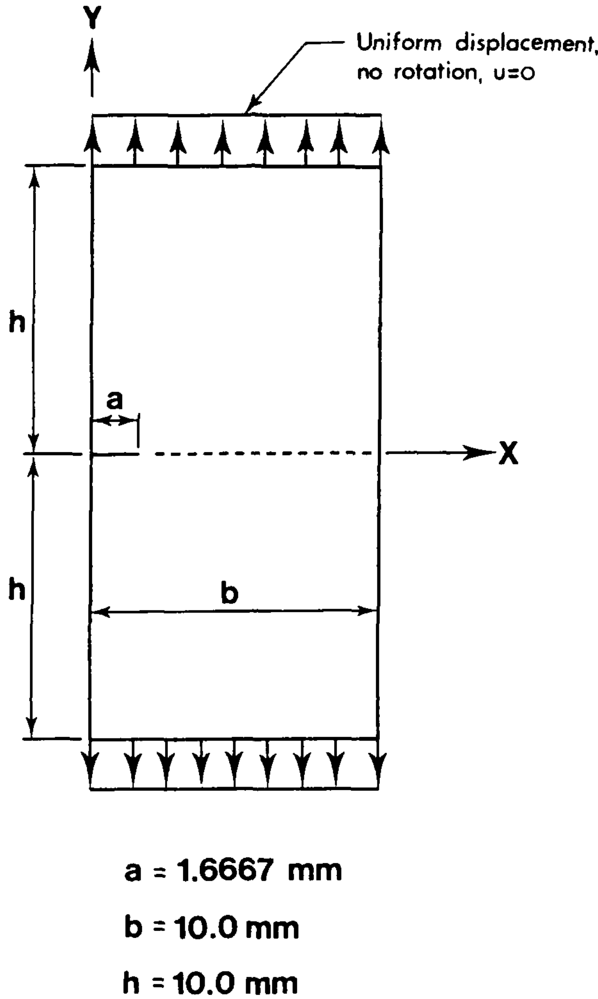

Fig. 1: Edge-cracked panel showing main dimensions.

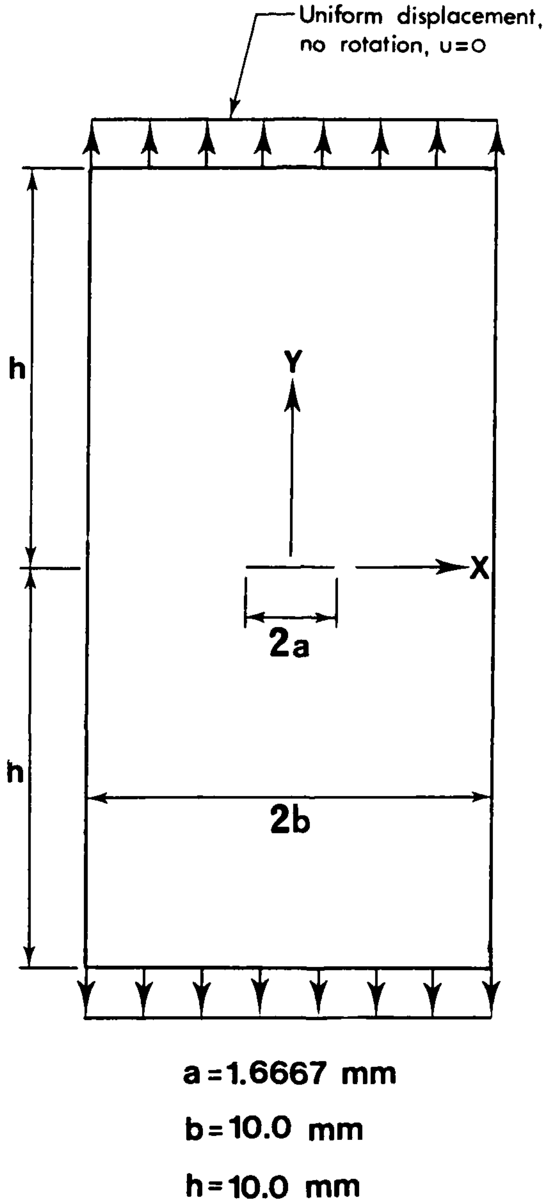

Fig. 2: Centre-cracked panel showing main dimensions.

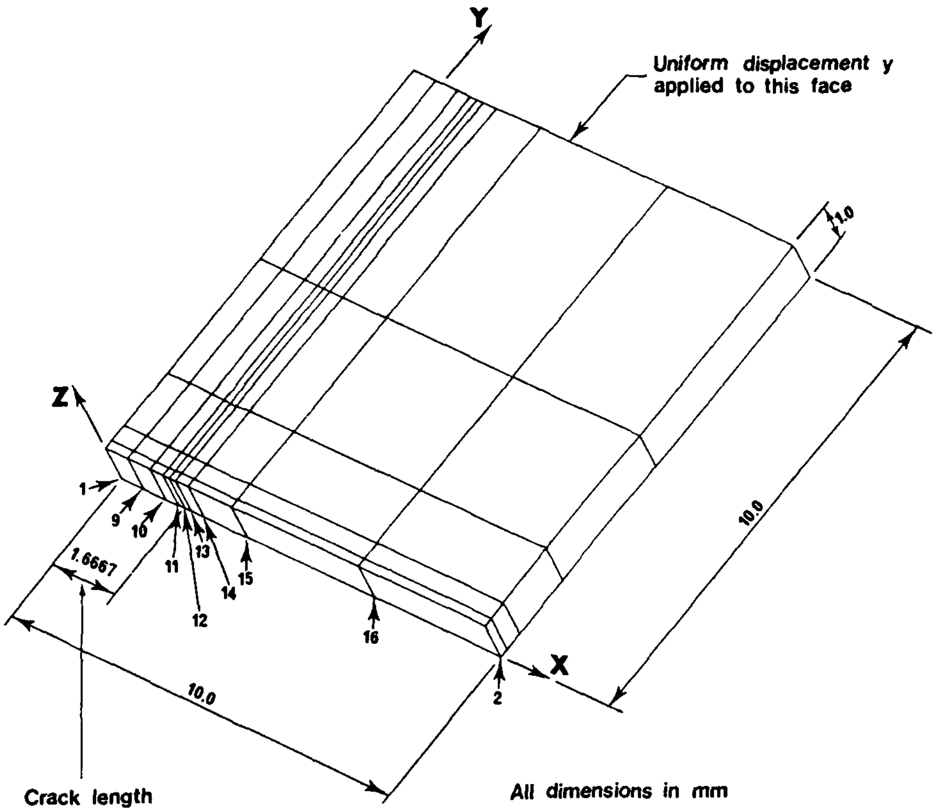

Fig. 3: Finite element mesh used to model the edge-cracked and centre-cracked panels.

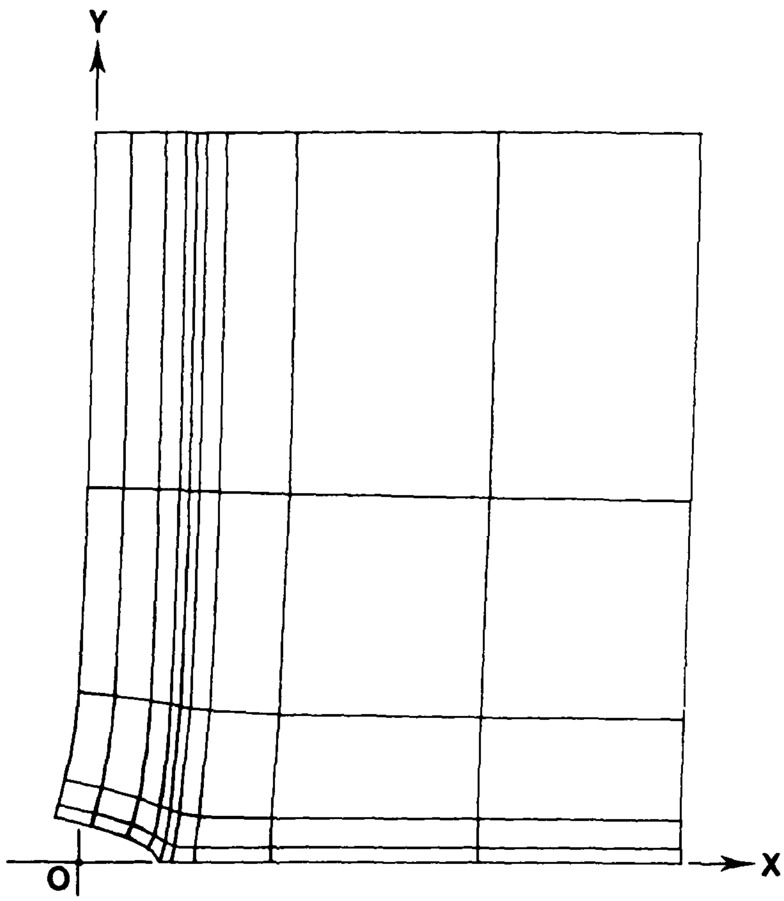

Fig. 4 : Typical deformed finite element mesh for the edge-cracked panel loaded by a uniform displacement.

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This report describes the formulation of a three-dimensional penalty element that uses the bulk stress at each node as a degree of freedom, in addition to the usual u, v, and w displacements. The penalty function method is used to define a modified energy functional that is constrained by the equation for bulk stress, and a solution is obtained by specifying stationarity of this modified functional. Edge-cracked and centre-cracked panels are modelled in order to investigate the practical implementation of the penalty

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